— Open Question —
Does the uncloneable bit exist?

Towards Unconditional Uncloneable Encryption [6]

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❶ Goal

Have a secure cryptographic scheme against cloning attacks.

❷ Idea

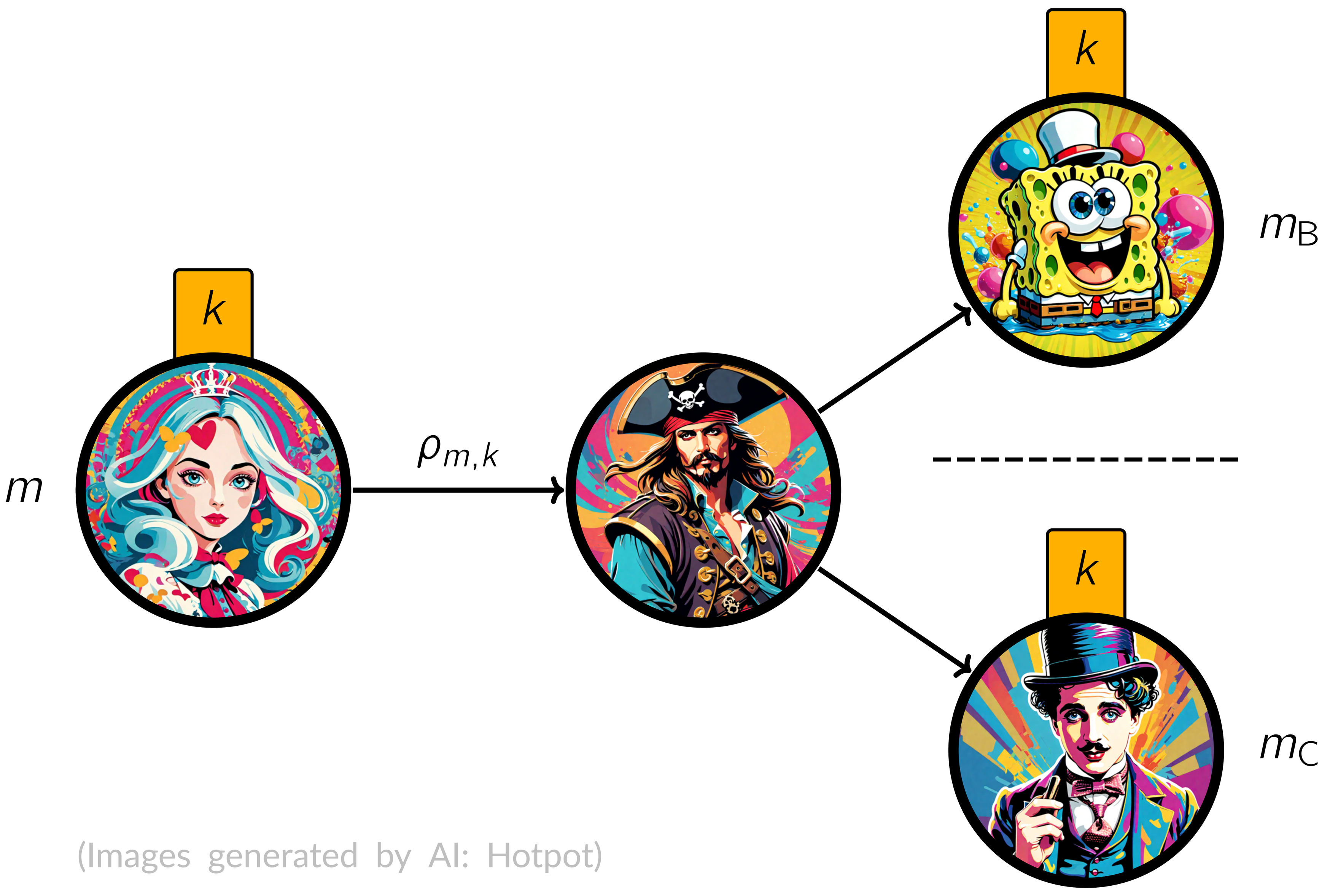
Laverage the quantum no-cloning theorem.

❸ Consequences

Applications to private-key quantum money [8], preventing storage attacks [8], quantum functional encryption [12], quantum copy-protection [2] and uncloneable decryption [10, 14, 11].

1. Uncloneable Bit

❹ **No-Cloning Game.** Alice encrypts a message $m \in \{0,1\}$ with a key $k \in \{1,..,K\}$ into a quantum state $\rho_{m,k}$. She sends it via a quantum channel but a pirate (P) intercepts it. Without knowing the key k, the pirate tries to share the information with two non-communicating parties, Bob (B) and Charlie (C), so that both of them may retrieve Alice's message m. We say that the adversary team (P, B, C) wins the game if both Bob's and Charlie's guesses are correct, *i.e.* if $m_{\mathrm{B}} = m_{\mathrm{C}} = m$.



(Images generated by AI: Hotpot)

❺ Definition (Correctness)

The encryption protocol $(m,k) \mapsto \rho_{m,k}$ is said to be *correct* if there exists a way to retrieve m from $\rho_{m,k}$ and k.

❻ Definition (Security)

A protocol $(m,k) \mapsto \rho_{m,k}$ is said to be *uncloneable-indistinguishable secure* if:

$$\mathbb{P}\Big((\mathrm{P},\mathrm{B},\mathrm{C}) \text{ win the game}\Big) \leqslant \frac{1}{2} + f(\lambda),$$

where $f(\lambda) \to 0$ as $\lambda \to \infty$, and where λ is the security parameter. Additionally, this security is said to be *strong* if moreover $f(\lambda) = \mathrm{negl}(\lambda)$.

❽ Former Work

Efforts have focused on its achievability under various models and definitions, including:

- in the quantum random oracle model (QROM) [8, 3, 4],
- in an interactive version of the scenario [7],
- in a device-independent variant with variable keys [11],
- assuming the existence of specific types of obfuscation [1, 9],
- in a variant with quantum keys [5],
- and a "succinct" variant [13].

❼ Open Question (Uncloneable Bit)

Is there an encryption scheme $(m,k) \mapsto \rho_{m,k}$ that is both correct and uncloneable-indistinguishable secure?

2. Candidate Scheme and Conjecture

❾ Candidate Scheme: Clifford Algebra

Let $\Gamma_1, \ldots, \Gamma_K$ be Hermitian unitaries that anti-commute. Consider the following encryption:

$$\rho_{m,k} := \frac{2}{d}\,\frac{I_d + (-1)^m \Gamma_k}{2},$$

where $m \in \{0,1\}$ and $k \in \{1,..,K\}$. This encryption protocol is correct since one can retrieve m from measuring $\rho_{m,k}$ in the eigenbasis of Γ_k. It remains to show the security.

❿ **Example.** It is possible to produce such Γ_k's using pairwise anti-commuting Pauli strings. Indeed, if $K = 2n$, consider:

$$\Gamma_k := X^{\otimes(k-1)} \otimes Y \otimes I^{\otimes(n-k)} \quad \text{and} \quad \Gamma_{n+k} := X^{\otimes(k-1)} \otimes Z \otimes I^{\otimes(n-k)},$$

for $k \in \{1,..,n\}$. Otherwise, if $K = 2n+1$, consider the same operators and add $\Gamma_{2n+1} := X^{\otimes n}$.

⓫ Conjecture

This scheme is uncloneable-indistinguishable secure:

$$\mathbb{P}\Big((\mathrm{P},\mathrm{B},\mathrm{C}) \text{ win the game}\Big) \leqslant \frac{1}{2} + \frac{1}{2\sqrt{K}}.$$

Remark. Here $K \sim 2\lambda$, but ideally $K \sim 2^\lambda$ (strong security).

3. Results

⓬ Proposition (Sufficient Formula)

To achieve the security of the Conjecture, it is sufficient to prove the following upper bound for all Hermitian unitaries $\{U_k\}$:

$$\Bigg\| \underbrace{\sum_{k=1}^{K} \Big(\Gamma_k \otimes U_k \otimes I + \Gamma_k \otimes I \otimes U_k + I \otimes U_k \otimes U_k \Big)}_{=:W_K} \Bigg\|_{\mathrm{op}} \leqslant K + 2\sqrt{K}.$$

⓭ **Remarks.** The value $K + 2\sqrt{K}$ is achieved when considering $U_k = I$ for all k. Moreover, the formula trivially holds if we assume that the operators U_k commute.

⓮ Theorem 1

The Conjecture is valid for $K \leqslant 7$.

Proof. When $K \leqslant 7$, we find the following sum-of-squares (SoS) decomposition:

$$\big(K + 2\sqrt{K}\big)\, I - W_K = \textstyle\sum_{k=1}^{K} \alpha_k A_k^2$$

for some explicit coefficients $\alpha_k \geqslant 0$ and operators A_k. Hence $\big(K + 2\sqrt{K}\big)\, I - W_K \succcurlyeq 0$ and therefore $K + 2\sqrt{K} \geqslant \|W_K\|_{\mathrm{op}}$. □

⓯ Numerical Results

The Conjecture is numerically confirmed for $K \leqslant 17$ (NPA level-2 algorithm) and $K \leqslant 18$ (Seesaw algorithm).

⓰ Theorem 2

Asymptotically, the winning probability of the no-cloning game for our candidate scheme is upper-bounded by $5/8$.

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