

# Topic 1: Introduction <sup>1</sup>

(Version of 20th August 2024)

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Department of Information Technology  
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Course 1DL442:  
Combinatorial Optimisation and Constraint Programming,  
whose part 1 is Course 1DL451:  
Modelling for Combinatorial Optimisation

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<sup>1</sup>Based partly on material by Guido Tack



# Optimisation

Constraint  
Problems

Combinatorial  
Optimisation

Modelling  
(in MiniZinc)

Solving

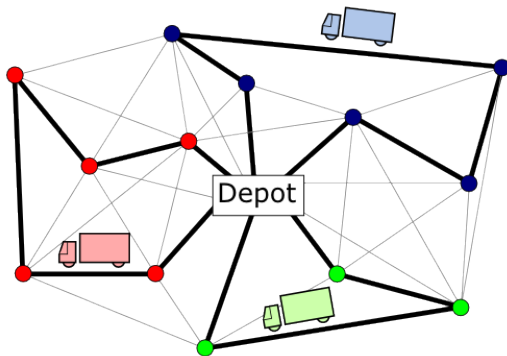
The MiniZinc  
Toolchain

Course  
Information

Part 1: Modelling for  
Combinatorial  
Optimisation

Part 2: Combinatorial  
Optimisation and CP

Contact



Optimisation is a science of **service**:  
to scientists, to engineers, to artists, and to society.



## MiniZinc Challenge 2015: Some Problems and Winners

Problem and Model	Backend and Solver	Technology
Costas array	Mistral	CP
capacitated VRP	iZplus	hybrid
GFD schedule	Chuffed	LCG
grid colouring	MiniSAT(ID)	hybrid
instruction scheduling	Chuffed	LCG
large scheduling	Google OR-Tools.cp	CP
application mapping	JaCoP	CP
multi-knapsack	mzn-cplex	MIP
portfolio design	fzn-oscar-cbls	CBLS
open stacks	Chuffed	LCG
project planning	Chuffed	LCG
radiation	mzn-gurobi	MIP
satellite management	mzn-gurobi	MIP
time-dependent TSP	G12.FD	CP
zephyrus configuration	mzn-cplex	MIP

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# Outline

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## 1. Constraint Problems

## 2. Combinatorial Optimisation

## 3. Modelling (in MiniZinc)

## 4. Solving

## 5. The MiniZinc Toolchain

## 6. Course Information

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## Example (Agricultural experiment design)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley							
corn							
millet							
oats							
rye							
spelt							
wheat							

### Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- 3 Balance: Every grain pair is grown in 1 common plot.

**Instance:** 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.



## Example (Agricultural experiment design)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	—	—	—	—
corn	✓	—	—	✓	✓	—	—
millet	✓	—	—	—	—	✓	✓
oats	—	✓	—	✓	—	✓	—
rye	—	✓	—	—	✓	—	✓
spelt	—	—	✓	✓	—	—	✓
wheat	—	—	✓	—	✓	✓	—

### Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- 3 Balance: Every grain pair is grown in 1 common plot.

**Instance:** 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.



## Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A							
Doctor B							
Doctor C							
Doctor D							
Doctor E							

### Constraints to be satisfied:

- 1 #on-call doctors / day = 1
- 2 #operating doctors / weekday  $\leq 2$
- 3 #operating doctors / week  $\geq 7$
- 4 #appointed doctors / week  $\geq 4$
- 5 day off after operation day
- 6 ...

**Objective function** to be minimised: Cost: ...





## Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A	call	none	oper	none	oper	none	none
Doctor B	appt	call	none	oper	none	none	call
Doctor C	oper	none	call	appt	appt	call	none
Doctor D	appt	oper	none	call	oper	none	none
Doctor E	oper	none	oper	none	call	none	none

### Constraints to be satisfied:

- 1 #on-call doctors / day = 1
- 2 #operating doctors / weekday  $\leq 2$
- 3 #operating doctors / week  $\geq 7$
- 4 #appointed doctors / week  $\geq 4$
- 5 day off after operation day
- 6 ...



### Objective function to be minimised: Cost: ...

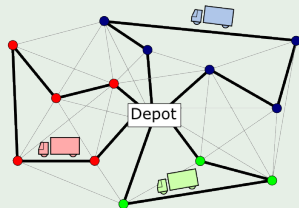


## Example (Vehicle routing: parcel delivery)

**Given** a depot with parcels for clients and a vehicle fleet,  
**find** which vehicle visits which client when.

**Constraints** to be **satisfied**:

- 1 All parcels are delivered on time.
- 2 No vehicle is overloaded.
- 3 Driver regulations are respected.
- 4 ...



**Objective function** to be **minimised**:

- Cost: the total fuel consumption and driver salary.

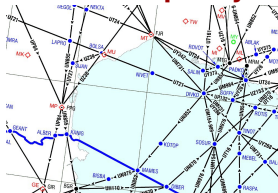
## Example (Travelling salesperson: optimisation TSP)

**Given** a map and cities,  
**find** a **shortest** route visiting each city once and returning to the starting city.



# Applications in Air Traffic Management

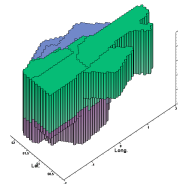
## Demand vs capacity



## Contingency planning

Flow	Time Span	Hourly Rate
From: Arlanda To: west, south	00:00 – 09:00	3
	09:00 – 18:00	5
	18:00 – 24:00	2
From: Arlanda To: east, north	00:00 – 12:00	4
	12:00 – 24:00	3
...	...	...

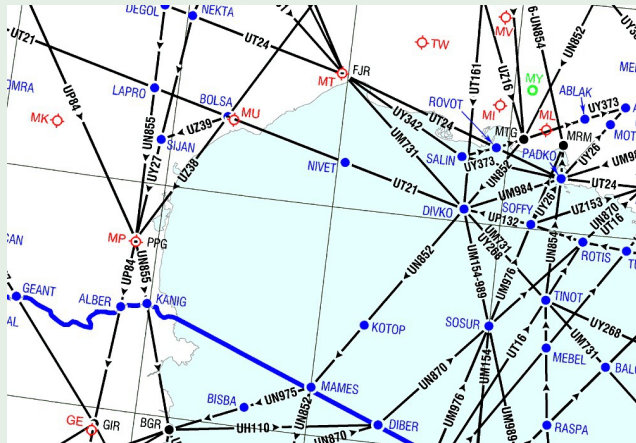
## Airspace sectorisation



## Workload balancing



Reroute flights, in height and speed, so as to balance the workload of air traffic controllers in a multi-sector airspace:

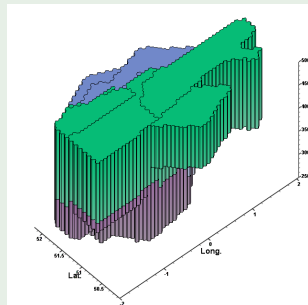
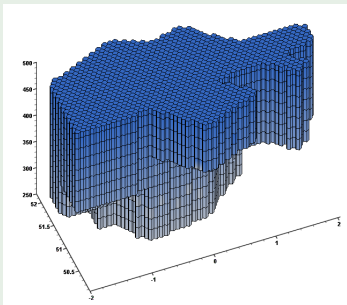




## Example (Airspace sectorisation)

**Given** an airspace split into  $c$  cells, a targeted number  $s$  of sectors, and flight schedules.

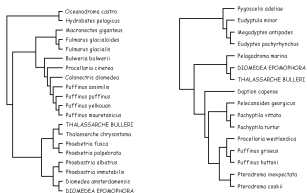
**Find** a colouring of the  $c$  cells into  $s$  connected convex sectors, with minimal imbalance of the workloads of their air traffic controllers.



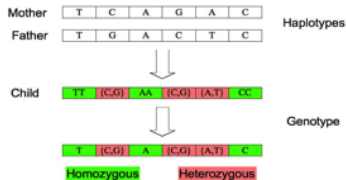
There are  $s^c$  possible colourings, but very few optimally satisfy the constraints: is **intelligent** search necessary?

## Applications in Biology and Medicine

## Phylogenetic supertree



## Haplotype inference



## Medical image analysis

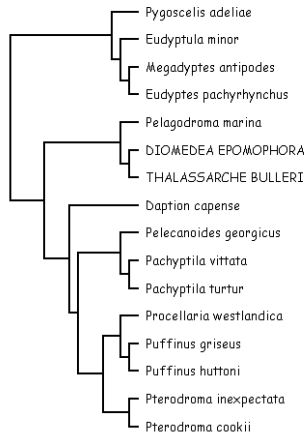
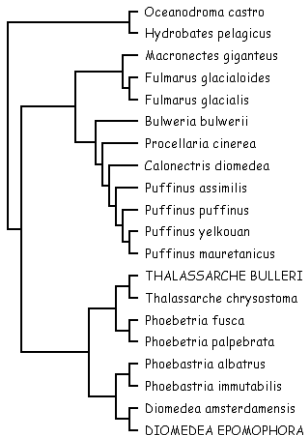


## Doctor rostering





# Example (What supertree is maximally consistent with several given trees that share some species?)





## Example (Haplotype inference by pure parsimony)

**Given**  $n$  child genotypes, with homo- and heterozygous sites:

...					
A	C / G	T	C	A / T	C
...					
A / T	G	T	C / G	A	C
...					

**find** a minimal set of (at most  $2 \cdot n$ ) parent haplotypes:

...					
A	C	T	C	T	C
...					
A	G	T	C	A	C
...					
T	G	T	G	A	C
...					

**so that** each given genotype conflates 2 found haplotypes.





# Applications in Programming and Testing

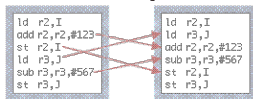
## Robot programming



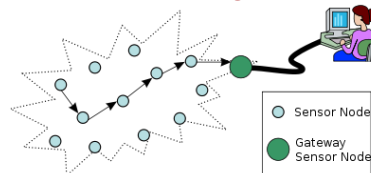
## Compiler design

COMPILERS  
FOR INSTRUCTION SCHEDULING

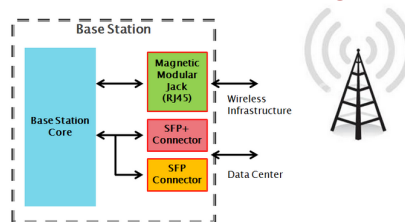
### C Compiler C++ Compiler



## Sensor-net configuration



## Base-station testing





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# Other Application Areas

## School timetabling

	Monday	Tuesday	Wednesday	Thursday	Friday
9.00	MT2302 Ordinary Differential Equations P781		LANC82072 Computer Graphics (I) Duel	MT2302 Numerical Analysis I Söderman, G03	
10.00	MT2302 Ordinary Differential Equations M310 / Reson, 2.3		LANC82072 Computer Graphics (I) Duel	MT2302 Ordinary Differential Equations Boman Engineering, Basement Theatre 3A	MT2302 Ordinary Differential Equations M310
11.00	CA2012 Algorithms and Data Structures 1.1		MT2313 Further Linear Algebra 1.1	MT2302 Ordinary Differential Equations Björkström, Theatre 1	MT2302 Ordinary Differential Equations Björkström, Theatre 1
12.00	MT2313 Further Linear Algebra Reson, Theatre 4	MT2302 Numerical Analysis I Söderman, G03	CA2012 Computer Graphics 1.1	MT2313 Further Linear Algebra Björkström, Theatre 1	MT2313 Further Linear Algebra Björkström, Theatre 1
1.00			PA25 Peer-Assisted Study MST / LPT / LPT / M08	MT2313 Further Linear Algebra Björkström, Theatre 1	MT2313 Further Linear Algebra Björkström, Theatre 1
1.40	CA2012 Computer Graphics 1.1			MT2313 Further Linear Algebra Björkström, Theatre 1	MT2313 Further Linear Algebra Björkström, Theatre 1
2.00		CA2012 Computer Graphics 1.1			
2.40		CA2012 Computer Graphics 1.1			

## Security: SQL injection?



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## Sports tournament design

svensk handboll



## Container packing



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## Definitions

In a **constraint problem**, values have to be **found** for all the unknowns, called **variables** (in the mathematical sense; also called **decision variables**) and ranging over **given** sets, called **domains**, so that:

- All the given **constraints** on the decision variables are **satisfied**.
- Optionally: A given **objective function** on the decision variables has an optimal value: either a **minimal** cost or a **maximal** profit.

A **candidate solution** to a constraint problem maps each decision variable to a value within its domain; it is:

- **feasible** if all the constraints are satisfied;
- **optimal** if the objective function takes an optimal value.

The **search space** consists of all candidate solutions.

A **solution** to a **satisfaction problem** is feasible.

An **optimal solution** to an **optimisation problem** is feasible and optimal.



# $P \stackrel{?}{=} NP$

(Cook, 1971; Levin, 1973)

This is one of the seven **Millennium Prize** problems of the Clay Mathematics Institute (Massachusetts, USA), each worth 1 million US\$.

Informally:

- $P$  = class of problems that need **no** search to be solved  
 $NP$  = class of problems that **might** need search to solve
- $P$  = class of problems with easy-to-**compute** solutions  
 $NP$  = class of problems with easy-to-**check** solutions

Thus: Can search always be avoided ( $P = NP$ ),  
or is search sometimes necessary ( $P \neq NP$ )?

Problems that are solvable in polynomial time (in the input size) are considered **tractable**, aka **easy**.

Problems needing super-polynomial time are considered **intractable**, aka **hard**.



# NP Completeness: Examples

Given a digraph  $(V, E)$ :

## Examples

- Finding a **shortest path** takes  $\mathcal{O}(V \cdot E)$  time and is thus in P.
- Determining the existence of a simple path (which has distinct vertices), from a given single source, that has *at least* a given number  $\ell$  of edges is NP-complete. Hence finding a **longest path** seems hard: increase  $\ell$  starting from a trivial lower bound, until answer is 'no'.

## Examples

- Finding an **Euler tour** (which visits each *edge* once) takes  $\mathcal{O}(E)$  time and is thus in P.
- Determining the existence of a **Hamiltonian cycle** (which visits each *vertex* once) is NP-complete.



# NP Completeness: More Examples

## Examples

- ***n*-SAT**: Determining the satisfiability of a conjunction of disjunctions of  $n$  Boolean literals is in P for  $n = 2$  but NP-complete for  $n = 3$ .
- **SAT**: Determining the satisfiability of a formula over Boolean literals is NP-complete.
- **Clique**: Determining the existence of a clique (complete subgraph) of a given size in a graph is NP-complete.
- **Vertex Cover**: Determining the existence of a vertex cover (a vertex subset with at least one endpoint for all edges) of a given size in a graph is NP-complete.
- **Subset Sum**: Determining the existence of a subset, of a given set, that has a given sum is NP-complete.



# Search spaces are often larger than the universe!



Many important real-life problems are NP-hard or worse: their real-life instances can only be solved exactly and fast enough by **intelligent** search, unless  $P = NP$ . **NP-hardness is not where the fun ends, but where it begins!**



## Example (Optimisation TSP over $n$ cities)

A brute-force algorithm evaluates all  $n!$  candidate routes:

- A computer of today evaluates  $10^6$  routes / second:

$n$	time
11	40 seconds
14	1 day
18	203 years
20	77k years

- Planck time is shortest useful interval:  $\approx 5.4 \cdot 10^{-44}$  second;  
a Planck computer would evaluate  $1.8 \cdot 10^{43}$  routes / second:

$n$	time
37	0.7 seconds
41	20 days
48	$1.5 \cdot$ age of universe

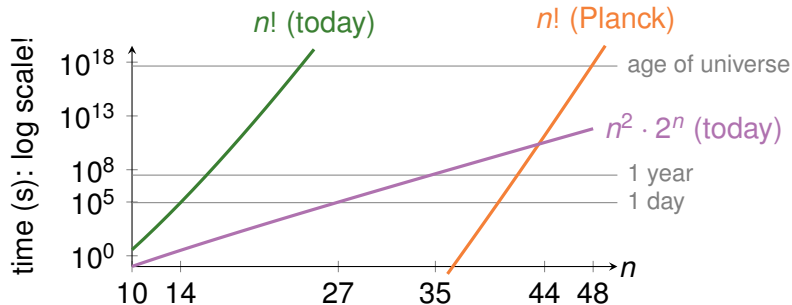
The dynamic program by Bellman-Held-Karp “only” takes  $\mathcal{O}(n^2 \cdot 2^n)$  time:  
 a computer of today takes a day for  $n = 27$ , a year for  $n = 35$ , the age of the  
 universe for  $n = 67$ , and beats the  $\mathcal{O}(n!)$  algo on Planck computer for  $n \geq 44$ .





# Intelligent Search upon NP-Hardness

Do not give up but try to stay ahead of the curve:  
there is an instance size until which an **exact** algorithm is fast enough!



**Concorde TSP Solver** beats the **Bellman-Held-Karp** exact algo: it uses local search & approximation algos, but sometimes proves exactness of its optima. The largest instance solved exactly, in 136 CPU years in 2006, has  $n = 85900$ .



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## 2. Combinatorial Optimisation

## 3. Modelling (in MiniZinc)

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A **solving technology** offers languages, methods, and tools for:

**what:** **Modelling** constraint problems in a **declarative** language.

and / or

**how:** **Solving** constraint problems **intelligently**:

- **Search**: Explore the space of candidate solutions.
- **Inference**: Reduce the space of candidate solutions.
- **Relaxation**: Exploit solutions to easier problems.

A **solver** is a program that takes a model and data as input and tries to solve that problem instance.

**Combinatorial (= discrete) optimisation** covers satisfaction *and* optimisation problems for variables ranging over *discrete* sets: **combinatorial problems**.

The ideas in this course extend to continuous optimisation, to soft optimisation, and to stochastic optimisation.



## Examples (Solving technologies)

With general-purpose solvers, taking model and data as input:

- Boolean satisfiability (SAT)
- SAT (resp. optimisation) modulo theories (SMT and OMT)
- (Mixed) integer linear programming (IP and MIP)
- Constraint programming (CP)
- ...
- Hybrid technologies (LCG = CP + SAT, ...)

👉 part 2 of 1DL442

Methodologies, *usually without* modelling and solvers:

- Dynamic programming (DP)
- Greedy algorithms
- Approximation algorithms
- Local search (LS)
- ...



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# What vs How

## Example

Consider the **problem** of sorting an array  $A$  of  $n$  numbers into an array  $S$  of increasing-or-equal numbers.

A **formal specification** is:

$$\text{sort}(A, S) \equiv \text{permutation}(A, S) \wedge \text{increasing}(S)$$

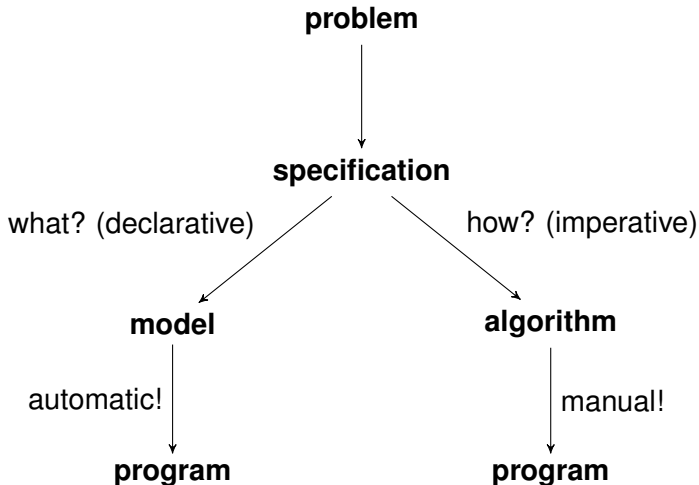
saying that  $S$  must be a permutation of  $A$  in increasing order.

Seen as a generate-and-test **algorithm**, it takes  $\mathcal{O}(n!)$  time, but it can be refined into the existing  $\mathcal{O}(n \log n)$  algorithms.

A **specification** is a **declarative** description of **what** problem is to be solved. An **algorithm** is an **imperative** description of **how** to solve the problem (fast).



# Modelling vs Programming





## Example (Sudoku)

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		

8	1	2	7	5	3	6	4	9
9	4	3	6	8	2	1	7	5
6	7	5	4	9	1	2	8	3
1	5	4	2	3	7	8	9	6
3	6	9	8	4	5	7	2	1
2	8	7	1	6	9	5	3	4
5	2	1	9	7	4	3	6	8
4	3	8	5	2	6	9	1	7
7	9	6	3	1	8	4	5	2

A Sudoku is a 9-by-9 array of integers in the range 1..9. Some of the elements are provided as parameters. The remaining elements are unknowns that have to satisfy the following constraints:

- 1 the elements in each row are all different;
- 2 the elements in each column are all different;
- 3 the elements in each 3-by-3 block are all different.





## Example (Sudoku)



Translate

Turn off instan

English Turkish Swedish English - detected ▾



MiniZinc Turkish Swedish ▾

Translate

A Sudoku is a 9-by-9 array of integers in the interval 1..9. ×  
Some of the elements are provided as parameters.  
The remaining elements are unknowns  
that have to satisfy the following constraints:  
- the elements in each row are all different;  
- the elements in each column are all different;  
- the elements in each 3-by-3 block are all different.

```
array[1..9,1..9] of var 1..9: Sudoku;  
solve satisfy;  
constraint forall(row in 1..9)  
  (alldifferent(Sudoku[row, ..]));  
constraint forall(col in 1..9)  
  (alldifferent(Sudoku[.., col]));  
constraint forall(i,j in {0,3,6})  
  (alldifferent(Sudoku[i+1..i+3, j+1..j+3]));
```



## Example (Sudoku )

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		

8	1	2	7	5	3	6	4	9
9	4	3	6	8	2	1	7	5
6	7	5	4	9	1	2	8	3
1	5	4	2	3	7	8	9	6
3	6	9	8	4	5	7	2	1
2	8	7	1	6	9	5	3	4
5	2	1	9	7	4	3	6	8
4	3	8	5	2	6	9	1	7
7	9	6	3	1	8	4	5	2

```

-2 array[1..9,1..9] of var 1..9: Sudoku;
-1 ... % load the hints
0 solve satisfy;
1 constraint forall(row in 1..9) (all_different(Sudoku[row,..]));
2 constraint forall(col in 1..9) (all_different(Sudoku[..,col]));
3 constraint forall(i,j in {0,3,6})
    (all_different(Sudoku[i+1..i+3,j+1..j+3]));

```



## Example (Agricultural experiment design, AED)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	✓	✓	✓	—	—	—	—
corn	✓	—	—	✓	✓	—	—
millet	✓	—	—	—	—	✓	✓
oats	—	✓	—	✓	—	✓	—
rye	—	✓	—	—	✓	—	✓
spelt	—	—	✓	✓	—	—	✓
wheat	—	—	✓	—	✓	✓	—

### Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- 3 Balance: Every grain pair is grown in 1 common plot.

**Instance:** 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.

General term: **balanced incomplete block design (BIBD)**.



## Example (Agricultural experiment design, AED)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	1	1	1	0	0	0	0
corn	1	0	0	1	1	0	0
millet	1	0	0	0	0	1	1
oats	0	1	0	1	0	1	0
rye	0	1	0	0	1	0	1
spelt	0	0	1	1	0	0	1
wheat	0	0	1	0	1	1	0

### Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
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- 3 Balance: Every grain pair is grown in 1 common plot.

**Instance:** 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.

General term: **balanced incomplete block design (BIBD)**.



In a BIBD, the plots are called **blocks** and the grains are called **varieties**:

Example (BIBD *integer* model :  $\checkmark \rightsquigarrow 1$  and  $- \rightsquigarrow 0$ )

```
-3 enum Varieties; enum Blocks;
-2 int: blockSize; int: sampleSize; int: balance;
-1 array[Varieties,Blocks] of var 0..1: BIBD; % BIBD[v,b]=1 iff v is in b
0 solve satisfy;
1 constraint forall(b in Blocks) (blockSize = sum(BIBD[..,b]));
2 constraint forall(v in Varieties) (sampleSize = sum(BIBD[v,..]));
3 constraint forall(v, w in Varieties where v < w)
    (balance = sum([BIBD[v,b]*BIBD[w,b] | b in Blocks]));
```

Example (Instance data for our AED )

```
-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Using the `count` abstraction instead of `sum`:

Example (BIBD *integer* model :  $\checkmark \rightsquigarrow 1$  and  $- \rightsquigarrow 0$ )

```
-3 enum Varieties; enum Blocks;
-2 int: blockSize; int: sampleSize; int: balance;
-1 array[Varieties,Blocks] of var 0..1: BIBD; % BIBD[v,b]=1 iff v is in b
0 solve satisfy;
1 constraint forall(b in Blocks) (blockSize = count(BIBD[..,b], 1));
2 constraint forall(v in Varieties) (sampleSize = count(BIBD[v,..], 1));
3 constraint forall(v, w in Varieties where v < w)
    (balance = count([BIBD[v,b]*BIBD[w,b] | b in Blocks], 1));
```

Example (Instance data for our AED )

```
-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Using the `count` abstraction over **linear** expressions:

Example (BIBD *integer* model :  $\checkmark \rightsquigarrow 1$  and  $- \rightsquigarrow 0$ )

```
-3 enum Varieties; enum Blocks;
-2 int: blockSize; int: sampleSize; int: balance;
-1 array[Varieties,Blocks] of var 0..1: BIBD; % BIBD[v,b]=1 iff v is in b
0 solve satisfy;
1 constraint forall(b in Blocks) (blockSize = count(BIBD[..,b], 1));
2 constraint forall(v in Varieties) (sampleSize = count(BIBD[v,..], 1));
3 constraint forall(v, w in Varieties where v < w)
    (balance = count([BIBD[v,b]+BIBD[w,b] | b in Blocks], 2));
```

Example (Instance data for our AED )

```
-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Reconsider the model fragment:

```
2 constraint forall(v in Varieties) (sampleSize = count(BIBD[v,...], 1));
```

This constraint is **declarative** (and by the way non-linear),  
so read it using only the verb “to be” or synonyms thereof:

*for all varieties  $v$ ,  
the count of occurrences of 1 in row  $v$  of BIBD  
must equal sampleSize*

The constraint is **not procedural**:

*for all varieties  $v$ ,  
we first count the occurrences of 1 in row  $v$   
and then check if that count equals sampleSize*

The latter reading is appropriate for solution **checking**,  
but solution **finding** performs no such procedural counting.





## Example (Idea for another BIBD model)

barley	{plot1, plot2, plot3}
corn	{plot1, plot4, plot5}
millet	{plot1, plot6, plot7}
oats	{plot2, plot4, plot6}
rye	{plot2, plot5, plot7}
spelt	{plot3, plot4, plot7}
wheat	{plot3, plot5, plot6}

### Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- 3 Balance: Every grain pair is grown in 1 common plot.



## Example (BIBD set model : a block set per variety)

```

-3 enum Varieties; enum Blocks;
-2 int: blockSize; int: sampleSize; int: balance;
-1 array[Varieties] of var set of Blocks: BIBD; % BIBD[v] = blocks for v
0 solve satisfy;
1 constraint forall(b in Blocks)
    (blockSize = sum(v in Varieties) (b in BIBD[v]));
2 constraint forall(v in Varieties)
    (sampleSize = card(BIBD[v]));
3 constraint forall(v, w in Varieties where v < w)
    (balance = card(BIBD[v] intersect BIBD[w]));

```

## Example (Instance data for our AED )

```

-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;

```



## Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A	call	none	oper	none	oper	none	none
Doctor B	appt	call	none	oper	none	none	call
Doctor C	oper	none	call	appt	appt	call	none
Doctor D	appt	oper	none	call	oper	none	none
Doctor E	oper	none	oper	none	call	none	none

### Constraints to be satisfied:

- 1 #on-call doctors / day = 1
- 2 #operating doctors / weekday  $\leq 2$
- 3 #operating doctors / week  $\geq 7$
- 4 #appointed doctors / week  $\geq 4$
- 5 day off after operation day
- 6 ...



### Objective function to be minimised: Cost: ...



## Example (Doctor rostering )

```
-5 set of int: Days;    % d mod 7 = 1 iff d is a Monday
-4 enum Doctors;
-3 enum ShiftTypes = {appt, call, oper, none};
-2 % Roster[i,j] = shift type of Dr i on day j:
-1 array[Doctors,Days] of var ShiftTypes: Roster;
0 solve minimize ...; % plug in an objective function
1 constraint forall(d in Days) (count(Roster[..,d],call) = 1);
2 constraint forall(d in Days where d mod 7 in 1..5)
   (count(Roster[..,d],oper) <= 2);
3 constraint count(Roster,oper) >= 7;
4 constraint count(Roster,appt) >= 4;
5 constraint forall(d in Doctors)
   (regular(Roster[d,..], "((oper none) | appt | call | none)*"));
6 ... % other constraints
```

## Example (Instance data for our small hospital unit )

```
-5 Days = 1..7;
-4 Doctors = {Dr_A, Dr_B, Dr_C, Dr_D, Dr_E};
```



Using decision variables as indices within arrays: **black magic?!**

### Example (Job allocation at minimal salary cost)

**Given** jobs `Jobs` and the salaries of work applicants `Apps`,

**find** a work applicant for each job

**such that** some constraints (on the qualifications of the work applicants for the jobs, on workload distribution, etc) are satisfied and the total salary cost is minimal:

```
1 array[Apps] of 0..1000: Salary; % Salary[a] = cost per job to appl. a
2 array[Jobs] of var Apps: Worker; % Worker[j] = appl. allocated job j
3 solve minimize sum(j in Jobs) (Salary[Worker[j]]);
4 constraint ...; % qualifications, workload, etc
```



Using decision variables as indices within arrays: **black magic?!**

## Example (Vehicle routing: backbone model)

```
enum Cities = {AMS, BRU, LUX, CDG}
```

Next: 

AMS	BRU	LUX	CDG





Using decision variables as indices within arrays: **black magic?!**

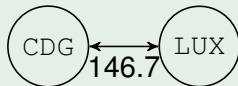
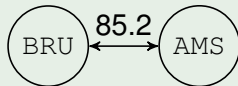
## Example (Vehicle routing: backbone model)

```
enum Cities = {AMS, BRU, LUX, CDG}
```

Next: 

AMS	BRU	LUX	CDG
BRU	AMS	CDG	LUX

So `all_different`(Next) is too weak!





Using decision variables as indices within arrays: **black magic?!**

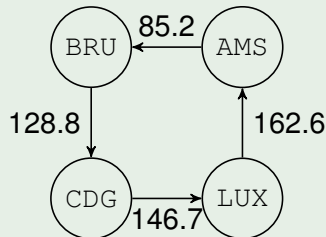
## Example (Vehicle routing: backbone model)

```
enum Cities = {AMS, BRU, LUX, CDG}
```

Next: 

AMS	BRU	LUX	CDG
BRU	CDG	AMS	LUX

Let us use `circuit` (Next) instead:







Using decision variables as indices within arrays: **black magic?!**

## Example (Vehicle routing: backbone model)

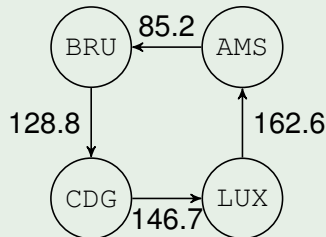
```
enum Cities = {AMS, BRU, LUX, CDG}
```

Next: 

AMS	BRU	LUX	CDG
BRU	CDG	AMS	LUX

Let us use `circuit` (Next) instead:

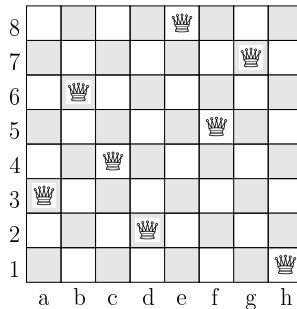
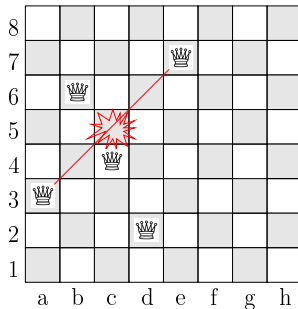
```
1 array[Cities,Cities] of float: Distance; % instance data
2 array[Cities] of var Cities: Next; % travel from c to Next[c]
3 solve minimize sum(c in Cities) (Distance[c,Next[c]]);
4 constraint circuit (Next);
5 constraint ...; % side constraints, if any
```





# Toy Example: 8-Queens

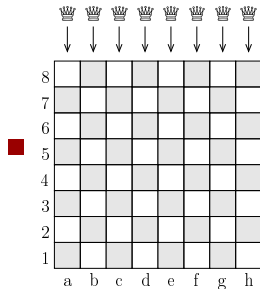
Can one place 8 queens onto an  $8 \times 8$  chessboard so that all queens are in distinct rows, columns, and diagonals?





# An 8-Queens Model

One of the many models, with **one** decision variable per queen:



Let **decision variable**  $\text{Row}[c]$ , of **domain**  $1 \dots 8$ , **denote** the row of **the** queen in column  $c$ , for  $c$  in  $\{a, b, c, \dots, h\}$ , which we rename into  $1 \dots 8$ . Example:  $\text{Row}[3] = 4$  means that the queen of column 3 (column  $c$  in the picture) is in row 4. The **constraint** that all queens must be in distinct columns is **satisfied** by the choice of variables!

■ The remaining **constraints** to be **satisfied** are:

- All queens are in distinct rows: the var.s  $\text{Row}[c]$  take distinct values for all  $c$
- All queens are in distinct diagonals:  
the expressions  $\text{Row}[c] + c$  take distinct values for all  $c$   
the expressions  $\text{Row}[c] - c$  take distinct values for all  $c$



# An 8-Queens Model in MiniZinc

Consider the following model [↗](#) in a file `8-queens.mzn`:

```
1 include "globals.mzn"; % ensures that lines 4 to 6 are understood
2 int: n = 8; % the given number of queens
3 array[1..n] of var 1..n: Row; % Row[c] = the unknown row of the queen
   in column c; % enforces that all queens are in distinct columns
4 constraint all_different( Row ); % distinct rows
5 constraint all_different( [Row[c]+c | c in 1..n] ); % distinct up-dia.
6 constraint all_different( [Row[c]-c | c in 1..n] ); % distinct down-dia.
7 solve satisfy; % solve to satisfaction of all the constraints
8 output [show(Row)]; % pretty-printing of solutions
```

The `all_different(X)` constraint holds if and only if all the expressions in the array `X` take different values.



# Modelling Concepts

---

- A **variable**, also called a **decision variable**, is an existentially quantified unknown of a problem.
- The **domain** of a decision variable  $x$ , here denoted by  $\text{dom}(x)$ , is the set of values in which  $x$  must take its value, if any.
- A **variable expression** takes a value that depends on the value of one or more decision variables.
- A **parameter** has a value from a problem description.
- Decision variables, parameters, and expressions are **typed**.

MiniZinc types are (arrays and sets of) Booleans, integers, floating-point numbers, enumerations, records, tuples, and strings, but not all these types can serve as types for decision variables.



# Decision Variables, Parameters, and Identifiers

- Decision variables and parameters in a model are concepts very different from programming variables in an imperative or object-oriented program.
- A decision variable in a model is like a variable in mathematics: it is *not* given a value in a model or a formula, and its value is only fixed in a solution, if a solution exists.
- A parameter in a model must be given a value, but only once: we say that it is *instantiated*.
- A decision variable or parameter is referred to by an *identifier*.
- An *index identifier* of an array *comprehension* takes on all its designated values in turn. Example: the index *c* in the 8-queens model.



# Parametric Models

---

- A parameter need not be instantiated inside a model.  
Example: drop “=8” from “`int : n=8`” in the 8-queens model to make it an `n-queens` model, and rename `8-queens.mzn` into `n-queens.mzn`.
- **Data** are values for parameters given outside a model:  
either in a **datafile** (`.dzn` suffix), or at the command line,  
or interactively in the integrated development environment (IDE).
- A **parametric model** has uninstantiated parameters.
- An **instance** is a pair of a parametric model and data.



# Modelling Concepts (end)

---

- A **constraint** is a restriction on the values that its decision variables can take together; equivalently, it is a Boolean-valued variable expression that must be true.
- An **objective function** is a numeric variable expression whose value is to be either minimised or maximised.
- An **objective** states what is being asked for:
  - find a first solution
  - find a solution minimising an objective function
  - find a solution maximising an objective function
  - find all solutions
  - count the number of solutions
  - prove that there is no solution
  - ...





# Constraint-Based Modelling

MiniZinc is a high-level **constraint-based** modelling language (*not* a solver):

- There are several **types** for decision variables: `bool`, `int`, `float`, `enum`, `string`, `tuple`, `record`, and `set`, possibly as elements of multidimensional matrices (`array`).
- There is a large vocabulary of **predicates** (`<`, `<=`, `=`, `!=`, `>=`, `>`, `all_different`, `circuit`, `regular`, ...), **functions** (`+`, `-`, `*`, `card`, `count`, `intersect`, `sum`, ...), and **logical connectives & quantifiers** (`not`, `/\`, `\/,` `->`, `<-`, `<->`, `forall`, `exists`, ...).
- There is support for *both* constraint **satisfaction** (`satisfy`) *and* constrained **optimisation** (`minimize` and `maximize`).

Most modelling languages are (much) lower-level than this!



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# Correctness Is Not Enough for Models

Constraint  
Problems

Combinatorial  
Optimisation

Modelling  
(in MiniZinc)

Solving

The MiniZinc  
Toolchain

Course  
Information

Part 1: Modelling for  
Combinatorial  
Optimisation

Part 2: Combinatorial  
Optimisation and CP

Contact





# Modelling is an Art!

---

There are good and bad models for each constraint problem:

- Different models of a problem may take different time on the same solver for the same instance.
- Different models of a problem may scale differently on the same solver for instances of growing size.
- Different solvers may take different time on the same model for the same instance.

**Good modellers are worth their weight in gold!**

**Use solvers: based on decades of cutting-edge research, they are very hard to beat on exact solving.**



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---

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## 2. Combinatorial Optimisation

## 3. Modelling (in MiniZinc)

## 4. Solving

## 5. The MiniZinc Toolchain

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Solutions to a problem instance can be found by running a MiniZinc **backend**, that is a MiniZinc wrapper for a particular solver, on a file containing a model of the problem.

## Example (Solving the 8-queens instance)

Let us run the solver Gecode, of CP technology, from the command line:

```
minizinc --solver gecode 8-queens.mzn
```

The result is printed on stdout:

```
[4, 2, 7, 3, 6, 8, 5, 1]
```

```
-----
```

This means that the queen of column 1 is in row 4 (note that MiniZinc uses 1-based indexing), the queen of column 2 is in row 2, and so on.

Use the command-line flag `-a` to ask for all solutions:

the line `-----` is printed after each solution, but the

line `=====` is printed after the last (the 92nd here) solution.



## Definition (Solving = Search + Inference + Relaxation)

- **Search**: Explore the space of candidate solutions.
- **Inference**: Reduce the space of candidate solutions.
- **Relaxation**: Exploit solutions to easier problems.

## Definition (Systematic Search: guarantees ultimately exact solving)

Progressively build a solution, and backtrack if necessary.

Use **inference** and **relaxation** in order to reduce the **search** effort.

It is used in most SAT, SMT, OMT, CP, LCG, and MIP solvers.

## Definition (Local Search: trades guarantee of exact solving for speed)

Start from a candidate solution and iteratively modify it a bit.

It is the basic idea behind LS and genetic algorithm (GA) technologies.

For some details, see Topic 7: Solving Technologies.



# There Are So Many Solving Technologies

---

- No technology universally dominates all the others.
- One should test several technologies on each problem.
- Some technologies have **no** modelling languages:  
LS, DP, and GA are rather methodologies.
- Some technologies have **standardised** modelling languages  
across all solvers: SAT, SMT, OMT, and (M)IP.
- Some technologies have **non-standardised** modelling languages  
across their solvers: CP and LCG.



# Model and Solve

---

## Advantages:

- + Declarative model of a problem.
- + Easy adaptation to changing problem requirements.
- + Use of powerful solving technologies that are based on decades of cutting-edge research.

## Disadvantages:

- Do I need to learn several modelling languages? **No!**
- Do I need to understand the used solving technologies in order to get the most out of them? **Yes, but ... !**





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# MiniZinc

MiniZinc is a declarative language (*not* a solver)  
for the constraint-based modelling of constraint problems:



- At Monash University, Australia
- Introduced in 2007; version 2.0 in 2014
- Homepage: <https://www.minizinc.org>
- Integrated development environment (IDE)
- Annual [MiniZinc Challenge](#) for solvers, since 2008
- There are also [courses at Coursera](#), also in Chinese



# MiniZinc Features

---

- Declarative language for modelling **what** the problem is
- Separation of problem **model** and instance **data**
- **Open-source** toolchain
- Much **higher**-level language than those of (M)IP and SAT
- Solver-**independent** language
- Solving-technology-**independent** language
- Vocabulary of **predefined** types, predicates and functions
- Support for **user-defined** predicates and functions
- Support for annotations with hints on **how** to solve
- Ever-growing number of users, solvers, and other tools



# MiniZinc Backends and Their Solvers

- SAT = Boolean satisfiability: Plingeling via PicatSAT, ...
- MIP = mixed integer programming: Cbc, FICO Xpress, Gurobi Optimizer, HiGHS, IBM ILOG CPLEX Optimizer, ...
- CP = constraint programming:  
Choco, Gecode, JaCoP, Mistral, SICStus Prolog, ...
- CBLS = constraint-based LS (local search), without exactness guarantee:  
Atlantis, OscaR.cblls via fzn-oscar-cblls, Yuck, ...
- LCG = lazy clause generation, a hybrid of CP and SAT:  
Chuffed, Google's CP-SAT of OR-Tools, ...
- Other hybrid technologies: iZplus, MiniSAT(ID), SCIP, ...
- ..., SMT, OMT, portfolios of solvers, ...



# MiniZinc Backends and Their Solvers

- SAT = Boolean satisfiability: **Plingeling** via **PicatSAT**, ...
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- ..., SMT, OMT, portfolios of solvers, ...

The backends installed on the IT department's ThinLinc hardware are in **red**.  
The commercial **Gurobi Optimizer** is under a free academic license:  
you may **not** use it for non-academic purposes.



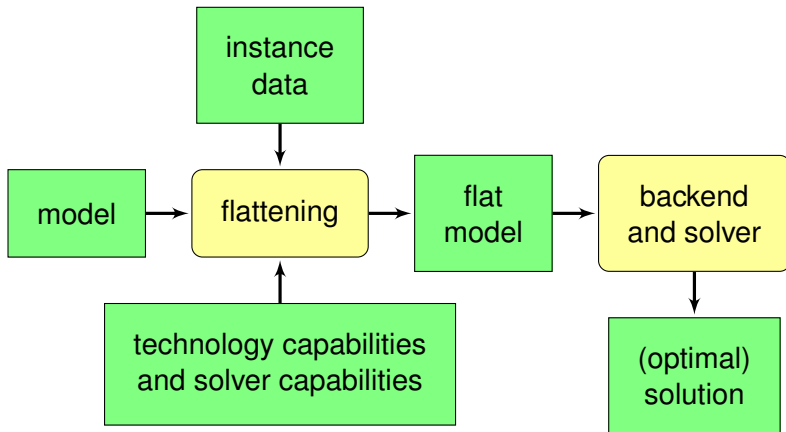
## MiniZinc Challenge 2015: Some Problems and Winners

Problem and Model	Backend and Solver	Technology
Costas array	Mistral	CP
capacitated VRP	iZplus	hybrid
GFD schedule	Chuffed	LCG
grid colouring	MiniSAT(ID)	hybrid
instruction scheduling	Chuffed	LCG
large scheduling	Google OR-Tools.cp	CP
application mapping	JaCoP	CP
multi-knapsack	mzn-cplex	MIP
portfolio design	<b>fzn-oscar-cbls</b>	CBLS
open stacks	Chuffed	LCG
project planning	Chuffed	LCG
radiation	mzn-gurobi	MIP
satellite management	mzn-gurobi	MIP
zephyrus configuration	mzn-cplex	MIP

(portfolio and parallel categories omitted)



# MiniZinc: Model Once, Solve Everywhere!



From a **single** language, one has access transparently to a wide range of solving technologies from which to choose.



# There Is No Need to Reinvent the Wheel!

Before solving, each decision variable of a **type** that is non-native to the targeted solver is replaced by decision variables of native types, using some well-known linear / clausal / ... encoding.

## Example (SAT)

The **order encoding** of integer decision variable `var 4..6: x` is

```
array[4..7] of var bool: B; % B[i] denotes truth of  $x \geq i$ 
constraint B[4];           % lower bound on x
constraint not B[7];        % upper bound on x
constraint B[4] \/\ not B[5]; % consistency
constraint B[5] \/\ not B[6]; % consistency
constraint B[6] \/\ not B[7]; % consistency
```

For an integer decision variable with  $n$  domain values, there are  $n + 1$  Boolean decision variables and  $n$  clauses, all 2-ary.





Before solving, each use of a non-native **predicate** or **function** is replaced by:

- either: its MiniZinc-provided default definition,  
stated in terms of a kernel of imposed predicates;

### Example (default; not to be used for IP and MIP)

```
all_different([x,y,z]) gives  $x \neq y \wedge y \neq z \wedge z \neq x$ .
```

- or: a backend-provided solver-specific definition,  
using some well-known linear / clausal / ... encoding.

### Example (IP and MIP)

A compact linearisation of  $x \neq y$  is

```
var 0..1: p;                                % p = 1 denotes that x < y holds
int: Mx = ub(x-y+1); int: My = ub(y-x+1);    % big-M constants
constraint x + 1 <= y + Mx * (1-p);          % either x < y and p = 1
constraint y + 1 <= x + My * p;              % or x > y and p = 0
```

One cannot naturally model graph colouring in IP,  
but the problem has integer decision variables (ranging over the colours).



# Benefits of Model-and-Solve with MiniZinc

---

- + Try many solvers of many technologies from 1 model.
- + A model improves with the state of the art of backends:
  - Type of decision variable: native representation or encoding.
  - Predicate: **inference**, **relaxation**, and definition.
  - Implementation of a solving technology.

More on this in Topic 7: Solving Technologies.

- + For most managers, engineers, and scientists, it is easier with such a model-once-and-solve-everywhere toolchain to achieve good solution quality and high solving speed, including for harder data, and this without knowing (deeply) how the solvers work, compared to programming from first principles.



# How to Solve a Constraint Problem?

---

**1** Model the problem

**2** Solve the problem

Easy, right?



# How to Solve a Constraint Problem?

## 1 Model the problem

- Understand the problem
- Choose the decision variables and their domains
- Choose predicates to formulate the constraints
- Formulate the objective function, if any
- Make sure the model really represents the problem
- Iterate!

## 2 Solve the problem

- Choose a solving technology
- Choose a backend
- Choose a search strategy, if not black-box search
- Improve the model
- Run the model and interpret the (lack of) solution(s)
- Debug the model, if need be
- Iterate!

Easy, right?



# How to Solve a Constraint Problem?

---

## 1 Model the problem

- Understand the problem
- Choose the decision variables and their domains
- Choose predicates to formulate the constraints
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- Choose a backend
- Choose a search strategy, if not black-box search
- Improve the model
- Run the model and interpret the (lack of) solution(s)
- Debug the model, if need be
- Iterate!

Not so easy, but much easier than without a modelling tool!



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---

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Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinatorial Optimisation and CP

Contact

Constraint  
Problems

Combinatorial  
Optimisation

Modelling  
(in MiniZinc)

Solving

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# Outline

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## 1. Constraint Problems

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# Content of Part 1 = M4CO (course 1DL451)

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The use of tools for solving a combinatorial problem, by

- 1 first modelling it in a solving-technology-independent constraint-based modelling language, and
- 2 then running the model on an off-the-shelf solver.





# Learning Outcomes of Part 1 = M4CO

In order to pass, the student must be able to:

- define the concept of combinatorial (optimisation or satisfaction) problem;
- explain the concept of constraint, as used in a constraint-based language;
- model a combinatorial problem in a solving-technology-independent constraint-based modelling language;
- compare empirically several models, say by introducing redundancy or by detecting and breaking symmetries;
- describe and compare solving technologies that can be used by the backends to a modelling language, including CP, LS, SAT, SMT, and MIP;
- choose suitable solving technologies for a new combinatorial problem, and motivate this choice;
- **present and discuss topics related to the course content, orally and in writing, with a skill appropriate for the level of education.**
  - ✎ written reports and oral resubmissions!



# Organisation & *Suggested* Time Budget of Part 1 = M4CO

Period 1: early September to early November, budget = 133.3 h:

- No textbook: slides, [MiniZinc](#) documentation, [Coursera](#)
- 1 **warm-up session** for learning the [MiniZinc](#) toolchain
- 3 teacher-chosen **assignments** with 3 **help sessions**, 1 **grading session**, and 1 **solution session** each, to be done in student-chosen duo team:  
*suggested* budget = average of 21 hours / assignment / student (3 credits)
- 1 student-chosen **project**, to be done in student-chosen duo team, and individual written **peer review** of another team's initial report:  
*suggested* budget = 49.5 hours / student (2 credits)
- 12 **lectures**, including a **mandatory** guest lecture: budget = 21 hours
- Prerequisites: basic concepts in algebra, combinatorics, logic, graph theory, set theory, and implementation of basic search algorithms



# No Exams in Part 1 and Part 2

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Both M4CO (1DL451) and COCP (1DL442) have no exam!

You must demonstrate — by writing **reports** — that you cannot only code, namely:

- correctly and efficiently solve a constraint problem via a model (in Part 1),
- design a correct and efficient **inference** algorithm or **search** algorithm for a CP solver (in Part 2),

but also motivate and explain your code in terms of **all** the course concepts, as well as experimentally demonstrate the correctness and efficiency of your code.



# Lecture Topics of Part 1 = M4CO

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- Topic 1: Introduction
- Topic 2: Basic Modelling
- Topic 3: Constraint Predicates
- Topic 4: Modelling (for CP and LCG)
- Topic 5: Symmetry
- Topic 6: Case Studies
- Topic 7: Solving Technologies
- Topic 8: Inference & Search in CP & LCG
- (Topic 9: Modelling for CBLS)
- (Topic 10: Modelling for SAT, SMT, and OMT)
- (Topic 11: Modelling for MIP)



## 3 Assignment Cycles of 2 to 3 Weeks in Part 1 = M4CO

Let  $D_i$  be the deadline day of Assignment  $i$ , with  $i \in 1..3$ :

- $D_i - 14$ : publication and all needed material was taught: start!
- $D_i - 8$ : help session a: participation strongly recommended!
- $D_i - 4$ : help session b: participation strongly recommended!
- $D_i - 2$ : help session c: participation strongly recommended!
- $D_i \pm 0$ : submission, by 13:00 Swedish time on a Friday
- $D_i + 5$  by 16:00: initial score  $a_i \in 0..5$  points
- $D_i + 6$ : teamwise oral grading session for some  $a_i \in \{1, 2\}$ :  
possibility of earning 1 extra point for final score;  
otherwise final score = initial score
- $D_i + 6 = D_{i+1} - 8$ : solution session and help session a



# Assignments (3 credits) and Overall Grade in Part 1

The final score on Assignment 1 is actually “pass” or “fail”.

Let  $a_i \in 0..5$  be the **final score** on Assignment  $i$ , with  $i \in 2..3$ :

- **20% threshold:**  $\forall i \in 2..3 : a_i \geq 20\% \cdot 5 = 1$   
No catastrophic failure on individual assignments
- **50% threshold:**  $m = a_2 + a_3 \geq 50\% \cdot (5 + 5) = 5$   
The formulae for the **modelling assignment grade** and **project grade** in 3..5 are at the course homepage
- **Worth going full-blast:** A **modelling assignment sum**  $m \in 5..10$  is combined with a **project score**  $p \in 5..10$  in order to determine the **overall grade** in 3..5 for 1DL451 according to a formula at the course homepage



# Project (2 credits) in Part 1 = M4CO

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## Topic:

- Model and solve a combinatorial problem that you are interested in, say for research, a course, a hobby, . . .
- See the Project page at the course homepage for ideas for projects and the format for a project proposal.

## Deadlines in 2024 (overlap with Assignments 2 and 3):

- Wed 18 Sep at 13:00: upload several proposals
- Wed 25 Sep at 13:00: secure our approval; start!
- Fri 18 Oct at 13:00: upload initial report
- Wed 23 Oct at 13:00: upload individual peer review
- Fri 1 Nov at 13:00: upload final report; score  $p \in 0..10$



# Project Guidelines

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- Start early, despite the time overlap with Assignments 2 and 3.
- Attend the project help sessions, some jointly for Assignment 3.
- Read the Rules and Grading Criteria at the Project page.
- An **approach** is either a model for the entire problem, or a script (consider using **MiniZinc Python**) with pre-processing + solving (possibly on a pipeline of multiple models) + post-processing: the final report is on **one** sufficiently complete and efficient approach.
- The initial report is on **one** approach, but it need be neither the final one, nor complete, nor efficient.
- Use the demo report with line 16 (instead of 15) for defining `\project`.





# Project Guidelines (end)

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- Model the constraints incrementally, and be prepared to backtrack to the choice of decision variables (aka viewpoint).
- If the instances are too easy, then you still need to demonstrate skills in the advanced concepts (49.5h!).
- If the instances are too hard, then relax the problem (say by some loss of precision on the objective value) or some instances (or both).
- Collaborate with other teams that work on the same problem for the parsing, generation, or simplification of shared instances, and so on (but *not* for modelling). There is *no* competition between such teams.
- Consider also using the powerful local-search backend Gecode-LNS for the experiments (see Assignment 3).



# Assignment and Project Rules

Register a **team** by Sun 8 Sep 2024 at 23:59 at Studium:

- **Duo team:** Two consenting teammates sign up.
- **Solo team:** Apply to the head teacher, who rarely agrees.
- **Random teammate?** Request from the helpdesk, else you are bounced.

Other considerations:

- **Why (not) like this? Why no email reply?** See FAQ.
- **Teammate swapping:** Allowed, but to be declared to the helpdesk.
- **Teammate scores may differ** if no-show or passivity at grading session.
- **No freeloader:** Implicit honour declaration in reports that each teammate can individually explain everything; random checks will be made by us!
- **No plagiarism:** Implicit honour declaration in reports; extremely powerful detection tools will be used by us; suspected cases of using **or providing** will be reported!



# Outline

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# Learning Outcomes of Part 2 = COCP

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In order to pass, the student must be able to:

- describe how a CP solver works,  
by giving its architecture and explaining the principles it is based on;
- augment a CP solver with a **propagator** for a new constraint predicate,  
and evaluate empirically whether the propagator is better than a **definition**  
based on the existing constraint predicates of the solver;
- devise empirically a (problem-specific) **search strategy** that can be used  
by a CP solver;
- design and compare empirically several constraint programs (with model  
and search parts) for a combinatorial problem;
- **present and discuss topics related to the course content,  
orally and in writing, with a skill appropriate for the level of education.**  
☞ **written reports!**



# Organisation and Time Budget of Part 2 = COCP

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Period 2: early November to mid January(!), budget = 133.3 h:

- 12 **lectures**, including a **mandatory** guest lecture:  
budget = 22.5 hours
- No textbook: slides and **MiniCP** teaching materials, with videos at [edX.org](https://edX.org)
- 1 **warm-up session** about the **MiniCP** code base, **INGInious**, and **GitHub**
- 3 teacher-chosen **assignments**, with 3 **help sessions** and 1 **solution session** each (but no grading session), done in student-chosen duo team:  
budget = average of 37 hours / assignment / student (5 credits)
- Prerequisites: Java; basic concepts in algebra, combinatorics, logic, graph theory, set theory, and implementation of basic search algorithms



# Lecture Topics of Part 2 = COCP

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- Topic 12: CP and the MiniCP Solver
- Module 1: TinyCSP
- Module 2: MiniCP: Domains, Variables, Constraints, Propagation, Fixpoint Algorithm, Views, State Management, Search, Backtracking
- Module 3: Sum Constraint, Element Constraint, Consistency
- Module 4: Table Constraint
- Module 5: AllDifferent Constraint
- Module 6: Circuit Constraint, Vehicle Routing, and LNS
- Module 7: Cumulative Scheduling
- Module 8: Disjunctive Scheduling
- Module 9: Black-Box Search
- Topic 18: Conclusion



## 3 Assignment Cycles of 2 to 3 Weeks in Part 2 = COCP

Let  $D_i$  be the deadline day of Assignment  $i$ , with  $i \in 4..6$ :

- $D_i - 14$ : **publication** and all needed material was taught: start!
- $D_i - 7$ : **help session a**: participation strongly recommended!
- $D_i - 4$ : **help session b**: participation strongly recommended!
- $D_i - 2$ : **help session c**: participation strongly recommended!
- $D_i \pm 0$ : **submission**, by 13:00 Swedish time on a Friday
- $D_i + 6$  by 16:00: **final score**  $a_i \in 0..5$  **points**
- No **initial grade** and no **grading session**!
- $D_i + 6 = D_{i+1} - 8$ : **solution session** and **help session a**



# Assignments (5 credits) in Part 2 and Overall Grade

The final score on Assignment 4 is actually “pass” or “fail”.

Let  $a_i \in 0..5$  be the **final score** on Assignment  $i$ , with  $i \in 5..6$ :

- **20% threshold:**  $\forall i \in 5..6 : a_i \geq 20\% \cdot 5 = 1$

No catastrophic failure on individual assignments

- **50% threshold:**  $c = a_5 + a_6 \geq \lceil 50\% \cdot (5 + 5) \rceil = 5$

The formula for the **programming assignment grade** in 3..5 is at the course homepage

- **Worth going full-blast:** A **modelling assignment sum**  $m \in 5..10$  is combined with a **project score**  $p \in 5..10$  and a **programming assignment sum**  $c \in 5..10$  in order to determine the **overall grade** in 3..5 for 1DL442 according to a formula at the course homepage





# Assignment Rules

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Register a **team**, if new, by Sun 10 Nov 2024 at 23:59:

- **Duo team:** Two consenting teammates inform the helpdesk.
- **Solo team:** Apply to the head teacher, who rarely agrees.
- **Random teammate?** Request from the helpdesk, else you are bounced.

Other considerations:

- **Why (not) like this? Why no email reply?** See FAQ
- **Teammate swapping:** Allowed, but to be declared to the helpdesk.
- **Teammate scores may differ**
- **No freeloader:** Implicit honour declaration in reports that each teammate can individually explain everything; random checks will be made by us!
- **No plagiarism:** Implicit honour declaration in reports; extremely powerful detection tools will be used by us; suspected cases of using **or providing** will be reported



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# How To Communicate by Email or Studium?

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- If you have a question about the **lecture material** or **course organisation**, then email the head teacher. An immediate answer will be given right before and after lectures, as well as during their breaks.

- If you have a question about the **assignments** or **infrastructure**, then contact the assistants at a help session or solution session for an immediate answer.

Short *clarification* questions (that is: *not* about modelling or programming issues) that are either emailed (see the address at the course website) or posted (at the Studium discussion) to the **COCP helpdesk** are answered as soon as possible during working days and hours.

No answer means that you should go to a help session:

**almost all the assistants' budgeted time is allocated to grading and to the help, grading, and solution sessions.**



# What Has Changed Since Last Time?

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## Changes made by the TekNat Faculty:

- Period 1 is one day shorter (now 9 weeks, again not 10): less time for the Project after Assignment 3, and you need to work on them in parallel.
- Period 1 starts 5 days later, hence Assignment 5 must be due 2 (not 3) weeks after Assignment 4, and we must begin teaching the material for Assignments 5 and 6 *before* the deadlines of Assignments 4 and 5.

## Changes triggered by the formal and informal course evaluations:

- Lectures of a week are not on consecutive days (except in the first week).
- Emphasis that many models are in [MiniZinc Benchmarks](#) and [Challenge](#).



# What To Do Now in Part 1?

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- Bookmark and read course website, especially FAQs.
- Read Sections 1 to 2.2 of the [MiniZinc Handbook](#).
- Get started on Assignment 1 and have questions ready for its first help session, which is on Fri 6 Sep 2024.
- Register a duo team by Sun 8 Sep 2024 at 23:59, possibly upon advertising for a teammate at a course event or the discussion at Studium, and requesting a random teammate from the helpdesk as a last resort.
- Install the MiniZinc toolchain on your hardware, if you have any.
- Be aware that few questions are tagged with MiniZinc at StackOverflow: you have to read the documentation.



# What To Do Now in Part 2?

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- Bookmark and re-read course website, especially FAQs.
- Inform us of a new duo team by Sun 10 Nov 2024 at 23:59, possibly upon advertising for a teammate at a course event or the discussion at Studium, and requesting a random teammate from the helpdesk as a last resort.
- Sign up at [edX](#) if you want to watch the MiniCP videos.
- Attend the warm-up session on [MiniCP](#), [INGInious](#), and [GitHub](#) on Fri 8 Nov 2024, and install MiniCP on your hardware, if you have any.
- Get started on Assignment 4 and have questions ready for its first help session, which is on Fri 15 Nov 2024.
- Get started on Assignment 5 before the deadline of Assignment 4: you can ask questions on Assignment 5 at the help sessions on Assignment 4.
- Be aware that there is no StackOverflow-like website for avoiding to have to read the MiniCP documentation.