



Topic 18: Conclusion

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Course 1DL451:
Modelling for Combinatorial Optimisation



Outline

1. Constraint Problems
2. Constraint Programming Technology
3. Constraint-Based Modelling
4. History, Success Stories, and Opportunities

Constraint
Problems

Constraint
Program-
ming
Technology

Constraint-
Based
Modelling

History,
Success
Stories, and
Opportunities



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Many important real-life problems are NP-hard or worse and can only be solved exactly and fast enough by **intelligent** search, unless $P = NP$:

- Assignment: personnel rostering, resource allocation, ...
- Configuration of products, design, experiment set-up, ...
- Packing: container or vehicle loading, carpet cutting, ...
- Routing of robots, vehicles, ...
- Scheduling, planning, ...
- ... hybrid problems, such as time-tabling and transportation logistics ...

Definition

In a **constraint problem**, values have to be **found** for all the decision variables within their **given** domains so that:

- All the given constraints on the decision variables are **satisfied**.
- Optionally: A cost is **minimal**, or a benefit is **maximal**.

Search spaces are often larger than the universe!

NP-hardness is not where the fun ends, but where it begins!



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Constraint programming (CP) offers languages, methods, and tools for:
what: Modelling constraint problems in a high-level declarative language.

how: Solving constraint problems intelligently,
either by strategy-guided systematic search plus inference,
or by strategy-guided local search plus inference.

Slogan of CP: Constraint Program = Model [+ Search]

CP solvers are complementary in strength to those of:

- Operations Research (OR): linear programming (LP), integer LP (ILP), mixed integer programming (MIP), ...
- Boolean satisfiability (SAT), satisfaction modulo theories (SMT), ...
- ...

This leads to hybrid solving technologies!

In Algorithms and Data Structures 3 (1DL481), taught in period 3 (January to March), there are assignments on local search and MIP, SAT, SMT modelling.



Scope of Constraint Programming

CP has a wide scope, because it addresses:

- satisfaction problems **and** optimisation problems
- discrete decision variables **and** continuous decision variables
- linear constraints **and** non-linear constraints

in principle in **any** combinations thereof, by:

- systematic search, if optimality is more crucial than speed
- local search, if speed is more crucial than optimality



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The **constraint predicates** (AllDifferent, Circuit, Table, ...) and **structured variable types** (sets, ...) allow us *both* to **model the structure** of a problem *and* to **exploit that structure when solving** the problem.

Dozens of **constraint predicates** (see the **Catalogue**) **declaratively** encapsulate complex **inference algorithms**.

There is no standardised CP modelling language: distinct CP solvers may support distinct predicates, possibly under distinct names and signatures, as well as distinct types.



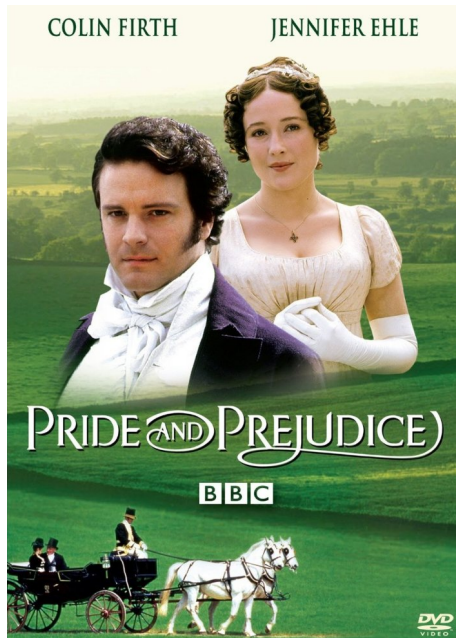
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Pride:

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*Constraint programming represents
one of the closest approaches computer science
has yet made to the Holy Grail of programming:
the user states the problem, the computer solves it.*

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Prejudice:

*The contribution of the article should be the reduction
of an engineering problem to a known optimization format.
[...] showcases pseudo code [...] submit this
work to a journal interested in code semantics [...].*

— Reviewer of a paper of ours at a prestigious OR journal



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Stand-Alone Languages and Solvers:

- **ALICE** by Jean-Louis Laurière, France, 1976
- **CHIP** at ECRC, Germany, 1987–1990; Cosytec.com, France, 1990–1992
- **OPL**, by P. Van Hentenryck, USA, and ILOG, France: modelling language for both **IBM ILOG CP Optimizer** and **IBM ILOG CPLEX Optimizer**
- **Comet**, by P. Van Hentenryck and L. Michel, USA
- **MiniZinc**, at Monash University, Australia
- ...

Libraries (the ones listed before “;” are open-source):

- Prolog: **ECLiPSe**, ...; **SICStus Prolog**, ...
- C++: **Gecode**, **Google CP-SAT**; **IBM ILOG CP Optimizer**, **CHIP**, ...
- Java: **Choco**, **Google CP-SAT**, **JaCoP**, **MiniCP**, ...; ...
- Objective-C: **Objective-CP**; ...
- Scala: **OscAR.cp**, **OscAR.cbis**; ...
- ...

**MiniZinc Challenge 2024: 16 (of 20) Problems and Winners**

Problem and Model	Backend and Solver	Solving Technology
aircraft-disassembly	CP-SAT (by Google)	portfolio: LCG, MIP, CBLS
cable-tree-wiring	CP-SAT (by Google)	portfolio: LCG, MIP, CBLS
community-detection	MZN/Gurobi	MIP
compression	PicatSAT	SAT
concert-hall-cap	MZN/Gurobi	MIP
fox-geese-corn	MZN/Gurobi	MIP
hoist-benchmark	Chuffed	hybrid: LCG = CP + SAT
monitor-placement-1id	CP Optimizer (by IBM)	CP
neighbours	CP-SAT (by Google)	portfolio: LCG, MIP, CBLS
peaceable_queens	Gecode-Dexter	portfolio: CP, LNS
portal	Chuffed	hybrid: LCG = CP + SAT
tiny-cvrp	MZN/Gurobi	MIP
train-scheduling	CP-SAT (by Google)	portfolio: LCG, MIP, CBLS
word-equations	PicatSAT	SAT
yumi-dynamic	CP-SAT (by Google)	portfolio: LCG, MIP, CBLS

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Success Stories by CP Users and Contributors:



cādence



FICO™

Google



ORACLE®



RedPrairie®



SIEMENS



THALES

XEROX®

...

Success stories: CP is the **technology of choice** in configuration, rostering, routing, scheduling (such as job shop), timetabling, ...



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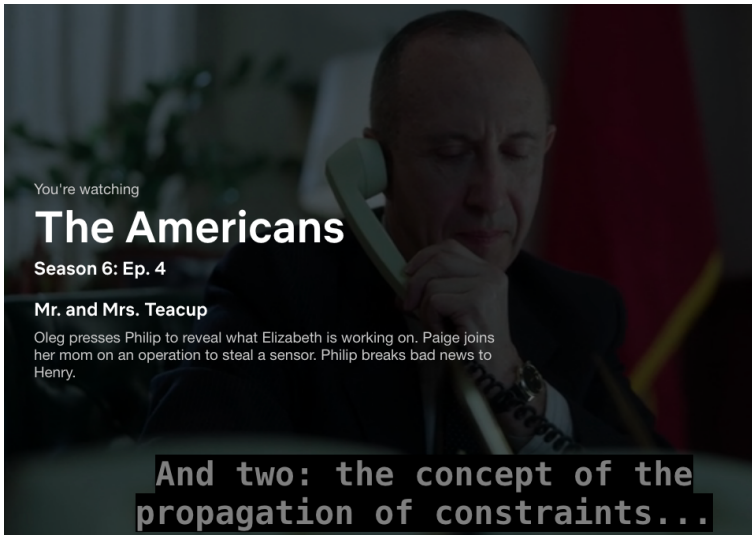
CP in Popular Culture

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You're watching

The Americans

Season 6: Ep. 4

Mr. and Mrs. Teacup

Oleg presses Philip to reveal what Elizabeth is working on. Paige joins her mom on an operation to steal a sensor. Philip breaks bad news to Henry.

And two: the concept of the propagation of constraints...



Opportunities for CP

Rapid prototyping (with high solving performance) when:

- The constraints are, still or again, subject to experiments.
- The partition into hard and soft constraints is not yet determined.

The combinatorial structure is impure, due to **side constraints**.

It is time to consider **all** or **more** problem constraints.

Domain knowledge is exploitable for **problem-specific search**.

It is a **configuration** problem.

It is a **personnel rostering** problem.

It is a **scheduling** (such as **job shop**) or **timetabling** problem.