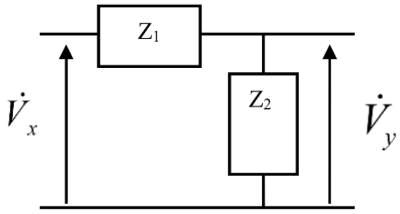
Calcolo delle funzioni di trasferimento di partitori non resistivi



$$\dot{V}_x = \dot{I}(Z_1 + Z_2)$$

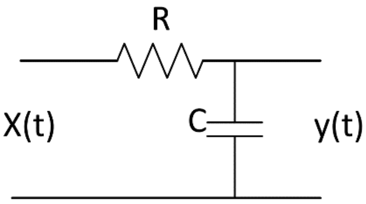
$$\dot{V}_y = \dot{I}Z_2$$

$$H(\omega) = \frac{\dot{V}_x}{\dot{V}_y} = \frac{Z_2}{Z_1 + Z_2}$$

$$\arg\{a+jb\} = \begin{cases} +\frac{\pi}{2} & se\ a = 0,\ \ b > 0 \\ -\frac{\pi}{2} & se\ a = 0,\ \ b < 0 \\ non\ definito & se\ a = 0,\ \ b = 0 \\ \tan^{-1}\left(\frac{b}{a}\right) & se\ a > 0 \\ \tan^{-1}\left(\frac{b}{a}\right) + \pi & se\ a < 0,\ \ b \geq 0 \\ \tan^{-1}\left(\frac{b}{a}\right) - \pi & se\ a < 0,\ \ b < 0 \end{cases}$$

Funzione di trasferimento circuito RC

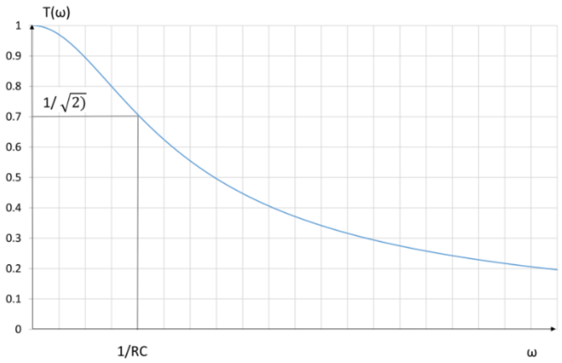
La rete si comporta come un filtro passa-basso non ideale. La pulsazione di taglio è $\omega_t = \frac{1}{RC}$.

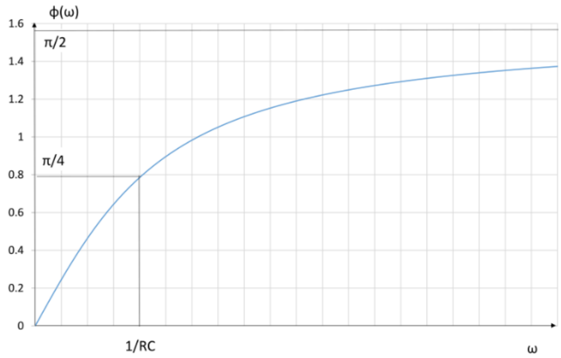


$$H(\omega) = \frac{\dot{V}_x}{\dot{V}_y} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR}$$

$$T(\omega) = |H(\omega)| = \frac{1}{|1 + j\omega CR|} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

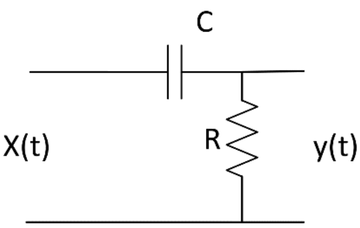
$$\beta(\omega) = -\arg\{H(\omega)\} = \arg\{1 + j\omega CR\} - \arg\{1\} = \tan^{-1}(\omega CR)$$





Funzione di trasferimento di un circuito CR

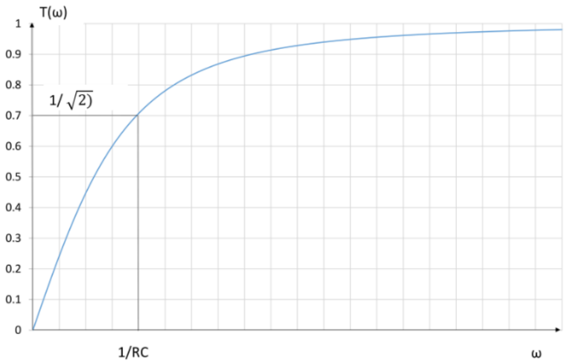
La rete si comporta come un filtro passa-alto non ideale. La pulsazione di taglio è $\omega_t = \frac{1}{RC}$.

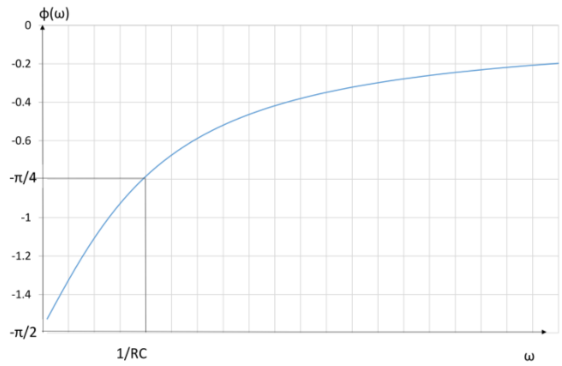


$$H(\omega) = \frac{\dot{V}_x}{\dot{V}_y} = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$$

$$T(\omega) = |H(\omega)| = \frac{|j\omega CR|}{|1 + j\omega CR|} = \frac{\omega CR}{\sqrt{1 + (\omega CR)^2}}$$

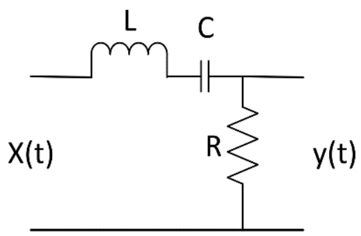
$$\beta(\omega) = -\arg\{H(\omega)\} = \arg\{1 + j\omega CR\} - \arg\{j\omega CR\} = \tan^{-1}(\omega CR) - \frac{\pi}{2}$$





Funzione di trasferimento di un circuito LCR

La rete si comporta come un filtro passa-banda non ideale. La pulsazione di risonanza è $\omega_0 = \frac{1}{\sqrt{LC}}$.

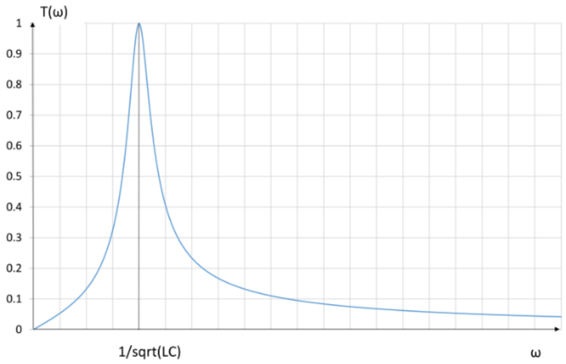


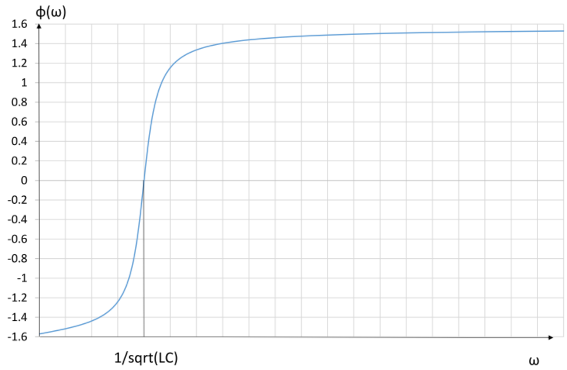
$$H(\omega) = \frac{\dot{V}_x}{\dot{V}_y} = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR}$$

$$T(\omega) = |H(\omega)| = \frac{|j\omega CR|}{|1 - \omega^2 LC + j\omega CR|} = \frac{\omega CR}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

$$\beta(\omega) = -\arg\{H(\omega)\} = \arg\{1 - \omega^2 LC + j\omega CR\} - \arg\{j\omega CR\}$$

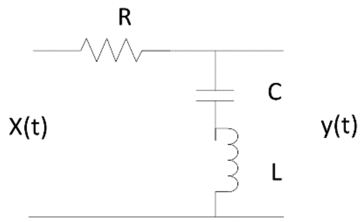
$$= \arg(1 - \omega^2 LC + j\omega CR) - \frac{\pi}{2} = \begin{cases} \tan^{-1}\left(\frac{\omega CR}{1 - \omega^2 LC}\right) & \omega > \omega_0 \\ \tan^{-1}\left(\frac{\omega CR}{1 - \omega^2 LC}\right) + \pi & \omega > \omega_0 \end{cases}$$





Funzione di trasferimento di un circuito RCL (serie)

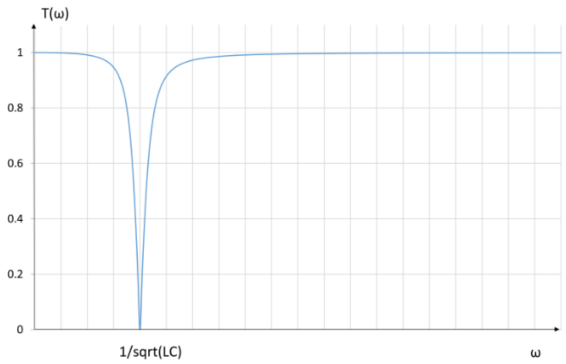
La rete si comporta come un filtro elimina-banda non ideale. La pulsazione di risonanza è $\omega_0 = \frac{1}{\sqrt{LC}}$.

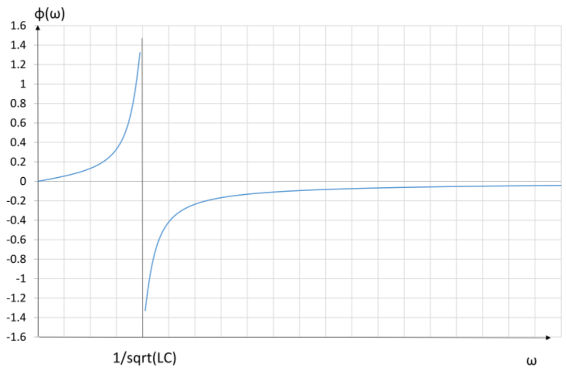


$$H(\omega) = \frac{\dot{V}_x}{\dot{V}_y} = \frac{Z_2}{Z_1 + Z_2} = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega CR}$$

$$T(\omega) = |H(\omega)| = \frac{|1 - \omega^2 LC|}{|1 - \omega^2 LC + j\omega CR|} = \frac{|1 - \omega^2 LC|}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

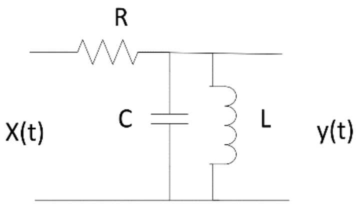
$$\beta(\omega) = -\arg\{H(\omega)\} = \arg\{1 - \omega^2 LC + j\omega CR\} - \arg\{1 - \omega^2 LC\} = \tan^{-1}\left(\frac{\omega CR}{1 - \omega^2 LC}\right)$$





Funzione di trasferimento di un circuito RCL (parallelo)

La rete si comporta come un filtro passa-banda non ideale. La frequenza di risonanza è $\omega_0 = \frac{1}{\sqrt{LC}}$.



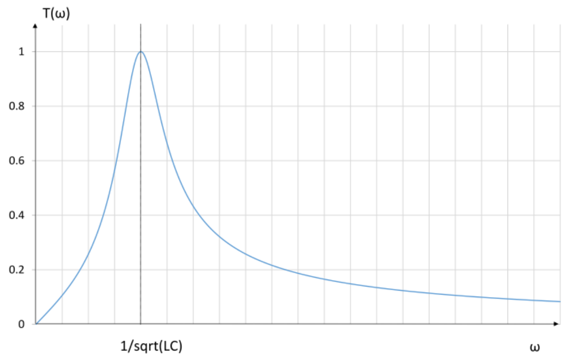
$$Z_2 = \frac{1}{Y_2} = \frac{1}{Y_L + Y_C} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{1-\omega^2 LC}$$

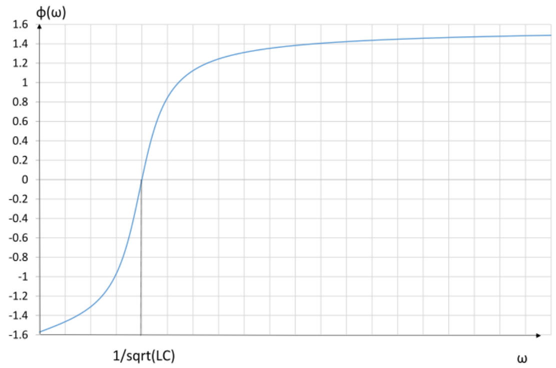
$$H(\omega) = \frac{\dot{V}_x}{\dot{V}_y} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{j\omega L}{1-\omega^2 LC}}{R + \frac{j\omega L}{1-\omega^2 LC}} = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L}$$

$$T(\omega) = |H(\omega)| = \frac{|j\omega L|}{|R(1-\omega^2 LC) + j\omega L|} = \frac{|\omega L|}{\sqrt{R^2(1-\omega^2 LC)^2 + (\omega L)^2}}$$

$$\beta(\omega) = -\arg\{H(\omega)\} = \arg\{R(1-\omega^2 LC) + j\omega L\} - \arg\{j\omega L\} =$$

$$= \begin{cases} \tan^{-1}\left(\frac{\omega L}{R(1-\omega^2 LC)}\right) - \frac{\pi}{2} & \omega > \omega_0 \\ \tan^{-1}\left(\frac{\omega L}{R(1-\omega^2 LC)}\right) + \frac{\pi}{2} & \omega > \omega_0 \end{cases}$$





$x(t) = V_x \cos(\omega_1 t - \varphi_x)$	$y(t) = V_x T(\omega_1)\cos[\omega_1 - \varphi_x - \beta(\omega_1)]$
$x(t) = A_0 + \sum_{n=1}^{+\infty} A_1 \cos(n\omega_0 t - \varphi_n)$	$y(t) = A_0 H_0 + \sum_{n=1}^{+\infty} A_n T(n\omega_0)\cos[n\omega_0 t - \varphi_n]$
$x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t}$	$y(t) = \sum_{-\infty}^{+\infty} c_n H(n\omega_0) e^{jn\omega_0 t}$