# Minimum MMD estimation

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# The Maximum Likelihood Estimator (MLE)

Let $X_1, \ldots, X_n$ be i.i.d in $\mathcal{X}$ from a probability distribution $P_0$.

Statistical inference :

- propose a model $(P_\theta, \theta \in \Theta)$, assume $P_0 = P_{\theta_0}$.
- compute $\hat{\theta}_n = \hat{\theta}_n(X_1, \ldots, X_n)$.

Letting $p_\theta$ denote the density of $P_\theta$, then

$$\hat{\theta}_n^{MLE} = \arg\max_{\theta \in \Theta} L_n(\theta), \text{ where } L_n(\theta) = \prod_{i=1}^{n} p_\theta(X_i).$$

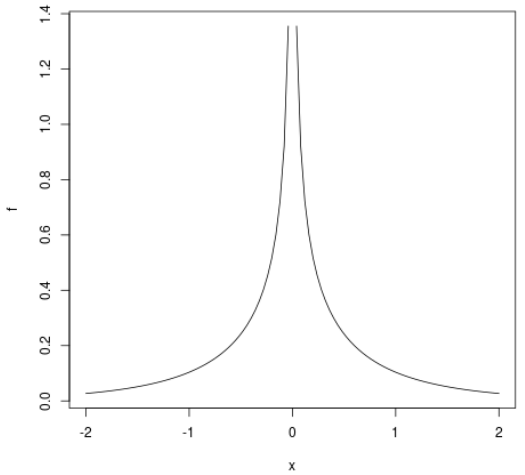
Example : $P_{(m,\sigma)} = \mathcal{N}(m, \sigma^2)$ then

$$\hat{m} = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \hat{m})^2.$$

# MLE not unique / not consistent

Example :

$$p_\theta(x) = \frac{\exp(-|x-\theta|)}{2\sqrt{\pi|x-\theta|}},$$



# MLE not unique / not consistent

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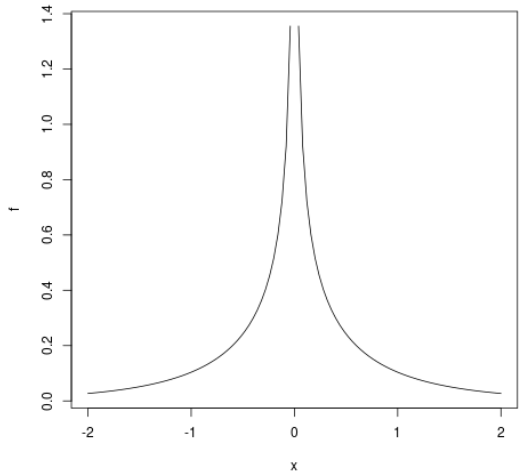
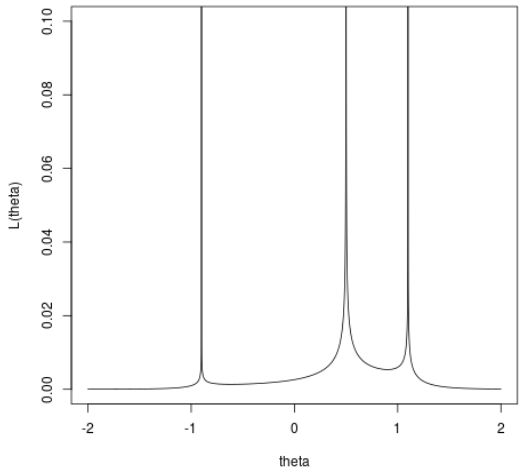



$$L_n(\theta) = \frac{\exp\left(-\sum_{i=1}^n |X_i-\theta|\right)}{(2\sqrt{\pi})^n \prod_{i=1}^n \sqrt{|X_i-\theta|}}.$$



# MLE fails in the presence of outliers

## What is an outlier ?

Huber proposed the contamination model : with probability $\varepsilon$, $X_i$ is not drawn from $P_{\theta_0}$ but from $Q$ that can be anything :

$$P_0 = (1-\varepsilon)P_{\theta_0} + \varepsilon Q.$$

Example : $P_\theta = \mathcal{U}nif[0,\theta]$, then

$$L_n(\theta) = \frac{1}{\theta^n}\prod_{i=1}^{n} \mathbb{1}_{\{0\leq X_i\leq\theta\}} \Rightarrow \hat{\theta} = \max_{1\leq i\leq n} X_i.$$

In the case of the following contamination, the MLE is extremely far from the truth :

$$P_0 = (1-\varepsilon).\mathcal{U}nif[0,1] + \varepsilon.\mathcal{N}(10^{10},1)...$$

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# Minimum Distance Estimation

$$\text{Empirical distribution : } \hat{P}_n := \frac{1}{n}\sum_{i=1}^{n} \delta_{X_i}.$$

## Minimum Distance Estimation (MDE)

Let $d(\cdot,\cdot)$ be a metric on probability distributions.

$$\hat{\theta}_d := \arg\min_{\theta\in\Theta} d\left(P_\theta, \hat{P}_n\right).$$

Wolfowitz, J. (1957). The minimum distance method. The Annals of Mathematical Statistics.

Idea : MDE with an adequate $d$ leads to robust estimation.

Bickel, P. J. (1976). Another look at robustness : a review of reviews and some new developments. Scandinavian Journal of Statistics. Discussion by Sture Holm.

Parr, W. C. & Schucany, W. R. (1980). Minimum distance and robust estimation. JASA.

Yatracos, Y. G. (1985). Rates of convergence of minimum distance estimators and Kolmogorov's entropy. Annals of Statistics.

# Integral Probability Semimetrics

## Integral Probability Semimetrics (IPS)

Let $\mathcal{F}$ be a set of real-valued, measurable functions and put

$$d_{\mathcal{F}}(P,Q) = \sup_{f\in\mathcal{F}} \left| \mathbb{E}_{X\sim P}[f(X)] - \mathbb{E}_{X\sim Q}[f(X)] \right|.$$

Müller, A. (1997). Integral probability metrics and their generating classes of functions. Applied Probability.

- assumptions required in order to ensure that $d_{\mathcal{F}}(P,Q) = 0 \Rightarrow P = Q$ (that is, $d_{\mathcal{F}}$ is a metric).
- assumptions required in order to ensure that $d_{\mathcal{F}} < +\infty$.

# Non-asymptotic bound for MDE

## Theorem 1

- $X_1, \ldots, X_n$ i.i.d from $P_0$,
- for any $f \in \mathcal{F}$, $\sup_{x \in \mathcal{X}} |f(x)| \leq 1$.

Then

$$\mathbb{E}\left[d_{\mathcal{F}}(P_{\hat{\theta}_{d_{\mathcal{F}}}}, P_0)\right] \leq \inf_{\theta \in \Theta} d_{\mathcal{F}}(P_\theta, P_0) + 4.\mathrm{Rad}_n(\mathcal{F}).$$

## Rademacher complexity

$$\mathrm{Rad}_n(\mathcal{F}) := \sup_P \mathbb{E}_{Y_1,\ldots,Y_n \sim P} \mathbb{E}_{\epsilon_1,\ldots,\epsilon_n} \left[\sup_{f \in \mathcal{F}} \frac{1}{n} \sum_{i=1}^n \epsilon_i f(Y_i)\right].$$

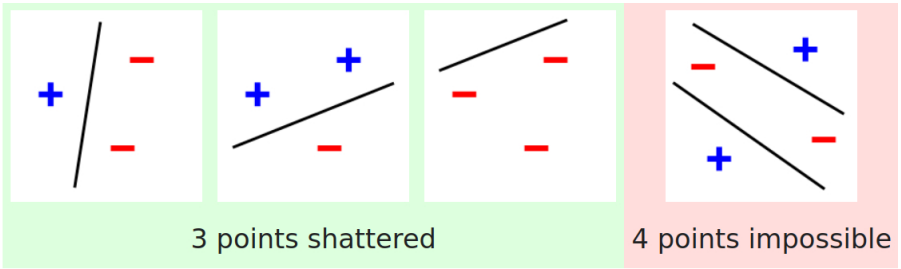
where $\epsilon_1, \ldots, \epsilon_n$ are i.i.d Rademacher variables :

$$\mathbb{P}(\epsilon_1 = 1) = \mathbb{P}(\epsilon_1 = -1) = 1/2.$$

# Example 1 : set of indicators

$$\mathbb{1}_A(x) = \begin{cases} 1 \text{ if } x \in A, \\ 0 \text{ if } x \notin A. \end{cases}$$

Image from Wikipedia.




## Reminder - Vapnik-Chervonenkis dimension

Assume that $\mathcal{F} = \{\mathbb{1}_A, A \in \mathcal{A}\}$ for some $\mathcal{A} \subseteq \mathcal{P}(\mathcal{X})$,

- $S_{\mathcal{F}}(x_1, \ldots, x_n) := \{(f(x_1), \ldots, f(x_n)), f \in \mathcal{F}\}$,
- $\mathrm{VC}(\mathcal{F}) := \max\{n : \exists x_1, \ldots, x_n, \ |S_{\mathcal{F}}(x_1, \ldots, x_n)| = 2^n\}$.

## Theorem (Bartlett and Mendelson)

$$\mathrm{Rad}_n(\mathcal{F}) \leq \sqrt{\frac{2.\mathrm{VC}(\mathcal{F}) \log(n+1)}{n}}.$$

Bartlett, P. L. & Mendelson, S. (2002). Rademacher and Gaussian complexities : Risk bounds and structural results. JMLR.

# Example 1 : KS and TV distances

Two classical examples :

- $\mathcal{A} = \{\text{all measurable sets in } \mathcal{X}\}$, then $d_{\mathcal{F}}(\cdot, \cdot)$ is the total variation distance $\mathrm{TV}(\cdot, \cdot)$.
  - $\mathrm{VC}(\mathcal{F}) = +\infty$ when $|\mathcal{X}| = +\infty$,
  - in general, $\mathrm{Rad}_n(\mathcal{F}) \not\to 0$.
- $\mathcal{X} = \mathbb{R}$, $\mathcal{A} = \{(-\infty, x], x \in \mathbb{R}\}$, then $d_{\mathcal{F}}(\cdot, \cdot)$ is the Kolmogorov-Smirnov distance $\mathrm{KS}(\cdot, \cdot)$.
  - KS distance was actually proposed by S. Holm for robust estimation,
  - $\mathrm{VC}(\mathcal{F}) = 1$, so :

$$\mathbb{E}\left[\mathrm{KS}(P_{\hat{\theta}_{\mathrm{KS}}}, P_0)\right] \leq \inf_{\theta \in \Theta} \mathrm{KS}(P_\theta, P_0) + 4.\sqrt{\frac{2\log(n+1)}{n}}.$$

# Example 2 : Maximum Mean Discrepancy (MMD)

- RKHS $(\mathcal{H}, \langle \cdot, \cdot \rangle_{\mathcal{H}})$ with kernel $k(x, y) = \langle \phi(x), \phi(y) \rangle_{\mathcal{H}}$.
- If $\|\phi(x)\|_{\mathcal{H}} = k(x, x) \leq 1$ then $\mathbb{E}_{X \sim \mu}[\phi(X)]$ is well-defined .
- The map $P \mapsto \mathbb{E}_{X \sim \mu}[\phi(X)]$ is one-to-one if $k$ is characteristic.
- Gaussian kernel $k(x, y) = \exp(-\|x - y\|^2/\gamma^2)$ satisfies these assumption.

$$\mathcal{F} = \{f \in \mathcal{H} : \|f\|_{\mathcal{H}} \leq 1\}.$$

$$\mathbb{D}_k(P, Q) := d_{\mathcal{F}}(P, Q) = \sup_{f \in \mathcal{F}} \left| \mathbb{E}_{X \sim P}[f(X)] - \mathbb{E}_{X \sim Q}[f(X)] \right|$$
$$= \left\| \mathbb{E}_{X \sim P}[\phi(X)] - \mathbb{E}_{X \sim Q}[\phi(X)] \right\|_{\mathcal{H}}.$$

# Example 2 : MMD

## Theorem (Bartlett and Mendelson)

$$\mathcal{F} = \{f \in \mathcal{H} : \|f\|_{\mathcal{H}} \leq 1\} \Rightarrow \mathrm{Rad}_n(\mathcal{F}) \leq \sqrt{\frac{\sup_x k(x,x)}{n}}.$$

## Corollary

$$\mathbb{E}\left[\mathbb{D}_k(P_{\hat{\theta}_{\mathbb{D}_k}}, P_0)\right] \leq \inf_{\theta \in \Theta} \mathbb{D}_k(P_\theta, P_0) + 4\sqrt{\frac{\sup_x k(x,x)}{n}}.$$

# Example 2 : MMD

We actually have

$$\mathbb{D}_k^2(P_\theta, \hat{P}_n) = \mathbb{E}_{X,X' \sim P_\theta}[k(X, X')] - \frac{2}{n}\sum_{i=1}^{n} \mathbb{E}_{X \sim P_\theta}[k(X_i, X)] + \frac{1}{n^2}\sum_{1 \leq i,j \leq n} k(X_i, X_j)$$

and so

$$\nabla_\theta \mathbb{D}_k^2(P_\theta, \hat{P}_n) = 2\mathbb{E}_{X,X' \sim P_\theta}\left\{ \left[ k(X, X') - \frac{1}{n}\sum_{i=1}^{n} k(X_i, X) \right] \nabla_\theta [\log p_\theta(X)] \right\}$$

that can be approximated by sampling from $P_\theta$.

# Example 3 : Wasserstein

Another classical metric belongs to the IPS family :

$$\mathrm{W}_\delta(P, Q) = \sup_{\substack{f\,:\,\mathcal{X}\to\mathbb{R} \\ \mathrm{Lip}(f)\leq 1}} \left| \mathbb{E}_{X\sim P}[f(X)] - \mathbb{E}_{X\sim Q}[f(X)] \right|$$

where $\mathrm{Lip}(f) := \sup_{x\neq y} |f(x) - f(y)|/\delta(x, y)$.

Bound on the Rademacher complexity when $\mathcal{X}$ is bounded :

Sriperumbudur, B.K., Fukumizu, K., Gretton, A., Schölkopf, B., Lanckriet, G.R. (2010). Non-parametric estimation of integral probability metrics. IEEE International Symposium on Information Theory.

Minimum Wasserstein estimation studied in :

Bernton, E., Jacob, P. E., Gerber, M. & Robert, C. P. (2019). On parameter estimation with the Wasserstein distance. Information and Inference : A Journal of the IMA.

# MDE and robustness

## Reminder

$$\mathbb{E}\left[d_{\mathcal{F}}(P_{\hat{\theta}_{d_{\mathcal{F}}}}, P_0)\right] \leq \inf_{\theta\in\Theta} d_{\mathcal{F}}(P_\theta, P_0) + 4.\mathrm{Rad}_n(\mathcal{F}).$$

Huber's contamination model : $P_0 = (1-\varepsilon)P_{\theta_0} + \varepsilon Q$.

$$\begin{aligned} d_{\mathcal{F}}(P_{\theta_0}, P_0) &= \sup_{f\in\mathcal{F}} \left|\mathbb{E}_{X\sim P_{\theta_0}} f(X) - (1-\varepsilon)\mathbb{E}_{X\sim P_{\theta_0}} f(X) - \varepsilon\mathbb{E}_{X\sim Q} f(X)\right| \\ &= \sup_{f\in\mathcal{F}} \left|\varepsilon\mathbb{E}_{X\sim P_{\theta_0}} f(X) - \varepsilon\mathbb{E}_{X\sim Q} f(X)\right| \\ &= \varepsilon.d_{\mathcal{F}}(P_{\theta_0}, Q) \leq 2\varepsilon \qquad \text{if for any } f\in\mathcal{F},\ \sup_x |f(x)| \leq 1 \end{aligned}$$

## Corollary - in Huber's contamination model

$$\mathbb{E}\left[d_{\mathcal{F}}(P_{\hat{\theta}_{d_{\mathcal{F}}}}, P_{\theta_0})\right] \leq 4\varepsilon + 4.\mathrm{Rad}_n(\mathcal{F}).$$

# MDE and robustness : toy experiment

Model : $\mathcal{N}(\theta, 1)$, $X_1, \ldots, X_n$ i.i.d $\mathcal{N}(\theta_0, 1)$, $n = 100$ and we repeat the exp. 200 times. Kernel $k(x, y) = \exp(-|x - y|)$.

| | $\hat{\theta}_{MLE}$ | $\hat{\theta}_{\mathrm{MMD}_k}$ | $\hat{\theta}_{\mathrm{KS}}$ |
|---|---|---|---|
| mean abs. error | 0.081 | 0.094 | 0.088 |

Now, $\varepsilon = 2\%$ of the observations drawn from a Cauchy.

| | | | |
|---|---|---|---|
| mean abs. error | 0.276 | 0.095 | 0.088 |

Now, $\varepsilon = 1\%$ are replaced by $1,000$.

| | | | |
|---|---|---|---|
| mean abs. error | 10.008 | 0.088 | 0.082 |

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# Improving the constant

From now, we assume that $\sup_x k(x,x) \leq 1$. We know :

$$\mathbb{E}\left[\mathbb{D}_k(P_{\hat{\theta}_{\mathbb{D}_k}}, P_0)\right] \leq \inf_{\theta\in\Theta} \mathbb{D}_k(P_\theta, P_0) + \frac{4}{\sqrt{n}}.$$

We will now prove a better result without using the Rademacher complexity :

**Theorem**

$$\mathbb{E}\left[\mathbb{D}_k(P_{\hat{\theta}_{\mathbb{D}_k}}, P_0)\right] \leq \inf_{\theta\in\Theta} \mathbb{D}_k(P_\theta, P_0) + \frac{2}{\sqrt{n}}.$$

# Proof of the theorem : preliminary lemma

## Lemma

For any $P_0$, when $X_1, \ldots, X_n$ are i.i.d from $P_0$,

$$\mathbb{E}\left[\mathbb{D}_k\left(\hat{P}_n, P^0\right)\right] \leq \frac{1}{\sqrt{n}}.$$

$$\begin{aligned}
\left\{\mathbb{E}\left[\mathbb{D}_k\left(\hat{P}_n, P^0\right)\right]\right\}^2 &\leq \mathbb{E}\left[\mathbb{D}_k^2\left(\hat{P}_n, P^0\right)\right] \\
&= \mathbb{E}\left[\left\|(1/n)\sum(\mu(\delta_{X_i}) - \mu(P_0))\right\|_{\mathcal{H}}^2\right] \\
&= (1/n)\mathbb{E}\left[\|\mu(\delta_{X_1}) - \mu(P_0)\|_{\mathcal{H}}^2\right] \\
&\leq 1/n.
\end{aligned}$$

# Proof of the theorem

$$\begin{aligned}\forall \theta,\ \mathbb{D}_k\left(P_{\hat{\theta}}, P^0\right) &\leq \mathbb{D}_k\left(P_{\hat{\theta}}, \hat{P}_n\right) + \mathbb{D}_k\left(\hat{P}_n, P^0\right) \\ &\leq \mathbb{D}_k\left(P_{\theta}, \hat{P}_n\right) + \mathbb{D}_k\left(\hat{P}_n, P^0\right) \\ &\leq \mathbb{D}_k\left(P_{\theta}, P^0\right) + 2\mathbb{D}_k\left(\hat{P}_n, P^0\right)\end{aligned}$$

$$\mathbb{E}\left[\mathbb{D}_k\left(P_{\hat{\theta}}, P_0\right)\right] \leq \inf_{\theta \in \Theta} \mathbb{D}_k(P_\theta, P_0) + \frac{2}{\sqrt{n}}.$$

# A bound in probability

Thanks to McDiarmid's inequality :

## Theorem

For any $P_0$, when $X_1, \ldots, X_n$ are i.i.d from $P_0$, with probability at least $1 - \delta$,

$$\mathbb{D}_k\left(P_{\hat{\theta}}, P^0\right) \leq \inf_{\theta \in \Theta} \mathbb{D}_k\left(P_\theta, P^0\right) + \frac{2 + 2\sqrt{2\log\left(\frac{1}{\delta}\right)}}{\sqrt{n}}.$$

Joint work with Badr-Eddine Chérief-Abdellatif (CNRS).

Chérief-Abdellatif, B.-E. and Alquier, P. Finite Sample Properties of Parametric MMD Estimation : Robustness to Misspecification and Dependence. Bernoulli, 2022.

# Example : Gaussian mean estimation

Example : $P_\theta = \mathcal{N}(\theta, \sigma^2 I)$ for $\theta \in \mathbb{R}^d$.
Using a Gaussian kernel $k(x,y) = \exp(-\|x - y^2\|/\gamma^2)$,

$$\mathbb{D}_k^2(P_\theta, P_{\theta'}) = 2\left(\frac{\gamma^2}{4\sigma^2+\gamma^2}\right)^{\frac{d}{2}}\left[1 - \exp\left(-\frac{\|\theta-\theta'\|^2}{4\sigma^2+\gamma^2}\right)\right].$$

Together with the previous result, this gives :

$$\|\hat{\theta}_n^{MMD} - \theta_0\|^2 \leq -(4\sigma^2+\gamma^2)\log\left[1 - 4\frac{(1+\sqrt{2\log 1/\delta})^2}{n}\left(\frac{4\sigma^2+\gamma^2}{\gamma^2}\right)^{\frac{d}{2}}\right].$$

$\gamma = 2d\sigma^2 \Rightarrow$

$$\|\hat{\theta}_n^{MMD} - \theta_0\|^2 \leq d\sigma^2 \frac{8e(1+\sqrt{2\log 1/\delta})^2}{n}(1+o(1)).$$

# Variance-aware bounds (1/2)

$$\left\{\mathbb{E}\left[\mathbb{D}_k\left(\hat{P}_n, P^0\right)\right]\right\}^2 \leq \mathbb{E}\left[\mathbb{D}_k^2\left(\hat{P}_n, P^0\right)\right]$$
$$= \mathbb{E}\left[\left\|(1/n)\sum(\mu(\delta_{X_i}) - \mu(P_0))\right\|_{\mathcal{H}}^2\right]$$
$$= (1/n)\underbrace{\mathbb{E}\left[\|\mu(\delta_{X_1}) - \mu(P_0)\|_{\mathcal{H}}^2\right]}_{=:v_k(P_0)}$$

## Lemma - variance-aware version

$$\mathbb{E}\left[\mathbb{D}_k\left(\hat{P}_n, P^0\right)\right] \leq \sqrt{\frac{v_k(P_0)}{n}} \leq \sqrt{\frac{1}{n}}.$$

# Variance-aware bounds (2/2)

## Theorem – bound in expectation

$$\mathbb{E}\left[\mathbb{D}_k(P_{\hat{\theta}}, P_0)\right] \leq \inf_{\theta\in\Theta} \mathbb{D}_k(P_\theta, P_0) + 2\sqrt{\frac{v_k(P_0)}{n}}.$$

## Theorem – bound in probability

With probability at least $1-\delta$,

$$\mathbb{D}_k\left(P_{\hat{\theta}}, P^0\right) \leq \inf_{\theta\in\Theta} \mathbb{D}_k\left(P_\theta, P^0\right) + 2\sqrt{\frac{v_k(P_0) 2\log\frac{1}{\delta}}{n}} + \frac{8\log\frac{1}{\delta}}{3n}.$$

Joint work with Geoffrey Wolfer (RIKEN AIP).

Wolfer, G. and Alquier, P. Variance-Aware Estimation of Kernel Mean Embedding. Preprint arXiv :2210.06672.

# Upper-bounding the variance $v_k(P_0)$

In the case of the Gaussian kernel

$$k(x,y) = \exp(-\|x-y\|^2/\gamma^2)$$

we have

$$v_k(P_0) \leq 1 - \exp\left[-\frac{2\mathrm{Tr}(Var_{P_0}(X))}{\gamma^2}\right] \leq \begin{cases} \frac{2\mathrm{Tr}(Var_{P_0}(X))}{\gamma^2} \\ 1. \end{cases}$$

Example : Gaussian mean estimation (continued).

## Using the variance aware bound

$$\gamma = \gamma_n \to +\infty \Rightarrow \|\hat{\theta}_n^{MMD} - \theta_0\|^2 \leq d\sigma^2 \frac{4\log 1/\delta}{n}(1+o(1)).$$

# Empirical bound

In practice, we can estimate $v_k(P_0)$ by

$$\hat{v}_k := \frac{1}{n-1} \sum_{i=1}^{n} \left( k(X_i, X_i) - \frac{1}{n} \sum_{j=1}^{n} k(X_i, X_j) \right).$$

We have $\mathbb{E}(\hat{v}_k) = v_k(P_0)$, and

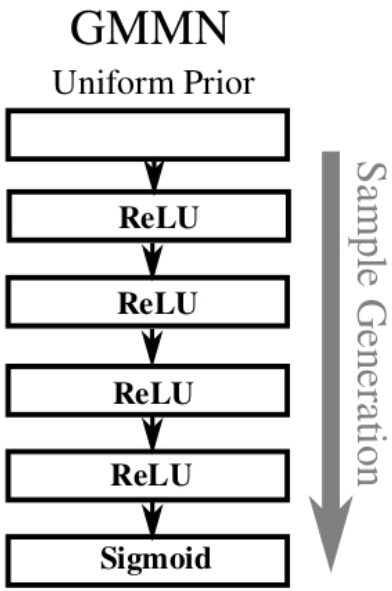
**Theorem – bound with empirical variance**

Assume that $k(x, y) = \psi(x - y) \in [a, b]$. Then, with probability at least $1 - \delta$,

$$\mathbb{D}_k\left(P_{\hat{\theta}}, P^0\right) \leq \inf_{\theta \in \Theta} \mathbb{D}_k\left(P_\theta, P^0\right) + 2\sqrt{\frac{\hat{v}_k 2 \log \frac{1}{\delta}}{n}} + \frac{32\sqrt{b-a} \log \frac{1}{\delta}}{3n}.$$

# Contents

# Generative Adversarial Networks (GAN, 1/2)




Generative model $X \sim P_\theta$ :

- $U \sim \text{Unif}[0,1]^d$,
- $X = F_\theta(U)$ where $F_\theta$ is some NN with weights $\theta$.

Dziugaite, G. K., Roy, D. M. & Ghahramani, Z. (2015). Training generative neural networks via maximum mean discrepancy optimization. UAI.

Li, Y., Swersky, K. & Zemel, R. (2015). Generative Moment Matching Networks. ICML.

$\rightarrow$ proposed to minimize the MMD to learn $\theta$.

# GAN (2/2)

Results from Dziugaite et al. (2015).



# Inference for Systems of SDEs (1/2)

This paper developped the asymptotic theory of MMD :

Briol, F. X., Barp, A., Duncan, A. B., & Girolami, M. (2019). Statistical Inference for Generative Models with Maximum Mean Discrepancy. Preprint arXiv :1906.05944.

They also applied the method to inference in SDEs :

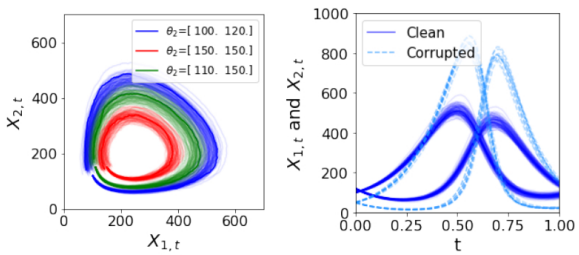
$$\mathrm{d}X_t = b(X_t, \theta_1)\mathrm{d}t + \sigma(X_t, \theta_2)\mathrm{d}W_t$$

- easy to sample from the model with a given $\theta = (\theta_1, \theta_2)$,
- they propose a method to approximate the gradient of the MMD criterion.

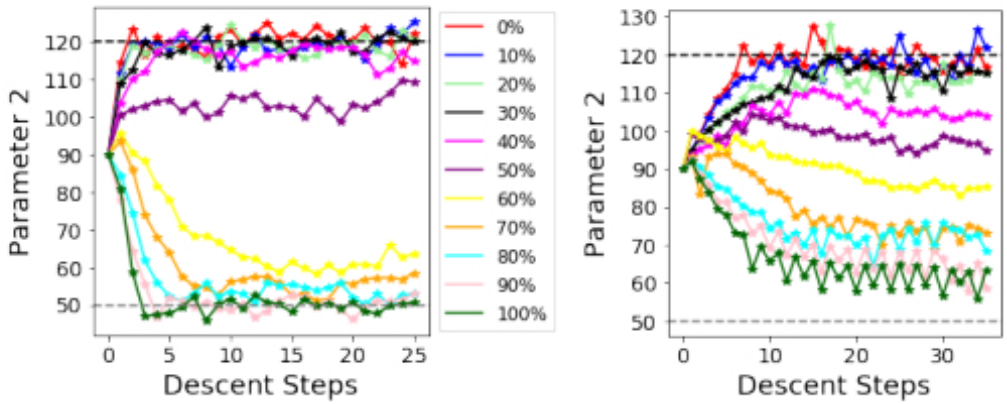
# Inference for Systems of SDEs (2/2)

Example in a (stochastic) Lotka-Volterra model.

$$\mathrm{d}\begin{pmatrix} X_{1,t} \\ X_{2,t} \end{pmatrix} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\theta_{11}X_{1,t} + \begin{pmatrix} -1 \\ 1 \end{pmatrix}\theta_{12}X_{1,t}X_{2,t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\theta_{13}X_{2,t}\right]\mathrm{d}t$$
$$+ \begin{pmatrix} 1 \\ 0 \end{pmatrix}\sqrt{\theta_{11}X_{1,t}}\mathrm{d}W_t^{(1)} + \begin{pmatrix} -1 \\ 1 \end{pmatrix}\sqrt{\theta_{12}X_{1,t}X_{2,t}}\mathrm{d}W_t^{(2)} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\sqrt{\theta_{13}X_{2,t}}\mathrm{d}W_t^{(3)},$$




Results from Briol et al. (2019) : compare MMD minimization to Wasserstein minimization.



# Regression

- problem with regression : we want to specify and estimate a parametric model $P_{\theta(X)}$ for $Y|X$. MMD requires to specify a model for $(X, Y)$.
- natural idea : estimate the distribution of $X$ by $\frac{1}{n}\sum_{i=1}^{n}\delta_{X_i}$ and use the MMD procedure on $P_{\theta(X)}$.
- the previous theory shows directly that we estimate the distribution of $(X, Y)$ consistently.
- it is far more difficult to prove that we estimate the distribution of $Y|X$.

Joint work with M. Gerber (Bristol).

Alquier, P. and Gerber, M. (2020). Universal Robust Regression via Maximum Mean Discrepancy. Preprint arXiv.

# Copulas

- another semi-parametric model : copulas.
- asymptotic theory + R package.


With B.-E. Chérief-Abdellatif (CNRS), J.-D. Fermanian (ENSAE Paris), A. Derumigny (TU Delft).

Alquier, P., Chérief-Abdellatif, B.-E., Derumigny, A. and Fermanian, J.-D. Estimation of copulas via Maximum Mean Discrepancy. JASA, to appear.

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**MMDCopula: Robust Estimation of Copulas by Maximum Mean Discrepancy**

Provides functions for the robust estimation of parametric families of copulas using minimization of the Maximum Mean Discrepancy, following the article Alquier, Chérief-Abdellatif, Derumigny and Fermanian (2020) <arXiv:2010.00408>.

| | |
|---|---|
| Version: | 0.1.0 |
| Depends: | R (≥ 3.6.0) |
| Imports: | VineCopula, cubature, pcaPP, randtoolbox |
| Suggests: | knitr, rmarkdown |
| Published: | 2020-10-13 |
| Author: | Alexis Derumigny [aut, cre], Pierre Alquier [aut], Jean-David Fermanian [aut], Badr-Eddine Chérief-Abdellatif [aut] |
| Maintainer: | Alexis Derumigny <a.f.f.derumigny at utwente.nl> |
| BugReports: | https://github.com/AlexisDerumigny/MMDCopula/issues |
| License: | GPL-3 |
| NeedsCompilation: | no |
| Materials: | README NEWS |
| CRAN checks: | MMDCopula results |

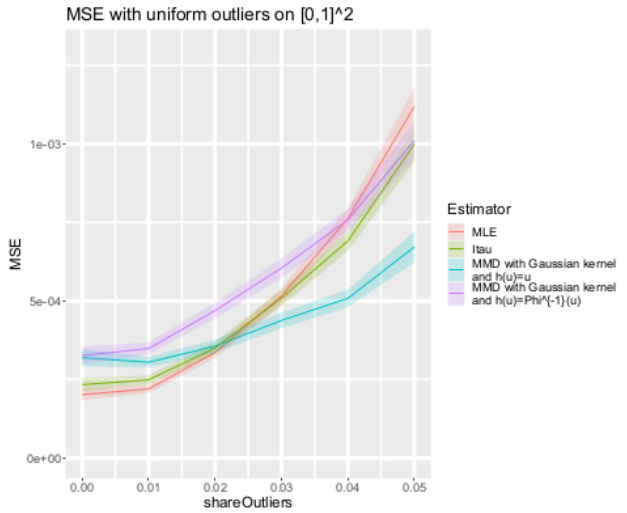
**Downloads:**

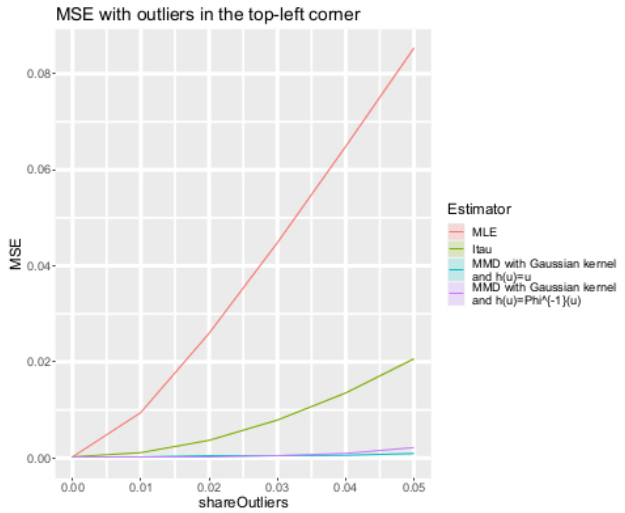
| | |
|---|---|
| Reference manual: | MMDCopula.pdf |
| Vignettes: | The MMD copula package: robust estimation of parametric copula models by MMD minimization |
| Package source: | MMDCopula_0.1.0.tar.gz |
| Windows binaries: | r-devel: MMDCopula_0.1.0.zip, r-release: MMDCopula_0.1.0.zip, r-oldrel: MMDCopula_0.1.0.zip |
| macOS binaries: | r-release: MMDCopula_0.1.0.tgz, r-oldrel: MMDCopula_0.1.0.tgz |

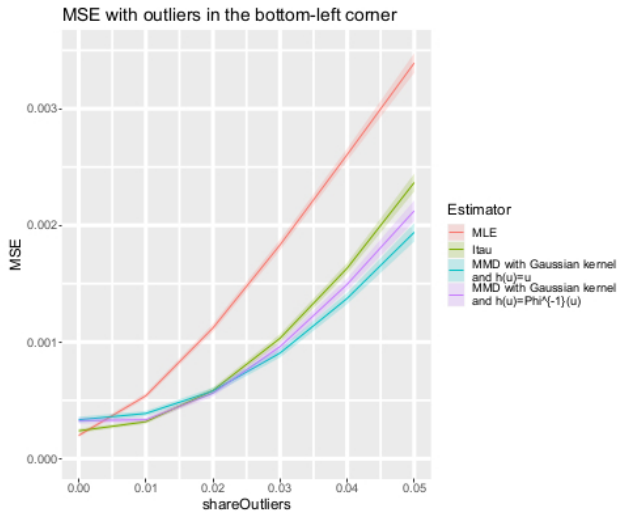
**Linking:**

Please use the canonical form https://CRAN.R-project.org/package=MMDCopula to link to this page.

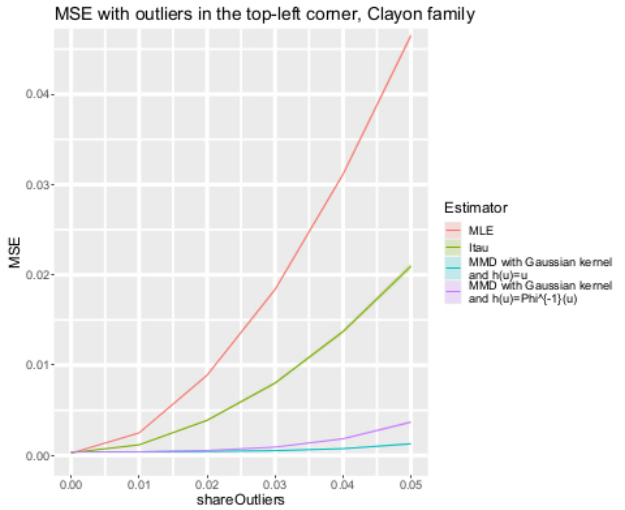
# Example : Gaussian copulas

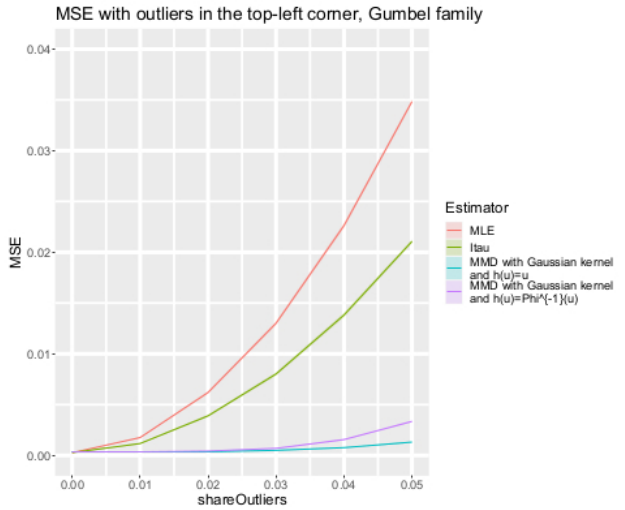


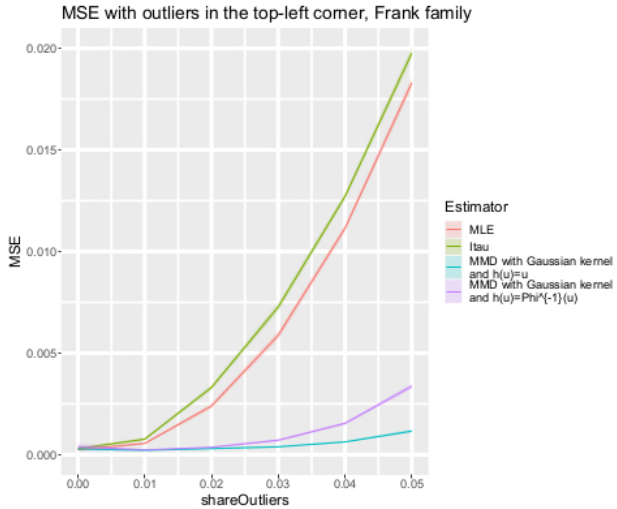



# Example : other models



# Bayesian estimation

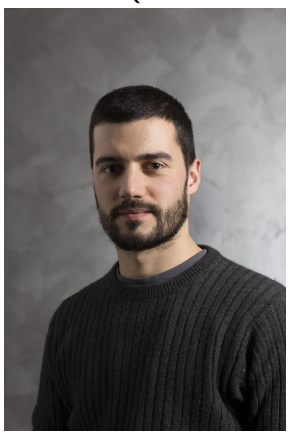
## Variational approximations :

Chérief-Abdellatif, B.-E. and Alquier, P. (2020). MMD-Bayes : Robust Bayesian Estimation via Maximum Mean Discrepancy. Proceedings of AABI.

## ABC :

S. Legramanti, D. Durante & P. Alquier (2022). Concentration and robustness of discrepancy–based ABC via Rademacher complexity. Preprint arXiv :2206.06991.

Sirio Legramanti (Univ. of Bergamo)



Daniele Durante (Bocconi University)

# La fin

終わり

ありがとう ございます。