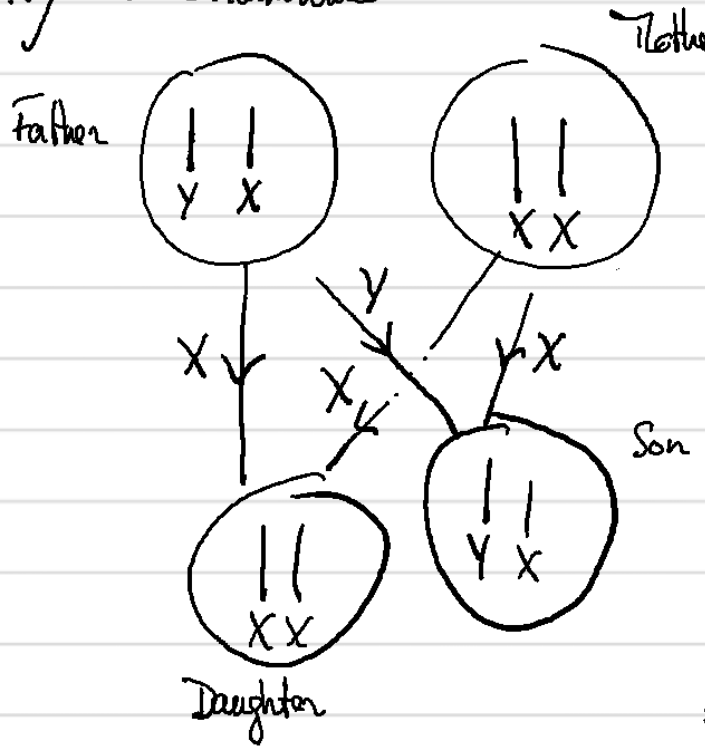
Exercises: Sexual reproduction

1.1) Sex chromosomes



Allele A, frequency p_f / p_m (on X chromosome)

1) Females:

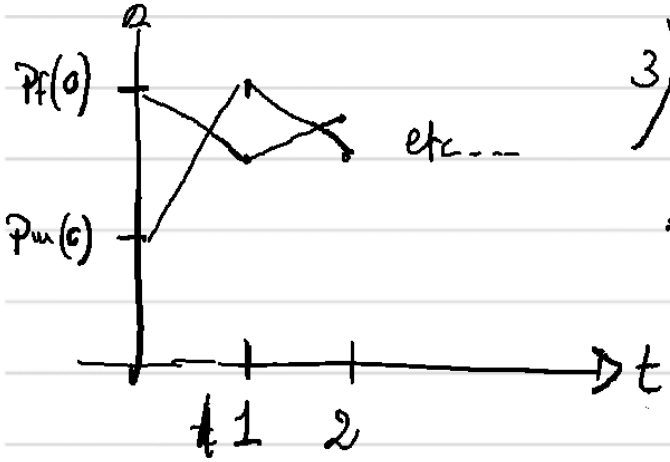
AA : $p_f \cdot p_m$

Aa : $p_f(1-p_m) + p_m(1-p_f)$

aa : $(1-p_f)(1-p_m)$

⇒ not HW proportions

2) $\begin{cases} p_f(t+1) = \frac{1}{2} p_f(t) + \frac{1}{2} p_m(t) & \text{(one X from father, one from mother)} \\ p_m(t+1) = p_f(t) & \text{(only X from mother)} \end{cases}$



3) $p_f(t+1) - p_m(t+1) = \frac{1}{2}(p_f - p_m)$

⇒ $p_f - p_m \xrightarrow[t\to\infty]{} 0$

⇒ At $t \to \infty$, $p_f = p_m = p^*$

Let $p(t) = \frac{2}{3} p_f(t) + \frac{1}{3} p_m(t)$

Then $p(t+1) = \frac{2}{3}\left(\frac{1}{2} p_f(t) + \frac{1}{2} p_m(t) \right) + \frac{1}{3} p_f(t)$

$= \frac{2}{3} p_f(t) + \frac{1}{3} p_m(t) = p(t)$

$\rightarrow$ p does not change through time, so $p(t) = p(0) = \frac{2}{3} p_f(0) + \frac{1}{3} p_m(0)$

For $t \to \infty$, $p \xrightarrow[t\infty]{} p^*$ $\Rightarrow p^* = \frac{2}{3} p_f(0) + \frac{1}{3} p_m(0)$

1.2) Overlapping generations

1) $x_{12}(t+dt) = x_{12}(t) + \gamma dt\left(2x_1x_2 - x_{12}\right) \dots$

$$\dot{x}_{11} = \gamma\left(x_1^2 - x_{11}\right)$$

2) $x_1(t+dt) = x_1(t) - \gamma dt \cdot x_1 + \gamma dt \cdot x_1 \to \dot{x}_1 = 0$

3) $\dot{x}_{12} = \gamma\left(2p(1-p) - x_{12}\right)$

$$\int_0^t \frac{dx_{12}}{2p(1-p) - x_{12}} = \int_0^t \gamma dt$$

$$\to -\ln\left(2p(1-p) - x_{12}\right) + \ln\left(2p(1-p) - \dot{x}_{12}\right) = \gamma t$$

$$\text{also} \quad x_{12} = 2p(1-p) + \left(\dot{x}_{12} - 2p(1-p)\right)e^{-\gamma t} \xrightarrow[t\to\infty]{} 2p(1-p)$$

4) Death + birth ~ one generation

Probability to not die | in time dt : $1 - \gamma dt$

in time t : $(1-\gamma dt)^{t/dt} = \exp\left(\frac{t}{dt}\ln(1-\gamma dt)\right)$

if $dt \to 0$, $\exp(-\gamma t)$: probability to not die in time t

~ exp. distribution, average lifetime $\frac{1}{\gamma}$