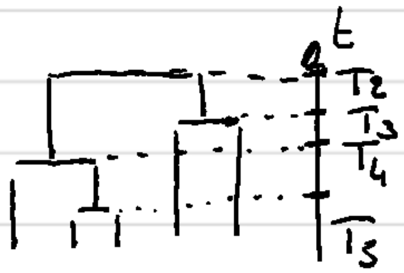
3.1) Length of a coalescent tree



During time T_5 : 5 lineages

$\rightarrow$ length $5 \cdot T_5$!

During time T_n, length $n T_n$

$$\Rightarrow L_5 = 5 \cdot T_5 + 4 T_4 + 3 T_3 + 2 T_2$$

$$\Rightarrow L_n = \sum_{i=2}^{n} i T_i$$

We want the average : $\langle L_n \rangle = \sum_{i=2}^{n} i \cdot \langle T_i \rangle$

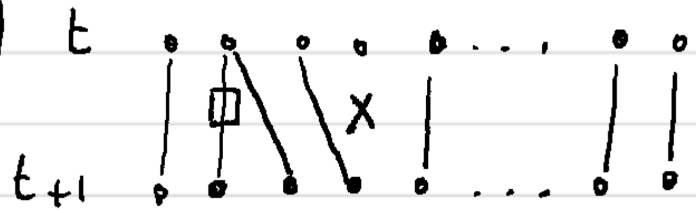
and $\langle T_i \rangle = \dfrac{N}{\binom{i}{2}} = \dfrac{2N}{i(i-1)}$

$$\rightarrow \langle L_n \rangle = 2N \sum_{i=2}^{n} \frac{1}{i-1} = 2N \sum_{i=1}^{n-1} \frac{1}{i} \sim 2N \left(\log(n-1) + \gamma \right)$$

$\Rightarrow$ length grows as $\log(n)$ where n is # of leaves!

and $\langle L_n \rangle \xrightarrow[n \infty]{} \infty$, but very slowly

3.2) Coalescent in the Moran model

1) 

X : chosen to die

□ : chosen for reproduction

Since only one □ : one coalescence at most!

2) Only one pair of lineages coalesces in the previous generation

There are in total $\binom{N}{2}$ pairs

The probability that the pair we picked is the coalescing one is

$$p = \frac{1}{\binom{N}{2}} = \frac{2}{N(N-1)}$$

$\Rightarrow$ prob. of <u>no</u> coalescence for a fixed pair is $1 - \frac{2}{N(N-1)}$

for t generations : $\left(1 - \frac{2}{N(N-1)}\right)^t$: geometric!

Therefore $T_2 = \frac{N(N-1)}{2}$

3) Prob. of not dying (for one individual) : $\left(1 - \frac{1}{N}\right)$

for t generations : $\left(1 - \frac{1}{N}\right)^t \rightarrow$ still geometric!

$\Rightarrow$ average lifetime is N

In units of generations : $T_2 = \frac{N-1}{2} \sim O(N)$, like WF