THÈSE DE DOCTORAT
DE SORBONNE UNIVERSITÉ

Spécialité : Physique

École doctorale nº564: Physique en Île-de-France

réalisée

au Laboratoire Kastler Brossel

sous la direction de Pierre-François Cohadon

présentée par

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pour obtenir le grade de :

DOCTEUR DE SORBONNE UNIVERSITÉ

Sujet de la thèse :

Progress towards cryogenic squeezed light optomechanics

soutenue le ?? ??? 2025

devant le jury composé de :

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Theory: background

This chapter will cover the elementary concepts required to describe an membrane based optomechanical system in a quantum regime. We will first recall basics on optical field quantization as well describing coherent and squeezed light field, to then turn to the more specific frequency dependent squeezed light field. Secondly, we will cover the mathematical description of a mechanical resonator interacting with a generic coherent optical field, highlighting the differences with the seminal optomechanical system of a mirror on a spring. Finally, we will derive the equations of motions of a membrane based optomechanical system with frequency dependent squeezed optical fields.

II.1 Quantum Optics Concepts

II.1.1 Quantum Description of Light

We introduce briefly field quantization concepts needed to describe monochromatic field propagation and measurements. key words : eigenmodes of the field, quantization, annihilation operators, quadratures, phase space, displacement operators, squeezing operators, coherent states, generic squeezed states,

Quantised Electromagnetic Field

Starting from Maxwell's equations in free space and applying canonical quantisation, the electric field operator can be written as:

$$\hat{\mathbf{E}}(\mathbf{r},t) = i\sum_{\ell} \mathcal{E}_l \left[\hat{a}_\ell(t)\mathbf{f}_\ell(\mathbf{r}) - \hat{a}_\ell^\dagger(t)\mathbf{f}_\ell^*(\mathbf{r})\right] \tag{II.1}$$

where $\mathcal{E}_l = \sqrt{\frac{\hbar\omega_l}{2\varepsilon_0 V}}$ is the field per photon in mode ℓ, $\mathbf{f}_\ell(\mathbf{r})$ are mode functions satisfying orthonormality, and $\hat{a}_\ell(t)$, $\hat{a}_\ell^\dagger(t)$ are the annihilation and creation operators associated with each mode (ℓ). Here, we describe

These operators satisfy the canonical commutation relations:

$$[\hat{a}_{\omega,\ell}(t), \hat{a}^\dagger_{\omega',\ell'}(t)] = \delta_{\omega,\omega'}\delta_{\ell,\ell'}, \quad [\hat{a}_{\omega,\ell}(t), \hat{a}_{\omega',\ell'}(t)] = 0 \tag{II.2}$$

The Hamiltonian for the electromagnetic field becomes a sum of harmonic oscillator energies:

$$\hat{H} = \sum_{\omega,\ell} \hbar\omega \left(\hat{a}^{\dagger}_{\omega,\ell}\hat{a}_{\omega,\ell} + \frac{1}{2} \right) \tag{II.3}$$

Quadrature Operators

To describe the phase space properties of a field mode, we define the quadrature operators:

$$\hat{q}_{\omega,\ell} = \frac{1}{\sqrt{2}}(\hat{a}_{\omega,\ell} + \hat{a}^{\dagger}_{\omega,\ell}) \tag{II.4}$$

$$\hat{p}_{\omega,\ell} = \frac{1}{\sqrt{2}i}(\hat{a}_{\omega,\ell} - \hat{a}^{\dagger}_{\omega,\ell}) \tag{II.5}$$

These are Hermitian operators corresponding to measurable observables and satisfy:

$$[\hat{q}_{\omega,\ell}, \hat{p}_{\omega,\ell}] = i \tag{II.6}$$

Generalised quadratures are defined via:

$$\hat{q}^{\omega,\ell}_{\theta} = \hat{q}_{\omega,\ell}\cos\theta + \hat{p}_{\omega,\ell}\sin\theta \tag{II.7}$$

Uncertainty Principle and Quantum Noise

From the commutation relation, the uncertainty principle follows:

$$\Delta q_{\omega,\ell}\Delta p_{\omega,\ell} \geq \frac{1}{2} \tag{II.8}$$

This defines the minimum amount of quantum noise (vacuum fluctuations) in the electromagnetic field.

II.1.2 *Coherent States

Coherent states $|\alpha_{\omega,\ell}\rangle$ are eigenstates of the annihilation operator:

$$\hat{a}_{\omega,\ell}|\alpha_{\omega,\ell}\rangle = \alpha_{\omega,\ell}|\alpha_{\omega,\ell}\rangle \tag{II.9}$$

They can be generated by displacing the vacuum:

$$|\alpha_{\omega,\ell}\rangle = \hat{D}(\alpha_{\omega,\ell})|0\rangle, \quad \hat{D}(\alpha) = \exp(\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a}) \tag{II.10}$$

They exhibit:

- Minimum uncertainty: $\Delta q = \Delta p = 1/\sqrt{2}$
- Classical-like dynamics
- Poissonian photon statistics

II.1.3 Squeezed States

Squeezed states reduce the variance of one quadrature below vacuum level:

$$|\xi_{\omega,\ell}\rangle = \hat{S}(\xi_{\omega,\ell})|0\rangle \tag{II.11}$$

$$\hat{S}(\xi) = \exp\left[\frac{1}{2}(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})\right], \quad \xi = re^{i\phi} \tag{II.12}$$

For phase quadrature squeezing ($\phi = 0$):

$$\Delta q_{\omega,\ell} = e^{-r}/\sqrt{2}, \quad \Delta p_{\omega,\ell} = e^{r}/\sqrt{2} \tag{II.13}$$

Squeezed light is a key resource for precision metrology and quantum information.

II.1.4 Field Operators

The quantised vector potential and electric field can be expressed as:

$$\hat{\mathbf{A}}(\mathbf{r}) = \sum_{\omega,\ell} \sqrt{\frac{\hbar}{2\varepsilon_0 \omega V}} \left[\hat{a}_{\omega,\ell}\mathbf{u}_{\omega,\ell}(\mathbf{r}) + \hat{a}^{\dagger}_{\omega,\ell}\mathbf{u}^{*}_{\omega,\ell}(\mathbf{r})\right] \tag{II.14}$$

$$\hat{\mathbf{E}}(\mathbf{r}) = i\sum_{\omega,\ell} \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} \left[\hat{a}_{\omega,\ell}\mathbf{u}_{\omega,\ell}(\mathbf{r}) - \hat{a}^{\dagger}_{\omega,\ell}\mathbf{u}^{*}_{\omega,\ell}(\mathbf{r})\right] \tag{II.15}$$

These operators are central to describing interactions between light and matter in cavity QED, optomechanics, and other quantum platforms.

This concludes our introduction to the quantum description of light, setting the stage for modelling interactions between quantum optical fields and mechanical resonators.

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Résumé : .

Mots clés : Optomecanique, Lumière comprimée, Cavité de grande Finesse, Interferométrie, Bruit thermique, Bruit de grenaille quantique, Resonateur de grand facteur de Qualité , Interféromètres pour la detection d'ondes gravitationnelles, Bruit de pression de radiation quantique

Subject : Optomechanics and squeezed light

Abstract:

Keywords : Optomechanics, Squeezing, High-Finesse cavity, Interferometry, Thermal Noise, Quantum Shot Noise, High-Q Resonator, Gravitational wave Interferometer,Quantum Radiation Pressure Noise