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Progress towards cryogenic squeezed light optomechanics

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Chapter I

Introduction

- I.1 Historical background**
- I.2 State of the art**
- I.3 Relevance of this work**

Chapter II

Theory: background

This chapter will cover the elementary concepts required to describe an membrane based optomechanical system in a quantum regime. We will first recall basics on optical field quantization as well describing coherent and squeezed light field, to then turn to the more specific frequency dependent squeezed light field. Secondly, we will cover the mathematical description of a mechanical resonator interacting with a generic coherent optical field, highlighting the differences with the seminal optomechanical system of a mirror on a spring. Finally, we will derive the equations of motions of a membrane based optomechanical system with frequency dependent squeezed optical fields.

II.1 Quantum Optics Concepts

II.1.1 Quantum Description of Light

We introduce briefly field quantization concepts needed to describe monochromatic field propagation and measurements. key words : eigenmodes of the field, quantization, annihilation operators, quadratures, phase space, displacement operators, squeezing operators, coherent states, generic squeezed states,

Quantised Electromagnetic Field

We consider the quantised electromagnetic field in volume V . The electric field operator can be expressed in the Heisenberg picture as:

$$\hat{\mathbf{E}}(\mathbf{r}, t) = i \sum_{\ell} \mathcal{E}_{\ell} \left[\hat{a}_{\ell} \mathbf{f}_{\ell}(\mathbf{r}) e^{-i\omega_{\ell} t} - \hat{a}_{\ell}^{\dagger} \mathbf{f}_{\ell}^*(\mathbf{r}) e^{i\omega_{\ell} t} \right] \quad (\text{II.1})$$

where $\mathcal{E}_{\ell} = \sqrt{\frac{\hbar\omega_{\ell}}{2\varepsilon_0 V}}$ is the field per photon in mode ℓ with \hbar the reduced Planck constant, ω_{ℓ} the angular frequency of mode ℓ and ε_0 the vacuum permittivity, $\mathbf{f}_{\ell}(\mathbf{r})$ are spatial mode functions satisfying orthonormality, and $(\hat{a}_{\ell}, \hat{a}_{\ell}^{\dagger})$ are the time dependent annihilation and creation operators associated with each mode ℓ satisfying the canonical commutation relations

$$[\hat{a}_{\ell}, \hat{a}_{\ell'}^{\dagger}] = \delta_{\ell\ell'}, \quad [\hat{a}_{\ell}, \hat{a}_{\ell'}] = 0, \quad [\hat{a}_{\ell}^{\dagger}, \hat{a}_{\ell'}^{\dagger}] = 0$$

Fock states

In this description of the optical field, each mode ℓ is modeled as a quantum harmonic oscillator with a discrete set of energy eigenstates known as *Fock states* or number states, denoted $|n_\ell\rangle$. These states form an orthonormal basis and satisfy $\hat{n}_\ell |n_\ell\rangle = n_\ell |n_\ell\rangle$, where \hat{n}_ℓ is the number operator defined by

$$\hat{n}_\ell = \hat{a}_\ell^\dagger \hat{a}_\ell.$$

The action of the creation and annihilation operators on these states is given by

$$\hat{a}_\ell |n_\ell\rangle = \sqrt{n_\ell} |n_\ell - 1\rangle, \quad \hat{a}_\ell^\dagger |n_\ell\rangle = \sqrt{n_\ell + 1} |n_\ell + 1\rangle.$$

They allow transitions between Fock states by lowering or raising the photon number in mode ℓ by one unit. The vacuum state $|0_\ell\rangle$ is annihilated by \hat{a}_ℓ , satisfying $\hat{a}_\ell |0_\ell\rangle = 0$. Thus, the Hamiltonian for the electromagnetic field becomes a sum of harmonic oscillator energies:

$$\hat{H} = \sum_\ell \hbar\omega_\ell \left(\hat{n}_\ell + \frac{1}{2} \right) \quad (\text{II.2})$$

Quasi monochromatic fields and slow variations

finite linewidth laser fields, slow variations of the eigenvalue of operator \hat{a} compared to the optical frequency, slow variations imply spectral decomposition of the field, make \hat{a} time dependent

Linearization of the optical field, mean field and fluctuations

We will often consider a single spatial mode of the electromagnetic field, as well as a strong optical field translating to $\hat{a}(t) = \bar{\alpha} + \delta\hat{a}(t)$. linearizing such that the field simplifies to

$$\hat{\mathbf{E}}(\mathbf{r}, t) = i\mathcal{E} \left(\hat{a}(t)\mathbf{f}(\mathbf{r})e^{-i\omega_\ell t} - \hat{a}^\dagger(t)\mathbf{f}^*(\mathbf{r})e^{i\omega_\ell t} \right) \quad (\text{II.3})$$

where we drop the ℓ subscript to simplify the notation. The field Hamiltonian then reduces to a single harmonic oscillator Hamiltonian $\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a}$ where we also drop the constant energy offset $\hbar\omega/2$.

Remarks:

Coherent and squeezed states

Quadrature Operators

To describe the phase space properties of a field mode, we define the Hermitian quadrature operators $X_{1,l}$ and $X_{2,l}$ as

$$\begin{aligned} \hat{X}_{1,l} &= \hat{a}_\ell + \hat{a}_\ell^\dagger \\ \hat{X}_{2,l} &= \hat{a}_\ell - \hat{a}_\ell^\dagger \end{aligned} \quad (\text{II.4})$$

More generally we can define arbitrary quadrature operators as

$$\begin{aligned}\hat{X}_{\theta,l} &= e^{i\theta}\hat{a}_l + e^{-i\theta}\hat{a}_l^\dagger \\ &= \cos\theta\hat{X}_{1,l} + \sin\theta\hat{X}_{2,l}\end{aligned}\quad (\text{II.5})$$

where we notice that $\hat{X}_{1,l} = \hat{X}_{\theta=0,l}$ and $\hat{X}_{2,l} = \hat{X}_{\theta=\pi/2,l}$. These are Hermitian operators corresponding to measurable observables and satisfy the commutation relation

$$[X_{\theta,l}, X_{\theta+\pi/2,l}] = 2i \quad (\text{II.6})$$

Uncertainty Principle and Quantum Noise

For two generic Hermitian operators \hat{A} and \hat{B} , the Heisenberg uncertainty principle reads as

$$\Delta\hat{A}\Delta\hat{B} > \frac{1}{2}|\langle[\hat{A}, \hat{B}]\rangle| \quad (\text{II.7})$$

where we defines $\Delta\hat{A} = \sqrt{|\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2|}$. This defines the minimum amount of quantum noise (vacuum fluctuations) in the electromagnetic field. Applying this equation to the quadratures defined above we get

$$\begin{aligned}\Delta\hat{X}_{1,l}\Delta\hat{X}_{2,l} &> 1 \\ \Delta X_{\theta,l}\Delta X_{\theta,l} &> 1\end{aligned}\quad (\text{II.8})$$

Coherent States

Coherent states $|\alpha_{\omega,\ell}\rangle$ are eigenstates of the annihilation operator:

$$\hat{a}_{\omega,\ell}|\alpha_{\omega,\ell}\rangle = \alpha_{\omega,\ell}|\alpha_{\omega,\ell}\rangle \quad (\text{II.9})$$

They can be generated by displacing the vacuum:

$$|\alpha_{\omega,\ell}\rangle = \hat{D}(\alpha_{\omega,\ell})|0\rangle, \quad \hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \quad (\text{II.10})$$

They exhibit:

- Minimum uncertainty: $\Delta q = \Delta p = 1/\sqrt{2}$
- Classical-like dynamics
- Poissonian photon statistics

Squeezed States

Squeezed states reduce the variance of one quadrature below vacuum level:

$$|\xi_{\omega,\ell}\rangle = \hat{S}(\xi_{\omega,\ell})|0\rangle \quad (\text{II.11})$$

$$\hat{S}(\xi) = \exp\left[\frac{1}{2}(\xi^*\hat{a}^2 - \xi\hat{a}^{\dagger 2})\right], \quad \xi = re^{i\phi} \quad (\text{II.12})$$

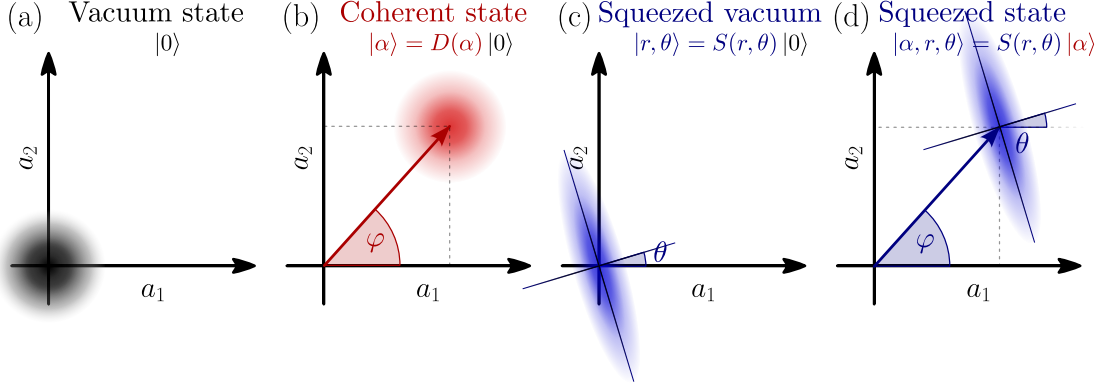


Fig. II.1 Phase-space representations of quantum states and transformations. (a) Wigner function of the vacuum state: a circular Gaussian centered at the origin, representing equal quantum fluctuations in both quadratures X_1 and X_2 . (b) Wigner function of a coherent state: a displaced circular Gaussian, showing a shift in phase space along an angle φ with unchanged, isotropic noise. (c) Wigner function of a squeezed vacuum state: an elliptical Gaussian centered at the origin, with reduced noise along a rotated quadrature X_θ and increased noise in the orthogonal direction. (d) Wigner function of a displaced squeezed state: an ellipse shifted away from the origin, combining anisotropic fluctuations and a nonzero mean amplitude. The displacement angle φ and squeezing angle θ are independent.

For phase quadrature squeezing ($\phi = 0$):

$$\Delta q_{\omega, \ell} = e^{-r} / \sqrt{2}, \quad \Delta p_{\omega, \ell} = e^r / \sqrt{2} \quad (\text{II.13})$$

Squeezed light is a key resource for precision metrology and quantum information.

This concludes our introduction to the quantum description of light, setting the stage for modelling interactions between quantum optical fields and mechanical resonators.

II.1.2 Optical Field Modulations

Linearization of the Electric Field and Modulation Sidebands

We consider a single optical mode with a strong coherent drive. The annihilation operator is linearized as:

$$\hat{a}(t) \rightarrow \bar{\alpha}(t) + \delta\hat{a}(t) \quad (\text{II.14})$$

where $\bar{\alpha}(t) \in \mathbb{C}$ is the classical coherent amplitude and $\delta\hat{a}(t)$ captures quantum fluctuations.

The classical part of the electric field is then:

$$\mathbf{E}_{\text{cl}}(\mathbf{r}, t) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0}} \left[\mathbf{f}(\mathbf{r}) \bar{\alpha}(t) e^{-i\omega t} - \mathbf{f}^*(\mathbf{r}) \bar{\alpha}^*(t) e^{i\omega t} \right] \quad (\text{II.15})$$

We now consider two types of sinusoidal modulation at frequency Ω :

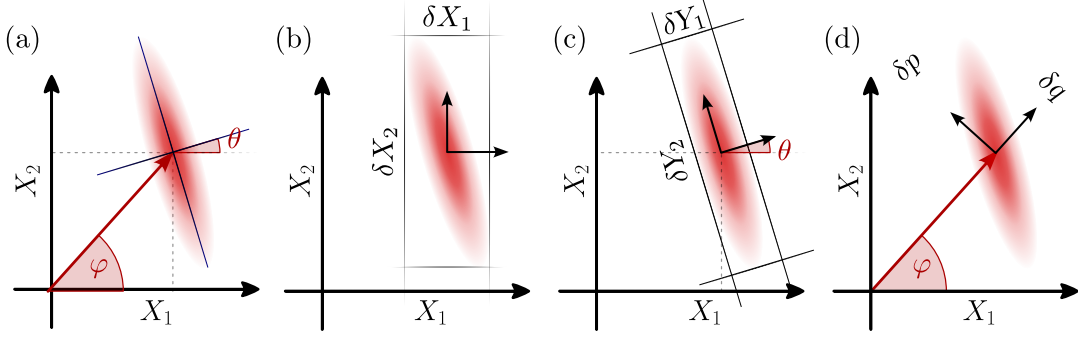


Fig. II.2 Phase-space representations of quantum states and transformations. (a) Wigner function of the vacuum state: a circular Gaussian centered at the origin, representing equal quantum fluctuations in both quadratures X_1 and X_2 . (b) Wigner function of a coherent state: a displaced circular Gaussian, showing a shift in phase space along an angle φ with unchanged, isotropic noise. (c) Wigner function of a squeezed vacuum state: an elliptical Gaussian centered at the origin, with reduced noise along a rotated quadrature X_θ and increased noise in the orthogonal direction. (d) Wigner function of a displaced squeezed state: an ellipse shifted away from the origin, combining anisotropic fluctuations and a nonzero mean amplitude. The displacement angle φ and squeezing angle θ are independent.

Amplitude Modulation (AM)

Let the coherent amplitude be modulated in amplitude:

$$\bar{\alpha}(t) = \bar{\alpha}_0 (1 + \epsilon_a \cos(\Omega t)) = \bar{\alpha}_0 \left(1 + \frac{\epsilon_a}{2} e^{i\Omega t} + \frac{\epsilon_a}{2} e^{-i\Omega t} \right) \quad (\text{II.16})$$

with $\epsilon_a \ll 1$. The conjugate is:

$$\bar{\alpha}^*(t) = \bar{\alpha}_0^* \left(1 + \frac{\epsilon_a}{2} e^{i\Omega t} + \frac{\epsilon_a}{2} e^{-i\Omega t} \right) \quad (\text{II.17})$$

Substituting into the field expression:

$$\mathbf{E}_{\text{cl}}^{(\text{AM})}(\mathbf{r}, t) = i \sqrt{\frac{\hbar \omega}{2 \epsilon_0}} \left[\mathbf{f}(\mathbf{r}) \bar{\alpha}_0 \left(e^{-i\omega t} + \frac{\epsilon_a}{2} e^{-i(\omega-\Omega)t} + \frac{\epsilon_a}{2} e^{-i(\omega+\Omega)t} \right) - \mathbf{f}^*(\mathbf{r}) \bar{\alpha}_0^* \left(e^{i\omega t} + \frac{\epsilon_a}{2} e^{i(\omega-\Omega)t} + \frac{\epsilon_a}{2} e^{i(\omega+\Omega)t} \right) \right] \quad (\text{II.18})$$

Phase Modulation (PM)

Now consider phase modulation of the coherent amplitude:

$$\bar{\alpha}(t) = \bar{\alpha}_0 e^{i\epsilon_\phi \cos(\Omega t)} \approx \bar{\alpha}_0 (1 + i\epsilon_\phi \cos(\Omega t)) = \bar{\alpha}_0 \left(1 + \frac{i\epsilon_\phi}{2} e^{i\Omega t} + \frac{i\epsilon_\phi}{2} e^{-i\Omega t} \right) \quad (\text{II.19})$$

and

$$\bar{\alpha}^*(t) \approx \bar{\alpha}_0^* \left(1 - \frac{i\epsilon_\phi}{2} e^{i\Omega t} - \frac{i\epsilon_\phi}{2} e^{-i\Omega t} \right) \quad (\text{II.20})$$

Substituting into the field:

$$\begin{aligned} \mathbf{E}_{\text{cl}}^{(\text{PM})}(\mathbf{r}, t) = i \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \Big[& \mathbf{f}(\mathbf{r}) \bar{\alpha}_0 \left(e^{-i\omega t} + \frac{i\epsilon_\phi}{2} e^{-i(\omega-\Omega)t} + \frac{i\epsilon_\phi}{2} e^{-i(\omega+\Omega)t} \right) \\ & - \mathbf{f}^*(\mathbf{r}) \bar{\alpha}_0^* \left(e^{i\omega t} - \frac{i\epsilon_\phi}{2} e^{i(\omega-\Omega)t} - \frac{i\epsilon_\phi}{2} e^{i(\omega+\Omega)t} \right) \Big] \end{aligned} \quad (\text{II.21})$$

Interpretation

In both cases, the field contains a carrier at frequency ω and two sidebands at $\omega \pm \Omega$. Amplitude modulation results in sidebands that are in phase with the carrier, while phase modulation produces sidebands with a $\pm\pi/2$ phase shift relative to the carrier.

II.1.3 Quantum Noise and Uncertainty

II.1.4 Sideband Representation

II.2 Optical Cavities : Basics

II.2.1 Cavity types and Resonance Conditions

II.2.2 Spatial and Longitudinal Modes

II.2.3 Static and Dynamical effects

II.3 Optical Cavities : Three Mirror Cavities

II.3.1

II.4 Cavity Optomechanics

II.4.1 Radiation Pressure Coupling

II.4.2 Quantum Langevin Equations

II.4.3 Mechanical Resonators

II.4.4 Noise spectra

II.4.5 Three Mirror Cavities as Novel Optomechanical Systems

II.5 Squeezed Light Theory

II.5.1 Single-mode Squeezing

II.5.2 Noise Spectra

II.5.3 Frequency-dependent Squeezing and its use

II.6 Numerical Simulations of FDS Optomechanical Experiments

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Sujet : Progress towards cryogenic squeezed light optomechanics

Résumé : .

Mots clés : Optomecanique, Lumière comprimée, Cavit  de grande Finesse, Interferom trie, Bruit thermique, Bruit de grenaille quantique, Resonateur de grand facteur de Qualit  , Interf rom tres pour la detection d'ondes gravitationnelles, Bruit de pression de radiation quantique

Subject : Optomechanics and squeezed light

Abstract:

Keywords : Optomechanics, Squeezing, High-Finesse cavity, Interferometry, Thermal Noise, Quantum Shot Noise, High-Q Resonator, Gravitational wave Interferometer, Quantum Radiation Pressure Noise