

$$H(\alpha) = -\alpha \log(\alpha) \quad (1)$$

$$\begin{aligned} E_C &= H(p_{11}) + H(p_{12}) + H(p_{21}) + H(p_{22}) \\ &= -p_{22} \log(p_{22}) - p_{21} \log(p_{21}) - p_{12} \log(p_{12}) - p_{11} \log(p_{11}) \end{aligned} \quad (2)$$

$$\begin{aligned} E_X &= H(p_{11} + p_{12}) + H(p_{21} + p_{22}) \\ &= (-p_{22} - p_{21}) \log(p_{22} + p_{21}) + (-p_{12} - p_{11}) \log(p_{12} + p_{11}) \end{aligned} \quad (3)$$

$$\begin{aligned} E_Y &= H(p_{11} + p_{21}) + H(p_{12} + p_{22}) \\ &= (-p_{22} - p_{12}) \log(p_{22} + p_{12}) + (-p_{21} - p_{11}) \log(p_{21} + p_{11}) \end{aligned} \quad (4)$$

$$\begin{aligned} I &= E_C - E_X - E_Y \\ &= (p_{22} + p_{21}) \log(p_{22} + p_{21}) + (p_{22} + p_{12}) \log(p_{22} + p_{12}) \\ &\quad - p_{22} \log(p_{22}) + (p_{21} + p_{11}) \log(p_{21} + p_{11}) - p_{21} \log(p_{21}) \\ &\quad + (p_{12} + p_{11}) \log(p_{12} + p_{11}) - p_{12} \log(p_{12}) - p_{11} \log(p_{11}) \end{aligned} \quad (5)$$

$$\frac{\partial^2 I}{\partial p_{11}^2} = \frac{1}{p_{21} + p_{11}} + \frac{1}{p_{12} + p_{11}} - \frac{1}{p_{11}} \quad (6)$$

$$\frac{\partial^2 I}{\partial p_{12}^2} = \frac{1}{p_{22} + p_{12}} + \frac{1}{p_{12} + p_{11}} - \frac{1}{p_{12}} \quad (7)$$

$$\frac{\partial^2 I}{\partial p_{21}^2} = \frac{1}{p_{22} + p_{21}} + \frac{1}{p_{21} + p_{11}} - \frac{1}{p_{21}} \quad (8)$$

$$\frac{\partial^2 I}{\partial p_{22}^2} = \frac{1}{p_{22} + p_{21}} + \frac{1}{p_{22} + p_{12}} - \frac{1}{p_{22}} \quad (9)$$

Négativité de 7 :

$$\frac{1}{p_{22} + p_{12}} + \frac{1}{p_{12} + p_{11}} - \frac{1}{p_{12}} < 0 \quad (10)$$

$$\frac{1}{p_{22} + p_{12}} + \frac{1}{p_{12} + p_{11}} < \frac{1}{p_{12}} \quad (11)$$

$$\frac{p_{12} + p_{11} + p_{22} + p_{12}}{(p_{22} + p_{12})(p_{12} + p_{11})} < \frac{1}{p_{12}} \quad (12)$$

$$\frac{p_{12}(2p_{12} + p_{11} + p_{22})}{p_{12}(p_{22} + p_{12})(p_{12} + p_{11})} < \frac{(p_{22} + p_{12})(p_{12} + p_{11})}{p_{12}(p_{22} + p_{12})(p_{12} + p_{11})} \quad (13)$$

$$p_{12}(2p_{12} + p_{11} + p_{22}) < (p_{22} + p_{12})(p_{12} + p_{11}) \quad (14)$$

$$2p_{12}^2 + p_{12}p_{11} + p_{12}p_{22} < p_{12}^2 + p_{12}p_{11} + p_{12}p_{22} + p_{22}p_{11} \quad (15)$$

$$p_{12}^2 < p_{22}p_{11} \quad (16)$$

$$p_{12} < \sqrt{p_{22}p_{11}} \quad (17)$$

Et même raisonnement pour les 3 autres dérivées secondes ...