Solution to Problem 1

a)

Assume that the cell radius is 1. The SIR can be approximated by:

$$\Gamma_0 = \frac{1}{I(3)\left(\sqrt{3\,K}\right)^{-3}} \Leftrightarrow K = \frac{1}{3}\,(\Gamma_0\,I(3))^{2/3} \approx \frac{1}{3}\left(10^{12/10} \times 11.03\right)^{2/3} \approx 10.4$$

We choose the next Loeschian number $K = 12 \Rightarrow \eta = 120/12 = 10$ ch. Now, we search for a radius r corresponding to $\Gamma_1 = 16$ dB

$$\Gamma_1 = \frac{r^{-3}}{I(3)\left(\sqrt{3\,K}\right)^{-3}} \Leftrightarrow r = \frac{\sqrt{3\,K}}{(\Gamma_1\,I(3))^{1/3}} \approx \frac{\sqrt{3 \times 12}}{\left(10^{16/10} \times 11.03\right)^{1/3}} \approx 0.789$$

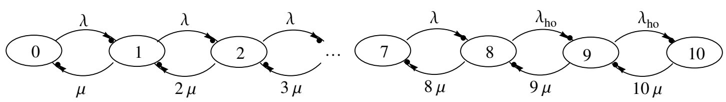
The probability of active handover algorithm is therefore:

$$p_{\text{ho}} = 1 - \frac{\pi\,r^2}{A_{\text{cell}}} = 1 - \frac{\pi\,r^2}{\frac{3}{2}\sqrt{3}} \approx 0.247$$

This means that $0.247 \times 10 \approx 2$ channels (or 3?) are reserved for handover and the capacity for new calls is $10 - 2 = 8$ channels per cell.

b)

The Markov chain with 2 reserved channels goes



The local equilibrium in each part of the chain results in

$$\begin{cases} \lambda\,p_{k-1} = k\,\mu\,p_k & k = 0,\,1,\,\ldots,\,8 \\ \lambda_{\text{ho}}\,p_{k-1} = k\,\mu\,p_k & k = 9,\,10 \end{cases} \Leftrightarrow \begin{cases} p_k = \frac{p_0}{k!}\,\rho^k & k = 0,\,1,\,\ldots,\,8 \\ p_k = \frac{p_0}{k!}\,\rho^8\,\rho_{\text{ho}}^{k-8} & k = 9,\,10 \end{cases}$$

with $\rho = \lambda/\mu$ and $\rho_{\text{ho}} = \lambda_{\text{ho}}/\mu$. We have

$$\lambda = \frac{1}{6} \times 1.247,\ \mu = \frac{1}{30} \Rightarrow \rho \approx 6.23 \ \text{ and } \ \lambda_{\text{ho}} = \frac{1}{6} \times 0.247 \Rightarrow \rho_{\text{ho}} = \frac{\lambda_{\text{ho}}}{\mu} = 1.23$$

The blocking probability is $\sum_{k=8}^{10} p_k$.

$$p_{\text{block}} = p_0\left(\frac{\rho^8}{8\,!} + \rho^8\,\frac{\rho_{\text{ho}}}{9\,!} + \rho^8\,\frac{\rho_{\text{ho}}^2}{10\,!}\right) \approx 65\,p_0$$

With a computer it is not difficult to compute

$$\sum_{k=0}^{10} p_k = 1 \Leftrightarrow p_0 = \left(\sum_{k=0}^{8}\frac{\rho^k}{k\,!} + \rho^8 \sum_{k=9}^{10}\frac{\rho_{\text{ho}}^{k-8}}{k\,!}\right)^{-1} \approx 0.002335 \Rightarrow p_{\text{block}} \approx 15\ \%$$

Solution - Problem 2

a) Determine the pole capacity *Mp* of this system if the assignment failure rate should not exceed 20%. (1.5p)

- Uplink DS-CDMA system with Omni directional antennas.
- Voice activity: q= 0.5
- $\frac{\lambda}{\mu} = 8user/cell$
- $f = 0.6$ then F= 1.6

Assignment failure should not exceed 20%, then

$$\Pr(M_c \geq K_o) \ \leq 20\%$$

Since the number of user Poisson distributed,

$$\Pr(M_c \geq K_o) \ = e^{-Fq\lambda/\mu} \sum_{n=K_o}^{+\infty} \frac{(Fq\lambda/\mu)^n}{n!} = e^{-6.4} \sum_{n=K_o}^{+\infty} \frac{(6.4)^n}{n!} \leq 0.2$$

Then, Ko =9 and we find Mpole from the following:

$$K_o = (1-\eta)M_{pole} = 0.5 * M_{pole} = 9$$

$$M_{pole} = 18$$

b) Determine the assignment failure rate if the CDMA system employs directional antennas with the antenna diagram shown in Figure and base stations placed on the corners of the cells.

When Omni directional antennas and only first tier is considered: $f = \frac{6.inter_{out}}{Inter_{Own}} = 60\%$

When directional antennas is considered:

$$f = \frac{Outer\ interference}{Own\ Interference} = \frac{(O.1*4+1*2)inter_{out}}{nter_{Own}} = 24\%$$

From (a), we know :

$$M_{pole} = 1 + \frac{1}{1.6*\ q}\frac{W}{R}\left(\frac{1}{\gamma_t}\right) = 18$$

Then, Mpole for directional is:

$$M_{pole} = 1 + \frac{1}{F\ q}\frac{W}{R}\left(\frac{1}{\gamma_t}\right) = 1 + \frac{1}{1.24\ q}\frac{W}{R}\left(\frac{1}{\gamma_t}\right) \approx 23$$

$$K_o = (1-\eta)M_{pole} = 0.5 * M_{pole} = 11$$

Assignment failure rate:

$$\Pr(M_c \geq K_o) = e^{-4.96} \sum_{n=K_o}^{+\infty} \frac{(4.96)^n}{n!} = 1{,}3\%$$

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- **PROBLEM 3:**

1) Let R denote the radius of the cell, P_t the transmit power and P_r the received power. Then

$$P_r(r) = \frac{cP_t}{r^4}, \tag{1}$$

where r is the distance between the terminal and base station, and c is the constant for pathloss. The CDF of P_r is defined as

$$\begin{aligned} F(p_r) &= \Pr(P_r \le p_r) \\ &= \Pr(\frac{cP_t}{r^4} r \le p_r) \\ &= \Pr(r \ge (\frac{cP_t}{p_r})^{\frac{1}{4}}) \\ &= \frac{\pi R^2 - \pi\sqrt{\frac{cP_t}{p_r}}}{\pi R^2} \\ &= 1 - \frac{1}{R^2}\sqrt{\frac{cP_t}{p_r}} \end{aligned} \tag{2}$$

2) The throughput is defined as

$$\lambda = \Pr(\text{no collision}) + \Pr(\text{two packets collision})P_{cap} \tag{3}$$

where

$$\Pr(\text{no collision}) = \frac{\gamma}{1!}e^{-\gamma} = \gamma e^{-\gamma} \tag{4}$$

$$\Pr(\text{two packets collision}) = \frac{\gamma^2}{2!}e^{-\gamma} = \frac{\gamma^2}{2}e^{-\gamma} \tag{5}$$

$$\begin{aligned} P_{cap} &= 2\Pr\frac{P_{rx1}}{P_{rx2}} \ge \gamma_0 \\ &= 2\Pr(\frac{R_2}{R_1})^{\alpha} \ge \gamma_0 \\ &= 2\int_0^R \Pr(\frac{R_2}{R_1}|R_2 = r_2)f_{r_2}(r_2)dr_2 \\ &= 2\int_0^R \frac{r_2^2(\gamma_0)^{-2/\alpha}}{R^2}\frac{2r_2}{R^2}dr_2 \\ &= \frac{1}{\gamma_0^{2/\alpha}} \approx 0.5 \end{aligned} \tag{6}$$

Hence

$$\lambda = \gamma e^{-\gamma} + \frac{\gamma^2}{2}e^{-\gamma}0.5 \tag{7}$$

The maximum throughput can be find by setting $\frac{d\lambda}{d\gamma} = 0$, solving which gives us $\gamma' = 1.24$ Therefore $\lambda_{max} = \gamma' e^{-\gamma'} + \frac{\gamma'^2}{2} e^{-\gamma'} 0.5 \approx 0.47$

3) The number of arrival packets follows Poisson distribution, therefore the probability of more than two packets collide is

$$\begin{aligned} \Pr(n > 2) &= \sum_{k=3}^{\inf} \frac{\gamma^k}{k!} e^{-\gamma} \\ &= 1 - \sum_{0}^{k=2} \frac{\gamma^k}{k!} e^{-\gamma} \end{aligned} \tag{8}$$

If we can tolerate at most 1% error in the results, then $\Pr(n > 2) \leq 0.01$. Thus we can find that $\gamma < 0.4$. If we require higher accuracy, then the assumption of neglecting more than two packets collide is only valid for even lower γ

- **PROBLEM 4:**

Let r denote the distance between the mobile and the access point, and D the radius of the coverage area. Then at the cell boundary, $r = D$, the achievable data rate is 400kbps, which corresponds to $\gamma(D) = 4 (> 4\text{dB})$. Assuming the transmission power is P.

$$\gamma(D) = \frac{P/D^\alpha}{I} = \frac{P/D^4}{I} = 5. \tag{9}$$

Therefore the interference constant is $I = (\frac{P}{4D^4})$.

Let d denote the distance at which $\gamma(d) = 20\text{dB}$.

$$\gamma(d) = \frac{P/d^4}{I} = 100. \tag{10}$$

Therefore $d = \frac{\sqrt{5}D}{5}$

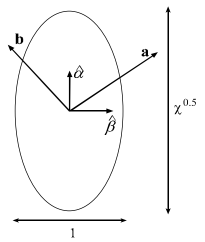
Now we can calculate the average date rate as follow:

$$\begin{aligned} E[R] &= \int_0^D R(r) f_r(r) dr \\ &= \int_0^d 10^4 \frac{2r}{D^2} dr + \int_d^D 400 \frac{P/r^4}{I} \frac{2r}{D^2} dr \\ &= \int_0^d 10^4 \frac{2r}{D^2} dr + \int_d^D 400 \frac{D^4}{r^4} \frac{2r}{D^2} dr \\ &= 3600\text{kbps} \end{aligned} \tag{11}$$

PROBLEM 5

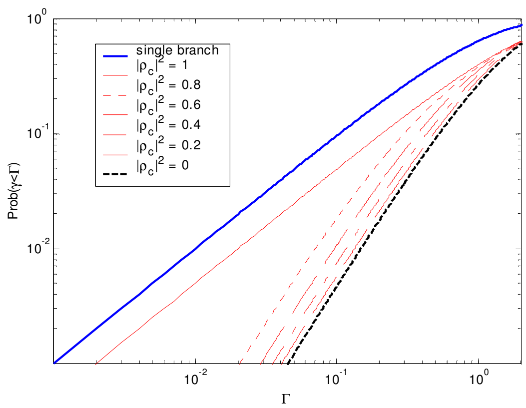
The network company *NetTwo* is to deploy a radio network within the rural parts of Sweden. The question for the company is whether to build space- or polarization diversity? Using the assumptions stated below:

a. With which antenna configuration will the company have to build the least no of sites to cover the same area? (1p)
b. How many more sites per area unit are required for the non-preferred solution if we assume a maximum error probability of 10^{-2}? (3p)



Assumptions:

1. Assume that the towers are high and that the channel cross polar discrimination χ *=9dB*
2. That the propagation constant α=2
3. Assume that the system is noise- and uplink limited
4. That the channel is subject to Rayleigh fading
5. That the distribution function for the SNR of the resulting combined signal for the two branch diversity system for different power correlation factors, ρ, is given by the figure below



Solution:

a) Space diversity results in the least no. of sites! (If they have been to the lecture they know that!)

b) <u>1. Polarization diversity:</u>
– χ=9dB -> ρ=0.77 (can be calculated geometrically or perhaps be given?) results in only 1dB diversity gain (according to figure)
<u>2. Space diversity</u>
- ρ=0 (uncorrelated branches) results in 11 dB diversity gain
- In addition space diversity has two receive antennas: +3dB
- In total, Space diversity is 13dB better. Hence, the system employing polarization diversity would result in 20 times more base stations in a rural environment