Solutions for IK2510 Exam 20121020

Problem 1

(a) Since we need to add 11dB fade margin, then $\gamma_{t,margin} = 10 + 11 = 21db$

$$\frac{\frac{1}{R^4}}{6\sum_{n=1}^{\infty}\frac{1}{(nD)^4}} \geq \gamma_{t,margin}$$

$$\frac{\left(R\sqrt{3K}\right)^4}{7.2126R^4} \geq 10^{2.1} \quad \rightarrow \quad K \geq 10.1 \rightarrow K = 12$$

The capacity of the system is:

$$\eta = \frac{C}{K} = \frac{120}{12} = 10ch/cell$$

(b)For a relative traffic load of 60%, we have $\rho/\eta = 0.6$

$$P_b = \frac{\frac{\rho^\eta}{\eta!}}{\sum_{k=1}^{\eta}\frac{\rho^k}{k!}} \approx 0.05$$

(c)

$$\Pr\left[\frac{G_o R^{-4}}{G_I D^{-4}} < 10\right] = \Pr\left[\frac{G_o}{G_I} < \frac{10}{3K^2}\right]$$

If G(dB) = Go(dB)-Gi(dB), then $\mu_G$=-11 and $\sigma_G$=6.7

$$\Pr\left[\frac{G-\mu_G}{\sigma_G} < \frac{10\log\left(\frac{10}{3K^2}\right)+11}{6.7}\right] = \phi(-0.8) = 22.2\%$$

Problem 2

If the transmission is successful, the data rate per user is 1024kbps. The number of supported users is denoted by K.

(a) 1024kbits/sec*Pr(successful transmission-Slotted Aloha)/K ≥ 10kbits/10sec
K = 1024kbps*0.368/1kbps = 376 users

(b) 1024kbits/sec*Pr(successful transmission-Slotted Aloha)*0.5 /K ≥ 10kbits/10sec
K = 1024kbps*0.368*0.5 /1kbps = 188 users

(c) K= 1024 users

Problem 3

In this problem, we have two power constraints:

- Maximum transmission power should not exceed 1W (30dBm)
- The minimum SIR at the TV receiver is 20dB to avoid distortion

$$SIR \geq 20dB$$
$$\frac{P_{rx}}{I_{total}} \geq 100 \quad \rightarrow I_{total} \leq -95dBm$$

The total interference at the TV receiver is:

$$I_{total} = Pt_1 * 10^{-10} + Pt_2 * 10^{-10} \leq 10^{-9.5}$$

$$Pt_1 + Pt_2 \leq 5dBm$$

Then, we conclude that the limiting factor is the interference at the TV receiver.

(a) For simultaneous transmissions, the equal power for both transmitters is considered. $Pt_1 = Pt_2 = 2dBm$

$$SIR_1 = \frac{Pt_1 * 10^{-7.5}}{Pt_2 * 10^{-9.5} + 10^{-10}} = 83.3662 \qquad \rightarrow R_1 = 12.7972 Mbits/s$$

$$SIR_2 = \frac{Pt_2 * 10^{-8.5}}{Pt_1 * 10^{-9.5} + 10^{-10}} = 8.33662 \qquad \rightarrow R_2 = 6.4458 Mbits/s$$

(b) For time-sharing, each transmitter can use the maximum allowed power. $Pt_1 = Pt_2 = 5dBm$

$$SIR_1 = \frac{Pt_1 * 10^{-7.5}}{10^{-10}} = 1000 \qquad \rightarrow R_1 = 20\ Mbits/s$$

$$SIR_2 = \frac{Pt_2 * 10^{-8.5}}{10^{-10}} = 100 \qquad \rightarrow R_2 = 13\ Mbits/s$$

Timesharing 50% of the time each yields

$$\overline{R} = 0.5R_1 + 0.5R_2 = 0.5 \cdot 20 + 0.5 \cdot 13 = 16.5\text{Mbit/s}$$

which is less than the total data rate in a) 12.8 + 6.4 = 19.2Mbit/s

Problem 4

Let transmission power be 1 without loss of generality. Also, let $d = 1$ without loss of generality. Let $N_b$ be the background noise. Then, $N_b = 1/4$. SINR of i-th scheme is denoted by $\gamma_i$. Achievable capacity of mobile is then $C_i = B\log_2(1+\gamma_i)$ , where $B$ is the bandwidth (or time portion) allocated to the mobile.

(a)

Scheme 1: $r_1 = \dfrac{1}{1+1+1+0.25} = 0.308$, $C_1 = 10\log_2(1+\gamma_1) = 3.87$ [Mbps]

Scheme 2: $r_2 = \dfrac{4}{0.25} = 16$, $C_2 = \dfrac{10}{4}\log_2(1+\gamma_2) = 10.21$ [Mbps]

Scheme 3: $r_3 = \dfrac{2}{2+0.25} = 0.888$, $C_3 = \dfrac{10}{2}\log_2(1+\gamma_3) = 4.58$ [Mbps]

Therefore, scheme 1 is the worst and scheme 2 is the best.

(b)

Distance between BS4 (or BS2) to point B is $\sqrt{0.5^2+1^2} = \sqrt{5}/2 = 1.118$.

Scheme 1: $r_1 = \dfrac{0.5^{-2}}{1.25^{-1}+1.25^{-1}+1.5^{-2}+0.25} = 1.743$, $C_1 = 10\log_2(1+\gamma_1) = 14.56$ [Mbps]

Scheme 2: $r_2 = \dfrac{0.5^{-2}+1.25^{-1}+1.25^{-1}+1.5^{-2}}{0.25} = 24.178$ , $C_2 = \dfrac{10}{4}\log_2(1+\gamma_2) = 11.64$ [Mbps]

Therefore, scheme 1 is better than scheme 2 at the point B.

(c)

Non-loaded BSs do not generate interference. However, they can participate in the joint transmission. Moreover, they do not require a radio resource to share.

Scheme 1: $r_1 = \dfrac{0.5^{-2}}{1.25^{-1}+0.25} = 3.809$, $C_1 = 10\log_2(1+\gamma_1) = 22.66$ [Mbps]

Scheme 3: $r_3 = \dfrac{0.5^{-2}+1.25^{-1}}{1.25^{-1}+0.25} = 4.571$, $C_3 = 2\dfrac{10}{2}\log_2(1+\gamma_3) = 24.78$ [Mbps]

Therefore, scheme 3 is better than scheme 1.

[Qualitative explanation] When BS4 is not loaded, BS1 does not have to share the radio resource with BS4. Thus, BS1 can utilize the full resource as if frequency reuse of 1 is employed. However, user SINR of scheme 3 is always better than that of scheme 1 because received signal power gets stronger with the same interference power. Therefore, scheme 3 is always better than scheme 1 provided that BSs 3&4 are not loaded.

Problem 5

(a) The optimum horizontal antenna beam width, when α=2 , is 90degrees or pi/2

(b) To calculate the gain of the antenna (G), we first calculate the directivity (D) and losses (L):

$$D = \frac{4\pi}{\theta_{vert}\theta_{horiz}} = \frac{4\pi}{\frac{\pi}{2} * \frac{2/3}{2}} = 24 = 13.8\ dB$$

$$L = 0.5\frac{dB}{m} * 2m = 1\ dB$$

$$G = D - L = 13.8 - 1 = 12.8dB$$

(c) For a space diversity to cover twice the area as polarization, the following must hold

$$R_{space} = \sqrt{2}\, R_{polarization}$$

The received power at the cell border should be the same for both configurations, then

$$P_r = \frac{PtxG_{polarization}}{R^2_{polarization}} = \frac{PtxG_{space}}{R^2_{space}} \quad \rightarrow \quad G_{space} = 2G_{polarization}$$

This means that the gain for space diversity should be 3dB higher than polarization diversity. From the figure, we see that this happens when $D/\lambda = 3$ , which give a minimum distance between antennas of 2m.

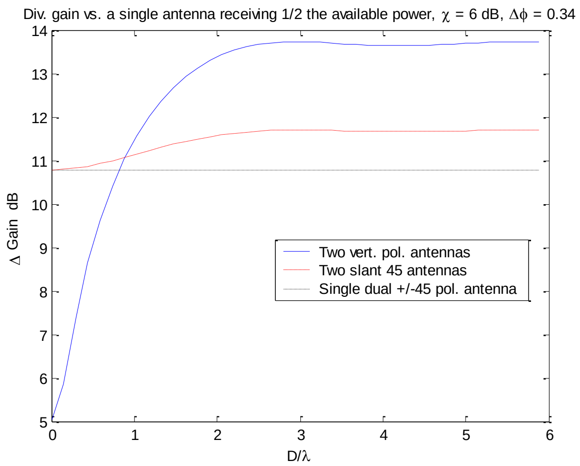


*Fig.3: Comparison of MRC diversity gain relative a single antenna receiving ½ the available power. $\chi$=6dB*