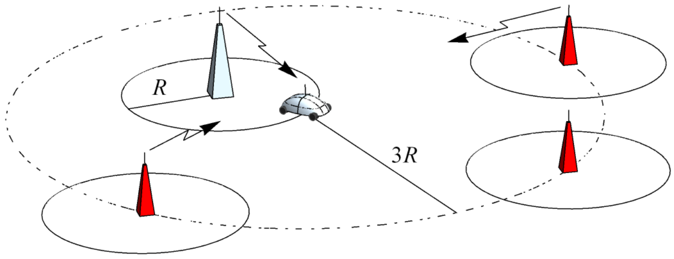
Problem 1

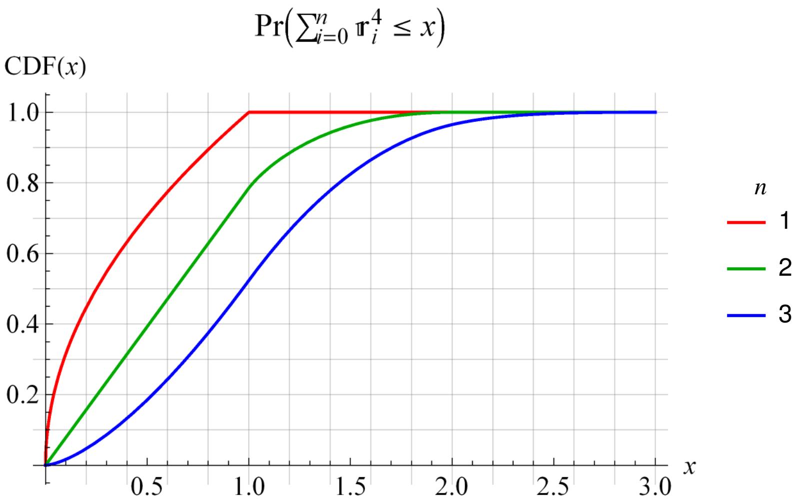


In a certain part of a network, with identical circular cells, we are considering a down link transmission to a mobile station (in focus) at the distance R from its base station. There are three potential interfering base stations at the distance $3\,R$ from the mobile station (in focus). The SNR at this mobile station is 30 dB and in order to run the service at the mobile station we need at least an SINR of 18 dB. The users are assumed to be uniformly distributed in the cells and the path loss exponent is assumed to be $\alpha = 4$. We assume that all base stations adjust their transmit power so that every mobile station gets the same received power.

a) Show that the service at the focused mobile station is running with 100 % probability if we have at most one transmitting interferer. (2p)

b) Let $q = 0.5$ be the probability that an interferer is transmitting. What is the probability that the service is running with exactly two transmitting interferers? (2p)

c) How large activity q can we accept if we want the service to run with 95 % probability? (2p)



The above diagram shows the cumulative probability distribution function of $\sum_{i=0}^{n} \mathbb{r}_i^4$, where $\mathbb{r}_i$ is the normalized user distance from the center of a circle cell, where the users are uniformly distributed.

Solution

a)

If we have no interferer the SINR = SNR = 30 dB which is clearly above the threshold. With one transmitting interferer we have

$$\mathrm{SIR} \geq \frac{P_{\mathrm{Rx}}}{P_{\mathrm{Tx}}(3\,R)^{-4}} = \frac{P_{\mathrm{Rx}}}{P_{\mathrm{Rx}}\,R^4(3\,R)^{-4}} = 81 \approx 19\,\mathrm{dB}$$

which also is above the 18 dB threshold. I.e. $\Pr(\mathrm{SINR} > 18\,\mathrm{dB}) = 1$.

b)

The SINR is computed as

$$\frac{1}{\mathrm{SINR}} = \frac{1}{\mathrm{SIR}} + \frac{1}{\mathrm{SNR}} \Rightarrow \mathrm{SINR} = \frac{1}{\frac{1}{\mathrm{SIR}} + \frac{1}{\mathrm{SNR}}} = \frac{1}{\sum_{i=0}^{\mathbb{n}} \mathbb{r}_i^4\, 3^{-4} + \frac{1}{1000}} = \frac{1}{\frac{1}{81}\sum_{i=0}^{\mathbb{n}} \mathbb{r}_i^4 + \frac{1}{1000}}$$

We want to compute $\Pr(\mathrm{SINR} > 18\,\mathrm{dB} \bigcap \mathbb{n} = 2)$. The probability that two interfering base stations transmit is $q^2(1-q)$ and there are 3 possibilities.

$$\Pr(\mathrm{SINR} > 18\,\mathrm{dB} \bigcap \mathbb{n} = 2) = \Pr(\mathrm{SINR} > 18\,\mathrm{dB} \mid \mathbb{n} = 2)\Pr(\mathbb{n} = 2)$$

$$= \Pr\left(\left(\frac{1}{81}\sum_{i=0}^{2} \mathbb{r}_i^4 + \frac{1}{1000}\right)^{-1} > 10^{1.8}\right) \times 3\,q^2(1-q)$$

$$\approx \Pr\left(\sum_{i=0}^{2} \mathbb{r}_i^4 \lesssim 1.2\right) \times 3 \times 0.5^3 = \mathrm{cdf}_2(1.2) \times 0.375 \overset{\text{diagram}}{\approx} 0.89 \times 0.375 \approx 0.33$$

c)

In general we have $\mathbb{n} \in \mathrm{Bin}(3, q)$ so

$$\Pr(\mathrm{SINR} > 18\,\mathrm{dB}) = \sum_{n=0}^{3}(\Pr(\mathrm{SINR} > 18\,\mathrm{dB} \mid \mathbb{n} = n)\Pr(\mathbb{n} = n)) = \sum_{n=0}^{3}\left(\Pr\left(\sum_{i=0}^{n} \mathbb{r}_i^4 \lesssim 1.2\right)\binom{3}{k} q^k(1-q)^{3-k}\right)$$

$$= 1 \times \binom{3}{0}(1-q)^3 + 1 \times \binom{3}{1} q(1-q)^2 + \mathrm{cdf}_2(1.2)\binom{3}{2} q^2(1-q) + \mathrm{cdf}_3(1.2)\binom{3}{3} q^3$$

$$\overset{\text{diagram}}{\approx} (1-q)^3 + 3\,q\,(1-q)^2 + 0.89 \times 3\,q^2\,(1-q) + 0.67\,q^3 = 1 - 0.33\,q^2$$

$$P(\mathrm{SINR} > 18\,\mathrm{dB}) > 0.95 \Rightarrow q \lesssim 0.39$$

Problem 2 (6 points):

In a slotted ALOHA system collided packets are all lost and single transmitted packets are assumed to be lost with probability 20% due to noise. Each transmitter needs to send a 15-kbit message on average every 10 seconds following the Poisson process, even if the previous messages have not yet been sent. Assume the compound process of retransmitted and new messages will also form a Poisson process. Assume the transmitters know if a packet is successfully received or not immediately after its transmission. When sending a packet, the transmission data rate of the transmitter is determined by R= 10*SNR kbps. The transmission power P_t is determined by $P_t = P_r * d^4 / z$, where P_r is the constant received power and P_r=15e-4 mW. z=0.2 is the power amplifier efficiency. The noise power is N_o*=1e-4 mW.* d is the distance between the transmitter and receiver of each communicating pair and is uniformly distributed between 10 and 100 meters. Each transmitter consumes 20 mW circuit power in the data transmission mode. If a transmitter is not transmitting data, it always sleeps and consumes 200 mW circuit power in the sleep mode. There is no delay or additional energy consumption in switching between the different modes.

(a). Determine the CDF of transmission power of the transmitters in the transmission mode (1 point).
(b). Determine the CDF of average power consumption of a transmitter if there is only one communicating pair (1 point).
(c). If there is only one communicating pair with d=50 meters, what is the energy efficiency of the transmitter (1 points)?
(d). How many communicating pairs can this system support (2 points)?
(e). When the maximum number of pairs exist in the network, given in (d), determine the energy efficiency of a transmitter whose distance to its receiver is 50 meters (1 points).

Solution: (a). $\Pr(P_t \le P) = \Pr(P_r * d^4/z \le P) = \Pr\left(d \le \sqrt[4]{\frac{Pz}{P_r}}\right) = \frac{\sqrt[4]{\frac{Pz}{P_r}} - 10}{90} = A\sqrt[4]{P} - 1/9$,

where $A = \frac{\sqrt[4]{\frac{z}{P_r}}}{90} = \frac{\sqrt[4]{\frac{0.2}{15*10^{-7}}}}{90} = 0.21$.

(b). SNR= P_r / N_o=15e-4 */1e-4=15*.

The average data rate is: R= $10\,SNR*(1-Pe) = 10*15*0.8$=120 kbps.

In every 10 s, the transmitter is in the active mode for t=15/120=0.125 s. It then stays in the sleep mode for 9.875 s. The average power consumption is given by

$\bar{P}$=(0.125*(P_t+0.02)+9.875*0.2)/10= 0.0125*P_t +1.9775.

$\Pr(\bar{P} \le P) = \Pr(0.0125*P_t + 1.9775 \le P) = \Pr(P_t \le 80*P - 158.2) = A\sqrt[4]{80*\mathrm{P} - 158.2} - 1/9$.

(c). $P_t = P_r * d^4/z$=15e-7 *50^4/0.2= 46.875;

The average power consumption is: $\bar{P}$=0.0125*P_t +1.9775=0.0125*46.875+1.9775=2.56.

The average data rate is: 15 kbits/10=1.5 kbps.

The energy efficiency: u=1500/2.56=586 b/Joule.

(d). The data rate requirement of each user is: 1.5kbps.

The highest efficiency of Slotted ALOHA is 36.8%. But the packet error rate is 20%.

The maximum number of users$\le$ R*(1-P_e)*36.8%/1.5=150*0.8*0.368/1.5=29.44.

At most 29 users can be supported.

(e). Observe the efficiency formula, $\lambda \le \max \gamma e^{\gamma} = e^{-1}$. The maximum number of users indicate that $\gamma = 1$. Therefore all users are always in the active mode for data transmission.

The average power consumption is:

$\bar{P}$=P_t+0.02=46.875+0.02=46.895.

The energy efficiency: u=1500/46.895= 32 b/Joule.

Problem 3 (6 points):

A cellular operator has installed a small mobile telephone system along a street. The system has only 4 base stations (BSs) located at the following distances (coordinates, in meters), measured from one end of the street:

$$(50,100,150,200).$$

The system uses only one channel. The propagation loss is proportional to the fourth power of the distance, i.e. $P_r = s*P_t/d^4$, where the fading s is uniformly distributed between 0.2 and 1. At one time, two terminals, A and B, at 75 and 140, are connected. BSs that do not serve any terminal are kept off. A terminal will be in outage if its SIR is below 10 dB.

(a) Determine the average link gain matrix for this channel set (2 points).
(b) Determine the uplink outage probabilities of the two terminals if A is connected to the second BS and B to the fourth. P_t=1 W (2 points).
(c) Determine the corresponding downlink outage probabilities of the two terminals in (b) (2 points).

Solution:

(a) Link gain $=E(\frac{s}{d^4})=\frac{E(s)}{d^4}=\frac{E(s)}{d^4}=\frac{0.6}{d^4}$.

In dB scale, link gain = 10 log10$\frac{0.6}{d^4}$=10 log10(0.6)-40log10 (d)

$G=-2.2185-40*\begin{pmatrix}\log_{10}25 & \log_{10}25 & \log_{10}75 & \log_{10}125\\ \log_{10}90 & \log_{10}40 & \log_{10}10 & \log_{10}60\end{pmatrix}$ dB.

(b) In the uplink:

The SIR of A is: $\gamma=\frac{\frac{s_A}{25^4}}{\frac{s_B}{40^4}}$=6.5536$\frac{s_A}{s_B}$.

The SIR of B is: $\gamma=\frac{\frac{s_B}{60^4}}{\frac{s_A}{125^4}}$=18.8380$\frac{s_B}{s_A}$.

When p>5, Pr($\frac{s_A}{s_B}<p)=1$.

When 0.2<=p<=1, the CDF of Pr($\frac{s_A}{s_B}<p)=\Pr\left(\frac{s_B}{s_A}<p\right)=\int_{0.2}^{1}\int_{0.2}^{1}\frac{1}{0.8}\frac{1}{0.8}I(s_A<ps_B)ds_Ads_B=\frac{1}{0.64}\int_{0.2}^{1}(ps_B-0.2)ds_B$=$\frac{1}{0.64}[0.5p\left(1-\frac{0.04}{p^2}\right)-0.2(1-0.2/p)]$=$\frac{1}{0.64}(0.5p-0.2+0.02/p)$.

where I(A)=1 if A is true.

When 1<=p<=5, the CDF of Pr($\frac{s_A}{s_B}<p)=\Pr\left(\frac{s_B}{s_A}<p\right)=1-Pr(\frac{s_A}{s_B}>p)=1-Pr(\frac{s_B}{s_A}<1/p)$=1-$\frac{1}{0.64}(0.5/p-0.2+0.02p)$.

The outage probability of A is:

Pr($\gamma=6.5536\frac{s_A}{s_B}<10$)=Pr($\frac{s_A}{s_B}<1.5259$)= 1-$\frac{1}{0.64}\left(\frac{0.5}{1.5259}-0.2+0.02/0.5259\right)=0.74$.

The outage probability of B is:

Pr($\gamma=18.8380\frac{s_B}{s_A}<10$)=Pr($\frac{s_B}{s_A}<0.5308$)=$\frac{1}{0.64}(0.5*0.5308-0.2+0.02/0.5308)$=0.16.

(c) In the downlink:

The SIR of A is: $\gamma=\frac{\frac{s_A}{25^4}}{\frac{s_B}{125^4}}$=625$\frac{s_A}{s_B}$.

The SIR of B is: $\gamma = \frac{\frac{s_B}{60^4}}{\frac{s_A}{40^4}}$=0.1975$\frac{s_B}{s_A}$.

The outage probability of A is:

Pr($\gamma = 625\frac{s_A}{s_B} < 10$)=Pr($\frac{s_A}{s_B} < 0.016$)=0.

The outage probability of B is:

Pr($\gamma = 0.1975\frac{s_B}{s_A} < 10$)=Pr($\frac{s_B}{s_A} < 50.6329$)=100%.

Problem 4 (6 points):

Consider a cell serving a homogeneous delay-sensitive application (e.g. video telephony). New and handover sessions arrive to the cell following independent Poisson processes with the intensities λ_{new} and λ_{ho}, respectively. The sojourn time of each session follows an exponential distribution with the parameter μ. The cell can accommodate N sessions in total.

In order to prioritize handover sessions, we consider a probabilistic reservation scheme described as follows:

- Handover sessions are always accepted as long as there is capacity.
- When there are currently n sessions in the cell, a new session is accepted with the probability of $\frac{1}{n+1}$ ($n < N$).

(a) Draw a state transition diagram depicting this reservation scheme. (for general N) (2 points)

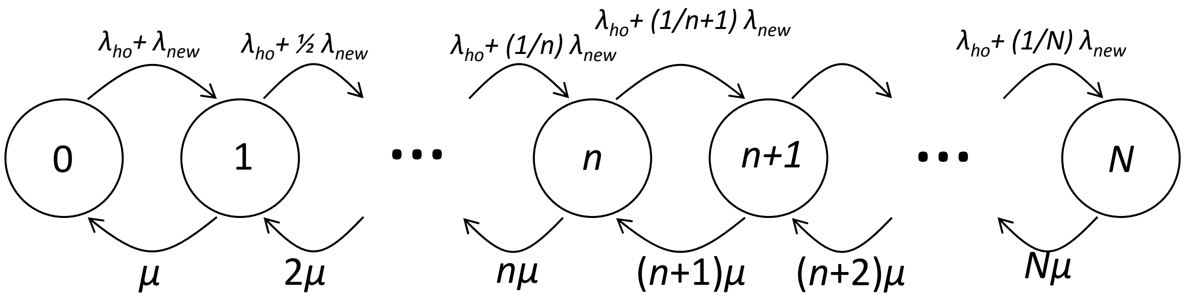
(b) Calculate steady-state probability when $N = 2$. Assume $\mu = 2\lambda_{new} = 2\lambda_{ho}$. (1.5 points)

(c) Express handover dropping probability and new call blocking probability in terms of steady-state probabilities. (for general N) (1.5 points)

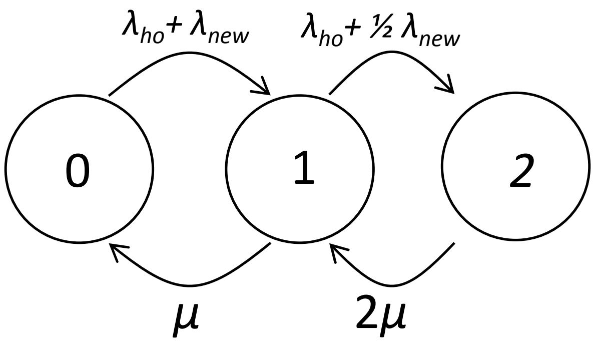
(d) Then, obtain these values by using the same parameters as (b). (1 point)

Solution:

(a)



(b) Without loss of generality, assume $\lambda_{new} = \lambda_{ho} = 1$ and $\mu = 2$



$2p_0 = 2p_1$

$1.5p_1 = 4p_2$

$p_0 + p_1 + p_2 = 1$

Then, $p_0 = p_1 = 8/19 = 0.4211$ and $p_2 = 3/19 = 0.1579$.

(c) $p_{drop} = p_N$

$$p_{block} = \sum_{i=0}^{N-1} \frac{i}{i+1} p_i + p_N$$

(d) $p_{drop} = p_2 = 0.1579$

$p_{block} = 0.5p_1 + p_2 = 0.3684$

Problem 5 (6 points):

We would like to plan a CDMA cellular system (we are interested in the uplink case). The considered DS-CDMA system has a processing gain (spreading factor) of $\frac{W}{R_S} = 256$, an information data rate of *Rs* = 10 kbps, and employing a perfect constant received power control. The maximum transmitted power of the mobile unit is *P*max = 24 dBm and the noise power spectral density is *N*0 = -197 dBW/Hz. The propagation path-loss is given by

LP = 37 + 40 log10(*r*), dB

where *r* is the distance between the transmitter and the receiver in *m*. The required signal energy-to-noise power spectral density for good signal quality is $\xi_t = 3\ \mathrm{dB}$.

a) Assuming hexagonal cells, determine the area capacity if the number of users per cell is *M* = 30 users/cell. (3 points)

b) Repeat part a) if the number of users per cell is *M* = 60 users/cell and compare! (3 points)

Solution:

Solution

a) With perfect constant received power control, the received SINR is

$$\Gamma = \frac{W}{R_s} \frac{P}{(M-1)FP + N_0 W} \geq \xi_t$$

Solving for P we get

$$P \geq \frac{N_0 W}{\frac{W}{R_s}\frac{1}{\xi_t} - (M-1)F}$$

To ensure P at the base station, the transmitted power should be

$$P_t = L_p P = 10^{3.7} R^4 P \leq P_{\max} \quad \rightsquigarrow \quad R \leq \left(\frac{P_{\max}}{10^{3.7} P}\right)^{1/4} = \left(\frac{P_{\max}}{10^{3.7}} \frac{\frac{W}{R_s}\frac{1}{\xi_t} - (M-1)F}{N_0 W}\right)^{1/4}$$

where R is the radius of the cell.

With $M = 30$ we get $R = 532$ m and the area capacity is

$$\eta_A = \frac{30}{\frac{3\sqrt{3}}{2} R^2} = 41 \text{ users/km}^2.$$

b) With $M = 60$ we get $R = 426$ m and the area capacity becomes

$$\eta_A = \frac{60}{\frac{3\sqrt{3}}{2} R^2} = 127 \text{ users/km}^2.$$