MVA for TRS – Direct approach

I – Introduction

The Initial Margin (IM) for uncleared OTC derivatives has become increasingly important because regulatory requirements. The purpose of this margin call is to reduce exposure to market risk for the period necessary for the liquidation/replacement of the traded portfolio following the default of a counterparty. To integrate IM into xVA calculations, it is necessary to simulate a Dynamic IM Forward throughout the duration of the portfolio. This paper aims to highlight a direct approach calculation for Margin Valuation Adjustment (MVA) through nested Monte-Carlo loops.

II – Pricing model

Let's assume a Bank B enters in total return swap (TRS) with its counterparty C. As uncleared OTC derivative, this TRS transaction implies an initial margin posting. This IM calculation won't be the purpose of this paper, but this calculation, based on Monte-Carlo VaR, will be also used for MVA, as we'll soon see.

The purpose of MVA is to quantify the total cost associated with posting the initial margin throughout the life of the traded portfolio, which only contains here one Total return swap. This calculation is based on future MI projections (also called Forward or Dynamic IM). The MVA is calculated as follows:

$$MVA^{TRS}(t_0) \ = \int_{t_0}^{T} ((1-R)\lambda_B(u) - s_{IM}(u))e^{-\int_{t_0}^{u}(\lambda_B(u)+\lambda_C(u))dx}\mathbb{E}^{\mathbb{Q}}\left(\frac{IM(u)}{B(t_0,u)}\right)du$$

$$\Rightarrow MVA^{TRS}(t_0) \ = \int_{t_0}^{T} ((1-R)\lambda_B(u) - s_{IM}(u))e^{-\int_{t_0}^{u}(\lambda_B(x)+\lambda_C(x))dx}\mathbb{E}^{\mathbb{Q}}\left(\frac{\mathcal{Q}_{MPOR,99\%}(\Delta TRS(u)|\mathcal{F}_u)}{B(t_0,u)}\right)du$$

With following notations:

- R the Bank's recovery rate
- s_{IM} the IM's remuneration spread.
- T the TRS maturity
- λ_B and λ_C default intensities for Bank B and its counterparty C
- $B(t_0,u) = e^{\int_{t_0}^{u} r(t)dt}$ the bank-account
- $IM(u)$ the forward initial margin that must be posted at time u.
- $\mathcal{Q}_{MPOR,99\%}$ the 99% quantile of the ΔTRS distribution during MPOR period, aka $VaR_{MPOR,99\%}$

Next analysis is done with following assumptions which will be discussed in last section:

- λ_B and λ_C are considered as constant.
- s_{IM} is also considered as constant.
- Interest rate r is considered as constant.
- Underlying's volatility σ is constant.
- TRS is bullet with fixed financing spread.
- IM dynamic will be based on risk-neutral risk factor (here only underlying) dynamic.

III – Algorithm

With previous assumption and after discretization, margin value adjustment for TRS follows next equations:

$$\begin{cases} MVA^{TRS}(t_0) \ \approx \displaystyle\sum_{k=0}^{T}((1-R)\lambda_B - s_{IM})e^{-\sum_{i=0}^{k}(\lambda_B+\lambda_C)}\left(\frac{1}{N_{outer}}\sum_{j=1}^{N_{outer}}\frac{\mathcal{Q}_{MPOR,99\%}{}^{j}(\Delta TRS(k)|\mathcal{F}_k)}{e^{\sum_{i=0}^{k} r}}\right) \\ \mathcal{Q}_{99\%}{}^{j}(\Delta TRS(k)|\mathcal{F}_k) \ = \left(\mathcal{Q}_{99\%}\left((TRS^i(k \ + \ MPOR) - TRS^i(k) \ |\mathcal{F}_k)_{i\,=\,[1,N_{inner}]}\right)\right)_j \end{cases}$$

$$\Rightarrow MVA^{TRS}(t_0) \ \approx \sum_{k=0}^{T}((1-R)\lambda_B - s_{IM})e^{-k\cdot(\lambda_B+\lambda_C+r)}\left(\frac{1}{N_{outer}}\sum_{j=1}^{N_{outer}}\left(\mathcal{Q}_{MPOR,99\%}\left((TRS^i(k \ + \ MPOR) - TRS^i(k) \ |\mathcal{F}_k)_{i\,=\,[1,N_{inner}]}\right)\right)_j\right)$$

$$\Rightarrow MVA^{TRS}(t_0) \ \approx ((1-R)\lambda_B - s_{IM})\cdot\sum_{k=0}^{T}e^{-k\cdot(\lambda_B+\lambda_C+r)}\left(\frac{1}{N_{outer}}\sum_{j=1}^{N_{outer}}\left(\mathcal{Q}_{MPOR,99\%}\left((TRS^i(k \ + \ MPOR) - TRS^i(k) \ |\mathcal{F}_k)_{i\,=\,[1,N_{inner}]}\right)\right)_j\right)$$

Then the algorithm will follow this path:

- For each $(j)_{j\,=\,[1,N_{outer}]}$, a path $\left(Q_{99\%}\left((TRS^i(k\,+\,MPOR) - TRS^i(k)\,|\mathcal{F}_k)_{i\,=\,[1,N_{inner}]}\right)\right)_j$ will be generated and used for the outer Monte-Carlo sum.
- The previous path is using a delta return vector $(TRS^i(k\,+\,MPOR) - TRS^i(k)\,|\mathcal{F}_k)_{i\,=\,[1,N_{inner}]}$ which will be generated for each $(i)_{i\,=\,[1,N_{inner}]}$, and then we'll take the quantile.
- $TRS(k)$ itself will follow:

$$\begin{cases} TRS(k)\;=\;(S(k,T)\;-\;K)\cdot DF(k,T)\;-\;s_{TRS}\cdot DCF(t_0,T)\cdot DF(k,T) \\ dS_t\;=\;S_t(rdt\;+\;\sigma dW_t) \end{cases}$$

$$\Rightarrow \begin{cases} TRS(k)\;=\;e^{-(T-k)r}\cdot(S(k,T)\;-\;K\;-\;s_{TRS}\cdot T \\ S(k,T)\;calculated\;with\;Euler\;scheme \end{cases}$$

IV – Model assumptions, limitations, and next research axis

- First, and as discussed, let's highlight the direct approach model assumptions we took:
  - Assuming $(\lambda_B, \lambda_C, r, \sigma)$ as constant variables is a huge assumption and are only useful for model simplicity overview, clearly not for a proper model:
    - Credit ranking of both B and C can lead to modifications of default intensities, and those should be calibrated, if possible, through B and C's issued Bonds or CDS, and default intensities should be themselves be modeled as stochastic processes (CIR).
    - Underlying's volatility should be calibrated with liquid Options for each future dates of dynamic IM if possible, or once again follows stochastic process (Heston).
    - r should be at least modeled through equilibrium short-rate model and even better in HJM framework.
    - Correlations factor should be added in the model then, by carefully choosing which variables need correlations (volatility and underlying is obvious but default intensities is less obvious, especially if companies are in very different market with loose financial connections).
  - Relaxing constant variables assumptions then leads to more complex code, but it doesn't change the whole logic of the model.
  - TRS itself can highlight several other numerous flavors which complexifies the dynamic IM without really changing the game:
    - Several payments period.
    - Underlying: Basket, Bonds etc.…
    - Floating financing spread
    - Etc.
  - The assumption of risk-neutrality for risk factors embedded in dynamic IM is quite impacting, as even if MVA is indeed a pricing object, it's based on initial margin quantity calculation methods, and this latter is in fact a real cash exchange at inception, based on projection over until MPOR period. This assumption should be discussed in next model, please refer to next bullet points.
- The direct approach we highlighted, based on nested Monte-Carlo loops, is expensive in terms of computation time, especially for complex and large portfolios (here we have only one TRS, and a vanilla one!)
- It's then highly sub-optimal and despite its simplicity, this model shouldn't be used.
- A next paper will introduce a well-known method based on estimators of the dynamic IM using Least Square Monte Carlo and regression by Gaussian processes in the context of quantile estimation, and we'll relax other assumptions too and analyze risk-neutral/real world dynamics impact.