Repo rate curve

I – Introduction

This paper aims to reach two goals, the latter implied by the former:

- Highlight several kinds of market data/transactions which involve repo rates. Thanks to this contribution, we'll have a transaction set which will be used for:
- Building the repo rate curve, in similar way than yield curve building. How to perform it will be highlighted.

II – Repo rate, definitions and disclaimer

Before switching to trades which embeds repo rate, let's start by some definition, which will be useful for next parts and make things clearer:

- **Repo rate**: one of accepted definition is: "the rate at which a Central Bank lends money to commercial banks or financial institutions against government securities". We should take care about this definition because it's quite general, also quite true in case of market with published secured overnight rate as we'll see, but subtleties lie in details :
    - First of all, this definition highlight the repo rate definition based on repo transactions, which is logic and totally true but as highlighted in next sections, this repo rate quantity can "appear" using another transactions, even if behind the scene a repo is always here.
    - This definition assumes that actors are on one side the Central Bank and on the other side financial institutions. Once again, its' totally true, but repo transaction can occur between different actors.
    - And, and **it's the most important topic**, the definition assumes that collateral is government securities (like Treasury Bonds in US market for example, with FED as Central Bank). This assumption needs two quotes, the first one is important but not "decisive", the second one is basically the most important idea of all this paper :
        - Repurchase agreements can embed other kind of collateral than high quality collateral ("special" repos, equity repos etc.…) but indeed most of those transactions use High quality Bond as collateral.
        - But the key point is repo rate is linked to collateral type. The repo rate, and hence repo curve, is then **local**: there's a repo curve for US treasuries, a repo curve for Iboxx, and in fact, a repo curve for each ISIN. But this last case is quite theoretical as for sure liquid repo or derivatives on each ISINs don't exist, then the repo curve is meaningless. That's why we'll tackle a macro-overview.
- **Implied repo rate**: Some derivatives, especially Futures, take repo rate into account for price calculation, such as $D = f(r^{repo})$. Assuming D is known, as market quote, we can reverse the formula in order to capture the repo rate through $r^{repo} = f^{-1}(D)$, following the classical implied quantity process (like implied volatility, implied hazard rate etc.). We'll detail in next sections this formula, but the key point is that through this process we can rely on existing observables in order to build the repo curve.

III – Building the repo curve, but why?

1 – Pricing and discounting

Derivatives pricing and specifically futures cash flows discounting is the main point, and in order to understand it, let’s analyze two very simple and quite abstract trading strategies, assuming CSA context:

<u>Strategy #1</u>

- Party A enters in some derivative with party B.
- The derivative has a value D and party B immediately post a **cash** collateral which value equals derivative’s value at inception.
- This cash collateral leads to interest payable A→B at OIS rate.

<u>Strategy #2</u>

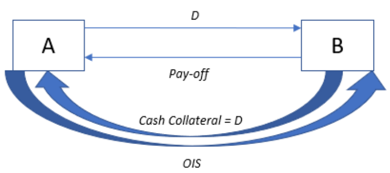
- Party A enters in some derivative with party B.
- The derivative has a value D and party B immediately post a **security** collateral (for example a Bond), which value equals derivative’s value at inception, and we assume that this security can be re-used in another transaction.
- Party A enters in repurchase agreement transaction with party C, receive cash in exchange and pay repo rate.

For sake of simplicity, we took two assumptions:

- A keeps the cash in book (it’s not really true, as A can refinance it in money-market)
- One unique currency

Relaxing those assumptions are not useful for our purpose, which is showing that proper discounting should be done in some cases with repo rate, and not with OIS.

Those strategies are summarized in the below table:

<table>
<tr><th>Strategy #1</th><th>Strategy #2</th></tr>
<tr><td>


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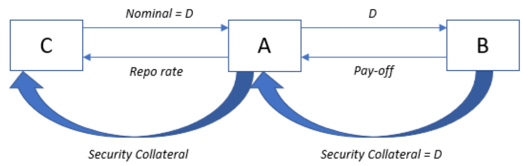

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The strategy #1 involves an auto-funding of the derivative through collateral transfer, itself financed at OIS rate. The effective cost of funding is then linked to OIS, and so OIS curve is the right curve for discounting.

The strategy #2 on the contrary leads to an effective cost of funding linked to repo rate, and then repo rate curve is this time the right curve for discounting.

That’s the key point and in fact the most impacting results : the repo rate curve is mandatory for correct pricing when collateral is done through securities. Discount factors should be calculated with this repo rate curve in order to price those kinds of transactions.

Please note that by allowing nominal lending in money market, the proper discount rate will be repo rate – money market rate, and the latter can be OIS too. But the interesting point is regardless the final resulting discount rate, it embeds repo rate.

By allowing mismatches between derivative's currency and collateral's currency, strategy involves cross-currency repurchase agreements and hence we should take care about repo rate conversion. We won't address this question in this paper, but we'll mention it in last section about research axis.

2 – Riding the repo curve

Some transactions, especially Total Return Futures (TRF) aim, by construction of the product, to ride the repo curve, in very similar manner that IRS are traded in order to ride the yield curve, with sometimes complex strategies involving short/long position on different tenors.

Those kind of trading strategies require the full repo rate term structure, in order to gain on repo curve direction/convexity/inversion.

3 – Rho repo greek

Using similar method than interest rate greek like assumed parallel shift DV01, traders and risk managers aims to observe impact of repo rate bump for their transactions. Rho repo should be observable by tenors, hence once again the need of the repo curve object.

This Greek is useful, at least mandatory, for trading context (pricing/hedging stand-alone and xVA SIMM with rate risk factors) and regulatory context like FRTB.

IV – Repo curve building blocks – Market with published secured overnight rate

We'll first focus on financial market where official *secured (*and then coming from repurchase agreements transactions*)* overnight rate are published, and hopefully with some liquid derivatives on this rate.

The reason is quite obvious : instead of implying repo rate from specific transactions, as we'll highlight in next section, the repo rate is a direct market observable and we just had to rely on available spot publication and chosen derivatives quotes in order to build the repo curve.

Next parts will focus on US market with the SOFR, but please note that the whole process can be used for SARON for example.

Now let's detail:

1 – SOFR published rates

Most obvious data for repo curve building is the secured overnight rate itself, here the SOFR. As its label highlights, it's an overnight rate then we can directly define our first block of the repo curve, with the little trick that this rate has a one-day lag:

$$r_{O/N}^{repo}(t) = SOFR(t-1)$$

2 – SOFR Futures

After the previous and very simple conclusion for the first block, we'll reach complexities.

First, in order to build the repo rate term structure, we'll need specific and liquid derivatives on this SOFR rate, and we rely here on SOFR Futures 1M and 3M, respectively noted as $SR1(t, T_k, T_{k+1})$ and $SR3(t, T_k, T_{k+1})$, with T_k dedicated reset/payment dates (tenors for the former are monthly, quarterly following IMM dates for the later, and we simplified by removing fixing/payment lags)

We have one major hurdle that we need to overcome: by design of the derivative, those futures "locks" SOFR average rate starting at T_k operator will have to pay at T_{k+1}.The period is then equivalent to a forward start repo's one, starting at T_k and matured at T_{k+1}. But please note that we're interested in term structure of repo rate, not forward repo curve.

As, contrary to up front fixing Libor Futures, SOFR futures implies tenor continuous calculation (averaging of forward SOFR rate during $[T_k, T_{k+1}]$), the switch from the resulting locked average rate and the term repo rate for each Future's maturity is not straightforward at all and in fact still under discussion after some ISDA's remarks. That's not the purpose of this paper to highlight this process but the reader may refer to https://www.cmegroup.com/market-data/files/cme-term-sofr-reference-rates-benchmark-methodology.pdf for more details.

Let's assume that we follow this methodology. according to Futures quotation, we can define another block for our repo curve with some function $f_{T_{k+1}}^{SRX}$:

$$r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{SRX}(SRX(t, T_k, T_{k+1})) = f_{T_{k+1}}^{SRX}(list\ of\ SOFR^{fwd}(t, \tau), \forall \tau \in [T_k, T_{k+1}])$$

3 – SOFR swaps

SOFR swaps, which spread is noted as SRSwap(t, T_{k+1}), are by design simpler than previous SOFR Futures, as quoted swap spread will directly represent the term expected average SOFR rate for some swap periods $[T_k, T_{k+1}]$. We don't have to perform the previous retro-engineering and then simply assume that:

$$r_{T_{k+1}}^{repo}(t) = SRSwap(t, T_{k+1})$$

Quite simple but unfortunately those derivatives are still not liquid enough, even cleared but still considered as OTC. It's then by design dangerous to rely on SOFR swap for the repo rate curve which aims to be a macro object, waiting for perhaps higher liquidity and volume in the future.

4 – Summary and disclaimers

We can summarize our building process with the next schema:

$$\begin{cases} SOFR\ repo \\ SOFR\ Futures \\ SOFR\ swaps \end{cases} \Rightarrow \begin{cases} Published\ SOFR\ rates \\ SOFR\ Futures\ quotes \\ SOFR\ swaps\ spreads \end{cases} \Rightarrow \begin{cases} r_{O/N}^{repo}(t) = SOFR(t-1) \\ r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{SRX}(SRX(t, T_k, T_{k+1})) \\ r_{T_{k+1}}^{repo}(t) = SRSwap(t, T_{k+1}) \end{cases} \Rightarrow \textit{Interpolation} \Rightarrow \textit{Repo curve}$$

Some disclaimers and remarks about this construction:

- Once again, the main assumption is it will be used for pricing/discounting if and only if the collateralization scheme of the target derivative involves Treasury Bonds, or at least securities with close risk profile. The discounting should be done using "risk-free" rate, which is indeed SOFR use case, but if the collateralization is done in parallel with very risky collateral, please be sure that there will be a big mismatch between discounting rate and the effective repo rate operator will have to pay with a low-grade security.
- As usual, the interpolation method is key as it can drastically change discount factor values.
- As already highlighted, we should be cautious about using SOFR swaps in this schema.

V – Repo curve building blocks – general

After the previous part which focused on the “easy” case with already published repo rate (through SOFR, SARON etc.), we’ll relax this assumption in this section, assuming that we have to rely on another transaction set. Several kinds of trades, all linked to repo rate, will be highlighted and their usage for repo curve building will be discussed.

1 – Repurchase agreements

What a surprise: the repurchase agreement is the obvious candidate for repo curve construction. Let’s assume a set of transactions built through liquid repurchase agreements, for every maturity and for every security: problem solved.

Unfortunately, this theoretical world doesn’t exist as repos are OTC products and liquidity and then “meaning” of the repo rate strongly depends on volume and targeted securities posted as collateral.

Liquid repurchase agreements are usually with close maturity and collateral is often HQLA Bond, which by design lead us to the repo rate curve block:

$$\begin{cases} r^{repo}_{T_{k+1}}(t) = \ Repo(t, T_{k+1}) \\ very\ short\ end\ of\ the\ repo\ curve \\ mostly\ Bonds\ as\ securities \end{cases}$$

2 – Futures

To highlight link between Futures pricing and repo rate, let’s focus on this very simple trading strategy:

- Trader takes a long position on asset.
- In parallel trader enters in short Future position, with no cost at inception.
- To buy this asset, trader borrows cash.
- He directly lends the security at repo rate through a stock loan transaction.

This strategy’s life cycle is highlighted in next tab, taking simple interest formula (the reasoning doesn’t change with simple compounding/exponential convention):

		t_0		T	
		Cash flows	*Security flows*	*Cash flows*	*Security flows*
π_1	*Long asset*	$-S_{t_0} \cdot 1 = -N$	S_{t_0}	*0*	*0*
	Short Future	*0*	*0*	$F(t_0, T)$	$-S_T$
	Bank account	*N*	*0*	$-r^{ZC}_{t_0 \to T} \cdot N \cdot DCF(t_0, T) - N$	*0*
π_2	*Repo*	*0*	$-S_{t_0}$	$r^{repo}_{t_0 \to T} \cdot N \cdot DCF(t_0, T)$	S_T
$\pi_1 + \pi_2$		***0***	***0***	$\boldsymbol{F(t_0, T) - r^{ZC}_{t_0 \to T} \cdot N \cdot DCF(t_0, T) - N + r^{repo}_{t_0 \to T} \cdot N \cdot DCF(t_0, T)}$	***0***

NFL leads to:

$$F(t_0, T) = r^{ZC}_{t_0 \to T} \cdot N \cdot DCF(t_0, T) + N - r^{repo}_{t_0 \to T} \cdot N \cdot DCF(t_0, T)$$

$$\underset{S_{t_0} = N}{\Longleftrightarrow} F(t_0, T) = S_{t_0} \cdot (1 + (r^{ZC}_{t_0 \to T} - r^{repo}_{t_0 \to T}) \cdot DCF(t_0, T))$$

$$\Rightarrow \boldsymbol{r^{repo}_{t_0 \to T} = r^{ZC}_{t_0 \to T} - \frac{\frac{F(t_0, T)}{S_{t_0}} - S_{t_0}}{DCF(t_0, T)}}$$

That’s the so-called implied repo rate we previously define but we should take care about too quickly using this quantity for our repo term structure as :

- It's a generic formula which can be applied to equity/Equity index Futures (even by including discrete dividend or dividend yield), in some extent for Bond Futures, but not directly for some commodity Futures, which NFL can't be directly applied (storage cost/convenience yield well-known "issues"). We should then be very cautious about underlying type.
- As usual, liquidity is the key point : this implied repo rate is totally meaningless if the Future is not liquid. But contrary to forward contract, we can rely on rightful assumption that those cleared Futures contracts are liquid enough to imply a meaningful implied repo rate.

Then we can define our second block with $f^{FUT}_{T_{k+1}}$ as the implied function:

$$\begin{cases} r^{repo}_{T_{k+1}}(t) = f^{FUT}_{T_{k+1}}(F(t, T_{k+1})) \\ short\ end\ and\ mid - curve \\ mostly\ all\ securities \end{cases}$$

3 – Total return swaps

TRS and repo rate are strongly linked as we can prove that TRS spread is equal to repo rate, let's do it:

- We'll assume that both TRS and repo are daily margined with for example €STR rate.
- We'll also assume that repurchase agreement transaction embeds opportunity cost : the possibility to replicate the repo position on cash+Future market is embedded in repo calculation

Those two assumptions lead to next formulas, assuming no income (for, calculus simplification):

<u>TRS</u>

$$s_{TRS,T}(t) = \frac{(F(t,T) - S_t) \cdot ZC(t,T) - \sum_{i=t}^{T-1} F(t,i) \cdot €STR(i) \cdot DCF(i,i+1) \cdot ZC(t,i)}{S_t \cdot DCF(t,T) \cdot ZC(t,T)}$$

<u>Repo market value</u>

$$\begin{cases} -S_t + S_t \cdot ZC(t,T) + r^{repo}_T(t) \cdot DCF(t,T) \cdot ZC(t,T) + \sum_{i=t}^{T-1} F(t,i) \cdot €STR(i) \cdot DCF(i,i+1) \cdot ZC(t,i) = Opportunity\ cost \\ Opportunity\ cost = F(t,T) \cdot ZC(t,T) - S_t \end{cases}$$

$$\Rightarrow r^{repo}_T(t) = -\frac{(F(t,T) - S_t) \cdot ZC(t,T) - \sum_{i=t}^{T-1} F(t,i) \cdot €STR(i) \cdot DCF(i,i+1) \cdot ZC(t,i)}{S_t \cdot DCF(t,T) \cdot ZC(t,T)} = -s_{TRS,T}(t)$$

We have then a direct link between TRS spread as repo but unfortunately we can't rely on this formula for the repo curve building for one basic reason : TRS are totally OTC, often "one shot" traded, no liquid at all, and often embeds exotic features which cause high divergences between them and repo market.

But as we'll soon see in next paragraph, there's an exit door.

4 – Standardized total return swaps

A specific flavor of the previous kind of trade with standardized TRS (STRS) and those have interesting features for our goal:

- Those contracts are standardized, without fancy embedded features, and with a fixed schedule following classical IMM dates.
- Those products are quoted by brokers and no more with pure OTC agreement.

- The special part is that contrary to classical TRS, quotes are for prices (strikes, also called settlement price), and not for the financing spread itself, which is standard and follow standardized floating rate + some conventional spread.
- Underlying is often index and especially Bond index like Iboxxes. And As Iboxx indexes embed several Bonds of the same type (Corporates etc.), it's clearly a perfect tool to capture a macro repo rate useful for many underlyings.

Those products are more liquid than classical OTC TRS and are then perfect candidate, the only subtlety is about this specific quotation through prices which should be explained in order to capture the repo rate :

Brokers quote their strike price considering the whole payment which will occur on financing leg, which please remember is theoretically linked to repo rate. Then the more performance receiver will pay through the financing leg, the more the whole repo amount is. It rings a bell as it's in fact exactly the Future price mechanism we previously observed: the STRS strike replicates Future price formula, which for example explain that STRS quotes decrease according to maturities, because expected repo rate increases, through higher financing leg amount.

We can then rely on the following formula too, assuming $r^{ZC}_{t_0\to T}$ is already known:

$$\Rightarrow r^{repo}_{t_0\to T} = r^{ZC}_{t_0\to T} - \frac{\frac{STRS(t_0,T)}{S_{t_0}} - S_{t_0}}{DCF(t_0,T)}$$

Then we can define our third block with $f^{STRS}_{T_{k+1}}$ as the implied function:

$$\begin{cases} r^{repo}_{T_{k+1}}(t) = f^{STRS}_{T_{k+1}}(STRS(t,T_{k+1})) \\ mid - curve \\ mostly\ Iboxx \end{cases}$$

5 – Total return Futures

By design a TRF replicates the performance of an OTC Total Return Swap. Both products TRF and TRS have a very similar payout, both allowing the transfer of the total economic exposure of an equity asset, including market and dividend risk, without actually having to own it.

But TRF, as listed Futures, are cleared and fungible transactions, and such as previous STRS, are good candidates in order to capture meaningful repo rate. Let's detail a bit the product:

<u>Quotation</u>:

$$TRF(t,T_{k+1}) = S_t + AccDistrib(t) - AccFund(t) + S_t \cdot s_{TRF,T_{k+1}}(t) \cdot DCF(t,T_{k+1})$$

With $AccDistrib(t)$ and $AccFund(t)$ two blackbox indices (provided and published) respectively embedding ongoing income distribution and ongoing funding.

<u>Theoretical spread:</u>

The theoretical spread is calculated following (still assuming here no income falls for performance part, for sake of simplicity):

$$s_{TRF,T_{k+1}}(t) = \frac{(F(t,T) - S_t) - \sum_{i=t}^{T-1} F(t,i) \cdot €STR(i) \cdot DCF(i,i+1)}{S_t \cdot DCF(t,T)}$$

We easily recognize, without discount effect, the TRS spread formula, and that's basically where the TRF VS repo rate lies, as we can then assume:

$$r_{T_{k+1}}^{repo}(t) \approx s_{TRF,T_{k+1}}(t)$$

$$\Rightarrow r_{T_{k+1}}^{repo}(t) \approx \frac{TRF(t,T_{k+1}) - S_t - AccDistrib(t) + AccFund(t)}{S_t \cdot DCF(t,T_{k+1})}$$

Please also note that underlying are mostly Equity indices (CAC40 etc.) and single equity, which lead us to the fourth block with $f_{T_{k+1}}^{TRF}$ as the implied function:

$$\begin{cases} r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{TRF}(TRF(t,T_{k+1})) \\ mid - curve\ and\ long\ term \\ mostly\ Equity\ indices\ and\ single\ equity \end{cases}$$

4 – Summary and disclaimers

As noticed, we can't (and we shouldn't) try to merge all kind of transactions in one unique Repo curve model, as underlying type/transactions liquidity automatically lead to several and split macro repo curve, which is our goal, let's detail it:

<u>Bond market</u>

- Iboxx indices of the STRS block should of course be chose in order to be in line with all blocks (a repo curve built in short end with Treasury Bonds and long end with STRS on Iboxx embedding high yield Bonds doesn't make sense)

$$\begin{cases} Bond\ Repos \\ Bond\ Futures \\ STRS \end{cases} \Rightarrow \begin{cases} Available\ repo\ rate \\ Futures\ quotes \\ STRS\ quotes \end{cases} \Rightarrow \begin{cases} r_{T_{k+1}}^{repo}(t) = Repo(t,T_{k+1}) \\ r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{FUT}(F(t,T_{k+1})) \\ r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{STRS}(STRS(t,T_{k+1})) \end{cases} \Rightarrow \textit{Interpolation} \Rightarrow \textit{Repo curve}$$

<u>Equity market</u>

- Equity repos are excluded due to lack of liquidity.
- In first approach preferred kind of underlying is Equity indices in order to capture macro repo rate

$$\begin{cases} Equity\ Futures \\ TRF \end{cases} \Rightarrow \begin{cases} Futures\ quotes \\ TRF\ quotes \end{cases} \Rightarrow \begin{cases} r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{FUT}(F(t,T_{k+1})) \\ r_{T_{k+1}}^{repo}(t) = f_{T_{k+1}}^{TRF}(TRF(t,T_{k+1})) \end{cases} \Rightarrow \textit{Interpolation} \Rightarrow \textit{Repo curve}$$

VI – Final thoughts, open areas and research axis

- Previous constructions took a macro point of view, which automatically means that for a derivatives pricing on specific ISIN, repo rates used for discounting and calculated from this kind of repo curve won't be the "exact" repo rate, never. But we can rightfully argue that those macro repo curve, built through a very large of transactions, themselves based on large scope of underlying (Indices), can lead to a proper proxy for discounting and "reality" of repo rates.
- This paper didn't highlight special repos use case, which highlight repo rate which is usually lower than the current "official" repo rate, and hence lead to what we called the repo spread. This analysis should be done, especially in SOFR market, as the SOFR specifically exclude special repos transaction for the published rate.
- This paper focused on calibration and didn't highlight a model for repo rate dynamics (following for example a kind of LMM model but for repo rate dynamic). It's perhaps interesting to dig a bit in this direction.
- Another interesting research axis and analysis is the prediction of the repo curve based on machine learning processes, which will be discussed in another paper.