TRS on loan - underlying pricing

I – Introduction

Such as classical TRS on Bond, operator may want to synthetically exchange performance of loan/mortgage kind of underlying. Specifications of the TRS itself don't really change with the underlying type, but prices remain the most important point.

Loan introduces very specific risk about prepayment, and this risk should be computed and embedded in price calculation, and this paper aims to highlight this model.

In this paper, we'll made the global assumption that underlying loan can't be sold/buy on the market at agreed price coming from supply/demand (illiquid loan), hence the need for a theoretical price calculation.

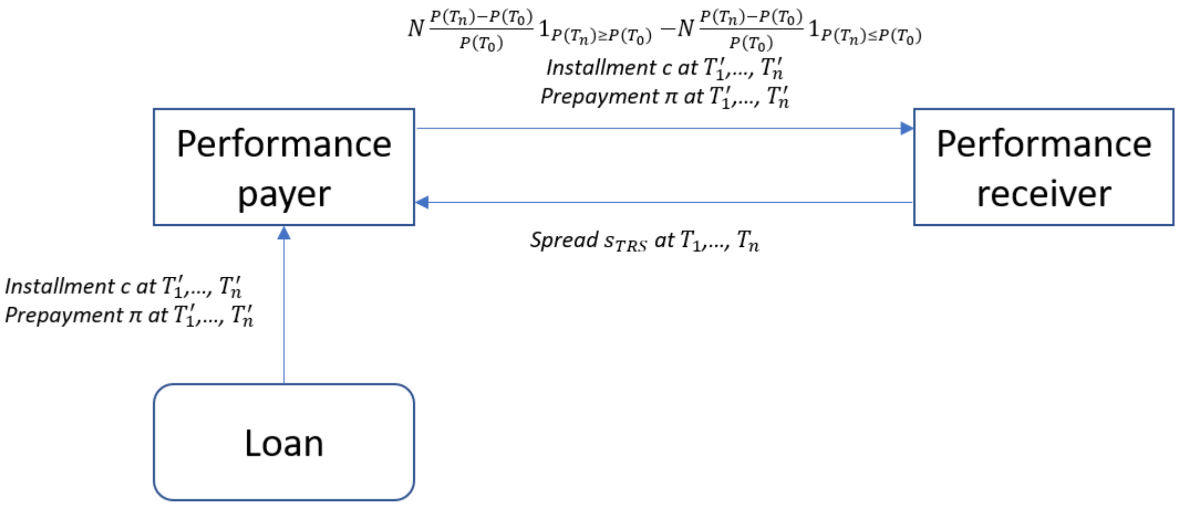
II – TRS on facilities, TRS on loans

First of all, we'll focus on this paper on very specific mechanism about loan generation :

- Initially a facility, also called credit line, is created. This structure doesn't directly lead to loans.
- Then user can drawdown loans from this credit line.
- This possible "basket" of loans is the underlying TRS user intends to transfer performance of.

Please also note that a more complex structure can be build : a bundle of facilities, each with their own set of loans. This kind of structure involves securitization kind of mechanisms and won't be discussed in this paper, but we'll mention it in research areas section.

In this paper, we'll assume that TRS exchange the performance of one unique loan, following the next schema:



Among several others, key metric for this kind of trade is of course the price set $P(T_0),\ldots,P(T_N)$ and how to calculate those, including theoretical spot price and forward prices.

Contrary to classical Bond, we can't rely on simple cash and carry mechanism because :

- In any case we need to calculate the theoretical spot price (it's not a market data), as we assume loan is not liquid.
- Prepayment specific mechanism introduces additional complexities that a simple cash and carry formula can't capture.

II – The loan spot price

1 – Framework and general ideas

In this section we'll especially focus on prepayment feature. The interest rate incentive is one (among ***several*** others) of the reasons to prepay.

If we think about interest-rate risk context, the only kind of loan which contains prepayment risk is fixed-rate loan, because floating-rate loan's coupon is directly adjusted to available market rates. The fixed-rate loan, on the contrary, remains "frozen" nevertheless possible prepayments (and hence lower nominal balances and then lower installments flows), which is a risk for bank's liquidity and profit. We'll denote r_{loan} this fixed loan rate.

We'll follow an already well-documented approach in order to calculate the loan price based on replication through another trade which involves same kind of risk, the amortized interest rate swap. Due to high liquidity of those instruments, we can, in first approach and with some approximations, assume that the loan's price is equal to available amortized swap rate $\mathrm{S}_{\mathrm{t,T}}^{\mathrm{amo}}$, using of course an IRS with same maturity and same payment frequency than loan.

Let's introduce some notations before continuing further:

$$\begin{cases} \text{amortized swap market value}: \; AS(t_0) \; = \; \mathbb{E}^{\mathbb{Q}}\left(\sum_{k=1}^{M} \tau_k \frac{N(T_{k-1})}{B(T_k)}(r_{loan} - l(T_{k-1}, T_{k-1}, T_k)\right) \\ \text{the amortized nominal}: \; N(T_k) = N(T_{k-1}) \cdot \omega(\pi(T_k)), \text{for some function } \omega \\ \text{the prepayment rate}: \; \pi(T_k) \end{cases}$$

We will enter details later about function ω and π, but the key point here is understanding that the amortized swap market value formula can't be used like that once the nominal itself is stochastically linked to interest rate:

- Loan prepayment schedule can differ from agreed AS payment schedule, and if fact it will be usually the case. Then the AS is no more a perfect hedge, and we can't rely anymore on its value for the loan pricing methodology.
- Technically speaking, a simple formula can't be reached through the measure change $\mathbb{E}^{\mathbb{Q}} \rightarrow \mathbb{E}^{\mathbb{T}_k}$ (here libor-like but we can extend the reasoning to RFR, only measure change will differ), because we'll reach functional form $\neq \alpha \cdot \; l(T_{k-1}, T_{k-1}, T_k)$ (for example with squared libor), and martingale reduction is not doable anymore.

It will lead us to another specific kind of proxy trade for our computation, the index amortizing swap (IAS).

2 – IAS and loan valuation

As highlighted in the previous section, in order to evaluate our loan, the simplest way is finding a proxy trade which highlights same risk profile. Let's zoom on the index amortized swap product:

- Just like classical amortized swap, the index amortized swap embeds a “melting” nominal, and this melting process is done through a deterministic function BUT on stochastic variable, here interest rate.
- The IAS, thanks to this interest rate dependency, is then acting as a kind of OTC option on that rate.
- And then the loan on one side and the IAS share the same embedded option on interest rate (the former through real prepayment cashflows, the latter through its nominal calculation, as we’ll see), and price one is equivalent (with approximations, the IAS is still a proxy) to price the other.

Now we’ll set the financial target, we’ll see in next section the model itself.

3 – IAS pricing

The IAS market value is:

$$IAS(t_0) \;=\; \mathbb{E}^{\mathbb{Q}}\left(\sum_{k=1}^{M} \tau_k \frac{N(T_{k-1}, \pi(T_{k\text{-}1}))}{B(T_k)} (r_{loan} - l(T_{k-1}, T_{k-1}, T_k)\right)$$

with the **mandatory** constraint that IAS notional at time T_k is equal to loan's one at $T_{k\text{-}1}$(unless all this proxy construction falls apart)

We need to choose a functional form for the prepayment rate, and we chose in this paper to model it as only linked to refinancing incentive *RI*. This latter *RI* has also a functional form and in order to allow some realistic (human-based decisions) prepayments, the refinancing incentive will be based on sigmoid function (different from a “pure rational” approach based on step function).

The full pricing model is then:

$$IAS(t_0) \;=\; \mathbb{E}^{\mathbb{Q}}\left(\sum_{k=1}^{M} \tau_k \frac{N(T_{k-1}, \pi(T_{k-1}))}{B(T_k)} (r_{loan} - l(T_{k-1}, T_{k-1}, T_k)\right)$$

$$N(T_k) = N(T_{k-1}) \cdot \omega(\pi(T_k)), \text{with here } \omega \text{ as annuity formula (no bullet)}$$

$$\pi = \pi(RI(T_k))$$

$$RI(T_k) = \alpha_1 + \alpha_2 \frac{1}{1 + e^{\alpha_3 \cdot \epsilon(T_k) + \alpha_4}}$$

$$\epsilon(T_k) = r_{loan} - S_{T_k}, \text{with swap rate } S_{T_k}$$

III – Implementation

The previous model will be implemented through following steps and for each functional block:

1 – Nominal part $\boldsymbol{N(T_{k-1}, \pi(T_{k-1}))}$

- The swap rate S_{T_k} dynamic will follow a Hull and White stochastic process.
- All intermediate steps are deterministic and will be embedded in dedicated functions.
- At the end the output is **one** path of $N(T_{k-1}, \pi(T_{k-1}))$

2 – Libor part $\boldsymbol{l(T_{k-1}, T_{k-1}, T_k)}$

- The libor dynamic will follow LMM.
- At the end the output is **one** path of $l(T_{k-1}, T_{k-1}, T_k)$

3 – Money-market account $\boldsymbol{B(T_k)}$

- The money-market account dynamic will use short rate coming from HJM Hull and white model.
- At the end the output is **one** path of $B(T_k)$

4 – Iteration to $\boldsymbol{IAS(t_0)}$

- Thanks to previous step we'll reach one realization of the inner sum.
- A number N of this simulation will be then realized in order to capture $IAS(t_0)$

IV – Model discussion and open areas

- The model should be extended to risky loans, and hence the proxy trade needs to be modified.
- The model should be extended to several loans coming from the **same** credit line. Possible links between trades should be discussed (for example prepayment will occur in FIFO mode for "best" loan with higher agreed r_{loan} etc…)
- The model should be extended to several loans coming from **different** credit lines. In that case possible correlation between those credit lines (assumed to be with different credit ranking for example) should be discussed.
- The model should be extended to RFR-based instead of libor-like IAS. In first approach, only libor dynamic need to be changed to RFR dynamic through FMM.