Standardized TRS converter

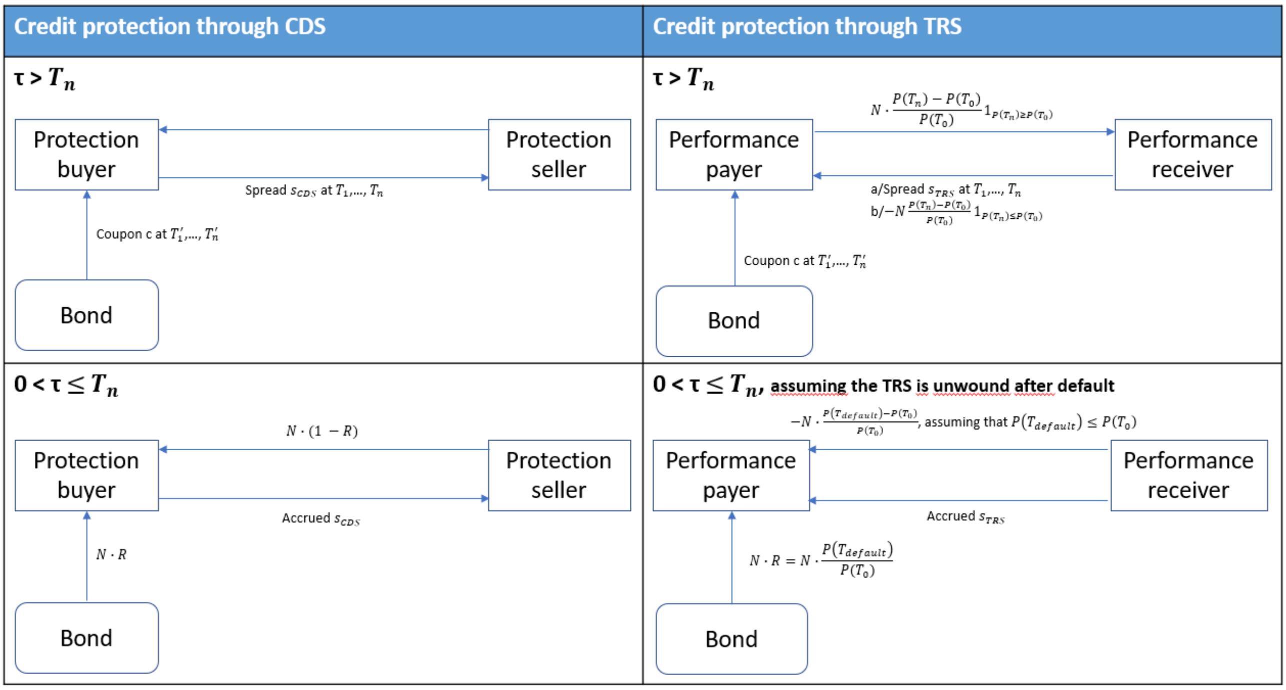
I – Introduction

This paper aims to highlight a possible new model for standardized TRS (STRS) new kind of conventional spread → up-front amount calculation. To reach this goal, the analysis will go through existing standardized CDS ISDA conversion mechanism and try to adapt it in STRS context.

II – CDS VS TRS

Before entering in conversion mechanisms, this chapter will compare CDS and TRS products, and highlight similarities and differences for those two-credit derivatives product, which will be fruitful in next sections.

Let's start by a summary tab about two possible strategies if one trader aims to hedge his credit risk coming from a risky Bond, with possible default on date τ:



We can observe several differences between those two strategies that should be highlighted:

- TRS is a linear product which implies that performance flow can be paid to one or another counterparty according to price levels. It's a key point in this default context as it explains the final $-N \cdot \frac{P(T_{default}) - P(T_0)}{P(T_0)}$ flow from performance receiver to performance payer, assuming that price after default will obviously less than price before default. This flow is the "equivalent" of CDS protection flow $N \cdot (1 - R)$ after default.
- As TRS performance payer must, contractually, pay performance if price will rise, and hence carries market risk other than only credit risk (this kind of obligation doesn't exist for CDS, which only focuses on credit risk), it leads to financial reward from performance receiver. That explains that contrary to CDS spread flows which fall from protection buyer to protection seller, TRS spread flow come from performance receiver to performance payer.

- As CDS only focuses on possible default, if this last one won't occur, the protection buyer is fully-entitled to Bond's performance. On the contrary on TRS context the full performance is directly shifted to performance receiver: that's a perfect hedge during life cycle, contrary to CDS which will only hedge credit risk.
- The previous mentioned perfect hedge for TRS after default is quite theoretical and assumes that $R = \frac{P(T_{default})}{P(T_0)}$, which is mathematically correct but market practice can be different : after default the underlying won't be traded anymore and hence $P(T_{default})$ may be not in line (but close) to the previous formula.

The main and most important point, which will be the "fil rouge" of all next sections, is then the nature of risk(s) embedded in both products' spread:

$$\boldsymbol{s_{CDS}\ embeds\ only\ Bond's\ credit\ risk}$$

$$\boldsymbol{s_{TRS}\ is\ holistic\ and\ embeds\ all\ Bond's\ risk\ (market\ +\ credit\ +\ liquidity)}$$

As Bond's price can rise or drop according to market conditions (interest rate for example), credit (obviously after default, but even before) and liquidity (a fly to quality can dramatically impacts specific Bond's price). Those effects are not autonomous and can impact each other, and that's basically which will lead to complexities, as well see in next sections.

III – CDS ISDA conversion reminder

CDS coupons are standardized (100 or 500bps) and in order to highlight a fair trade $(CDS(t_0) = 0)$ taking account this constraint, a cash settlement amount need to be exchanged between protection buyer and seller. In this part we'll focus on conventional spread ways.

The ISDA standard CDS is based on following important inputs for our analysis:

- IMM dates for maturities, which accrual due to full first coupon feature
- Standard coupon rate
- Conventional spread s_{CDS}^{conv}
- Recovery rate
- Interest rate curve

The locked interest rate curve is a key point as once again in CDS context we only focus on credit risk, and output credit curve will be built through conventional spread, independently from interest rate curve then. We'll see in TRS context that we should relaxed this assumption.

Following notations will be used:

$$\begin{cases} \lambda(t) \ = \ \dfrac{\mathbb{P}(\tau < t + dt | \tau \geq t)}{dt} \ the\ hazard\ rate \\ S(t,T) \ = \ e^{-\int_t^T \lambda(s) ds}\ the\ survival\ probability \end{cases}$$

The CDS market value is then:

$$\mathrm{CDS}(\mathrm{t_0}) = \mathrm{s_{CDS}} \cdot \sum_{\mathrm{k=1}}^{\mathrm{N}} \mathrm{DCF}(\mathrm{t_{k-1}}, \mathrm{t_k}) \mathrm{ZC}(\mathrm{t_0}, \mathrm{t_k}) \mathrm{S}(\mathrm{t_0}, \mathrm{t_k}) \ - \ (1 - \mathrm{R}) \cdot \int_{\mathrm{t_0}}^{\mathrm{t_N}} \mathrm{ZC}(\mathrm{t_0}, \mathrm{s}) \mathrm{S}(\mathrm{t_0}, \mathrm{s}) \lambda(\mathrm{s}) \mathrm{ds}$$

$$\xrightarrow[discrete]{} CDS(t_0) = s_{CDS} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\,(t_0, t_k) \cdot S(t_0, t_k) - (1-R) \cdot \sum_{k=1}^{M \times t_N} ZC\,(t_0, t_k)\big(S(t_0, t_{k-1}) - S(t_0, t_k)\big)$$

with M = number of discrete points per year on which we assume a credit event can happen. t_N is the maturity of the swap. $m = 1, \ldots, M \times t_N$ is the index the discrete point on which a credit event can happen between the valuation date and the maturity of the CDS.

The first step of the up-front amount calculation is solving the following equation, assuming a constant hazard rate unknown and through a proxy and fictitious $\widetilde{CDS}$:

$$\begin{cases} \widetilde{CDS}(t_0, \tilde{\lambda}) \;=\; -Accrued, calculated\ using\ coupon\ = s_{CDS}^{conv} \\ \qquad with\ unknown\ constant\ \tilde{\lambda}\ to\ be\ solved \\ and\ \widetilde{CDS}(t_0, \tilde{\lambda}) \;=\; \boldsymbol{s_{CDS}^{conv}} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\,(t_0, t_k) \cdot \boldsymbol{\tilde{S}(t_0, t_k)} - (1-R) \cdot \sum_{k=1}^{M \times t_N} ZC\,(t_0, t_k)\big(\boldsymbol{\tilde{S}(t_0, t_{k-1})} - \boldsymbol{\tilde{S}(t_0, t_k)}\big) \end{cases}$$

Once $\tilde{\lambda}$ is calculated, the second and final step in order to calculate the upfront amount is:

$$\begin{cases} Up-front\ amount \;=\; \widetilde{CDS}(t_0, \tilde{\lambda}) \\ with\ \widetilde{CDS}(t_0, \tilde{\lambda}) \;=\; s_{CDS} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\,(t_0, t_k) \cdot \boldsymbol{\tilde{S}(t_0, t_k)} - (1-R) \cdot \sum_{k=1}^{M \times t_N} ZC\,(t_0, t_k)\big(\boldsymbol{\tilde{S}(t_0, t_{k-1})} - \boldsymbol{\tilde{S}(t_0, t_k)}\big) \end{cases}$$

IV – TRS possible ISDA conversion model

After the quite previous technical section, let's go back to TRS trade and let's analyse if a similar analysis can be propagated. But before entering in model discussion, the first part will remind TRS pricing formula.

1 – TRS pricing equation

Taking same notations, TRS market value is equal to:

$$\begin{cases} TRS(t_0) = s_{TRS} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\,(t_0, t_k) - \sum_{k=1}^{N} ZC\,(t_0, t_k)\big(ZC^{risky}(t_0, t_k) - ZC^{risky}(t_0, t_{k-1})\big) \\ \qquad with\ ZC^{risky}(t_0, t) = \; \mathbb{E}^{\mathbb{Q}}\left(e^{-\int_{t_0}^{t}(r(s) + \lambda(s) + \gamma(s))ds}\right) \end{cases}$$

Previous formula took following assumptions:

- The Financing leg's nominal is assumed to be constant and scaled to 1. Relaxing this assumption by considering nominal based on Bond's reset price won't change the reasoning, it's for sake of calculation simplicity.
- The financing leg cash flows are assumed to be riskless. Once again relaxing this assumption by adding the $S(t_0, \mathrm{t_k})$ is doable and doesn't change the whole reasoning.
- The underlying is assumed the be a risky zero-coupon Bond in order to remove coupon effects in calculation.
- Forward and discounting curve are assumed to be the same (no multi-curve framework)

The Bond underlying pricing formula can be also detailed, but in order to make it simple, it's assumed to be linked to following risk factors (possible all stochastic, following their own risk-neutral dynamics, for example using actuarial LMN model):

$$\begin{cases} the\ risk - free\ interest\ rate\ r(t) \\ the\ hazard\ rate\ \lambda(t) \\ the\ liquidity\ factor\ \gamma(t) \end{cases}$$

We won't detail here those dynamics, as it's not the purpose of this document, but key point to understand is that contrary to CDS framework which highlights $CDS(t_0) = \mathrm{f}(\lambda)$ (as please remember that the CDS ISDA conversion model assumes interest rate curve is an input), the TRS model leads to $TRS(t_0) = f(r, \lambda, \gamma)$ in first approach, and of course it will have an impact of the financial meaning of the possible TRS conventional spread.

In all next sections, we'll assume (which is not the current market practice) that in addition of existing standardized TRS functionalities (IMM dates, settlement prices quote per maturities etc…) that it will be possible to define, such as standardized CDS, an up-front payment through a new kind of conventional TRS spread s_{TRS}^{conv}

2 – Model #1

This first model assumes that:

$$\begin{cases} interest\ rate\ curve\ is\ still\ an\ input \\ \gamma(t) = 0 \end{cases}$$

Or, which leads to same conclusion:

$$\begin{cases} interest\ rate\ curve\ is\ still\ an\ input \\ liquidity\ curve\ is\ build\ through\ market\ datas\ and\ considered\ as\ an\ input\ too \end{cases}$$

In that case, exactly like standardized CDS model, the only remaining factor is the hazard rate, which embeds credit risk, and the previous CDS up-front model can be directly implemented for TRS too.

But of course, it has a major drawback: acting like that leads to implicit assumption that s_{TRS}^{conv} is a quantity which only embeds credit risk (a s_{CDS}^{conv} kind of "doppelganger"), which is, as we saw in section II, not the case at all.

Despite its "simplicity", this model is then wrong and should not be used.

3 – Model #2

Another way is splitting TRS market value per risk factors, following the algorithm:

- $s_{TRS}^{conv} = s_{TRS}^{conv,r} + s_{TRS}^{conv,\lambda} + s_{TRS}^{conv,\gamma}$
- $Accrued = DCF(last\ IMM, t_0) \cdot s_{TRS}^{conv} = DCF(last\ IMM, t_0) \cdot (s_{TRS}^{conv,r} + s_{TRS}^{conv,\lambda} + s_{TRS}^{conv,\gamma}) = Accrued^r + Accrued^\lambda + Accrued^\gamma$
- $for\ each\ x = (r, \lambda, \gamma),\ solving:$

$$\begin{cases} \widetilde{TRS}(t_0, \tilde{x}) = -Accrued^x \\ with\ unknown\ constant\ \tilde{x}\ to\ be\ solved\ and\ other\ risk\ factors\ (curves)\ considered\ as\ inputs \\ and\ \widetilde{TRS}(t_0, \tilde{x}) = \boldsymbol{s_{TRS}^{conv,r}} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\ (t_0, t_k) - \sum_{k=1}^{N} ZC\ (t_0, t_k) \left(\boldsymbol{\widetilde{ZC^{risky,x}}}(t_0, t_k) - \boldsymbol{\widetilde{ZC^{risky,x}}}(t_0, t_{k-1})\right) \end{cases}$$

And again once $\tilde{x}$ is calculated, the second and final step in order to calculate the upfront amount is:

$$\begin{cases} Upfront\ amount^x = \widetilde{TRS}(t_0, \tilde{x}) \\ with\ \widetilde{TRS}(t_0, \tilde{x}) = s_{TRS} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\ (t_0, t_k) - \sum_{k=1}^{N} ZC\ (t_0, t_k) \left(\boldsymbol{\widetilde{ZC^{risky,x}}}(t_0, t_k) - \boldsymbol{\widetilde{ZC^{risky,x}}}(t_0, t_{k-1})\right) \end{cases}$$

And actually:

$$Upfront\ amount = Upfront\ amount^{r} + Upfront\ amount^{\lambda} + Upfront\ amount^{\gamma}$$

The major drawback of this second model is obviously linked to the first step:

$$s_{TRS}^{conv} = s_{TRS}^{conv,r} + s_{TRS}^{conv,\lambda} + s_{TRS}^{conv,\gamma}$$

As it's totally abstract and doesn't follow the market practice: which trader on earth will / is able to split a holistic metric like TRS spread in several components? Based on which weights/which heuristics?

This model indeed embeds this time all risk factors, but unfortunately assumes an unrealistic/artificial mathematical artefact, which can't be in line with market practice.

V – Open areas and research axes

1. Next models investigation should consider the main constraint, which is a unique input s_{TRS}^{conv}. Perhaps it won't be possible, and in that case the converter target for TRS is probably doomed.
2. If another model can be found, it should also be linked to the other standardization feature we didn't discuss in this paper: price quotes per maturities. We should avoid incoherences.
3. A link with the previous point should be investigated: perhaps the converter should lie on **price** specific input and not on **rate**.