Haircut models

I – Introduction

This paper aims to highlight several possible models about haircut calculations. This quantity remains a user-defined/tailored one and it’s unlikely trader will choose a theoretical value, whatever the “correctness” of the model. But this theoretical quantity is a financial metric which may act as financial indicator for traders.

II – Haircut definition

Let’s follow ICMA definition : “A *haircut* is the difference between the initial market value of an asset and the purchase price paid for that asset at the start of a repo.” It’s then basically “something” which is subtracted to real price for a financial reason, let’s investigate this.

The best way, in our opinion, to understand the haircut, is analyzing an extreme use case (unrealistic) but which will make things clearer:

- Let’s assume a repo trade which will be daily margined.
- Let’s also assume a theoretical world where, if the counterparty defaults, you can instantaneously sell the collateral, without margin period of risk

Due to perfect daily margining and the possibility to immediately sell the collateral after default, this trade doesn’t highlight a risk related to the value of the collateral VS lent cash, and hence we can highlight two points which will be fruitful for next sections:

- Haircut and initial margin are strongly related metrics.
- Haircut’s calculation, such as initial margin, is linked, to collateral price.

In next section we’ll propose several possible models based on different paradigms.

III – Historical data-driven models

1 – Min/max model

The most straightforward/obvious model is the once which considers the haircut as the relative difference of collateral prices, according to a certain period:

$$\begin{cases} H(t_0) \ = \ \dfrac{\underset{\tau\in[t_0\,-\,\delta,t_0]}{max} Coll(\tau) \ - \ \underset{\tau\in[t_0\,-\,\delta,t_0]}{min} Coll(\tau)}{\underset{\tau\in[t_0\,-\,\delta,t_0]}{min} Coll(\tau)} \\ Coll(\tau) \text{ as collateral price} \\ \delta \text{ as defined past period} \end{cases}$$

This methodology takes the assumption that the amplitude of the possible loss due to collateral price movement will follow the worst observed amplitude in the past.

A **special** attention should be done on income (dividend/coupon) effect, as it can lead to spurious result: indeed, once the ex-div date is reached, price automatically drops. **BUT** as, if the repo counterparty may default, the collateral is now fully ours (we don’t have to repay the income to counterparty), **in fact** there's no more income effect tin that case. We should then take care of $\underset{\tau\in[t\,-\,\delta,t]}{min}\, Coll(\tau)$ value in those case and perhaps think about a way to neutralize income effect.

Unfortunately, as for every data-driven method, this representation is not so obvious due to δ factor: how can we choose it? The equivalent question is until which past date can we consider we captured the "correct" amplitude? And "reverse" issue can be raised if δ is too large: the haircut metric may be captured in that case crisis effect which occurred years/decade in the past, and assuming it's in line with current state of the market is quite optimistic.

Here heuristics and possible rule of thumbs may help, using the following way:

- Capture a set of traded repos with their haircut values and set a proxy δ (large enough)
- Calculate theoretical $H(t_0)$ for each trade for each $\delta_i \in [t_0\, -\, \delta, t_0]$
- Catching, if possible, the best date (unrealistic) or at least approximations of range $[t_0\, -\, \delta_{max}, t_0\, -\, \delta_{min}]$ which the closest mean of the target amplitude.
- That's a statistical analysis and it should provide best mean and standard deviation (at least), useful for trader (a "best" mean with a standard deviation of 50% will may indicate that perhaps this metric should not be used…)

2 – Historical VaR

The repurchase agreement can be theoretically split between a portfolio with the loan π_{loan} and another portfolio with collateral π_{coll}

An alternative approach can use the historical VaR metric on π_{coll}, which will capture the maximum collateral loss (based on its return) which should not be exceeded during a specific time period with a given probability level.

This loss metric, relative to the current collateral price, may then be used as proxy for the haircut value.

Once again, such as the previous model, the issue will be about the VaR time period, and similar statistical approach need to be done in order to be close to the best δ.

IV – Forward models

Contrary to previous historical model, we won't reach here the issue of range δ's choice, as we'll took the assumption for all model that as, in case of counterparty's default, the collateral may be sold at the end of margin period of risk, then $\delta = \text{MPOR}$. But we'll encounter other issues/constraints, as we'll highlight.

1 – Min/max model

We'll adapt the previous model with T as repo maturity:

$$H(t_0) \;=\; \underset{t\in[t_0\,,T]}{max}\left(\frac{\underset{\tau\in[t\,,t+MPOR]}{max}\, Coll(\tau)\; -\; \underset{\tau\in[t\,,t+MPOR]}{min}\, Coll(\tau)}{\underset{\tau\in[t,t+MPOR]}{min}\, Coll(\tau)}\right)$$

The outer max is chosen to capture the worst-case scenario once again, with maximum amplitude.

The straightforward approach is the following one:

- 1/Define a specific dynamic for the underlying.
- 2/For a fixed $t \in [t_0, T]$, simulate n paths for period $[t, t + MPOR]$
- 3/Capture the ratio and store it.
- 4/Doing it again for next t
- 5/Capture the outer maximum.

Subtleties can occur about the first step:

- We can model the stochastic dynamic with both real and risk-neutral drift and as the target remains risk/collateral management and not pricing, the real world seems to be the accurate one (and in line with previous historical data-driven model)
- If the collateral is an Equity (which is usually not the case), we can assume lognormality.
- But if the collateral is a Bond, we should be extremely cautious about its dynamic, which can be ***in first approach*** also seen as lognormal if Bond's maturity is far away, but it should follow other kind of process (possibly with par reversion) if the Bond's maturity is close, due to pull to par effect, for example:

	Equity collateral	Bond collateral
Distant Bond's maturity	$\delta S = \mu S \delta t + \sigma S \varphi \sqrt{\delta t}$	$\delta P = \mu P \delta t + \sigma P \varphi \sqrt{\delta t}$
Close Bond's maturity		$\delta P = \alpha(100 - P)\delta t + \sigma P \varphi \sqrt{\delta t}$

- The "***in first approach"*** note is important for Bond case here are even if we took into account in previous tab the possible pull to par effect, we still assumed that the Bond is just like the equity a real underlying, but the correct approach, following here classical literature, is considering it at derivative on interest rate and in that case the dynamic will be more like:

$$\begin{cases} \delta P = \dfrac{\partial P}{\partial r}\delta r + \dfrac{\partial^2 P}{\partial r^2}(\delta r)^2 + \dfrac{\partial P}{\partial t}\delta t \\ \delta r = u(r,t)\delta t + w(r,t)\varphi\sqrt{\delta t}, \text{with u as } \boldsymbol{real\ word} \text{ drift} \end{cases}$$

$$\Longrightarrow \delta P = \frac{\partial P}{\partial r} w\varphi\sqrt{\delta t} + \delta t\left(\frac{\partial P}{\partial r}u + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}w^2\varphi^2 + \frac{\partial P}{\partial t}\right)$$

$$\Longrightarrow \delta P = -PDur_P w\varphi\sqrt{\delta t} + \delta t\left(P\left(\frac{1}{2}Conv_P w^2\varphi^2 - Dur_P u\right) + \Theta\right)$$

2 – Monte-Carlo VaR

As we previously highlighted in section II, haircut and initial margin metric are strongly linked. The main idea of this paragraph is relying on existing MVA methodology and adapt it to our current haircut problem.

With this view, haircut can be defined as, still with our autonomous collateral portfolio π_{coll}

$$H(t_0) = \int_{t_0}^{T} \mathbb{E}^{\mathbb{P}}(VaR_c^{MPOR}(\pi_{coll}(s))\, ds$$

$$\Rightarrow H(t_0) = \int_{t_0}^{T} \mathbb{E}^{\mathbb{P}}\big(Q_c(\delta\pi_{coll}(s)|\mathcal{F}_s)\big)\, ds, \text{with } Q \text{ as quantile notation}$$

$$\Longrightarrow H(t_0) \ = \int_{t_0}^{T} \mathbb{E}^{\mathbb{P}}\big(Q_c(\pi_{coll}(s + \mathrm{MPOR})\text{-}\pi_{coll}(s)|\mathcal{F}_s)\big)\, ds$$

Some remarks about this formula:

- The expectation is under real world measure then portfolio dynamic (underlying dynamic) should follow previously mentioned real world processes
- It’s linked to the previous remark but as we focus here on risk/collateral management in real world, there’s no discount effect in this formula (not a pricing context)
- Once again we have inner/outer summation, the latter about the Monte-Carlo computation itself, the former about the summation all the trade‘s life cycle

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V – Open areas and research axis

- Different models should be generalized in case of generalized collateral repo (also called basket repo), with several underlying in basket as collateral. Correlation between assets should be considered, especially with the Monte-Carlo VaR model. As correlation are difficult to measure, it will be clearly a pain point of this kind of model.
- VaR had been chosen for many models for its simplicity, other risk measure like expected shortfall need to be analyzed too, and perhaps may lead to more accurate results.
- Highlighted forward model assumes “simple” asset dynamics, especially based on gaussian distributions. Other kind of distribution coming from extreme value theory are very interesting, especially in this risk/collateral management context, for the analysis.