TRS on credit line – undistributed amount pricing

I – Introduction

This paper aims to propose a pricing model in order calculate credit line price, when this product is seen as tradable underlying.

Let's assume a credit line $Crl(t)$ with total amount N^{Crl}, the purpose is pricing this credit line seen as underlying.

If total amount N^{Crl} is drawn, under one unique real loan L or K loans L^i, the pricing of this credit line is quite straightforward, assuming loans' prices are already available or calculated (taking into account all effects including pre-payment evaluations cf. III - TRS on loans - underlying's prepayment features document):

$$\begin{cases} Crl(t) = \sum_{i=1}^{K} L^i(t) \\ \sum_{i=1}^{K} N^{L^i} = N^{Crl} \end{cases}$$

The purpose of this paper is about another possibility which highlight a partial drawdown of the full N^{Crl}. In that case an additional quantity $\alpha(t)$ should be priced, and we can't use loan's prices, as there are by design not issued yet (even if as we'll see, we'll in fact use one fake/proxy loan price, but with specific weighting). The pricing scheme become:

$$\begin{cases} Crl(t) = \alpha(t) + \sum_{i=1}^{\tilde{K}} L^i(t) \\ \sum_{i=1}^{\tilde{K}} N^{L^i} = N^{\tilde{K}} < N^{Crl} \\ \alpha(t)\ ? \end{cases}$$

The main idea in order to find this $\alpha(t)$, which will be developed in next sections, is relying on pre-payment pricing "ideas", but with a reverse point of view.

II – Pricing model

For this first model, we'll assume that the undistributed amount doesn't generate additional fees, it's simply a frozen amount which **can**, by definition, be "switched" to real loan.

We also still assume that loan pricing, even using a theoretical way, is already available.

All the thing is about this "can", it's then a possibility, hence a probability of issuing, let's denote it $\mathbb{P}^{draw}(t)$

If we're 100% sure the remaining amount will be drawn in one block right now, pricing is straightforward:

$$\alpha(t) = \mathbb{P}^{draw}(t) \cdot L^{N^{Crl}-N^{\tilde{K}}}(t) = 1 \cdot L^{N^{Crl}-N^{\tilde{K}}}(t) = L^{N^{Crl}-N^{\tilde{K}}}(t)$$

Of course, the main challenge is evaluation this probability as once we'll know how to calculate this quantity, we'll simply have to use it as weight, let's detail the reasoning then:

- Let's focus on pre-payment mechanism: it's useful for already issued loan in order to model the borrower's incentive to pay some capital portion in advance.
- This incentive, and then the pre-payment schedule, is not something that can be agreed at inception, like instalments schedule.
- It's modelled through a refinancing incentive function, often a sigmoid in order to catch so-called irrational behaviours (occurred pre-payments when market rate increase or reverse etc.). The loan itself is then evaluated through index amortized swap pricing, please once again refer to III - TRS on loans - underlying's prepayment features document for more details)

We're not interested here in the loan pricing itself, as once again we took the assumption this price is an input. But the refinancing incentive is very interesting as it exactly highlights what we're looking for, a probability:

$$\begin{cases} RI(t) = \alpha_1 + \alpha_2 \dfrac{1}{1+e^{\alpha_3 \varepsilon(t) + \alpha_4}} \\ \varepsilon(t) = r_{Crl} - s_t \end{cases}$$

With r_{Crl} the credit line agreed rate for to be issued loan and S_t the available market swap rate or any other market rate considered as main borrowing/lending rate.

And now let's "reverse" the pre-payment mechanism reasoning:

- Let's assume that credit line borrower is keen to draw loan when market rate is increasing, as he can borrow it at r_{Crl} and for example lend it at s_t.
- In that case the incentive for draw downing loan will raise, following the sigmoid curve. It's equivalent to say that the probability for next drawdowns will raise.

As proxy (as of course it's still a proxy of expected borrower behaviour) we can then define, by redefining refinancing incentive $RI(t)$ to draw downing incentive $DI(t)$*:*

$$\begin{cases} \mathbb{P}^{draw}(t) = DI(t) = \alpha_1 + \alpha_2 \dfrac{1}{1+e^{\alpha_3 \varepsilon(t) + \alpha_4}} \\ \varepsilon(t) = r_{Crl} - s_t \\ \alpha(t) = \left(\alpha_1 + \alpha_2 \dfrac{1}{1+e^{\alpha_3 \varepsilon(t) + \alpha_4}} \right) \cdot L^{N^{Crl} - N^{\tilde{K}}}(t) \end{cases}$$

III – Model assumptions and limitations

- If the not issued amount generates specific fees, those should be considered in calculation, but the amount of discounted fees should be weighted by $(1 - \mathbb{P}^{draw}(t))$, as if the remaining nominal is drawn, specific fees will disappear ("switched" to real loan instalments).
- The model is a proxy based on same kind of huge assumptions than classical pre-payment pricing, aka related to market interest rate movements. Using a sigmoid instead of a step function to model "human" behaviour remains a proxy.
- The major pain point is obviously the choice of $(\alpha)_i$ for the draw downing incentive because:
    - Capturing those through a statistical analysis on past draw down may be difficult or at least impossible if we don't have enough similar credit lines products traded in the past. Moreover, if interest rate levels were stable, we unlikely manage to capture something interesting.
    - Capturing those through existing transactions seems to be difficult too, for same liquidity reasons: finding a similar credit line product which may enable us to calibrate those factors is a lost cause.
- The liquidity incentive should also be embedded in the model, the interest rate incentive can't be the unique driver and it's perhaps the less important of both.