Funding cost

I – Introduction

This paper aims to highlight a model for funding cost calculation in inventory management context. This metric is a useful tool for a proper and dedicated handling of cash/securities transaction for a securities finance trader, to compare in being transactions with actual cost of funding for the whole Bank. This paper focus on two main models, a local one and a holistic one.

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II – High-level overview

Components of bank funding costs are:

<u>Risk-free rates</u>

In post-Libor transition world, this quantity is highlighted in RFR published rates (SOFR, ESTR, etc.…) and RFR term rates through derivatives like RFR Futures.

<u>Credit risk premium</u>

This quantity is equivalent to hazard rate that market transaction like CDS on Bank’s issued Bond embed.

<u>Liquidity risk premium</u>

This quantity is the most complex (and discussed) to catch, but as we’ll soon see we can use issued Corporate Bonds as proxy vehicle to calculate this metric.

III – Local Model

In previous section, we basically highlighted that the funding cost embeds three main quantities:

- RFR
- Credit risk premium
- Liquidity risk premium

And the aggregation of those three terms lead to the whole funding cost. To model it, we’ll use in this paper the well-known LMN model, which perfectly suits our target, as we’ll highlight.

This model needs two strong assumptions:

- The bank can issue corporate Bonds and those bonds are liquid enough. Those securities will be our “core” to highlight the cost of funding.
- Liquid credit default swap exists on those issued Bonds.
- RFR data are available or liquid instrument exists which highlights these quantities.

As previously mentioned, the perfect candidate to catch the real funding cost of a specific Bank is issued corporate Bond, which theoretical pricing formula is:

$$P(t_0,T) = \mathbb{E}^{\mathbb{Q}}\left(c\cdot\sum_{t_c}^{T} e^{-\int_{t_0}^{t_c}(r_s+\lambda_s+\gamma_s)ds} + e^{-\int_{t_0}^{T}(r_s+\lambda_s+\gamma_s)ds} + (1-R)\cdot\sum_{t_d=t_0+1}^{T}\lambda_{t_d}e^{-\int_{t_0}^{t_d}(r_s+\lambda_s+\gamma_s)ds}\right)$$

With following notations and quantities definition:

- $cof(t)$ the cost of funding
- r_s the risk-free rate
- λ_s the hazard rate
- γ_s the liquidity factor
- c the coupon, assumed to be fixed
- t_c coupon payment dates
- R the recovery rate, considered as a model input

This Bond security represents the main tool for Bank financing and the most liquid one. The cost of funding is then like this Bond’s whole yield (sum of interest rate + hazard rate + liquidity factor), and equal to coupon value if the Bond is issued at par.

A watchful reader may directly notice that, for every $t \in [t_0, T]$, $cof(t) = r_t + \lambda_t + \gamma_t$, and the main challenge is then capturing those sub-components.

<u>1 – Interest rate factor</u>

The first risk-factor (risk-free interest rate term curve) is assumed to be already known, through RFR spot market and derivatives (published RFR, Futures/swap on RFR). This metric is not linked to the Bank itself (or at least with a ***very*** marginal view through, for example, repurchase agreements using Bank's Bond as collateral and used for SOFR/SARON RFR calculation), and we'll consider in all models that this risk-free rate curve is an input.

<u>2 – Hazard rate factor</u>

We'll use for this section our assumption about CDS issuance and liquidity of those products.

As a reminder

$$\begin{cases} \lambda(t) \; = \dfrac{\mathbb{P}(\tau < t + dt | \tau \geq t)}{dt} \; \textit{the hazard rate} \\ S(t,T) \; = \; e^{-\int_t^T \lambda(s) ds} \; \textit{the survival probability} \end{cases}$$

The CDS market value is then:

$$CDS(t_0) = s_{CDS} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) ZC\,(t_0, t_k) S(t_0, t_k) \; - \; (1 - R) \cdot \int_{t_0}^{t_N} ZC(t_0, s) S(t_0, s) \lambda(s) ds$$

$$\xrightarrow[discrete]{} CDS(t_0) = s_{CDS} \cdot \sum_{k=1}^{N} DCF(t_{k-1}, t_k) \cdot ZC\,(t_0, t_k) \cdot S(t_0, t_k) - (1 - R) \cdot \sum_{k=1}^{M \times t_N} ZC\,(t_0, t_k) \big(S(t_0, t_{k-1}) - S(t_0, t_k)\big)$$

With ZC zero-coupon Bonds (aka discount factor) calculated thanks to the previous step and with M = number of discrete points per year on which we assume a credit event can happen. t_N is the maturity of the swap. $m = 1, \dots, M \; \times t_N$ is the index the discrete point on which a credit event can happen between the valuation date and the maturity of the CDS.

Calibration of the model imply finding a hazard rate (non-cumulative hazard rate) function that matches the market CDS spreads.

The construction of the hazard rate term structure is done by an iterative process called bootstrapping. Let's assume we have quotes for xY for a given issuer. From the 1Y CDS spread s_{CDS}^{1Y}, we will find the hazard rate $\lambda(O, 1Y)$ which equates $CDS(t_0) = 0$. By iterating this process, we obtain the hazard rates: $\lambda(O, 1Y)$, $\lambda(1Y, 2Y)$ etc…., and we can imply the whole term curve.

<u>3 – Liquidity factor</u>

Once both previous steps are done, we can then use the previous $P(t_0, T)$ formula to imply the liquidity factor γ, with similar bootstrapping mechanism than CDS/hazard rate step:

- Discretization of the formula
- Catching the first maturity issued corporate Bond and imply the first liquidity factor.
- Iterate on second maturity issued Corporate Bond
- Etc.…

The whole process is summarized on next tab:

Steps #	Involved product	Calculation	Output (cumulative)
#1: RFR	Published RFR, RFR derivatives	Market data catching	r_t
#2: hazard rate	CDS	bootstrapping	r_t, λ_t
#3: liquidity factor	Corporate Bond	bootstrapping	r_t, λ_t, γ_t
#4: aggregation			$cof(t) = r_t + \lambda_t + \gamma_t$

IV – Holistic Model

The previous model took two strong assumptions about CDS and Corporate Bonds existence, which may don't exist for some Banks. But as the funding cost still exists and need to be monitored, another approach is needed, let's review it.

Let's assume that the Bank didn't issued Bonds and there's no CDS market on those too.

As proxy and holistic aside metrics, we can use following quantities to catch, respectively, hazard rate and liquidity premium:

- CDS index like Itraxx, by carefully choosing it to match Bank's characteristics (credit rating etc…)

- Bond index like Iboxx, once again with a careful picking

Once those kinds of indices are in the loop, we can then use those to catch, for sure, spot components for our funding cost target. A possible painful limitation will be about term quantities, as we'll need specific derivatives on those indices, which may don't exist.

Contrary to the previous model, which was only linked to market transactions, this model then implies what we call expertise about indices choice, and this step can't be easily automated. An exit door may exists thinking about machine learning tool here, which would automatically assign best indices through a comparison/regression according to Bank's characteristics, assuming learning algorithm and been already trained on previous set of Banks.

Annex – LMN model

Several previous ideas come from a classical model which aims to calibrate stochastic processes embedded in corporate Bond calculation, the LMN model. It's not useful per se for our approach as we don't need stochastic metric for our funding cost but a "fixed" (fixed is a big word, it's dynamic according to bootstrapping results, but the result itself depends on market data, which are not stochastic). The interested reader can look at this model:

For this model, we'll set next dynamics for two remaining risk-factors:

$$\begin{cases} d\lambda_t = \beta(\alpha - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t^{\lambda}, \lambda_0 \\ d\gamma_t = \omega dW_t^{\gamma}, \gamma_0 \end{cases}$$

With additional notations:

- α the hazard rate long-term mean
- β the hazard rate speed reversion to long-term mean
- σ the hazard rate volatility
- λ_0 the spot hazard rate
- ω the liquidity factor volatility
- γ_0 the spot hazard rate
- $(W_t^{\lambda}, W_t^{\gamma})$ independent Brownian motions

Several assumptions deserve specific comments:

- Hazard rate is modelled following CIR processes, which indicates mean reversion and non-negativity features
- Liquidity factor follows a normal distribution, which allows negative values
- All those two factors are assumed to be uncorrelated

The calibration process can be then seen, in first approach (calculation will be detailed later in section III), like:

Steps #	Involved product	Calculation	Output (cumulative)
#1: interest rate		Forward market model for RFR dynamic	dr_t
#2: hazard rate	CDS	$(\alpha^*, \beta^*, \sigma^*, {\lambda_0}^*) = \underset{\alpha,\beta,\sigma,\lambda_0}{\operatorname{argmin}} \sum_T (s_{CDS}^{theo}(\alpha, \beta, \sigma, \lambda_0) - s_{CDS}^{mkt})^2$	$dr_t, d\lambda_t$
#3: liquidity factor	Corporate Bond	$(\omega^*, {\gamma_0}^*) = \underset{\omega,\gamma_0}{\operatorname{argmin}} \sum_T (P^{theo}(\omega, \gamma_0) - P^{mkt})^2$	$dr_t, d\lambda_t, d\gamma_t$
#4: aggregation			$dcof(t) = dr_t + d\lambda_t + d\gamma_t$