Homework 5

February 26th
Due March 6th

Your assignment may be handwritten or typeset, but in any case it should be neat and readable. Your name, class number and assignment number should be clearly visible (like on this document for example). Multipage assignment must be stapled.

You are encouraged to work in groups for this assignment. However, the redaction should be done on one's own: do not copy some other student's work, or give your assignment to some other student. To consult textbooks or online resources is fair game; on the other hand, to look up the exact exercise and its solution is not. I will be available at my office hours to discuss the homework.[1]

Reading

The class textbook is *Notes on Elementary Probability,* by Liviu I. Nicolaescu. You will find it on the course website.

Sections 2.1., 2.2.1. We have gone through these sections in class, please read through it (pay little attention to the quantile function, it is of little use).

Section 2.2.2. We will skip this section entirely, and wait until (much) later in the course.

Section 2.2.3. We are making our way through this section. It will be required reading in the next homework.

Exercises from the book

Included in this homework are exercises 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 2.9, 2.10, 2.11.

Exercise 1

This is a variation on Exercise S from the review.

A company sells 1,000 mechanical pieces to a client. Each of these pieces may be defective or not.

[1] Monday and Thursday, from 10:00 to 11:30, or by appointment.

1. For this question and the next one, assume that exactly two pieces are defective. The client will use only 850 pieces, chosen uniformly at random. What is the distribution and parameter(s) of the number of unfit pieces they will use? What is the probability that they will not use any defective piece?

2. As above, assume exactly two pieces are defective. The client has a quality control service, but it is incredibly lazy. They will check only one piece at random, and say that the whole batch is either perfect or dreadful, according to the condition of the piece. What is the distribution and parameter(s) of the number of defects that quality control will detect?

3. For the rest of the exercise, assume that each piece is defective with a probability 0.1%, independently of the others. What is the distribution and parameter(s) of the number of items with a defect? What is the expected number of defective pieces?

4. For the next contract (same model, same quantity, same probability of defect), the seller wants to be sure that no piece is defective. She will personally inspect each and every item, and throw away the defective ones. What is the distribution and parameter(s) of the number of pieces, defective or not, that the company will produce? What is the expected number of defective pieces they will have to produce?

Bonus exercise

1. How many ways are there to choose $k+\ell-1$ elements of $\{1, 2, \cdots n-1\}$?

2. How many ways are there to choose $k+\ell-1$ elements of $\{1, 2, \cdots n-1\}$, knowing that the kth element is precisely m?

3. Deduce that
$$\sum_{m=k}^{n-\ell} \binom{m-1}{k-1}\binom{n-1}{\ell-1} = \binom{n+m-1}{k+\ell-1}.$$

4. Prove that, assuming N and M are independent, distributed according to negative binomial distributions with respective parameters (k, p) and (ℓ, p), the sum $N+M$ has distribution $\mathcal{N}eg\mathcal{B}in(k+\ell, p)$.