



PGD for low frequency dynamics problems involving localized non-linearities



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Main Subprojects

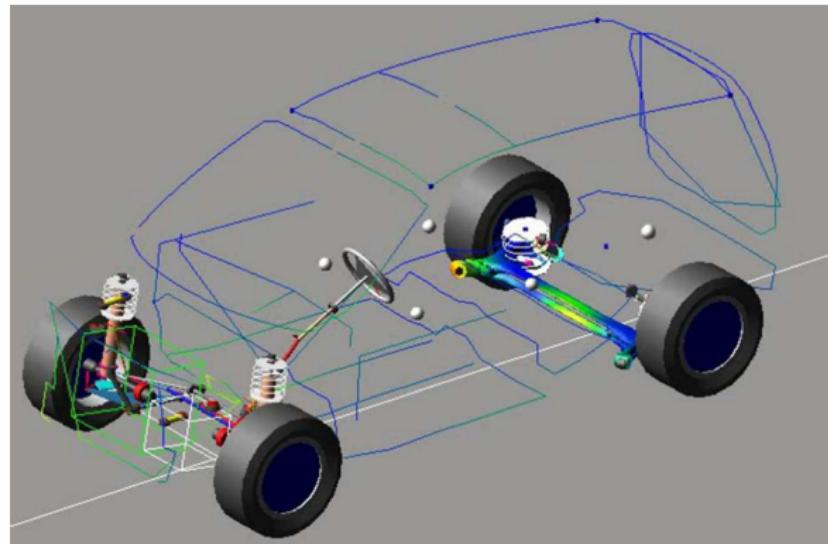
-  Project Leading
-  Transversal Methods
-  Non Linear Vibrations
-  Fast Dynamics
-  Fluid Dynamics
-  Promoting & Broadcasting



Non Linear Vibrations work packages :

- Stakes and Goals
- Use cases
- Non-Intrusive Methods
- Intrusive Methods

Non Linear Vibrations - Use case



- Calculation time
- Set of parameters
- Non-linearity

Presentation plan

- 1 MECASIF Project
- 2 Model Order Reduction
- 3 Different methods
 - POD
 - PGD
- 4 A few results
 - Error
 - MAC
- 5 Prospects

Model order reduction

Reducing calculation time by reducing the dimension of the problem

Stakes of time saving

- Reducing the development cost
- Optimisation / Creation of probabilistic models
- Solving of problems reaching calculating machine's limits

Reduced order modeling : Solving projected problems

How to obtain the basis to project on :

Reduction methods

- POD - Proper Orthogonal Decomposition [A. Chatterjee - 2000]
- PGD - Proper Generalized Decomposition [P. Ladevèze - 1989]

Construction of the Reduced-Order Basis, bibliography



offline/online techniques

- Proper Orthogonal Decomposition (POD)
[Sirovich 87] [Holmes et al. 93] [Krysl et al. 00] [Kunisch,Wolkwein 02]
[Willcox et al. 02] [Picinbono et al. 03] [Bergmann et al. 05] [Lieu et al. 06]
[Gunzburger et al. 06] [Niroomandi et al. 08] [Farhat et al. 08]
[Matthies et al. 10] ... also known as KLD [Karhunen 43] [Loeve 55], PCA
[Pearson 1901], [Hotteling 33]. In finite dimension SVD [Ekardt & Young39], and
if more than 2 variables HOSVD [Baranyi et al.06]
- Reduced-Basis (RB)
[Maday et al. 02] [Patera et al. 02] [Rozza et al. 07] [Haasdonk et al. 08]
[Boyaval et al. 09] ...
- On-the-fly techniques - PGD - review article [Chinesta, Ladevèze, Cueto11]
- Radial approximation (PGD time/space)
[Ladevèze 85, 99] ... [Ladevèze et al. 99-11] [Nouy et Ladevèze 03, 04]
[Ladevèze, Dureisseix, Néron 04] [Ladevèze et al. 08, 09, 10] [Néron and
Dureisseix 08] [Boucard and Néron 11, 12] ...
- Generalized spectral decomposition
[Nouy et al. 07-12]
- Multidimensional separation - PGD time/space/parameter
[Ammar and Chinesta 06] [Ryckelynck 06] [Chinesta et al. 08-11]
[Beringhier et al. 10] ...



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Different methods

POD

- a posteriori or offline/online technique
- give modes in:
 - space
 - time

- Principle :

Projection on a basis of modes extracted from truncated SVD

PGD

- modes calculation on the fly
- give modes in :
 - space
 - time
 - parameter

- Principle :

Modes calculated using an iterative method (fixed point) at the same time as the resolution.

Proper Orthogonal Decomposition

A dynamic problem is solved on the system, and the SVD gives :

$$U(X, t) = \sum_{k=1}^{\dim} V_{Sk} f_k(X) g_k(t)$$

The basis is obtained by truncating the result

The n modes $f_k(X)$ associated to the highest singular values are chosen to make the reduced basis.

The SVD guarantees the best truncated basis such as :

$$U(X, t) - U_n(X, t) = U(X, t) - \sum_{k=1}^n V_{Sk} f_k(X) g_k(t)$$

is minimal for a fixed n (in Frobenius norm).

These couples are the most representative ones of the studied response.

Solving
(Snapshot)

→ Getting the
reduced basis

→ Projection on
the RB

→ Solving

Proper Generalized Decomposition

Approximation using separable variables

$$U_n(X, t, \lambda) = \sum_{k=1}^n f_k(X)g_k(t)h_k(\lambda)$$

The variational formulation gives a problem for each type of function

$$f_k = F(U_{k-1}, g_k, h_k), \quad g_k = G(U_{k-1}, f_k, h_k), \quad h_k = H(U_{k-1}, f_k, g_k)$$

Algorithm

```
for k = 1 à n
    for j = 1 à jmax
        fk = F(Uk-1, gk, hk)
        gk = G(Uk-1, fk, hk)
        hk = H(Uk-1, fk, gk)
    end
end
```

Details

Iterating on the modes
Using a fixed point loop

$$\begin{cases} f_k = F(U_{k-1}, g_k, h_k) \\ g_k = G(U_{k-1}, f_k, h_k) \\ h_k = H(U_{k-1}, f_k, g_k) \end{cases}$$

to solve the non-linear problem.
until you reach n modes.

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Error indicators

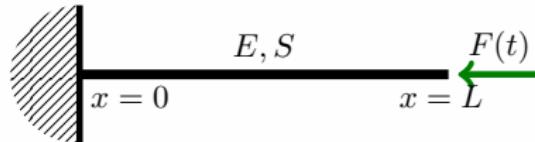


Figure : Academic problem

Relative error (obtained with reference solution) :

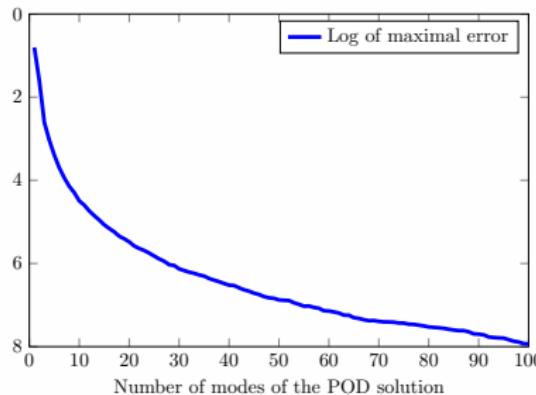


Figure : POD Solution

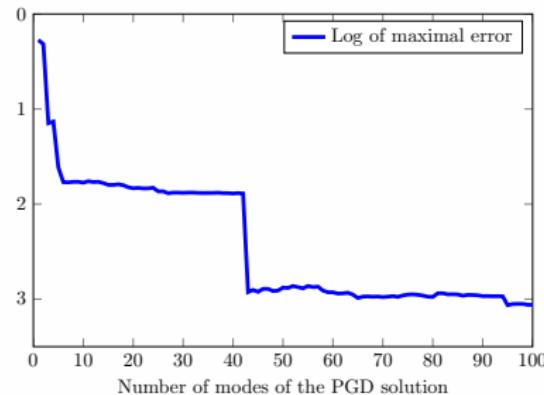


Figure : PGD Solution

MAC Analysis

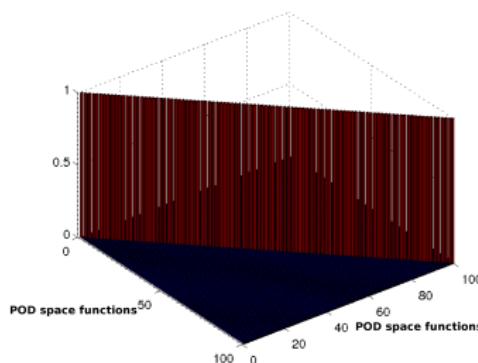
$$\frac{(\varphi_i \cdot \psi_j)^2}{\varphi_i \cdot \varphi_i \times \psi_j \cdot \psi_j}$$


Figure : POD space functions



POD : Orthogonality



PGD : Correlation between mods

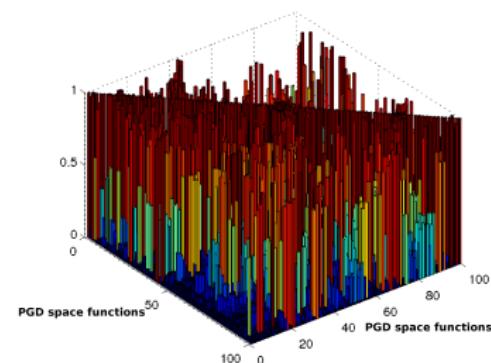


Figure : PGD space functions

MAC Analysis

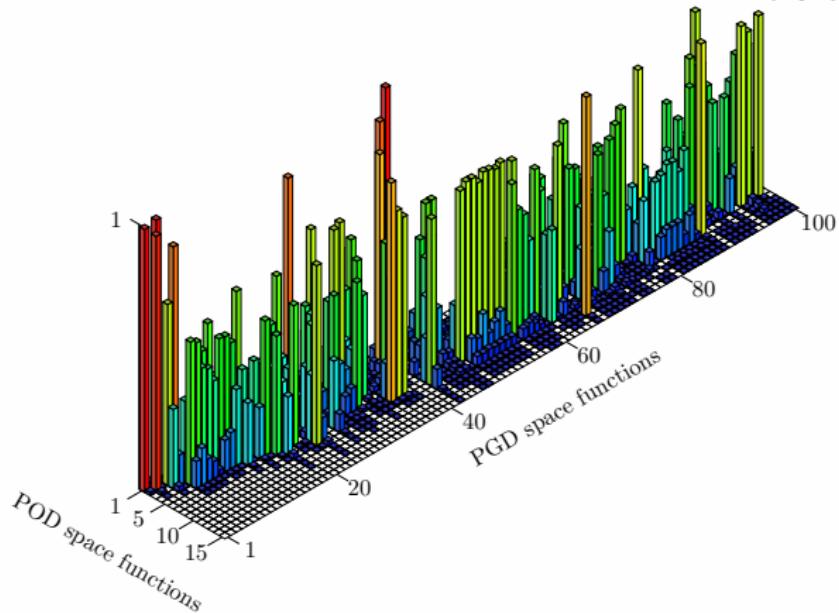
$$\frac{(\varphi_i \cdot \psi_j)^2}{\varphi_i \cdot \varphi_i \times \psi_j \cdot \psi_j}$$


Figure : Comparison between POD and PGD space functions

- PGD mods only representing the first POD mods

Orthogonalization - 2 variables

Algorithm

```
for k = 1 à n
    for j = 1 à jmax
        fk = F(Uk-1, gk)
        gk = G(Uk-1, fk)
    end
    [fk, g1..k-1] = otho(f1..k, gk)
end
```

Details

Iterating on the modes
Using a fixed point loop
$$\begin{cases} f_k = F(U_{k-1}, g_k) \\ g_k = G(U_{k-1}, f_k) \end{cases}$$
 to solve the non-linear problem.
Orthogonalizing
until you reach n modes.

Gram-Schmidt

$$\begin{aligned} U_k &= \sum_{i=1}^{(k-1)} f_i g_i & + f_k g_k \\ &\quad + \sum_{i=1}^{(k-1)} (f_i \cdot f_k) f_i g_k & - \sum_{i=1}^{(k-1)} (f_i \cdot f_k) f_i g_k \\ U_k &= \sum_{i=1}^{(k-1)} f_i (g_i + g_k (f_i \cdot f_k)) & + (f_k - \sum_{i=1}^{(k-1)} (f_i \cdot f_k) f_i) g_k \\ U_k &= \sum_{i=1}^{(k-1)} f_i g_i & + f_k g_k \end{aligned}$$

Orthogonalization - 3 variables

Gram-Schmidt

$$U_k = \begin{aligned} & \sum_{i=1}^{(k-1)} f_i g_i h_i \\ & + \sum_{i=1}^{(k-1)} (f_i \cdot f_k) f_i g_k h_k \end{aligned} \quad \begin{aligned} & + f_k g_k h_k \\ & - \sum_{i=1}^{(k-1)} (f_i \cdot f_k) f_i g_k h_k \end{aligned}$$
$$U_k = \begin{aligned} & \sum_{i=1}^{(k-1)} f_i (g_i h_i + g_k h_k (f_i \cdot f_k)) \\ & + (f_k - \sum_{i=1}^{(k-1)} (f_i \cdot f_k) f_i) g_k h_k \end{aligned}$$

Possibilities



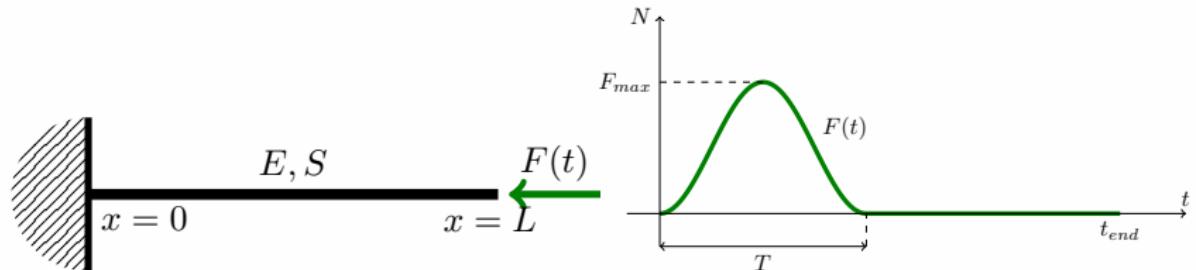
Finding missing functions thanks to a minimization problem



Orthogonalization afterwards or use the HOSVD

Post processing of PGD modes

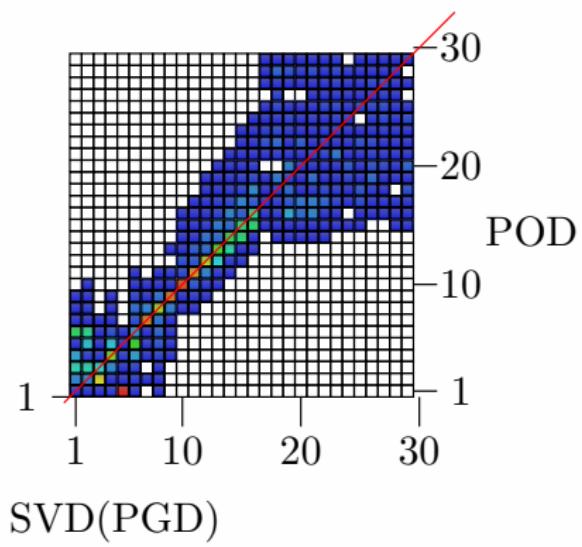
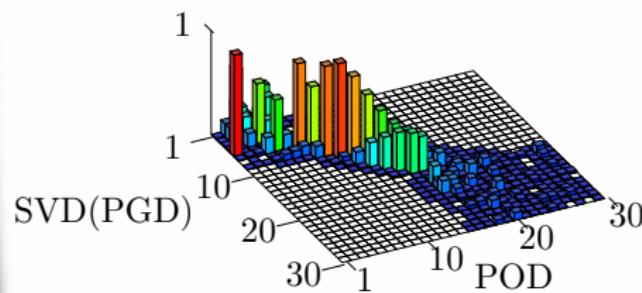
3 methods - MAC Analysis



Post processing of PGD modes

3 methods - MAC Analysis

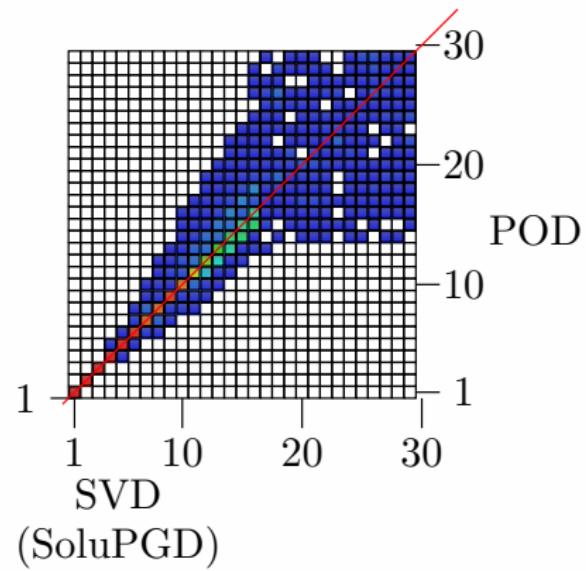
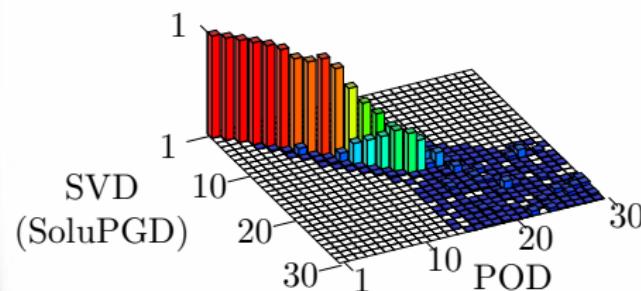
SVD(PGD) - POD



Post processing of PGD modes

3 methods - MAC Analysis

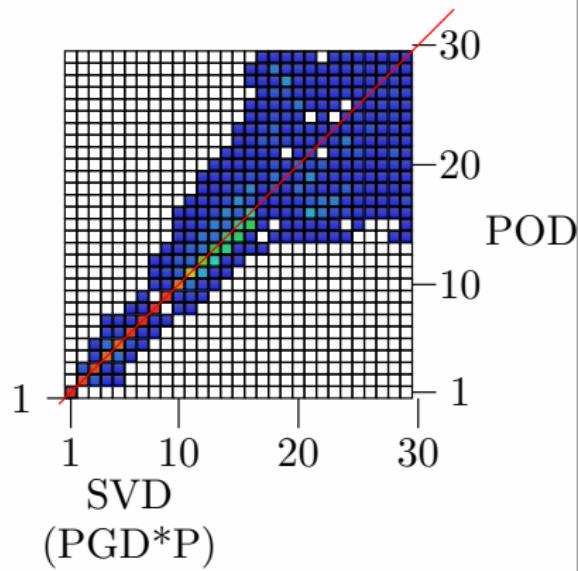
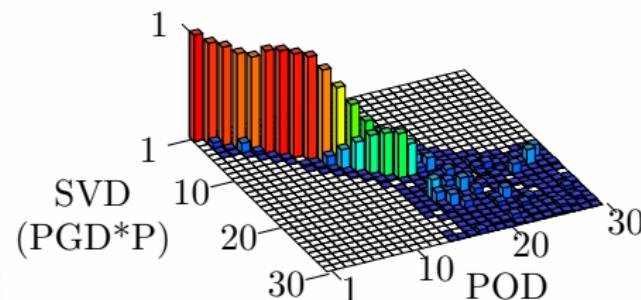
SVD(PGD Solution) - POD



Post processing of PGD modes

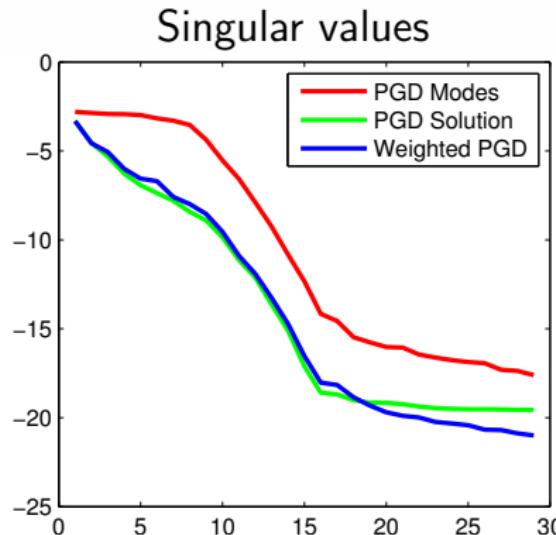
3 methods - MAC Analysis

SVD(weighted PGD) - POD



Post processing of PGD modes

3 methods - MAC Analysis

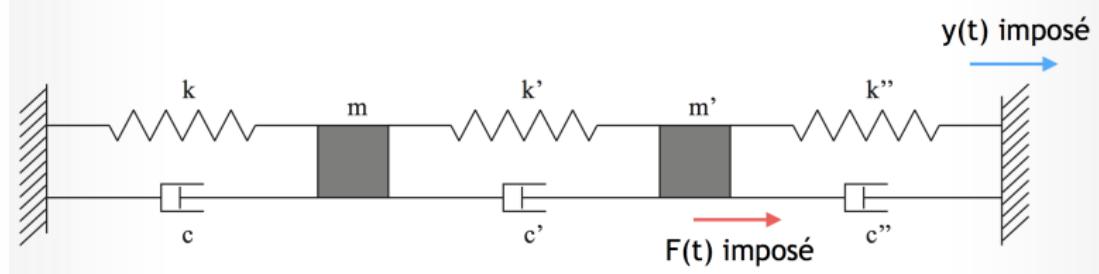


- Lack of meaning from the SVD of the set of PGD modes
- Closes results for SVD of solution and SVD of weighted set
- < 15 meaningful modes

Prospects



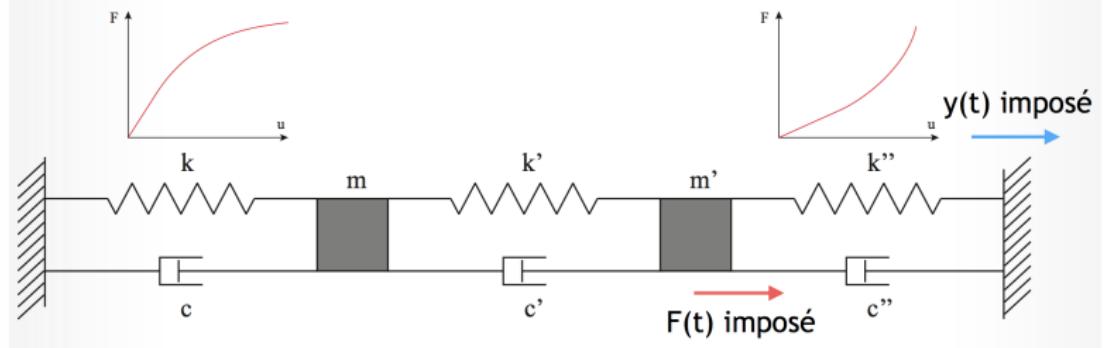
Prepare program for Mecasif project test cases.



Prospects



Prepare program for Mecasif project test cases.

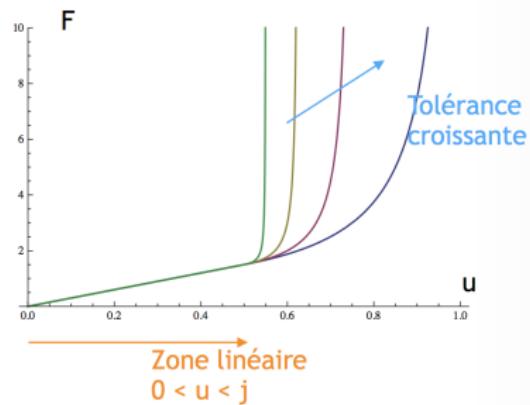


Prospects



Prepare program for Mecasif project test cases.

$$\begin{aligned} F(u) &= ku \quad \text{si } u \leq j \\ &= -\frac{k\Delta j^2}{(u-j)-\Delta j} + k(j-\Delta j) \quad \text{sinon} \end{aligned}$$

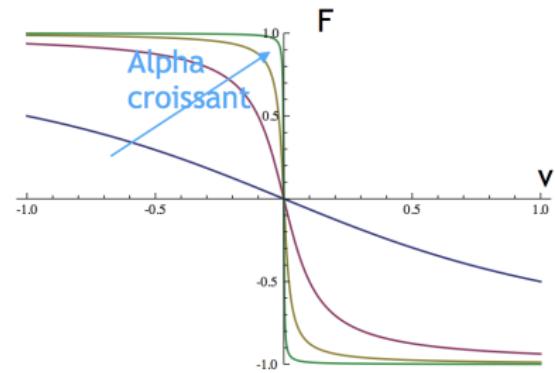


Prospects



Prepare program for Mecasif project test cases.

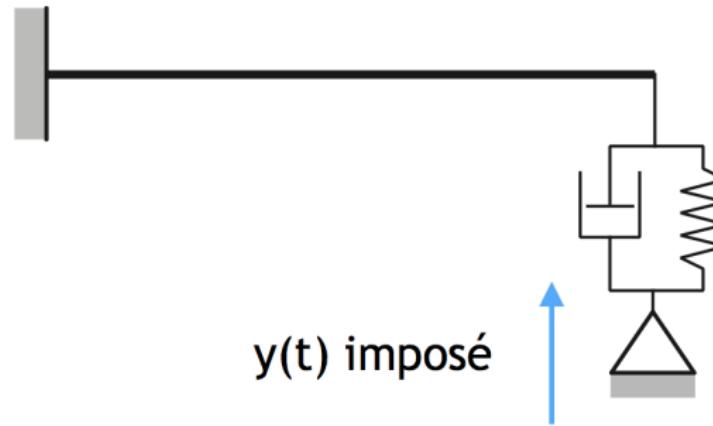
$$F(v) = -\frac{2F_{max}}{\pi} \arctan[\alpha v]$$



Prospects



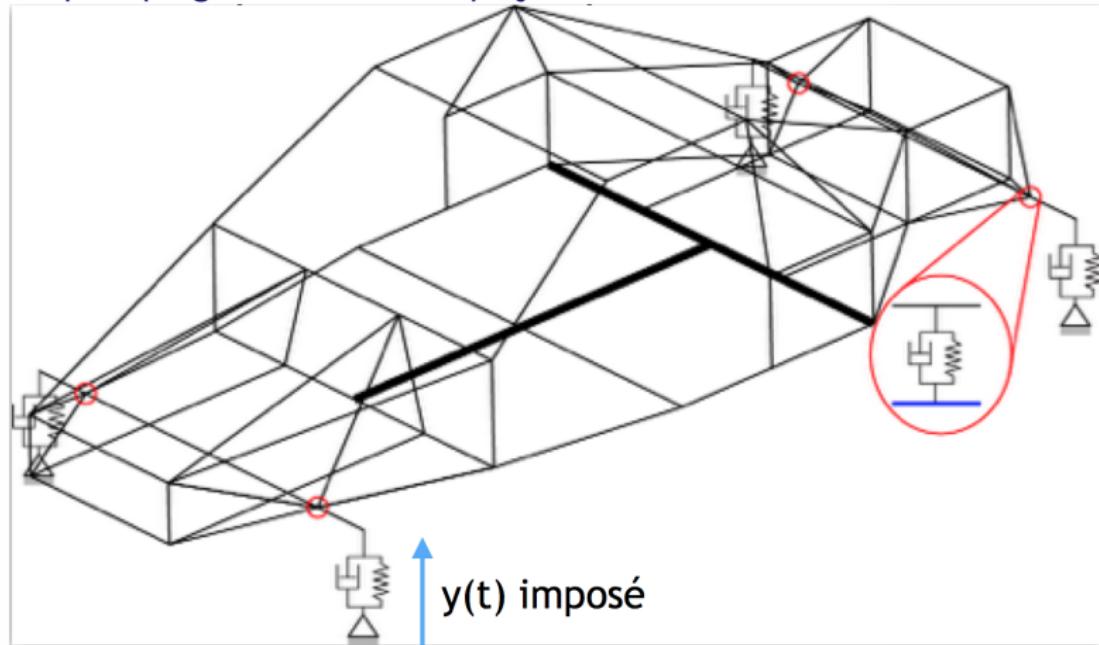
- Prepare program for Mecasif project test cases.



Prospects



Prepare program for Mecasif project test cases.



Prospects



Integration Scheme

- Influence of scheme dumping compared to problem dumping on PGD Convergence



- Use SVD within the resolution to "clean" the PGD basis



- Integration in an open FE software



Solving non-linear problems

- Use the test cases non-linearities
- Allow PGD non-linearities

