PGD for low frequency dynamics problems involving localized non-linearities

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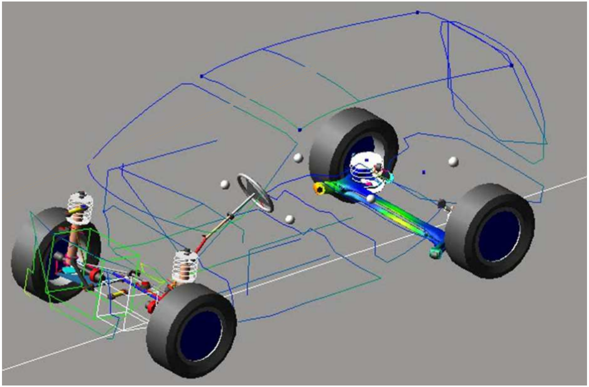
Main Subprojects

- Project Leading
- Transversal Methods
- Non Linear Vibrations
- Fast Dynamics
- Fluid Dynamics
- Promoting & Broadcasting

Non Linear Vibrations work packages :

- Stakes and Goals
- Use cases
- Non-Intrusive Methods
- Intrusive Methods

Non Linear Vibrations - Use case



- Calculation time
- Set of parameters
- Non-linearity

Presentation plan

Model order reduction

Reducing calculation time by reducing the dimension of the problem

Stakes of time saving

- Reducing the developement cost
- Optimisation / Creation of probabilistic models
- Solving of problems reaching calculating machine's limits

Reduced order modeling : Solving projected problems

How to obtain the basis to project on :

Reduction methods

- POD - Proper Orthogonal Decomposition [A. Chatterjee - 2000]
- PGD - Proper Generalized Decomposition [P. Ladevèze - 1989]

Construction of the Reduced-Order Basis, bibliography

offline/online techniques

- Proper Orthogonal Decomposition (POD)
  [Sirovich 87] [Holmes et al. 93] [Krysl et al. 00] [Kunisch,Wolkwein 02] [Willcox et al. 02] [Picinbono et al. 03] [Bergmann et al. 05] [Lieu et al. 06] [Gunzburger et al. 06] [Niroomandi et al. 08] [Farhat et al. 08] [Matthies et al. 10] ... also known as KLD [Karhunen 43] [Loeve 55], PCA [Pearson 1901], [Hotteling 33]. In finite dimension SVD [Ekardt & Young39], and if more than 2 variables HOSVD [Baranyi et al.06]
- Reduced-Basis (RB)
  [Maday et al. 02] [Patera et al. 02] [Rozza et al. 07] [Haasdonk et al. 08] [Boyaval et al. 09] ...

On-the-fly techniques - PGD - review article [Chinesta, Ladevèze, Cueto11]

- Radial approximation (PGD time/space)
  [Ladevèze 85, 99] ... [Ladevèze et al. 99-11] [Nouy et Ladevèze 03, 04] [Ladevèze, Dureissex, Néron 04] [Ladevèze et al. 08, 09, 10] [Néron and Dureisseix 08] [Boucard and Néron 11, 12] ...
- Generalized spectral decomposition
  [Nouy et al. 07-12]
- Multidimensional separation - PGD time/space/parameter
  [Ammar and Chinesta 06] [Ryckelynck 06] [Chinesta et al. 08-11] [Beringhier et al. 10] ...

Presentation plan

Different methods

POD

- a posteriori or offline/online technique
- give modes in:
  - space
  - time
- Principle :

Projection on a basis of modes exctracted from truncated SVD

PGD

- modes calculation on the fly
- give modes in :
  - space
  - time
  - parameter
- Principle :

Modes calculated using an iterative method (fixed point) at the same time as the resolution.

Proper Orthogonal Decomposition

A dynamic problem is solved on the system, and the SVD gives :

$$U(X,t) = \sum_{k=1}^{dim} V_{Sk}\ f_k(X) g_k(t)$$

The basis is obtained by truncating the result

The n modes $f_k(X)$ associated to the highest singular values are chosen to make the reduced basis.

The SVD guarantees the best truncated basis such as :

$$U(X,t) - U_n(X,t) = U(X,t) - \sum_{k=1}^{n} V_{Sk}\ f_k(X) g_k(t)$$

is minimal for a fixed n (in Frobenius norm).

These couples are the most representative ones of the studied response.

Solving (Snapshot) → Getting the reduced basis → Projection on the RB → Solving

Proper Generalized Decomposition

Approximation using separable variables

$$U_n(X,t,\lambda) = \sum_{k=1}^{n} f_k(X) g_k(t) h_k(\lambda)$$

The variational formulation gives a problem for each type of function

$$f_k = F(U_{k-1}, g_k, h_k)\ , \qquad g_k = G(U_{k-1}, f_k, h_k)\ , \qquad h_k = H(U_{k-1}, f_k, g_k)$$

Algorithm

for $k = 1$ à n
    for $j = 1$ à j_{max}
        $f_k = F(U_{k-1}, g_k, h_k)$
        $g_k = G(U_{k-1}, f_k, h_k)$
        $h_k = H(U_{k-1}, f_k, g_k)$
    end
end

Details

Iterating on the modes
Using a fixed point loop

$$\begin{cases} f_k = F(U_{k-1}, g_k, h_k) \\ g_k = G(U_{k-1}, f_k, h_k) \\ h_k = H(U_{k-1}, f_k, g_k) \end{cases}$$

to solve the non-linear problem.
until you reach n modes.

Presentation plan

1. MECASIF Project
2. Model Order Reduction
3. Different methods
   - POD
   - PGD
4. A few results
   - Error
   - MAC
   - Loading velocity
5. Prospects

Error indicators

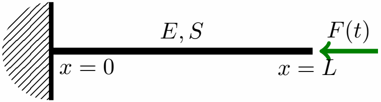


Figure : Academical problem

Relative error (obtained with reference solution) :

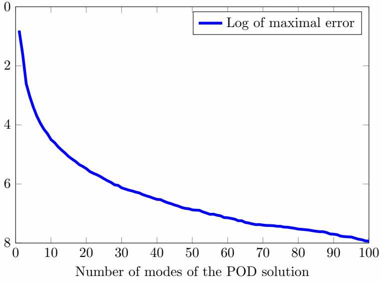


Figure : POD Solution

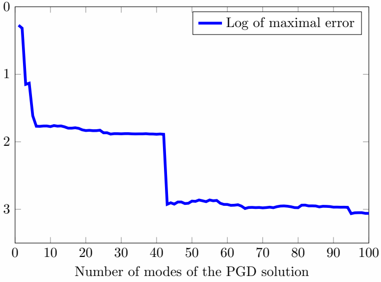


Figure : PGD Solution

MAC Analysis $\dfrac{\left(\varphi_i.\psi_j\right)^2}{\varphi_i.\varphi_i \times \psi_j.\psi_j}$

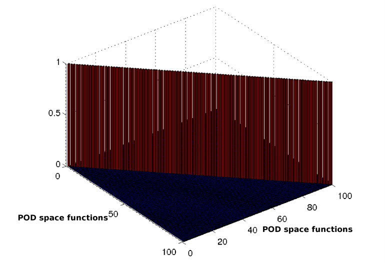


Figure : POD space functions

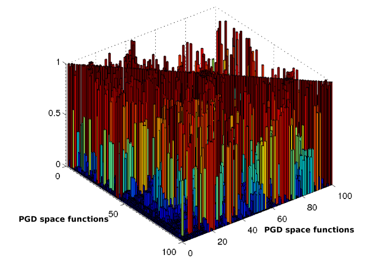


Figure : PGD space functions

- POD : Orthogonality
- PGD : Correlation between mods

MAC Analysis $\frac{(\varphi_i . \psi_j)^2}{\varphi_i . \varphi_i \times \psi_j . \psi_j}$

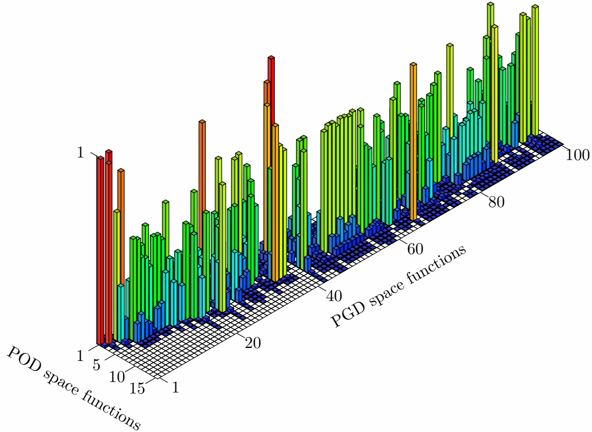


Figure : Comparison between POD and PGD space functions

PGD mods only representing the first POD mods

Orthogonalization - 2 variables

Algorithm

for $k = 1$ à n
for $j = 1$ à j_{max}
$f_k = F(U_{k-1}, g_k)$
$g_k = G(U_{k-1}, f_k)$
end
$[f_k, g_{1..k-1}] = otho(f_{1..k}, g_k)$
end

Details

Iterating on the modes
Using a fixed point loop
$$\begin{cases} f_k = F(U_{k-1}, g_k) \\ g_k = G(U_{k-1}, f_k) \end{cases}$$
to solve the non-linear problem.
Orthogonalizing
until you reach n modes.

Gram-Schmidt

$$\begin{aligned}
U_k &= \sum_{i=1}^{(k-1)} f_i g_i && + f_k g_k \\
&\quad + \sum_{i=1}^{(k-1)} (f_i.f_k) f_i g_k && - \sum_{i=1}^{(k-1)} (f_i.f_k) f_i g_k \\
U_k &= \sum_{i=1}^{(k-1)} f_i (g_i + g_k (f_i.f_k)) && + (f_k - \sum_{i=1}^{(k-1)} (f_i.f_k) f_i) g_k \\
U_k &= \sum_{i=1}^{(k-1)} f_i g_i && + f_k g_k
\end{aligned}$$

Orthogonalization - 3 variables

Gram-Schmidt

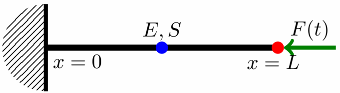
$$
\begin{aligned}
U_k = \; & \sum_{i=1}^{(k-1)} f_i g_i h_i & + f_k g_k h_k \\
& + \sum_{i=1}^{(k-1)} (f_i.f_k) f_i g_k h_k & - \sum_{i=1}^{(k-1)} (f_i.f_k) f_i g_k h_k \\
U_k = \; & \sum_{i=1}^{(k-1)} f_i (g_i h_i + g_k h_k (f_i.f_k)) & + (f_k - \sum_{i=1}^{(k-1)} (f_i.f_k) f_i) g_k h_k
\end{aligned}
$$

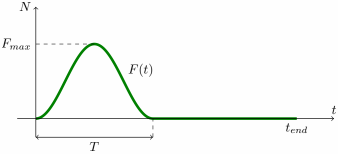
Possibilities

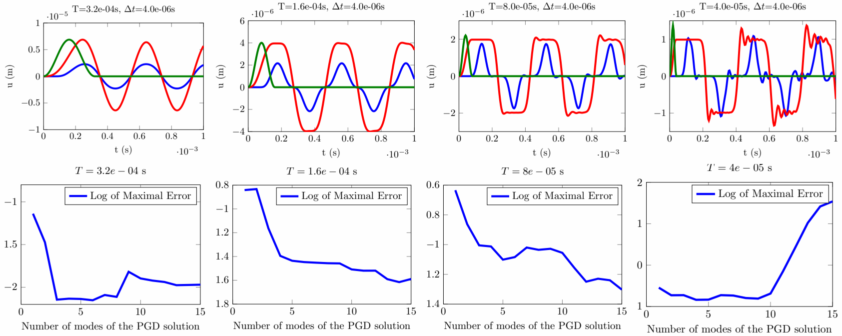
- Finding missing functions thanks to a minimization problem
- Orthogonalization afterwards and use the HOSVD

Different Loading Velocities

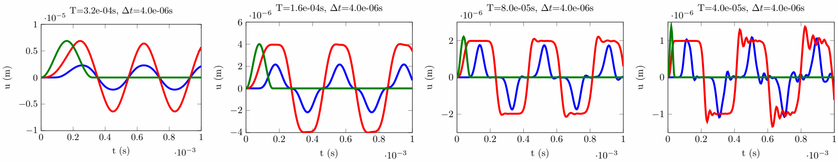
Loading in Sinus Verse for different period lengths :



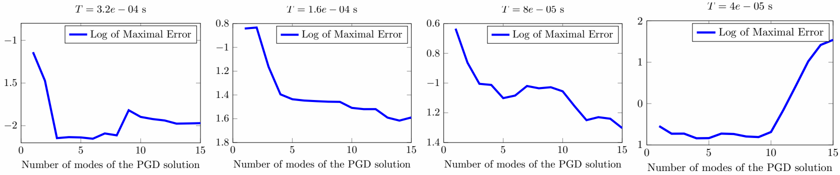




Different Loading Velocities



- Getting closer to a choc problem : presence of waves
- Presence of numerical perturbations of high frequencies



- Convergence depending on loading velocity / frequency

Prospects

- Integration Scheme
  - Use of the Galerkin discontinous in time method for the PGD
  - Influence of the choice of integration scheme on PGD Convergence
- Integration in an open FE software
- Add features present in other coworkers codes
  - Adding of parameter variables to the PGD
  - Using minimization instead of Galerkin in the fixed point
- Solving non-linear problems
  - The usual solver can be used on non-linear problems
  - Yet to be implemented for the PGD