PGD for low frequency dynamics problems involving localized non-linearities

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Project origin

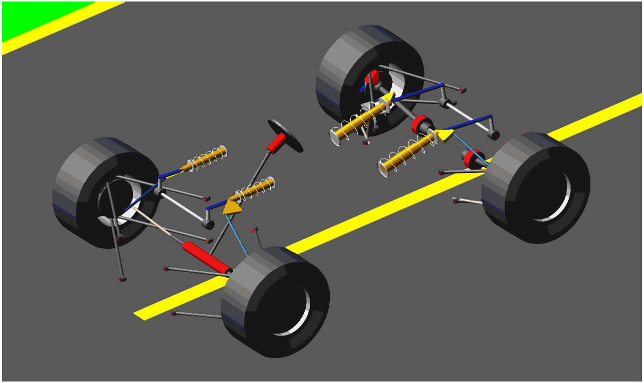


Figure : Industrial problem

- Calculation time
- Set of parameters
- Non-linearity

Presentation plan

Model order reduction

Reducing calculation time by reducing the dimension of the problem

Stakes of time saving

- Reducing the developement cost
- Optimisation / Creation of probabilistic models
- Solving of problems reaching calculating machine's limits

Reduced order modeling : Solving projected problems
How to obtain the basis to project on :

Reduction methods

- POD - Proper Orthogonal Decomposition [A. Chatterjee - 2000]
- PGD - Proper Generalized Decomposition [P. Ladevèze - 1989]

Presentation plan

Different methods

POD

- a posteriori
- give modes in :
  - space
  - time
- Principle :

Projection on a basis of modes exctracted from truncated SVD

PGD

- modes calculation "on the fly"
- give modes in :
  - space
  - time
  - parameter
- Principle :

Modes calculated using an iterative method (fixed point) at the same time as the resolution.

Proper Orthogonal Decomposition

A dynamic problem is solved on the system, and the SVD gives :

$$U(X,t) = \sum_{k=1}^{dim} V_{Sk}\ f_k(X) g_k(t)$$

The basis is obtained by truncating the result

The n modes $f_k(X)$ associated to the highest singular values are chose to make the reduced basis.
The SVD guarantees the best truncated basis such as :

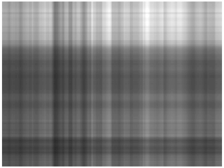
$$U(X,t) - U_n(X,t) = U(X,t) - \sum_{k=1}^{n} V_{Sk}\ f_k(X) g_k(t)$$

is minimal for a fixed n (in Frobenius norm).
These couples are the more representative of the studied response.

Solving (Snapshot) → Get the reduced basis → Projection on the reduced basis → Solving

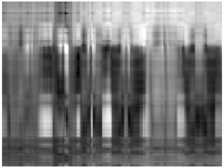
Picture Compression - Different number of singular values

Original

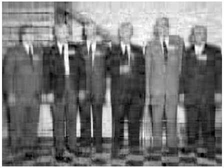


1 value

2 values



4 values



16 values

64 values

256 values

Total (480)

Proper Generalized Decomposition

Approximation using separable variables

$$U_n(X,t,\lambda) = \sum_{k=1}^{n} f_k(X) g_k(t) h_k(\lambda)$$

The variational formulation gives a problem for each type of function

$$f_k = F(U_{k-1}, g_k, h_k)\ , \qquad g_k = G(U_{k-1}, f_k, h_k)\ , \qquad h_k = H(U_{k-1}, f_k, g_k)$$

Algorithm

for $k = 1$ à n
 for $j = 1$ à j_{max}
  $f_k = F(U_{k-1}, g_k, h_k)$
  $g_k = G(U_{k-1}, f_k, h_k)$
  $h_k = H(U_{k-1}, f_k, g_k)$
 end
end

Iterating on the modes.
Solving the non-linear problem

$$\begin{cases} f_k = F(U_{k-1}, g_k, h_k) \\ g_k = G(U_{k-1}, f_k, h_k) \\ h_k = H(U_{k-1}, f_k, g_k) \end{cases}$$

with a fixed point loop.
Until you reach n modes.

Presentation plan

Error indicators

$$e_{Max} = \left| \frac{max(s_{Ref} - s_{Cal})}{max(s_{Ref}) - min(s_{Ref})} \right| \quad (1)$$

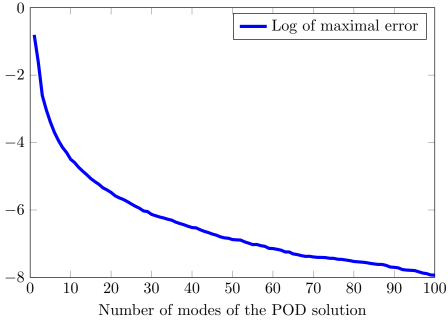


Figure : Evolution of error indicators for a POD Solution

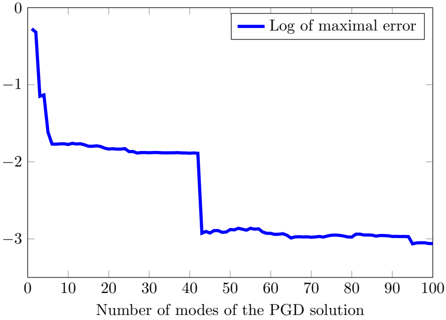


Figure : Evolution of error indicators for a PGD Solution

MAC Analysis $\frac{(\varphi_i.\psi_j)^2}{\varphi_i.\varphi_i \times \psi_j.\psi_j}$

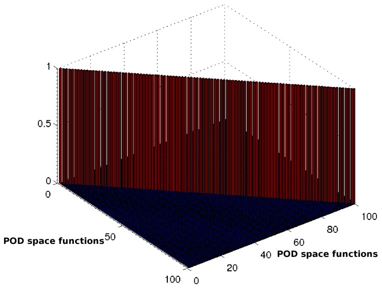


Figure : POD space functions

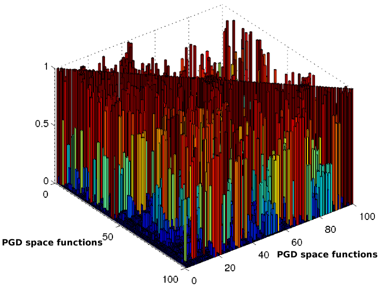


Figure : PGD space functions

MAC Analysis $\frac{(\varphi_i.\psi_j)^2}{\varphi_i.\varphi_i \times \psi_j.\psi_j}$

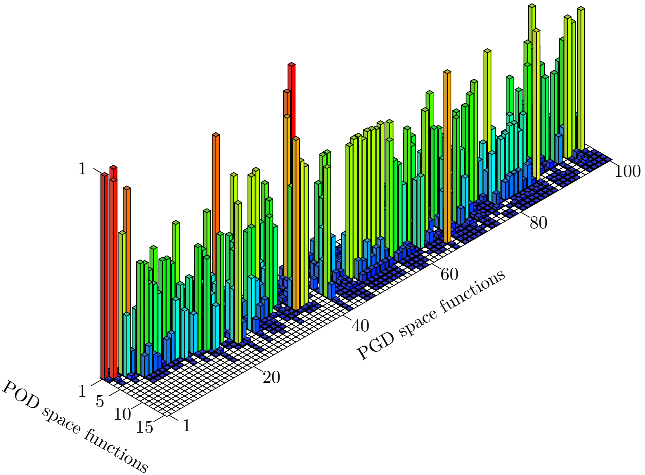
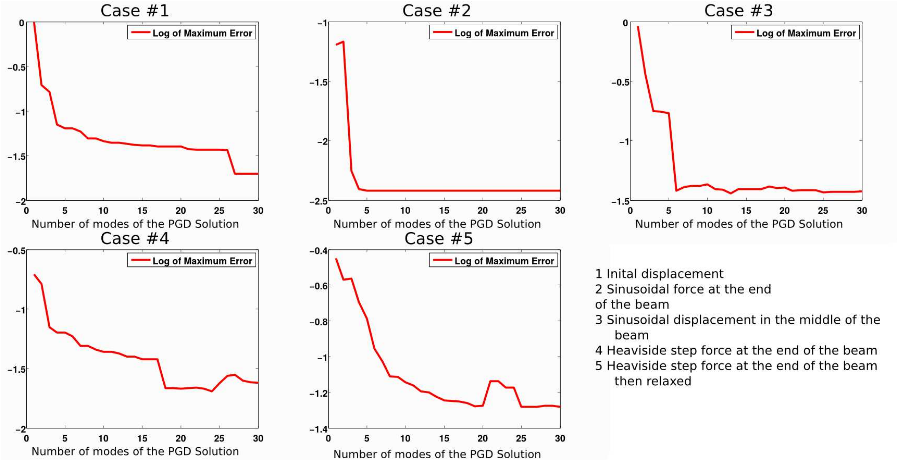


Figure : Comparison between POD and PGD space functions

Loading velocity - Different Cases



1 Inital displacement
2 Sinusoidal force at the end of the beam
3 Sinusoidal displacement in the middle of the beam
4 Heaviside step force at the end of the beam
5 Heaviside step force at the end of the beam then relaxed

Different Cases - Sinus Verse on a beam

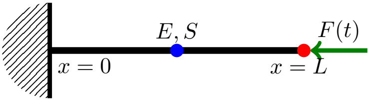


Figure : Beam Problem

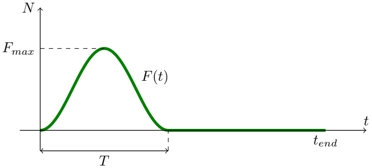
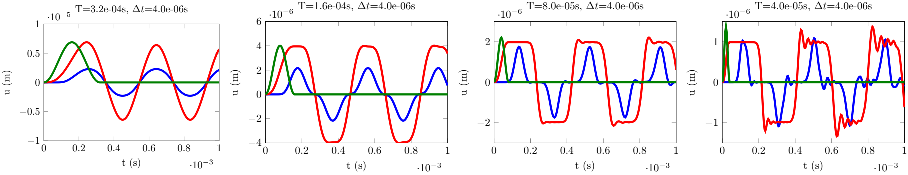
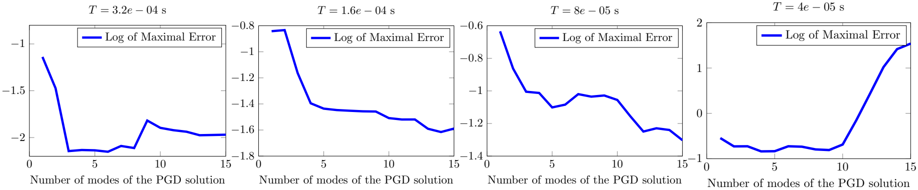


Figure : Loading

Different Cases - Different Loading Velocities

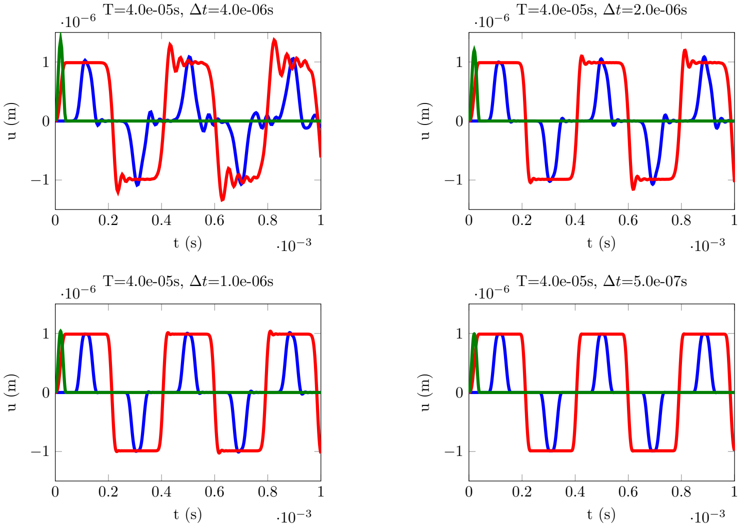
Loading in Sinus Verse for different period length :





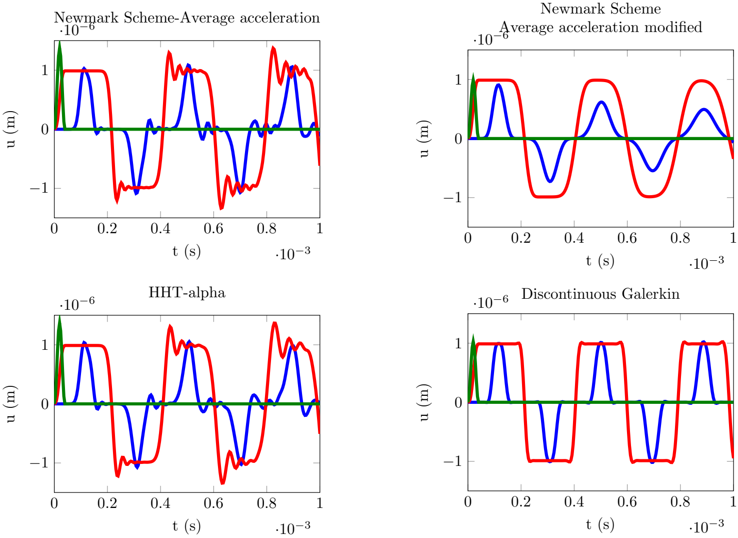
Different Time Steps

Violent Case solved with different time steps for the integration scheme :



Different Integration Schemes

Violent Case solved with different Schemes :



Time Discontinuous Galerkin Method

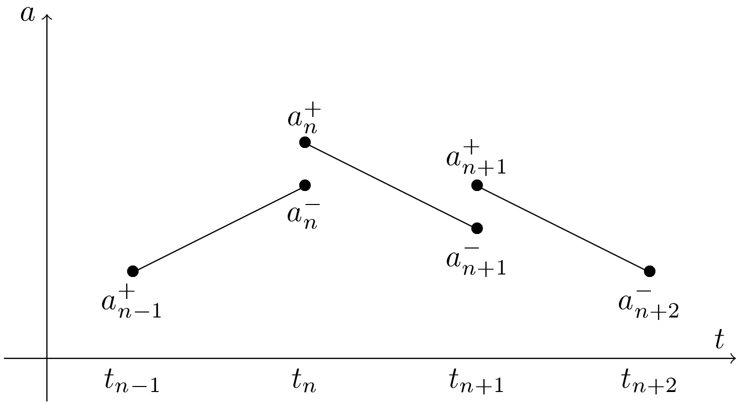


Figure : Representation allowing discontinuities

Perspectives

- Integration Scheme
  - Use of the Galerkin discontinous in time method for the PGD
  - Influence of the choice of integration scheme on PGD Convergence
- Integration in an open FE software
- Putting together a work environment for the PGD team
  - Adding of parameter variables to the PGD
  - Using minimization instead of fixed point
- Solving non-linear problems
  - The usual solver can be used on non-linear problems
  - Yet to be implemented for the PGD