Sommaire

0.1 Système de départ, TDG en dynamique

À l'instant m :

$$\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}\frac{\Delta t}{6} & \mathbf{K}\frac{\Delta t}{6} \\ 0 & \mathbf{K} & -\mathbf{K}\frac{\Delta t}{2} & -\mathbf{K}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{3} + \mathbf{C}\frac{\Delta t}{2} & \mathbf{K}\frac{(\Delta t)^2}{6} + \mathbf{M} + \mathbf{C}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{12} - \mathbf{M}\frac{1}{2} & \mathbf{K}\frac{(\Delta t)^2}{12} + \mathbf{M}\frac{1}{6} + \mathbf{C}\frac{\Delta t}{6} \end{bmatrix} \begin{bmatrix} \mathbf{u}_m^+ \\ \mathbf{u}_{m+1}^- \\ \mathbf{v}_m^+ \\ \mathbf{v}_{m+1}^- \end{bmatrix} = \begin{bmatrix} \mathbf{K}\mathbf{u}_m^- \\ \mathbf{K}\mathbf{u}_m^- \\ \left(\mathbf{M}\mathbf{v}_m^- + \frac{\Delta t}{2}(\mathbf{f}_m + \mathbf{f}_{m+1})\right) - \Delta t.\left(\mathbf{K}\mathbf{u}_m^-\right) \\ -\frac{\Delta t}{6}\left(\mathbf{K}\mathbf{u}_m^- - \mathbf{f}_{m+1}\right) - \frac{1}{3}.\mathbf{M}\mathbf{v}_m^- \end{bmatrix} \tag{1}$$

Chapitre 1

Formulation à variable séparée

Note : Les indices de sommation à l'intérieur de Σ ne sont pas indiqués pour éviter une surcharge des équations. Il s'agit de la somme des produits fournissant la solution PGD trouvée à l'itération précédente.

-Pertinence de la représentation des vitesses avec φ ?

$$\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}\frac{\Delta t}{6} & \mathbf{K}\frac{\Delta t}{6} \\ 0 & \mathbf{K} & -\mathbf{K}\frac{\Delta t}{2} & -\mathbf{K}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{3}+\mathbf{C}\frac{\Delta t}{2} & \mathbf{K}\frac{(\Delta t)^2}{6}+\mathbf{M}+\mathbf{C}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{12}-\mathbf{M}\frac{1}{2} & \mathbf{K}\frac{(\Delta t)^2}{12}+\mathbf{M}\frac{1}{6}+\mathbf{C}\frac{\Delta t}{6} \end{bmatrix} \begin{bmatrix} \varphi\mathbf{g}_m^{u}{}^{+}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{+}h \\ \varphi\mathbf{g}_{m+1}^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_{m+1}^{u}{}^{h} \\ \varphi\mathbf{g}_m^{v}{}^{+}h + \Sigma\varphi\mathbf{g}_m^{v}{}^{+}h \\ \varphi\mathbf{g}_{m+1}^{v}{}^{-}h + \Sigma\varphi\mathbf{g}_{m+1}^{v}{}^{-}h \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h) \\ \mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h) \\ \big(\mathbf{M}(\varphi\mathbf{g}_m^{v}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{v}{}^{-}h) + \frac{\Delta t}{2}(\mathbf{f}_m + \mathbf{f}_{m+1})\big) - \Delta t.\left(\mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h)\right) \\ -\frac{\Delta t}{6}\left(\mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h) - \mathbf{f}_{m+1}\right) - \frac{1}{3}.\mathbf{M}(\varphi\mathbf{g}_m^{v}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{v}{}^{-}h) \end{bmatrix} \tag{1.1}$$

$$\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}\frac{\Delta t}{6} & \mathbf{K}\frac{\Delta t}{6} \\ 0 & \mathbf{K} & -\mathbf{K}\frac{\Delta t}{2} & -\mathbf{K}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{3}+\mathbf{C}\frac{\Delta t}{2} & \mathbf{K}\frac{(\Delta t)^2}{6}+\mathbf{M}+\mathbf{C}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{12}-\mathbf{M}\frac{1}{2} & \mathbf{K}\frac{(\Delta t)^2}{12}+\mathbf{M}\frac{1}{6}+\mathbf{C}\frac{\Delta t}{6} \end{bmatrix} \left(\left[\varphi h \begin{bmatrix} \mathbf{g}_m^{u}{}^{+} \\ \mathbf{g}_{m+1}^{u}{}^{-} \\ \mathbf{g}_m^{v}{}^{+} \\ \mathbf{g}_{m+1}^{v}{}^{-} \end{bmatrix} + \begin{bmatrix} \Sigma\varphi\mathbf{g}_m^{u}{}^{+}h \\ \Sigma\varphi\mathbf{g}_{m+1}^{u}{}^{-}h \\ \Sigma\varphi\mathbf{g}_m^{v}{}^{+}h \\ \Sigma\varphi\mathbf{g}_{m+1}^{v}{}^{-}h \end{bmatrix} \right] \right)$$
$$= \begin{bmatrix} \mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h) \\ \mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h) \\ \big(\mathbf{M}(\varphi\mathbf{g}_m^{v}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{v}{}^{-}h) + \frac{\Delta t}{2}(\mathbf{f}_m + \mathbf{f}_{m+1})\big) - \Delta t.\left(\mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h)\right) \\ -\frac{\Delta t}{6}\left(\mathbf{K}(\varphi\mathbf{g}_m^{u}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{u}{}^{-}h) - \mathbf{f}_{m+1}\right) - \frac{1}{3}.\mathbf{M}(\varphi\mathbf{g}_m^{v}{}^{-}h + \Sigma\varphi\mathbf{g}_m^{v}{}^{-}h) \end{bmatrix} \tag{1.2}$$

L'équation doit être scalaire :

$$
(\varphi h)^T \times \begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}\frac{\Delta t}{6} & \mathbf{K}\frac{\Delta t}{6} \\ 0 & \mathbf{K} & -\mathbf{K}\frac{\Delta t}{2} & -\mathbf{K}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{3} + \mathbf{C}\frac{\Delta t}{2} & \mathbf{K}\frac{(\Delta t)^2}{6} + \mathbf{M} + \mathbf{C}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{12} - \mathbf{M}\frac{1}{2} & \mathbf{K}\frac{(\Delta t)^2}{12} + \mathbf{M}\frac{1}{6} + \mathbf{C}\frac{\Delta t}{6} \end{bmatrix} \left(\left[\varphi h \begin{bmatrix} \mathbf{g}_m^{u\,+} \\ \mathbf{g}_{m+1}^{u\,-} \\ \mathbf{g}_m^{v\,+} \\ \mathbf{g}_{m+1}^{v\,-} \end{bmatrix} + \begin{bmatrix} \Sigma\varphi\mathbf{g}_m^{u\,+}h \\ \Sigma\varphi\mathbf{g}_{m+1}^{u\,-}h \\ \Sigma\varphi\mathbf{g}_m^{v\,+}h \\ \Sigma\varphi\mathbf{g}_{m+1}^{v\,-}h \end{bmatrix} \right] \right)
$$
$$
= (\varphi h)^T \times \begin{bmatrix} \mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h) \\ \mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h) \\ \left(\mathbf{M}(\varphi\mathbf{g}_m^{v\,-}h + \Sigma\varphi\mathbf{g}_m^{v\,-}h) + \frac{\Delta t}{2}(\mathbf{f}_m + \mathbf{f}_{m+1})\right) - \Delta t.\left(\mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h)\right) \\ -\frac{\Delta t}{6}\left(\mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h) - \mathbf{f}_{m+1}\right) - \frac{1}{3}.\mathbf{M}(\varphi\mathbf{g}_m^{v\,-}h + \Sigma\varphi\mathbf{g}_m^{v\,-}h) \end{bmatrix} \tag{1.3}
$$

$$
(\varphi h)^T \times \begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}\frac{\Delta t}{6} & \mathbf{K}\frac{\Delta t}{6} \\ 0 & \mathbf{K} & -\mathbf{K}\frac{\Delta t}{2} & -\mathbf{K}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{3} + \mathbf{C}\frac{\Delta t}{2} & \mathbf{K}\frac{(\Delta t)^2}{6} + \mathbf{M} + \mathbf{C}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{12} - \mathbf{M}\frac{1}{2} & \mathbf{K}\frac{(\Delta t)^2}{12} + \mathbf{M}\frac{1}{6} + \mathbf{C}\frac{\Delta t}{6} \end{bmatrix} \left(\varphi h \begin{bmatrix} \mathbf{g}_m^{u\,+} \\ \mathbf{g}_{m+1}^{u\,-} \\ \mathbf{g}_m^{v\,+} \\ \mathbf{g}_{m+1}^{v\,-} \end{bmatrix} \right)
$$
$$
= (\varphi h)^T \times \begin{bmatrix} \mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h) \\ \mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h) \\ \left(\mathbf{M}(\varphi\mathbf{g}_m^{v\,-}h + \Sigma\varphi\mathbf{g}_m^{v\,-}h) + \frac{\Delta t}{2}(\mathbf{f}_m + \mathbf{f}_{m+1})\right) - \Delta t.\left(\mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h)\right) \\ -\frac{\Delta t}{6}\left(\mathbf{K}(\varphi\mathbf{g}_m^{u\,-}h + \Sigma\varphi\mathbf{g}_m^{u\,-}h) - \mathbf{f}_{m+1}\right) - \frac{1}{3}.\mathbf{M}(\varphi\mathbf{g}_m^{v\,-}h + \Sigma\varphi\mathbf{g}_m^{v\,-}h) \end{bmatrix}
$$
$$
-(\varphi h)^T \times \begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}\frac{\Delta t}{6} & \mathbf{K}\frac{\Delta t}{6} \\ 0 & \mathbf{K} & -\mathbf{K}\frac{\Delta t}{2} & -\mathbf{K}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{3} + \mathbf{C}\frac{\Delta t}{2} & \mathbf{K}\frac{(\Delta t)^2}{6} + \mathbf{M} + \mathbf{C}\frac{\Delta t}{2} \\ 0 & 0 & \mathbf{K}\frac{(\Delta t)^2}{12} - \mathbf{M}\frac{1}{2} & \mathbf{K}\frac{(\Delta t)^2}{12} + \mathbf{M}\frac{1}{6} + \mathbf{C}\frac{\Delta t}{6} \end{bmatrix} \begin{bmatrix} \Sigma\varphi\mathbf{g}_m^{u\,+}h \\ \Sigma\varphi\mathbf{g}_{m+1}^{u\,-}h \\ \Sigma\varphi\mathbf{g}_m^{v\,+}h \\ \Sigma\varphi\mathbf{g}_{m+1}^{v\,-}h \end{bmatrix} \tag{1.4}
$$

$$
\begin{gathered} \varphi h^T \times \mathbf{f} = f \\ \varphi h^T \times \mathbf{K} \times \varphi h = K \\ \varphi h^T \times \mathbf{K} \times \varphi_i h_i = K_i \end{gathered} \tag{1.5}
$$

Où $_i$ est l'indice de sommation. Même chose pour **M** et **C**. Ceci permet de n'avoir plus que des scalaires.

$$
\begin{aligned}
&\begin{bmatrix}
K & 0 & -K\frac{\Delta t}{6} & K\frac{\Delta t}{6} \\
0 & K & -K\frac{\Delta t}{2} & -K\frac{\Delta t}{2} \\
0 & 0 & K\frac{(\Delta t)^2}{3} + C\frac{\Delta t}{2} & K\frac{(\Delta t)^2}{6} + M + C\frac{\Delta t}{2} \\
0 & 0 & K\frac{(\Delta t)^2}{12} - M\frac{1}{2} & K\frac{(\Delta t)^2}{12} + M\frac{1}{6} + C\frac{\Delta t}{6}
\end{bmatrix}
\begin{bmatrix}
\mathbf{g}_m^{u\,+} \\
\mathbf{g}_{m+1}^{u\,-} \\
\mathbf{g}_m^{v\,+} \\
\mathbf{g}_{m+1}^{v\,-}
\end{bmatrix} \\
&= \begin{bmatrix}
K\mathbf{g}_m^{u\,-} + \Sigma K_i(\mathbf{g}_m^{u\,-})_i \\
K\mathbf{g}_m^{u\,-} + \Sigma K_i(\mathbf{g}_m^{u\,-})_i \\
M\mathbf{g}_m^{v\,-} + \Sigma M_i(\mathbf{g}_m^{v\,-})_i + \frac{\Delta t}{2}(f_m + f_{m+1}) - \Delta t\left(K\mathbf{g}_m^{u\,-} + \Sigma K_i(\mathbf{g}_m^{u\,-})_i\right) \\
-\frac{\Delta t}{6}\left(K\mathbf{g}_m^{u\,-} + \Sigma K_i(\mathbf{g}_m^{u\,-})_i\right) - f_{m+1}) - \frac{1}{3}(M\mathbf{g}_m^{v\,-} + \Sigma K_i(\mathbf{g}_m^{v\,-})_i)
\end{bmatrix} \\
&\quad - \sum^{i}
\begin{bmatrix}
K_i & 0 & -K_i\frac{\Delta t}{6} & K_i\frac{\Delta t}{6} \\
0 & K_i & -K_i\frac{\Delta t}{2} & -K_i\frac{\Delta t}{2} \\
0 & 0 & K_i\frac{(\Delta t)^2}{3} + C_i\frac{\Delta t}{2} & K_i\frac{(\Delta t)^2}{6} + M_i + C_i\frac{\Delta t}{2} \\
0 & 0 & K_i\frac{(\Delta t)^2}{12} - M_i\frac{1}{2} & K_i\frac{(\Delta t)^2}{12} + M_i\frac{1}{6} + C_i\frac{\Delta t}{6}
\end{bmatrix}
\begin{bmatrix}
\mathbf{g}_m^{u\,+} \\
\mathbf{g}_{m+1}^{u\,-} \\
\mathbf{g}_m^{v\,+} \\
\mathbf{g}_{m+1}^{v\,-}
\end{bmatrix}_i
\end{aligned} \tag{1.6}
$$