



PGD for low frequency dynamics problems involving localized non-linearities



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Main Subprojects

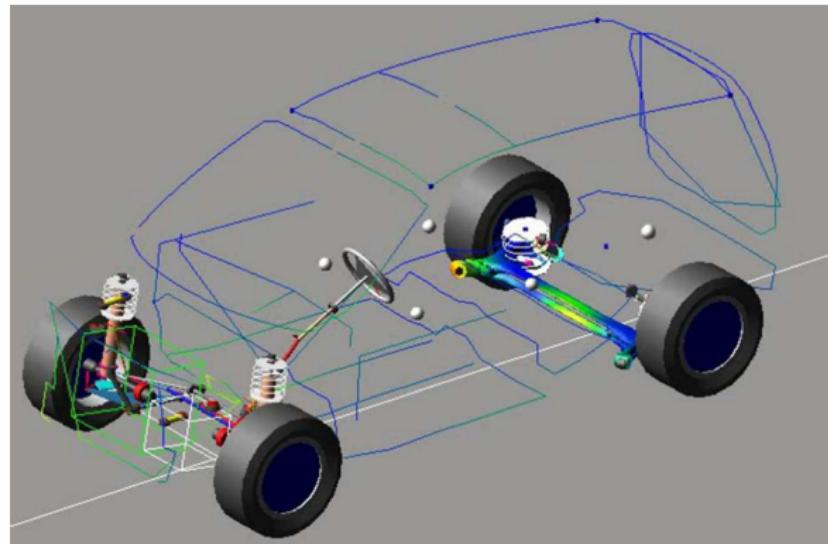
-  Project Leading
-  Transversal Methods
-  Non Linear Vibrations
-  Fast Dynamics
-  Fluid Dynamics
-  Promoting & Broadcasting



Non Linear Vibrations work packages :

- Stakes and Goals
- Use cases
- Non-Intrusive Methods
- Intrusive Methods

Non Linear Vibrations - Use case



- Calculation time
- Set of parameters
- Non-linearity

Presentation plan

- 1 MECASIF Project
- 2 Model Order Reduction
- 3 Different methods
 - POD
 - PGD
- 4 A few results
 - Error
 - MAC
 - Loading velocity
- 5 Prospects

Model order reduction

Reducing calculation time by reducing the dimension of the problem

Stakes of time saving

- Reducing the development cost
- Optimisation / Creation of probabilistic models
- Solving of problems reaching calculating machine's limits

Reduced order modeling : Solving projected problems

How to obtain the basis to project on :

Reduction methods

- POD - Proper Orthogonal Decomposition [A. Chatterjee - 2000]
- PGD - Proper Generalized Decomposition [P. Ladevèze - 1989]

Construction of the Reduced-Order Basis, bibliography



offline/online techniques

- Proper Orthogonal Decomposition (POD)
[Sirovich 87] [Holmes et al. 93] [Krysl et al. 00] [Kunisch,Wolkwein 02]
[Willcox et al. 02] [Picinbono et al. 03] [Bergmann et al. 05] [Lieu et al. 06]
[Gunzburger et al. 06] [Niroomandi et al. 08] [Farhat et al. 08]
[Matthies et al. 10] ... also known as KLD [Karhunen 43] [Loeve 55], PCA
[Pearson 1901], [Hotteling 33]. In finite dimension SVD [Ekardt & Young39], and
if more than 2 variables HOSVD [Baranyi et al.06]
- Reduced-Basis (RB)
[Maday et al. 02] [Patera et al. 02] [Rozza et al. 07] [Haasdonk et al. 08]
[Boyaval et al. 09] ...
- On-the-fly techniques - PGD - review article [Chinesta, Ladevèze, Cueto11]
- Radial approximation (PGD time/space)
[Ladevèze 85, 99] ... [Ladevèze et al. 99-11] [Nouy et Ladevèze 03, 04]
[Ladevèze, Dureisseix, Néron 04] [Ladevèze et al. 08, 09, 10] [Néron and
Dureisseix 08] [Boucard and Néron 11, 12] ...
- Generalized spectral decomposition
[Nouy et al. 07-12]
- Multidimensional separation - PGD time/space/parameter
[Ammar and Chinesta 06] [Ryckelynck 06] [Chinesta et al. 08-11]
[Beringhier et al. 10] ...



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Different methods

POD

- a posteriori or offline/online technique
- give modes in:

- space
- time

- Principle :

Projection on a basis of modes extracted from truncated SVD

PGD

- modes calculation on the fly
- give modes in :
 - space
 - time
 - parameter

- Principle :

Modes calculated using an iterative method (fixed point) at the same time as the resolution.

Proper Orthogonal Decomposition

A dynamic problem is solved on the system, and the SVD gives :

$$U(X, t) = \sum_{k=1}^{\dim} V_{Sk} f_k(X) g_k(t)$$

The basis is obtained by truncating the result

The n modes $f_k(X)$ associated to the highest singular values are chosen to make the reduced basis.

The SVD guarantees the best truncated basis such as :

$$U(X, t) - U_n(X, t) = U(X, t) - \sum_{k=1}^n V_{Sk} f_k(X) g_k(t)$$

is minimal for a fixed n (in Frobenius norm).

These couples are the more representative of the studied response.

Solving
(Snapshot)

→ Getting the
reduced basis

→ Projection on
the RB

→ Solving

Proper Generalized Decomposition

Approximation using separable variables

$$U_n(X, t, \lambda) = \sum_{k=1}^n f_k(X)g_k(t)h_k(\lambda)$$

The variational formulation gives a problem for each type of function

$$f_k = F(U_{k-1}, g_k, h_k), \quad g_k = G(U_{k-1}, f_k, h_k), \quad h_k = H(U_{k-1}, f_k, g_k)$$

Algorithm

```
for k = 1 à n
    for j = 1 à jmax
        fk = F(Uk-1, gk, hk)
        gk = G(Uk-1, fk, hk)
        hk = H(Uk-1, fk, gk)
    end
end
```

Details

Iterating on the modes
Using a fixed point loop

$$\begin{cases} f_k = F(U_{k-1}, g_k, h_k) \\ g_k = G(U_{k-1}, f_k, h_k) \\ h_k = H(U_{k-1}, f_k, g_k) \end{cases}$$

to solve the non-linear problem.
until you reach n modes.

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Error indicators

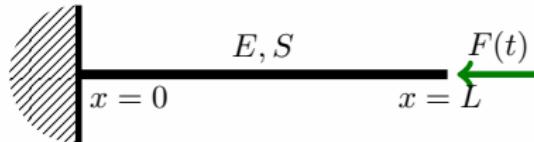


Figure : Academic problem

Relative error (obtained with reference solution) :

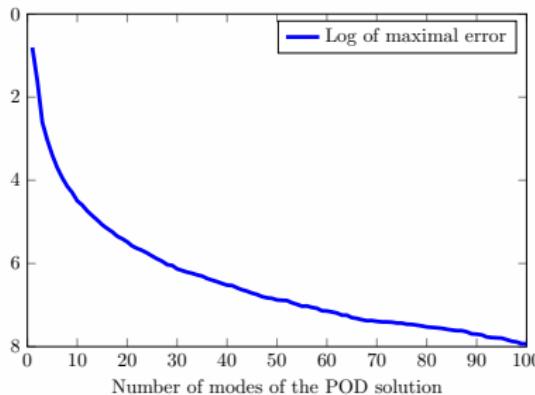


Figure : POD Solution

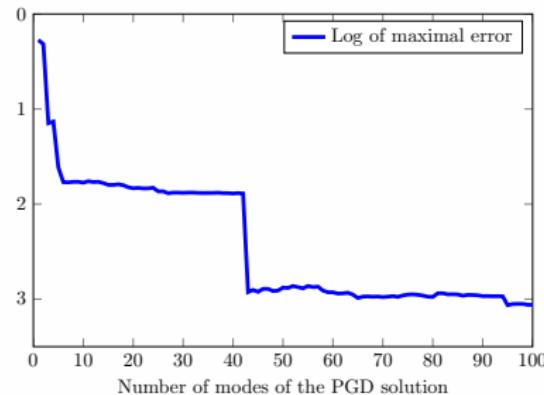


Figure : PGD Solution

MAC Analysis

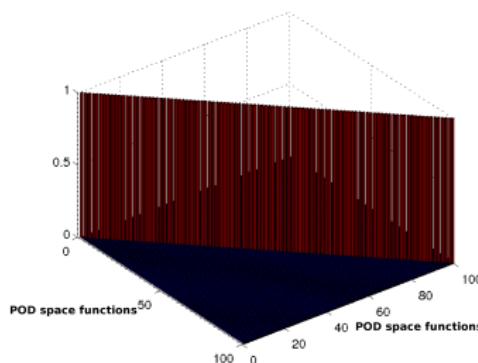
$$\frac{(\varphi_i \cdot \psi_j)^2}{\varphi_i \cdot \varphi_i \times \psi_j \cdot \psi_j}$$


Figure : POD space functions



POD : Orthogonality



PGD : Correlation between mods

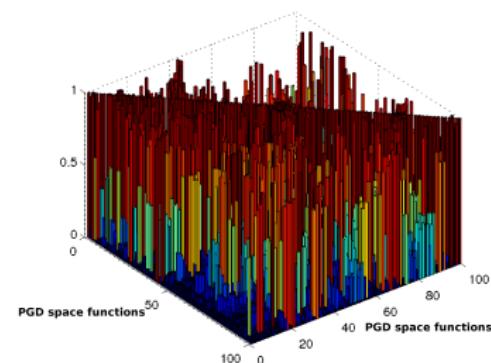


Figure : PGD space functions

MAC Analysis

$$\frac{(\varphi_i \cdot \psi_j)^2}{\varphi_i \cdot \varphi_i \times \psi_j \cdot \psi_j}$$

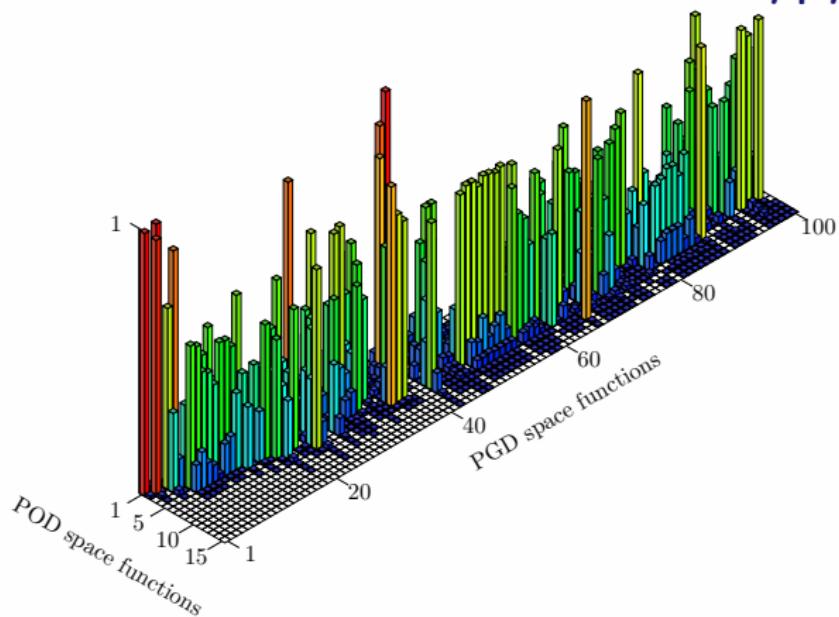
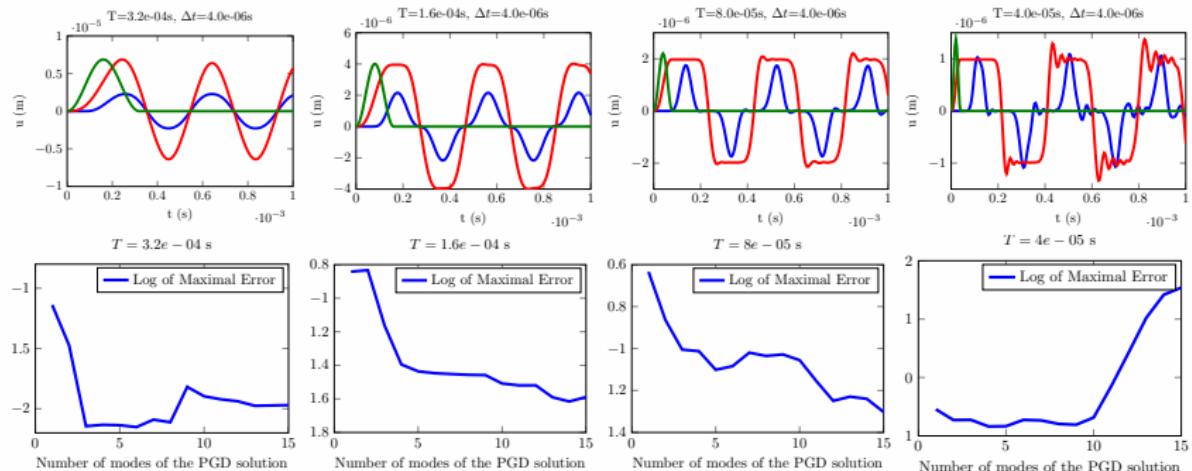
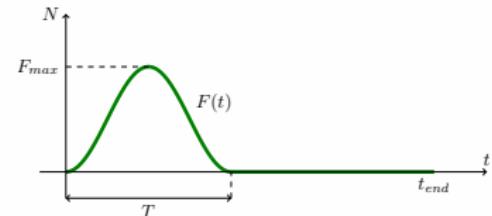
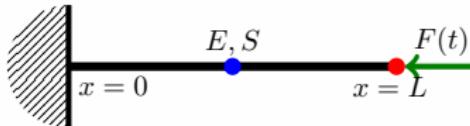


Figure : Comparison between POD and PGD space functions

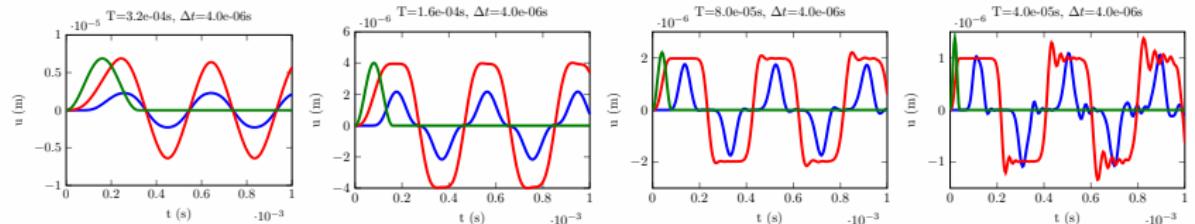
- PGD mods only representing the first POD mods

Different Loading Velocities

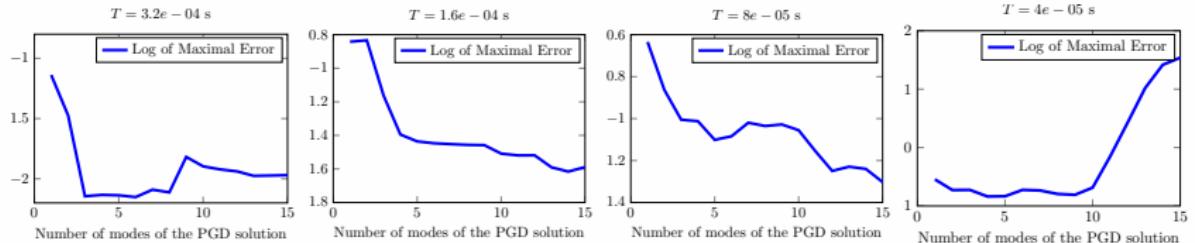
Loading in Sinus Verse for different period lengths :



Different Loading Velocities



- Getting closer to a choc problem : presence of waves
- Presence of numerical perturbations of high frequencies



- Convergence depending on loading velocity / frequency

Prospects



Integration Scheme

- Use of the Galerkin discontinuous in time method for the PGD
- Influence of the choice of integration scheme on PGD Convergence



Integration in an open FE software



Putting together a work environment for the PGD team

- Adding of parameter variables to the PGD
- Using minimization instead of Galerkin



Solving non-linear problems

- The usual solver can be used on non-linear problems
- Yet to be implemented for the PGD