Equilibrium Search Models

Franck Malherbet [1]
(CREST-ENSAE)

2023-2024

1. https://sites.google.com/view/franck-malherbet/home/teaching/labor-economics

- Until now, we have focussed solely on the *behavior of job seekers*, *i.e.* restricting attention to labour supply.

- In doing so, we have taken the wage distribution as given.

- This approach leaves the setting of wages unexplained.

- Equilibrium search models are taking a step forward in making endogenous the wage distribution.

- In a *perfectly competitive world*, individuals with identical abilities and working conditions get identical wages.

- Empirical facts about wage differentials suggest however that such workers might be paid differently.

- But why are similar workers paid differently ?

- We will see how equilibrium search models can explain this phenomenon, *i.e.*, how individuals with identical productive abilities and preferences and with identical jobs can receive different wages.

- The basic Job search model is based on the assumption that there exists a non degenerate distribution of wages from which some offers are drawn randomly at rate λ.
- But why should there be a distribution of wages for *perfectly identical* workers ?
- Diamond (1971) emphasized that if the reactions of employers were introduced in the basic model, then the equilibrium distribution of wages would be *concentrated at a single point*.[2]
- This is the famous Diamond's critique/paradox.

2. Diamond (1971), A Model of Price Adjustment, *Journal of Economic Theory*.

- Reconsider the expression of the reservation wage in the basic model :

$$x = z + \frac{\lambda}{r+q} \int_x^{\infty} (w - x) \, \mathrm{d}H(w) \tag{1}$$

- If any wage above *or equal* to x can be accepted, why should employers pay more than x ? *No incentive to pay more than x !*

- The equilibrium distribution of wages should reduce to a single point x, then we get from (1) that $x = z$, i.e. :
  - Workers are paid their reservation wage,
  - Firms reap the entire surplus of the match (i.e. *the marginal product minus the labor cost*).

- The equilibrium distribution of wages is degenerate : this casts some doubts on the validity of the basic job search model.

- A potential answer to the critique is first formulated by Albrecht and Axell (1984).[3]

- This approach consists of extending the basic model by introducing heterogeneity among the workers.

- They argue that the distribution of wages results from the existence of a distribution of reservation wages.

- This explanation is not entirely satisfactory : a large part of the variance of observed wages is still left unexplained even when individual heterogeneity is taken into account.

3. Albrecht and Axell (1984), An equilibrium model of search employment, *Journal of Political Economy*.

1. Numerous studies reveal that part of the variance in wages always remains unexplained even when individual heterogeneity is taken into account.

- Consider the following Mincerian *(log-)wage* equation for individual i :

$$\begin{aligned}\log(\text{earnings}_i) &= \alpha_i + \beta_1.\text{schooling}_i + \beta_2.\text{experience} + \beta_3.\text{experience}^2 \\ &\quad + \beta_4.\text{gender and race}_i + \beta_5.\text{sector}_i + \beta_6.\text{location}_i\end{aligned}$$

- Mincer-type equations applied to individual worker data only explain a fraction (typically, less than 50%) of observed difference in earnings.
  - difference in workers' unobserved heterogeneity ?
  - difference in firms' salary policies ?

1. (cont'd)

- The recent advent of **matched employer-employee panel data** (MEED) has considerably improved our understanding of wage differentials.

- In the recent years, following Abowd, Kramarz and Margolis's (AKM) initial push, many matched employer-employee datasets have been constructed in Denmark, Italy, Sweden, Austria, etc., to estimate wage equations. [4]

- Matched employer-employee data are obtained by merging two different data sources :

    1. one panel of worker data (*index* i, t),
    2. one panel of firm data (*index* j, t).

4. Abowd, Kramarz and Margolis (1999), High wage workers and high wage firms, *Econometrica*.

1. (cont'd)
   - What matched employer-employee data show is that there are systematic differences across workers and across firms that *cannot be explained by classical individual or market attributes*.
   - Even after controlling for unobserved heterogeneity a sizeable share of the wage dispersion is left unexplained (more than one third).
   - Postel-Vinay and Robin (2002) : [5]

Category	Individual FE	Firm FE	**Remaining**
Managers and Engineers	38%	20%	42%
Professionals	19%	28%	53%
Technicians and Associate Professionals	8%	33%	59%
Clerks	11%	35%	54%
Skilled Plant Workers	0%	42%	58%
Unskilled Employees	3%	39%	58%
Unskilled Plant Workers	0%	44%	56%

5. Postel-Vinay and Robin, 2002, Equilibrium wage dispersion with worker and employer heterogeneity, *Econometrica*.

2. The solution to Diamond's paradox based on different reservation wages is not satisfactory.

   Hornstein et al. (2011) have shown that it cannot generate large wage differentials for identical individuals for plausible values of preferences parameters. [6]

- They show that the basic model predicts that the ratio of mean wage to reservation wage is very small.
- (Again) the reservation wage, x, verifies :

$$x = z + \frac{\lambda}{r+q}\int_x^{+\infty}(w-x)\mathrm{d}H(w)$$

- Let us define by :
  - $\rho \in (0;1)$, $z = \rho\bar{w}$
  - $\bar{w}$, $\bar{w} = \int_x^{+\infty}\frac{w}{1-H(x)}\mathrm{d}H(w)$
  - λ_u^*, $\lambda_u^* = \lambda[1-H(x)]$

6. Hornstein, Krusell, & Violante, 2011, Frictional wage dispersion in search models : A quantitative assessment, *American Economic Review*.

2. (cont'd)

- The reservation wage rewrites :

$$x = z + \frac{\lambda_u^*}{r+q}(\bar{w} - x)$$

- The mean-min wage (Mm) ratio is :

$$Mm \equiv \frac{\bar{w}}{x} = \frac{\frac{\lambda_u^*}{r+q} + 1}{\frac{\lambda_u^*}{r+q} + \rho}$$

- The Mean-min ratio is a new metric for frictional wage dispersion, *i.e.* wage differentials are entirely determined by luck in the search process.

- The greater this ratio, the wider the wage dispersion.

2. (cont'd)

- This measure has one important property : it does not depend *directly* on the wage offer distribution H.
- Why ? Because all that is relevant about H is captured by λ_u^* which can be directly measured through labor market transitions.
- The mean-min ratio is merely a function of 4 parameters :
  - λ_u^*
  - q
  - r
  - ρ

2. (cont'd)

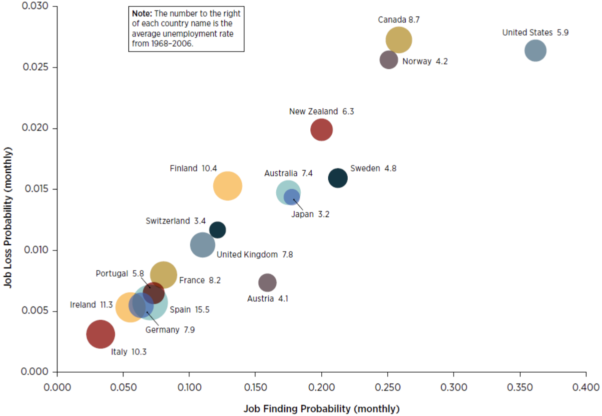
Some *unpleasant search arithmetic* :

- Hornstein et al. (2011) calibrates the model on the US economy over the period 1991-2007 (monthly basis) :

US Calibration		
average monthly job separation rate	q	0.03
average monthly job finding rate	λ_u^*	0.43
interest rate	r	0.0041
replacement ratio	ρ	0.4

- The Mean-min ratio is equal to :

$$Mm = \frac{\frac{\lambda_u^*}{r+q} + 1}{\frac{\lambda_u^*}{r+q} + \rho} \simeq 1.05$$



Source : Hornstein and Lubik (2012)

- In Europe unemployment spells last much longer than in the US.
- Does this observation mean that the search model would predict much higher frictional wage dispersion for European labor markets ?

2. (cont'd)

- Not necessarily because what matters is :
  1. the $\frac{\lambda_u^*}{q}$ ratio (unemp. duration relative to job duration), and in European countries both variables are longer than in the US.
  2. the replacement ratio, ρ.[7]

- Taking a conservative approach (i.e. with $\rho = 0.4$ as in the US), the mean-min ratio in Europe satisfies :

$$Mm = \frac{\frac{\lambda_u^*}{r+q} + 1}{\frac{\lambda_u^*}{r+q} + \rho} \simeq 1.09$$

- Hence, the baseline job search model predicts that there is very little wage dispersion both in the U.S. and European cases.

7. Remark that the mean-min ratio is decreasing in ρ.

- Hornstein et al. (2011) convincingly show that the basic job search model predicts a narrow dispersion of wages and this conclusion is robust to most changes in the assumptions.

- They extend the analysis to a number of alternative specifications including *on-the-job search models*.

- In this case, the Mean-min ratio satisfies :[8]

$$Mn = \frac{\frac{\lambda_u - \lambda_e}{r+q+\lambda_e} + 1}{\frac{\lambda_u - \lambda_e}{r+q+\lambda_e} + \rho}$$

- The Mn ratio is increasing in λ_e, hence the model with OTJS generates more frictional wage inequality than the baseline model.[9]

8. See Hornstein et al. (2011), *op. cit.*
9. For instance in the case where $\lambda_u = \lambda_e$, the Mn ratio equals $1/\rho$ and is worth 2.5 (using the same parameter value than in the baseline U.S. calibration).

- For reasonable values in the range of U.S. data, the model produces Mn ratios between 1.16 and 1.27, ratios much more in line with the empirical observations.
- From this standpoint equilibrium search models pioneered by Burdett and Mortensen (1998) are good candidates to explain wage dispersion, with advantage of being :[10]
  - from a theoretical point of view, reasonably realistic,
  - from an empirical point of view, easily empirically implementable.

10. Burdett and Mortensen (1998), Wage differentials, employer size, and unemployment, *International Economic Review*.

- Mortensen (2003) argues that equilibrium search models meet these latter two requirements and offer a natural framework in which to analyze the multiform wage dispersion evident in MEED.

- *Equilibrium search models* rest upon **two basic principles** :

  1. Labor market competition between employers is the fundamental determinant of wages,
  2. Competition is limited by search frictions reflecting information imperfection on the location of job offers.

- Equilibrium search models provide an acceptable framework to understand :
  - wage inequality (across individuals)
  - wage mobility/dynamics (over time)

- *Equilibrium search models* range between **two polar cases** :
  - *Competitive wage equilibrium*,
  - *Monopsony wage equilibrium*.[11]

- Apart from these two polar cases, *equilibrium search models* offer simple explanations of why...
  - wages vary across workers,
  - wages vary across firms,

- ... why *some residual wage dispersion remains* once heterogeneity has been accounted for.

11. See Diamond (1971), *op. cit.*

- Why do firms pay different wages to identical workers ?
  - Some firms will offer lower (resp. higher) wages : they get a higher (lower) profit from the job, $y - w$,
  - but when they find a worker :
    1. their wage offer is less (more) often accepted,
    2. they also bear a larger (lower) probability that their worker will quit their job for a higher wage firm.
  - This occurs because when the wage offer is lower (higher) , there are more (less) chance of finding a better wage offer elsewhere.
  - Firms offering low (high) wages will be smaller (larger) : their employees quit more (less) often, and it is more (less) difficult for them to recruit.
- This trade-off gives rise to pure wage dispersion.

- Jolivet, Postel-Vinay and Robin (2006) examine empirical features of worker turnover and wage distributions across European countries and the US.[12]

- They argue that a successful theory should account for the following stylized facts :

  1. Workers transit from *job-to-job* or *in-and-out of employment*,
  2. Most job-to-job transitions are associated with a *wage increase*, yet a sizeable fraction of those transitions (20-40%) are still associated with a *wage cut*,
  3. Job separation hazards exhibit (slightly) negative duration dependence,
  4. Wages are dispersed,
  5. The distribution of wages in a cross-section of employed workers *first-order stochastically dominates* the distribution of entry wages.[13]

12. Jolivet, Postel-Vinay, & Robin (2006), The empirical content of the job search model : Labor mobility and wage distributions in Europe and the US, *European Economic Review*.

13. A distribution F first-order stochastically dominates (FOSD) a distribution G, if and only if, $F(x) \leq G(x)$ for all x.

The canonical Burdett-Mortensen model

- In what follows, we build on Burdett and Mortensen (1998).[14]
- Time is continuous. We focus on steady-states.[15]
- The economy is composed of a *continuum of firms and of (infinitely lived) workers of unitary mass*.
- Workers are either employed or unemployed and they search both *off-and-on-the-job*.
- The labor market is affected by search frictions in that workers can only sample job offers sequentially at rate :
  - $\lambda_u > 0$, for the unemployed worker,
  - $\lambda_e > 0$, for the employed worker.

14. Burdett and Mortensen (1998), Wage differentials, employer size, and unemployment, *International Economic Review*.
15. See e.g. Moscarini and Postel-Vinay (2016) for an out-of-steady-state model.
Moscarini and Postel-Vinay (2016), Wage Posting and Business Cycles : a Quantitative Exploration, *Review of Economic Dynamics*.

- A job offer is a commitment to pay a constant wage, w.
- It is a *take-it-or-leave-it* offer.
- The wage is hence fixed *ex-ante*, *i.e.*, there is no *ex-post* bargaining (see next lecture).
- Firms' wage posting strategy are summarized by the sampling distribution $H(w)$ where $w \in [\underline{w}, \overline{w}]$.
- Firm-worker matches are dissolved at rate $q > 0$ (exogeneous).
- Upon match dissolution, the worker moves back into the **pool of the unemployed workers**.

- The expected utility of an unemployed worker satisfies :

$$rV_u = z + \lambda_u \int_x^{\overline{w}} \left(V_e\left(w\right) - V_u\right) \mathrm{d}H\left(w\right)$$

- The expected utility of an employed worker satisfies :

$$rV_e\left(w\right) = w + q\left(V_u - V_e\left(w\right)\right) + \lambda_e \int_w^{\overline{w}} \left(V_e\left(w'\right) - V_e\left(w\right)\right) \mathrm{d}H\left(w'\right)$$

- The job search behavior of job-seekers is identical to that in the job-search model with OTJS :[16]

$$x = z + (\lambda_u - \lambda_e) \int_x^{\overline{w}} \frac{\bar{H}(\xi)}{r + q + \lambda_e \bar{H}(\xi)} \mathrm{d}\xi$$

where $\bar{H} \equiv 1 - H$.

- Let us distinguish :
  - $G(w)$, the CDF of wages among employees (*i.e. accepted/earned wages*),
  - $H(w)$, the CDF of wage offers (*i.e.* the *sampling distribution*).

- Thus, due to OTJS, the cross section distribution of wages, $G(.)$, will differ from the offers sampling distribution, $H(.)$.

16. see lecture 2.

- The two distributions $H(.)$ and $G(.)$ are linked using labor market flows, as will (hopefully) become clear below.

- Let us first denote by u the unemployment rate.

- The law of motion of unemployment is given by :

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \underbrace{q\,(1-u)}_{\text{destruction}} - \underbrace{\lambda_u\,[1-H(x)]\,u}_{\text{creation}} = q\,(1-u) - \lambda_u u$$

using the fact that x is the lower bound of the *sampling distribution.*[17]

- The stationary unemployment rate reads :

$$u = \frac{q}{\lambda_u + q} \tag{2}$$

17. It is never optimal to make wage offers that are always rejected !

- The flow of entries into the set of workers earning w or less verifies :

$$u.H(w).\lambda_u$$

- The flow of exits from the set of workers earning w or less verifies :

$$(1-u).G\left(w\right).\left(q+\lambda_e\left[1-H(w)\right]\right)$$

- At stationary equilibrium, the *equality of the flows of entries and exits* in jobs offering wages lower than w implies :

$$\lambda_u.u.H(w) = (1-u).G(w).\left[\lambda_e \bar{H}(w) + q\right] \tag{3}$$

- Making use of the unemployment rate (2) in the flow equation (3) we get :

$$G(w) = \frac{H(w)}{1 + \kappa \bar{H}(w)} \text{ where } \kappa = \frac{\lambda_e}{q} \tag{4}$$

- The previous equation implies that $G(w) < H(w)$ when $\lambda_e > 0$, *i.e.* the probability of occupying a job with a wage less than w is lower than those of receiving a job offer with a wage less than w.

- The gap between the two distributions depends on parameter κ :
  - It counts the average number of outside offers an **employed worker** can expect to receive before becoming unemployed.
  - It is a measure of the intensity of interfirm competition on the labor market.

- The higher κ, the higher the intensity of interfirm competition and the lower the *search frictions* for workers. [18]

18. Jolivet et al. (2006) provide estimates for κ to be between 0.27 and 2.03 (see table 2).

Table 2
Constrained model estimates (per annum)

Country	BEL	DNK	ESP	FRA	GBR	GER	IRL	ITA	NLD	PRT	USA
δ^c	0.0353 (0.0040)	0.0504 (0.0049)	0.0878 (0.0044)	0.0129 (0.0013)	0.0803 (0.0049)	0.0394 (0.0026)	0.0716 (0.0060)	0.0526 (0.0032)	0.0324 (0.0025)	0.0548 (0.0034)	0.0547 (0.0035)
λ_1^c	0.0522 (0.0104)	0.0520 (0.0114)	0.0512 (0.0067)	0.0476 (0.0050)	0.0764 (0.0110)	0.0660 (0.0069)	0.0840 (0.0131)	0.0259 (0.0045)	0.0666 (0.0073)	0.0205 (0.0049)	0.1028 (0.0103)
λ_2^c	0.0080 (0.0023)	0.0577 (0.0060)	0.0136 (0.0021)	0.0105 (0.0012)	0.0822 (0.0058)	0.0189 (0.0019)	0.0318 (0.0050)	0.0110 (0.0019)	0.0220 (0.0022)	0.0194 (0.0029)	0.0320 (0.0032)
κ^c	1.2046 (0.2682)	0.4814 (0.1187)	0.5053 (0.0704)	2.0300 (0.2158)	0.4698 (0.0754)	1.1326 (0.1265)	0.8119 (0.1412)	0.4073 (0.0765)	1.2259 (0.1435)	0.2768 (0.0721)	1.1853 (0.1308)
λ_0^c	0.3367 (0.0253)	0.6483 (0.0442)	0.5971 (0.0201)	0.5614 (0.0227)	0.7195 (0.0330)	0.7705 (0.0303)	0.4455 (0.0260)	0.4140 (0.0152)	0.4552 (0.0231)	0.6373 (0.0293)	1.7143 (0.0885)

Source : Jolivet et. al (2006)

- We have shown that the distribution of wages among employees dominates the distribution of wages offered.

- Jolivet et al. (2006) have compared the distribution of wages of persons holding jobs with the wages at which unemployed persons are hired in 11 countries.

- They show that the cumulative distribution of wages in the population of workers as a whole *systematically stochastically dominates* the distribution of entry wages.

- This is a particular materialization of the broad idea of positive returns to labor market experience.

- Mobility decreases with the experience of the worker as he climbs the wage ladder, *i.e.*, the model generates positive returns to experience that depends on search frictions.

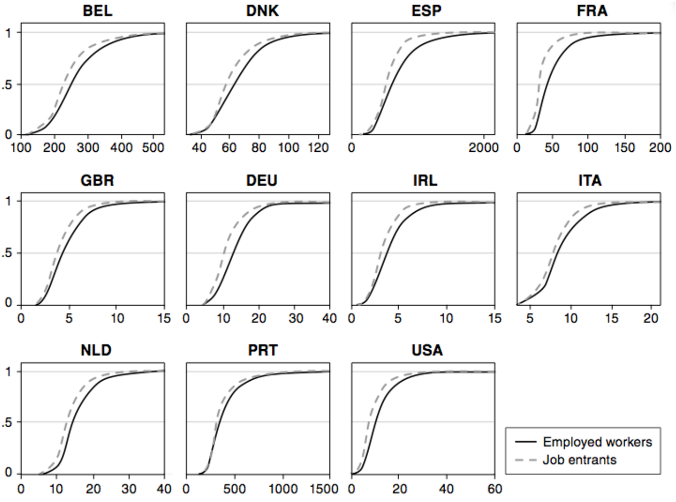


FIGURE 5.9
Cumulative distribution functions of hourly wages (in local currency) in 10 European countries (1994–1997) and in the United States (1993–1996).

Source: Jolivet et al. (2006).

- To explain wage distribution, one needs to specify *firms' behavior* :
  - Firms compete by posting wages to attract workers.
  - Firms make *take-it* or *leave-it* offers to worker.[19]
  - A wage offer is a **commitment to pay a constant wage** flow w until the worker leaves the match or is hit by a q-shock.

- The firm chooses the wage to post so as to maximize their *stationary instantaneous profit* (assuming $r \approx 0$) :[20]

$$\max_{w \geq z} \quad \pi(w) = (y - w) \underbrace{\ell(w)}_{\text{size of the firm}}$$

- The optimal wage satisfies :

$$\frac{\ell'(w)}{\ell(w)} = (y - w)^{-1}$$

19. In this benchmark equilibrium search model firms can not counter outside offers.
20. This assumption, which amounts to assume that r is close from 0, is grounded on the fact that r is small wrt to λ_u, λ_e, q.

- The wage-setting policy of firms allows us to obtain a *relation between the wage and employment*.

1. There are $\ell(w)$ employees in each firm that pays wage w and there is a mass $H'(w)$ of firms that pay wage w.

   Therefore, the total mass of employees paid wage w is :

$$H'(w)\ell(w)$$

2. By definition the mass of employees paid wage w is also equal to :

$$G'(w)(1-u)$$

- Hence, it follows that :

$$G'(w)(1-u) = H'(w)\ell(w) \tag{5}$$

- Making use of (4) and (5), we get :

$$\frac{\ell'(w)}{\ell(w)} = \frac{2\kappa H'(w)}{1+\kappa\bar{H}(w)} \qquad (6)$$

Appendix

- Recalling that the optimal behavior of firms implies $\frac{\ell'(w)}{\ell(w)} = (y-w)^{-1}$, we have that :

$$2(y-w)H'(w) + H(w) = \frac{1+\kappa}{\kappa} \qquad (7)$$

- This *differential equation* defines the distribution of wage offers that is compatible with :
  - the equilibrium flows,
  - the strategic wages setting behavior of firms.

- For any constant A, the general solution of (7) satisfies :

$$H(w) = A\sqrt{y-w} + \frac{1+\kappa}{\kappa}$$

Appendix

- The constant A can be determined using the fact that firms have no interest in offering a wage smaller than the reservation wage x of unemployed job seekers.

- Hence, making use of the fact that $H(x) = 0$, we find the equilibrium wage distribution :

$$H(w) = \frac{1+\kappa}{\kappa}\left[1 - \sqrt{\frac{y-w}{y-x}}\right] \tag{8}$$

- The wage distribution H is defined over the interval $[\underline{w}, \overline{w}]$:

$$\begin{aligned} \overline{w} &= y - (y-x)\left(\tfrac{1}{1+\kappa}\right)^2, \quad && \textit{for} \quad H(\overline{w}) = 1 \\ \underline{w} &= x, && \textit{for} \quad H(\underline{w}) = 0 \end{aligned}$$

- Hence, pure wage dispersion thus obtains even though workers and firms are all *ex-ante* identical.

- What if $\lambda_e = 0$?

    1. From equation (4), it follows that the sampling distribution and the observed distribution collapse in a single distribution :

    $$G(w) = H(w)$$

    2. From equation (8), the sampling distribution collapses in a single mass point at the reservation wage :

    $$\overline{w} = x$$
    $$\underline{w} = x$$

- Hence we are back to the Diamond's paradox, *i.e.*, search frictions are not sufficient to generate wage dispersion.

- Differentiation of $H(w)$ wrt w yields :

$$H'(w) = \frac{1+\kappa}{2\kappa} \frac{1}{\sqrt{(y-x)(y-w)}}$$

- The equilibrium density of the sampling distribution $H'(.)$ is an *increasing* function of the wages.

- This result is a consequence of the following properties :
  - all agents are homogeneous, and
  - the firms' strategy consists in offering a constant wage.

- Under these conditions, a firm that raises its wage w increases its volume of employment to the detriment of employment in other firms.

- This movement leads to an increasing relationship between the wage and the size of the firms.

- The model developed by Burdett and Mortensen (1998) delivers a number of empirical implications :

  1. The lower bound of the equilibrium wage distribution being equal to the reservation wage, an unemployed job seeker accepts all the offers.

  2. The wage of an employed individual rises when she moves from one job to another.

     The wages rise as workers gain experience, *i.e.*, the first wage is at least the reservation wage (when exiting unemployment), but every time they change firms, the wage rises, *i.e.* workers climb the jobs' ladder.

  3. The Mean-min ratio is much more higher with on-the-job search (about 1.2, see Hornstein et al., 2011) but is still below the empirical Mean-min ratio.

- Empirical implications (cont'd)

    4. The model predicts negative duration dependence.

       The *hazard rate of a job spell* is :

$$\lambda_e.[1 - H(w)]$$

       Hence, jobs spell with longer elapsed durations (and higher wages) reduce the option value to move for the worker.

- In this model, workers only move-up the wage ladder.

  It implies that the model does not explain the wage growth within the same firm.[21]

21. See e.g. Burdett and Coles (2003) or Stevens (2004) for such an extension.
Burdett and Coles (2003), Equilibrium Wage-tenure Contracts, Econometrica.
Stevens (2004), Wage-Tenure Contracts in a Frictional Labour Market : Firms Strategies for Recruitment and Retention, Review of Economic Studies. For recent empirical evidence see e.g. Bayer and Kuhn (2019), Which Ladder to Climb ? Decomposing Life Cycle Wage Dynamics, unpublished.

- Early attempts to estimate BM type models :
  - Bowlus, Kieffer and Neumann (1995) [22]
  - van den Berg and Ridder (1998) [23]
  - Bontemps, Robin, van den Berg (1999, 2000) [24]

- **Data :** Usual data sources to estimate BM type models are panel data e.g. :
  - France : *Enquete emploi*
  - UK : *British Household Panel Survey (BHPS)*
  - US : *Current Population Survey (CPS)*
  - ...

22. Bowlus et al. (1995), Estimation of Equilibrium Wage Distributions with Heterogeneity, *Journal of Applied Econometrics*.
23. van den Berg and Ridder (1998), An empirical equilibrium search model of the labor market, *Econometrica*.
24. Bontemps et. al (1999), Equilibrium Search with Continuous Productivity Dispersion : Theory and Nonparametric Estimation, *International Economic Review*.

- The estimation of the model requires to have information on :
  1. wages
  2. unemployment spells
  3. employment spells

- The model features {UE,EU,EE} transitions. All (random) transitions are Poisson processes, hence durations are exponentially distributed.

- As all job offers are accepted by the unemployed, the exit from unemployment hazard rate is λ_u.

- The job separation hazard rate is made up of two terms :
  - q (layoff)
  - $\lambda_e.[1 - H(w)]$ (quit)

  Hence, we get $q + \lambda_e.[1 - H(w)]$.

- Assume for simplicity no censoring and for an individual i denote by :
    - t_{u_i}, duration of an unemployment spell
    - t_{e_i}, duration of an employment spell
    - w_i, wage
    - $\mathbb{I}_{J2J}$, indicator variable for transition for employed workers :

$$\mathbb{I}_{J2J} = \begin{cases} 1 \text{ for } \textit{job-to-job (quit)} \\ 0 \text{ for } \textit{job-to-unemployment (layoff)} \end{cases}$$

    - $\mathbb{I}_e$, indicator variable for initial employment status :

$$\mathbb{I}_e = \begin{cases} 1 \text{ for } \textit{employment} \\ 0 \text{ for } \textit{unemployment} \end{cases}$$

- Conditional on being unemployed, the individual contribution of an unemployed worker to the likelihood is :

$$\ell(w_i, t_{u_i} | \mathbb{I}_e = 0) = \underbrace{\lambda_u . e^{-\lambda_u . t_{u_i}}}_{\text{Prob. to have an unemp. duration } t_u} \times \underbrace{h(w_i)}_{\text{Prob. to get a wage offer } w_i}$$

or using the log

$$\log \ell(w_i, t_{u_i} | \mathbb{I}_e = 0) = \log(\lambda_u) - \lambda_u . t_{u_i} + \log(h(w_i))$$

- Conditional on being employed, the individual contribution of an employed worker to the likelihood is :

$$
\begin{aligned}
&\ell(w_i, t_{e_i}, \mathbb{I}_{J2J} | \mathbb{I}_e = 1) = \\
&\underbrace{g\left(w_i\right)}_{\text{density of wages}} \times \underbrace{\left(q + \lambda_e.[1 - H(w_i)]\right).e^{-(q+\lambda_e.[1-H(w_i)]).t_{e_i}}}_{\text{Prob. to have an emp. duration } t_e} \\
&\times \underbrace{\left(\frac{\lambda_e.[1 - H(w_i)]}{q + \lambda_e.[1 - H(w_i)]}\right)^{\mathbb{I}_{J2J}}}_{\text{quit}} . \underbrace{\left(\frac{q}{q + \lambda_e.[1 - H(w_i)]}\right)^{1-\mathbb{I}_{J2J}}}_{\text{layoff}}
\end{aligned}
$$

or using the log

$$
\begin{aligned}
\log \ell(w_i, t_{e_i}, \mathbb{I}_{J2J} | \mathbb{I}_e = 1) \quad = \quad & \log(g\left(w_i\right)) - \left(q + \lambda_e.[1 - H(w_i)]\right).t_{e_i} \\
& + \mathbb{I}_{J2J} \log(\lambda_e.[1 - H(w_i)]) \\
& + (1 - \mathbb{I}_{J2J}) \log(q)
\end{aligned}
$$

- In a steady-state, the distribution of initial employment (resp. unemployment) statuses satisfies :

$$\Pr\{\mathbb{I}_e = 0\} = \frac{q}{q+\lambda_u} \; (\textit{unemployment})$$
$$\Pr\{\mathbb{I}_e = 1\} = \frac{\lambda_u}{q+\lambda_u} \; (\textit{employment})$$

- The likelihood of an individual observation then satisfies :

$$\ell_i = \left(\frac{q}{q+\lambda_u}.\ell(w_i, t_{u_i}|\mathbb{I}_e = 0)\right)^{1-\mathbb{I}_e} \times \left(\frac{\lambda_u}{q+\lambda_u}.\ell(w_i, t_{e_i}, \mathbb{I}_{J2J}|\mathbb{I}_e = 1)\right)^{\mathbb{I}_e}$$

  or using the log

$$\log \ell_i = (1-\mathbb{I}_e).\left(\log\left(\frac{q}{q+\lambda_u}\right) + \log \ell(w_i, t_{u_i}|\mathbb{I}_e = 0)\right) + \mathbb{I}_e.\left(\log\left(\frac{\lambda_u}{q+\lambda_u}\right) + \log \ell(w_i, t_{e_i}, \mathbb{I}_{J2J}|\mathbb{I}_e = 1)\right)$$

- Remark that the (log)-likelihood function ℓ_i is a function of the structural parameters $(q, \lambda_u, \lambda_e)$ and of the sampling distribution $H(w)$ and the cross-sectional distribution $G(w)$.

- Following Bontemps et al. (1999, 2000), an usual route to estimate BM type models is to use a sequential two-steps procedure.

- Sketch of the estimation procedure :
  - Estimate the cdf G and pdf g of the cross-sectional distribution using a non parametric estimator (e.g. empirical cdf for G and a kernel density estimator for g).[25]
  - Use the fact that the sampling distribution and the cross-sectional distribution are interrelated to get an estimate of H :

$$\widehat{H}(w,q,\lambda_e) = \frac{(1+\kappa)\widehat{G}(w)}{1+\kappa\widehat{G}(w)} \text{ and } \widehat{h}(w,q,\lambda_e) = \frac{1+\kappa}{\left(1+\kappa\widehat{G}(w)\right)^2}\widehat{g}(w)$$

    where $\kappa \equiv \frac{\lambda_e}{q}$.

  - Replace H and h with $\widehat{H}(w,q,\lambda_e)$ and $\widehat{h}(w,q,\lambda_e)$ into the likelihood.
  - Maximize the sample log-likelihood $\mathcal{L}$:

$$\max_{\{q,\lambda_e,\lambda_u\}} \mathcal{L}(q,\lambda_e,\lambda_u) = \sum_i \log \ell_i \left(q,\lambda_e,\lambda_u,\widehat{H}(w,q,\lambda_e)\right)$$

25. In Matlab `ecdf` calculates the Kaplan-Meier estimate of the cumulative distribution function (cdf) and `ksdensity` compute the kernel density or distribution estimate.

- The search equilibrium model presents one major flaw : the density of wage distribution is an increasing function of wage.
- This unpleasant property also applies to the cross-section wage density $G'(w)$.
- In practice, the wage distribution has a *log-normal* form, *i.e.* the density is increasing then decreasing. Appendix
- Some additional degree of firm and/or worker heterogeneity is needed to accommodate the typical hump shape of observed wage densities.

Burdett and Mortensen with heterogeneous firms

- It is possible to remedy this problem by assuming that firms are heterogeneous, *i.e.*, they operate different more or less productive technologies. [26]

- Let us assume that firms differ in their (constant) marginal productivity of labor y and that workers are still homogeneous.

- We assume that y does not depend on the number of workers at the firm, and consequently, we will refer to this firm as a firm of type-y and to y as the (labor) productivity of this firm.

- Upon receiving a job offer, workers draw the type of the firm from which the offer comes from in an exogeneous sampling distribution, $\Gamma(y)$, with support on $[\underline{y}, \overline{y}]$.

- Let $H(w)$ denote the corresponding equilibrium sampling distribution of job offers.

26. Bontemps, Robin and van den Berg (2000), *Op. Cit.*

- Each type-y firm offers a wage $w(y)$ that maximizes the steady-state profit flow :

$$\pi(w, y) = (y - w)\ell(w)$$

where $\ell(w)$ obtains from (2), (4) and (5) :

$$\ell(w) = (1-u)\frac{1+\kappa}{\left[1+\kappa\overline{H}(w)\right]^2} = \frac{\lambda_u}{\lambda_u + q}\frac{1+\kappa}{\left[1+\kappa\overline{H}(w)\right]^2}$$

- Hence under the same set of assumptions than in the *benchmark model*, the profit of a type-y firm paying a wage w satisfies :

$$\pi(w, y) = (y - w)\ell(w) = (y - w)\frac{\lambda_u}{\lambda_u + q}\frac{1+\kappa}{\left[1+\kappa\overline{H}(w)\right]^2}$$

- In equilibrium :
  1. Each type-y firm offers a wage $w(y)$ that maximizes the steady-state profit flow :

$$w(y) = \arg\max_{w} \quad \pi(w, y)$$

- Equilibrium (cont'd)

  The equilibrium level of profit reached by a type-y firm is therefore :

$$\pi\left(w(y), y\right) = \left(y - w(y)\right) \frac{\lambda_u}{\lambda_u + q} \frac{1+\kappa}{\left[1 + \kappa \overline{H}\left(w\left(y\right)\right)\right]^2}$$

2. The sampling distribution of wages and firm types must be identical :

$$H(w(y)) = \Gamma(y)$$

- The fraction of offers that pay $w(y)$ or less is simply equal to the fraction of offers made by employers with productivity y or less.

- This comes from the fact that there is a direct one-to-one mapping between the productivity, y, and the wage $w(y)$.

- The wage function, $w(y)$, must satisfy the first order condition for an interior solution, $\frac{\partial \pi(w(y),y)}{\partial w(y)} = 0$, which can be written as :

$$\frac{2\kappa H'(w(y))(y - w(y))}{1 + \kappa \overline{H}(w(y))} = 1$$

- Using the fact that $H'(w(y))\, w'(y) = \Gamma'(y)$, it follows that the optimal wage relationship between w and y, $w(y)$, solves the ordinary differential equation :

$$w'(y) = \frac{2\kappa \Gamma'(y)(y - w(y))}{1 + \kappa (1 - \Gamma(y))}$$

with the boundary condition $w(\underline{y}) = z$.[27] Closed form solution

- As profit is positive for any participating firm, we have that more productive employers pay higher wages.[28]

27. The firm with the smallest productivity, $\underline{y}$, offers unemployed workers their reservation wage x and hires workers only from the unemployment pool.

28. Note also that due to firms' market power, wages are lower than productivity.

- Given the equilibrium wage function $w(y)$, the associated equilibrium offer and wage densities can be written as :

$$h\left(w(y)\right) = H'\left(w(y)\right) = \frac{\Gamma'\left(y\right)}{w'\left(y\right)} = \frac{1+\kappa\left(1-\Gamma\left(y\right)\right)}{2\kappa\left(y-w(y)\right)}$$

- Similarly, making use of $G\left(w\right) = \frac{H(w)}{1+\kappa[1-H(w)]}$, it follows that :

$$g\left(w\left(y\right)\right) = G'\left(w\left(y\right)\right) = \frac{\left(1+\kappa\right)}{\left(1+\kappa\left[1-H\left(w\right)\right]\right)^2}.H'\left(w(y)\right)$$

- As $\Gamma(y)$ is increasing in y, a sufficient condition for a declining wage offer density, h, is that the profit per worker, $y-w(y)$ increases with y.

- One can rationalize (almost) any wage distribution, H, as an equilibrium outcome by an appropriate choice of the underlying productivity distribution Γ.

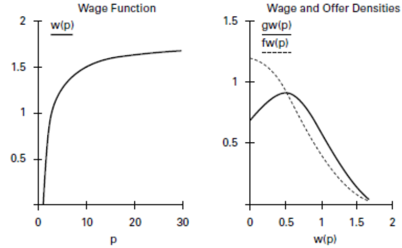
- Based on Christensen et al. (2001), Mortensen (2003) proposes the following *(rough) parametrization* of the model :[29]

q	λ_e	λ_u	z
0.287	0.207	0.207	0

together with a Pareto distribution of productivity, given by

$$\Gamma(y) = 1 - y^{-\alpha}$$

and shape parameter $\alpha = 2$.



29. Christensen, Lentz, Mortensen, Neumann, and Werwatz (2001, 2005), On-the-Job-Search and the Wage Distribution, *Journal of Labor Economics*.

- Heterogeneity in firms' productivity considerably improves the empirical predictions of equilibrium search models.

- The wage posting model provides an excellent description of the wage dispersion and the selection process of workers into jobs.

- Still it reveals some counterfactual predictions :

  - Predicts zero downward wage mobility between job spells,

  - Predicts zero wage mobility within spells,

  - Predicts (*implied by the observed distribution* $G(w)$), a distribution of firm productivity with an implausibly long right tail.

    Intuition :

    - In the model firms have too much market power because the wage-setting mechanism limits competition between firms.
    - To explain high wages, productivities need to be higher than those observed in the data.

Sequential auctions and bargaining

- A limitation of the Burdett and Mortensen (1998) framework is that there is too little between-firm competition.

- Postel-Vinay and Robin (2002) dampen the monopsonistic power of the employers by allowing them to counter outside offers through Bertrand competition. [30]

- Two-sided heterogeneity :

  - firms' heterogeneity (permanent firm effect), y
  - workers' heterogeneity (permanent worker effect), ε
  - match productivity, $y \times \varepsilon$
  - flow income while unemployed, $z \times \varepsilon$

- The *productivity of firms* and the *status of workers* (employed/unemployed) are perfectly observed.

30. Postel-Vinay and Robin (2002), Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*.

- Let us denote by :
  - $V_u(\varepsilon)$, lifetime utility of unemployment for a type-ε worker
  - $V_e(w, \varepsilon, y)$, lifetime utility of a job paid w for a type-ε worker in a type-y firm
- Firms make take-it or leave-it offers to workers and renegotiations are by mutual consent only.
- When an unemployed worker meets a firm, the firm offers him a wage, $x(\varepsilon, y)$, that makes him indifferent between employment and unemployment :

$$V_e\left(\underbrace{x(\varepsilon, y)}_{\text{reservation wage}}, \varepsilon, y\right) = V_u(\varepsilon)$$

- When a worker paid w in a type-y firm get an offer from a type-y' firm, the two employers compete to hire the same worker, *i.e.*, there is Bertrand competition between the *incumbent* and the *poacher*.

- Several cases may arise :
    1. If $y' > y$, the highest wage the incumbent may offer is $w = y \times \epsilon$ and the poacher offer the lowest wage $\omega(\varepsilon, y, y')$ that allow to attract the worker. Hence :

$$V_e\left(\omega\left(\varepsilon, y, y'\right), \varepsilon, y'\right) = V_e\left(\underbrace{y \times \epsilon}_{\text{max wage the incumbent may offer}}, \varepsilon, y\right)$$

    In this case the worker moves to the high productivity firm and reaps the full surplus of the incumbent, *i.e.* from the less productive employer. [31]

31. The bargaining outcome looks like a second-price auction as the worker gets the full surplus from the second highest bidder (the incumbent).

2. If $y' < y$, two possibilities must be considered :

- If $V_e(w, \varepsilon, y) > V_e(y' \times \epsilon, \varepsilon, y')$ where $w = y' \times \epsilon$ is the *maximum wage* the poacher may offer. The worker has no credible threat to leave the match.

  In this case, the worker stays and the the status quo prevails.

- If $V_e(w, \varepsilon, y) < V_e(y' \times \epsilon, \varepsilon, y')$, where $w = y' \times \epsilon$ is the *maximum wage* the poacher may offer. The worker has a credible threat to leave the match.

  The incumbent (counter)-offers a wage $\omega^c(\varepsilon, y', y)$ such that :

$$V_e\left(\omega^c(\varepsilon, y', y), \varepsilon, y\right) = V_e\left(y' \times \epsilon, \varepsilon, y'\right)$$

  In this case, the worker stays with a wage increase, and extracts the full surplus of the less productive employer.

- The sequential auctions model improves upon the existing literature in addressing the issue of wage dynamics.[32]

- In particular, the model allows to understand how wage earners may :

  - obtain a wage increase within the same firm, *i.e.* explains the returns to tenure.

  - accept a wage-cut while moving to another firm.

    Intuition : Let $y' > y$. A worker paid w in a type-y firm can move to a type-y' firm with a wage-cut such that $\omega(\varepsilon, y, y') < w$ because the higher productivity in the new firm allows to get larger wage rises in the future.

    This is **consistent with the empirical observations** reported in Jolivet et al. (2006) where JTJ transitions with a wage-cut account for more than 20% of JTJ transitions.

32. Cahuc, Postel-Vinay and Robin (2006) relax the assumption of take-it or leave-it offers and introduce wage bargaining into the model to better account for wage dynamics.
Cahuc, Postel-Vinay and Robin (2006), Wage bargaining with on-the-job search : Theory and evidence, *Econometrica*.

The Godfather

- Remark that wage-cut obtains more simply assuming a Godfather shock.
- This trick is used in e.g. Jolivet et al. (2006) where employed workers receive an offer at rate $\lambda_2 > 0$ which they cannot reject.
- In such a context, the job separation hazard rate rewrites :

$$q + \lambda_e.[1 - H(w_i)] + \underbrace{\lambda_2}_{\text{Godfather shock}}$$

- Following such a shock, an employed worker is thus forced to leave her current job for another job, with a new wage drawn from the sampling distribution.

Conclusion

- We have so far mainly focussed on partial equilibrium model (with the exception of equilibrium search models) where the behavior of firms is left unexplained.
- Equilibrium search models have focussed in endogenizing the wage-offer distribution but have mostly ignored the endogeneity of the offer arrival rate.
- In addition, an operational description of the labor market would also require that parameters governing workers' transitions (such as the exit rate from unemployment or job destruction) be made endogeneous.
- Search and matching models partly fill these gaps.
- These models are particularly well suited to help us understand the equilibrium (natural) rate of unemployment.
- In addition, they are attractive for studying labor market institutions.

Appendix : Burdett-Mortensen (I)

- The derivative of (4) wrt w reads :

$$
\begin{aligned}
G'(w)\left(1+\kappa\overline{H}(w)\right) &= H'(w)\left[1+\kappa G(w)\right] \\
G'(w) &= \frac{H'(w)\left[1+\kappa G(w)\right]}{1+\kappa\overline{H}(w)}
\end{aligned}
$$

- Reinserting into $G'(w)(1-u)=\ell(w)H'(w)$, we get :

$$
\begin{aligned}
\frac{H'(w)\left[1+\kappa G(w)\right]}{1+\kappa\overline{H}(w)}(1-u) &= \ell(w)H'(w) \\
(1+\kappa G(w))(1-u) &= \ell(w)\left(1+\kappa\overline{H}(w)\right)
\end{aligned}
$$

- Taking the log and differentiating wrt to w, we get :

$$
\frac{\kappa G'(w)}{1+\kappa G(w)}=\frac{\ell'(w)}{\ell(w)}-\frac{\kappa H'(w)}{1+\kappa\overline{H}(w)}
$$

- Then using the expression of $G'(w)$, equation (6) follows.

Go Back

Appendix : Burdett-Mortensen (II)

- The homogeneous equation associated with the differential equation satisfies :

$$\frac{H'(w)}{H(w)} = -\frac{1}{2(y-w)}$$

- Integrating both sides wrt w, one gets :

$$\begin{aligned}
\int \frac{H'(w)}{H(w)} \mathrm{d}w &= \int \frac{\mathrm{d}\log H(w)}{\mathrm{d}w} \mathrm{d}w = \int \mathrm{d}\log H(w) = \log H(w) + c_0 \\
-\frac{1}{2}\int \frac{1}{y-w} \mathrm{d}w &= \frac{1}{2}\int \frac{\mathrm{d}\log(y-w)}{\mathrm{d}w} \mathrm{d}w = \frac{1}{2}\int \mathrm{d}\log(y-w) = \frac{1}{2}\log(y-w) + c_1
\end{aligned}$$

- It follows that :

$$\begin{aligned}
\log H(w) &= \frac{1}{2}\log(y-w) + c_1 - c_0 \\
H(w) &= \exp^{(c_1 - c_0)} \exp^{\frac{1}{2}\log(y-w)} = A\exp^{\frac{1}{2}\log(y-w)}
\end{aligned}$$

where $A = \exp^{(c_1 - c_0)}$ is an arbitrary constant.

- Using the fact that $x^a = \exp^{a\log(x)}$, the general solution to the homogeneous equation satisfies :

$$H(w) = A(y-w)^{\frac{1}{2}} = A\sqrt{y-w}$$

Go Back

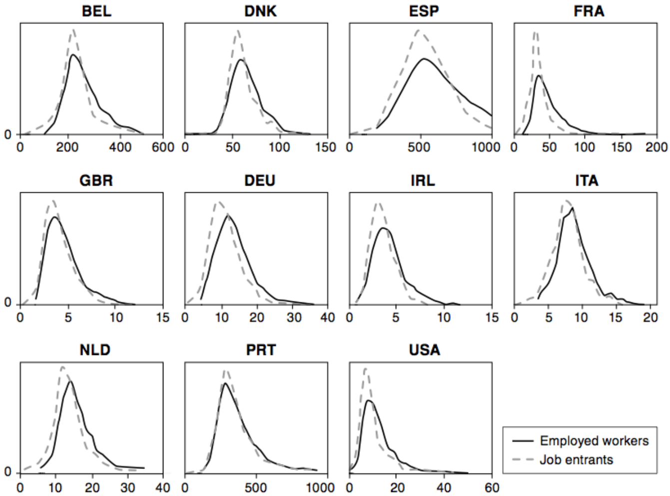


FIGURE 5.10
Density functions of hourly wages (in national currency) in 10 European countries (1994–1997) and in the United States (1993–1996).

Source: Jolivet et al. (2006).

Go Back

Appendix : Burdett-Mortensen with heterogeneous firms

- Making use of the envelope theorem :

$$\frac{\mathrm{d}\pi}{\mathrm{d}y}\left(y, w\left(y\right)\right) = \frac{\partial \pi}{\partial y}\left(y, w\left(y\right)\right) = \frac{\lambda_u\left(1+\kappa\right)}{\left(\lambda_u+q\right)\left[1+\kappa\overline{\Gamma}\left(y\right)\right]^2}$$

- Assuming free entry such that the profit of the firm with the weakest productivity is zero, i.e. $\pi\left(w\left(\underline{y}\right), \underline{y}\right) = 0$, and integrating, the profit rewrites as :

$$\pi(w\left(y\right), y) = \int_{\underline{y}}^{y} \frac{\lambda_u\left(1+\kappa\right)}{\left(\lambda_u+q\right)\left[1+\kappa\overline{\Gamma}\left(\varsigma\right)\right]^2}\mathrm{d}\varsigma$$

- Making use of $\pi(w, y) = (y-w)\,\ell\left(w\right)$ and $\ell\left(w\right)$, the wage equation follows :

$$w\left(y\right) = y - \left[1+\kappa\overline{\Gamma}\left(y\right)\right]^2 \int_{\underline{y}}^{y} \frac{1}{\left[1+\kappa\overline{\Gamma}\left(\varsigma\right)\right]^2}\mathrm{d}\varsigma$$

Go Back