IV Methods in Macro

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IV in Macroeconomics

$$\boxed{z_t = \alpha u_t^i + \eta_t \qquad \eta_t \sim \mathcal{WN}(0, \sigma_\eta^2)}$$

- Wealth on **new instruments** and expanding Macro literature:
  - **oil shocks** (e.g. Hamilton, 2003; Kilian, 2008; Känzig, 2018)
  - **fiscal spending shocks** (e.g. Ramey, 2011; Ricco et al., 2016; Ramey and Zubairy, 2018)
  - **tax shocks** (e.g. Romer and Romer, 2010; Leeper et al., 2013; Mertens and Ravn, 2012; Mertens and Montiel-Olea, 2018)
  - **conventional/unconventional monetary policy shocks** (e.g. Romer and Romer, 2004; Gürkaynak et al., 2005; Gertler and Karadi, 2015; Miranda-Agrippino and Ricco, 2017; Jarocinski and Karadi 2017; Altavilla et al, 2019; Ambrogio Cesa-Bianchi et al 2020; Swanson 2020)
  - **technology news** (e.g. Cascaldi-Garcia and Vukotic, 2019)

IV in Macroeconomics

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IV in Macroeconomics

$$z_t = \alpha u_t^i + \underbrace{(\ldots \qquad \ldots)}_{\text{contamination}} + \eta_t \qquad \eta_t \sim \mathcal{WN}(0, \sigma_\eta^2)$$

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  - **technology news** (e.g. Cascaldi-Garcia and Vukotic, 2019)

Intuition

- Isolate exogenous variation in the innovation of the ‘indicator’ variable using ‘external’ information
- The **external instruments** provide a measure of unobserved structural shocks
- The contemporaneous transmission coefficients are consistently estimated using moments of observables

Usual Conditions for SVAR-IV

Stock (2008), Stock and Watson (2012, 2018) and Mertens and Ravn (2013)

Reduced-Form VAR

$$A(L)Y_t = e_t$$

z_t is an **instrument** for a shock of interest u_t^1

Conditions – Identification in SVAR-IV

(i) $E[u_t^1 z_t] = \alpha$ *(Relevance)*

(ii) $E[u_t^{2:n} z_t] = 0$ *(Contemporaneous Exogeneity)*

(iii) $u_t = Proj(u_t | Y_t, Y_{t-1}, \ldots)$ *(Fundamentalness/Invertibility)*

External Instrument for the Shock

① Given condition (iii) (Invertibility) we have

$$e_t = \underset{[n\times n]}{B_0^{-1}} u_t$$

$$\begin{pmatrix} e_t^1 \\ e_t^{2:n} \end{pmatrix} = \left(\begin{array}{c:c} \underset{[n\times 1]}{b_1} & \underset{[n\times (n-1)]}{b_2} \end{array} \right) \begin{pmatrix} u_t^1 \\ u_t^{2:n} \end{pmatrix}$$

that we can rewrite as

$$\begin{pmatrix} e_t^1 \\ e_t^{2:n} \end{pmatrix} = \left(\begin{array}{c:c} \underset{[1\times 1]}{b_{11}} & b_{21} \\ \hdashline \underset{[(n-1)\times 1]}{b_{12}} & b_{22} \end{array} \right) \begin{pmatrix} u_t^1 \\ u_t^{2:n} \end{pmatrix}$$

External Instrument for the Shock

- Let's consider

$$\begin{pmatrix} \mathbb{E}(e_t^1 z_t) \\ \mathbb{E}(e_t^{2:n} z_t) \end{pmatrix} = \begin{pmatrix} b_1 & \vdots & b_2 \end{pmatrix} \begin{pmatrix} \mathbb{E}(u_t^1 z_t) \\ \mathbb{E}(u_t^{2:n} z_t) \end{pmatrix} = b_1 \alpha$$

$$\Big\downarrow$$

$$\mathbb{E}(e_t^1 z_t)^{-1} \mathbb{E}(e_t z_t) = \begin{pmatrix} 1 \\ b_{11}^{-1} b_{12} \end{pmatrix}$$

that is true given

② (**Relevance**) z_t correlated with the shock of interest

$$\mathbb{E}[z_t u_t^1] = \alpha$$

③ (**Validity**) z_t is orthogonal to all other structural shocks at time t

$$\mathbb{E}[z_t u_t^{2:n}] = 0$$

Contemporaneous Transmission Coefficients

$$\mathbb{E}(e_t^1 z_t)^{-1}\mathbb{E}(e_t^{2:n} z_t) = b_{11}^{-1} b_{12}$$

- b_1 is consistently estimated up to a scale
- Equivalent to regressing $e_t^{2:n}$ on e_t^1 using z_t as external instrument
- Method:
  1. Estimate a VAR(p) and get estimates of e_t – i.e. residuals
  2. Regress $\hat{e}_t$ on z_t
  3. Calculate $b_{11}^{-1} b_{12}$ as ratio of regression coefficients
  4. Choose normalisation – e.g. $b_{11} = 1$
- To gain efficiency we can instrument $Y_{1,t}$ in the VAR

Usual Conditions for IV-LP$^{\perp}$

Stock and Watson (2018)

Local Projections-IV$^{\perp}$

$$Y_{i,t+h} = F_{h,i1}\widehat{Y}_{1,t} + \gamma_h' W_t + \nu_{i,t+h}^h \ ,$$

Let's consider z_t an instrument for a shock of interest u_t^1

Conditions – Identification in LP-IV$^{\perp}$

(i) $E[u_t^{1,\perp} z_t^{\perp}] = \alpha$ *(Relevance)*

(ii) $E[u_t^{2:n,\perp} z_t^{\perp}] = 0$ *(Contemporaneous Exogeneity)*

(iii) $E[u_{t-j}^{\perp} z_t^{\perp}] = 0$ *for all* $j \neq 0$ *(Lead-Lag Exogeneity)*

where $x_t^{\perp} = x_t - Proj(x_t|\mathcal{W}_t)$ *for a given* x_t*, and* $\mathcal{W}_t = \mathrm{span}\{W_t\}$

Usual Conditions for IV-LP

Stock and Watson (2018)

Local Projections without controls

$$Y_{i,t+h} = F_{h,i1}\widehat{Y}_{1,t} + \nu_{i,t+h}$$

Conditions – Identification in LP-IV

(i) $E[u_t^1 z_t] = \alpha$ *(Relevance)*

(ii) $E[u_t^{2:n} z_t] = 0$ *(Contemporaneous Exogeneity)*

(iii) $E[u_{t+j}^k z_t] = 0$ *for all* $\{j, k\}$ *(Lead-Lag Exogeneity)*

Usual Conditions for IV-LP

Stock and Watson (2018)

Proposition – Relationship between SVAR-IV, LP-IV$^{\perp}$ and invertibility

Let $\mathcal{Z}$ denote the set of scalar stochastic processes (instruments) such that for all $Z \in \mathcal{Z}$, Z satisfies LP-IV Conditions (i), (ii) and (iii) for $j > 0$, but not (iii) for $j < 0$.

Let $W_t = \{Y_{t-1}, Y_{t-2}, \ldots\}$. Then LP-IV$^{\perp}$ is satisfied for all $Z \in \mathcal{Z}$ if and only if

(a) *Z satisfies condition SVAR-IV and*

(b) *the invertibility condition holds*

Partial Invertibility

Invertibility

A shock is **invertible** if it is a contemporaneous linear combination of the VAR residuals

$$u_t^1 = \lambda' e_t$$

Partial Invertibility

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Invertibility (II)

A shock u_t^1 is invertible if

$$u_t^1 = Proj(u_t^1 | Y_t, Y_{t-1}, \ldots)$$

Semi-structural MA representation

Proposition – Semi-structural Representation

Let the Wold representation of a covariance stationary vector process Y_t be

$$Y_t = C(L)e_t \qquad e_t \sim \mathcal{WN}(0, \Sigma) \tag{1}$$

where Σ is the positive definite variance-covariance matrix of Wold innovations. If the system is partially invertible in the shocks u_t^i, for $i = 1, \ldots, m$, i.e. there exist m vectors λ_i such that $\lambda_i' e_t = u_t^i$, then Y_t admits a semi-structural moving average representation of the form

$$Y_t = C(L)\Sigma \sum_{i=1}^{m} \lambda_i u_t^i + C(L)\Sigma\tilde{\lambda}\xi_t \tag{2}$$

where ξ_t is an $(n-m) \times 1$ vector of linear combinations of Wold innovations that is orthogonal to all u_t^i for $i = 1, \ldots, m$, i.e. $E(u_t^i \xi_t') = 0$.

Semi-structural MA representation

- The Wold representation factorise in two **orthogonal** terms:
  - the first depends only on the **partially-invertible** shocks at **time** t
  - the second on **past, current and future** other **non-invertible shocks**

- **Intuition:**

$$e_t = \Theta_0 B(L) u_t = \widetilde{B}(L) u_t = \left(b_1 \;\; b_2(L)\right) u_t$$

  where $B(L)$ is a Blaschke matrix (Lippi, Reichlin, 1994)

- The IRFs of a **partially identified** SVARs are the the dynamic causal responses to **partially invertible** shocks

Identification in SVAR-IV under Partial Invertibility

Conditions – Identification in SVAR-IV

Let $u_t^{1:m}$ denote the m invertible structural shocks in the system, and $u_t^{m+1:n}$ the remaining $n-m$ non-invertible shocks. Let z_t be an instrument for the shock of interest u_t^1, and define $z_t^{\perp} = z_t - Proj\left(z_t | \mathcal{H}_{t-1}^{Y}\right)$.

The impact effects of u_t^1 onto Y_t and the (relative) IRFs are correctly identified in a SVAR-IV if z_t satisfies the following conditions:

(i) $E[u_t^1 z_t] = \alpha$ *(Relevance)*

(ii) $E[u_t^{2:n} z_t^{\perp}] = 0$ *(Contemporaneous Exogeneity)*

(iii) $E[u_{t-j}^{m+1:n} z_t^{\perp}] = 0$ *for all* $j \neq 0$ *for which* $E[u_{t-j}^{m+1:n} e_t'] \neq 0$. *(Limited Lead-Lag Exogeneity)*

Identification in SVAR-IV under Partial Invertibility

Condition (iii) arises because of the dynamics

- If **all the shocks are invertible**
  $\Longrightarrow$ Condition (iii) is trivially satisfied

- If **some** of the shocks are **non-invertible**
  $\Longrightarrow$ The instrument can 'safely' incorporate past and future invertible shocks only

- If **all of the other** shocks are **non-invertible**
  $\Longrightarrow$ The instrument can 'safely' incorporate only past/future of the shock of interest

Identification in SVAR-IV under Partial Invertibility

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What if **Condition (iii)** is **violated**?

Identification in SVAR-IV under Partial Invertibility

Remark – Violation of the Exogeneity Conditions

Let z_t be an instrument that satisfies Condition (i) but possibly fails Condition (ii) and Condition (iii), i.e.

$$z_t = \alpha u_t^1 + \sum_k \beta_k u_{t-k}^{\not 1} \ , \tag{3}$$

where $u_t^{\not 1}$ is a non-invertible shock, for $k \in \mathbb{Z}$. The Wold representation can be mapped into the structural shocks employing Blaschke factors

$$e_t = \left(b_1 \;\; b_2(L)\right) u_t \ , \tag{4}$$

The estimated IRFs for variable i, to shock 1, at horizon h, are biased and of the form

$$\widetilde{IRF}_{i1}^{h} = IRF_{i1}^{h} + \left[C_h \sum_{j \in J} \sum_{k \in K} b_{2,j,\not 1} \frac{\beta_k}{\alpha} \delta_{jk} \right]_i \ , \tag{5}$$

Identification in SVAR-IV and LP-IV

Proposition – Relation between SVAR-IV under Partial Invertibility and LP-IV$^{\perp}$

Let Z be the set of scalar stochastic processes z_t that satisfy LP-IV Conditions (i) and (ii) – i.e. $E[u_t^1 z_t] = \alpha$ and $E[u_t^{2:n} z_t] = 0$ –, but satisfy Condition LP-IV (iii) $E[u_{t-j} z_t] = 0$ only for $j < 0$ and not for $j > 0$. Let us also assume that $Proj(u_t|\mathcal{H}_{t-1}^{Y}) = 0$. Let $\widetilde{Z} \subseteq Z$ be such that any $z_t \in \widetilde{Z}$ satisfies the LP-IV$^{\perp}$ conditions for $\mathcal{W}_t \equiv \mathcal{H}_{t-1}^{Y}$. z_t is an element of $\widetilde{Z}$ if and only if it identifies the shock of interest in a Structural VAR in Y_t either as external or internal instrument.

Identification in SVAR-IV and LP-IV

	u_t^1 invertible	u_t^1 non-invertible
Strong Lead-Lag Exogeneity		
$E[u^i_{t-j} z_t] = 0 \; \forall \; i \; \& \; j \neq 0$	LP-IV SVAR-IV SVAR-H	LP-IV SVAR-H
Limited Lead-Lag Exogeneity but Contamination by Past Shocks		
$E[u^i_{t-j} z_t] \neq 0$ for some $j > 0$ ($= 0$ for $j < 0$) but $E[u^i_{t-j} z_t^{\perp}] = 0$ and $E[u^i_{t-j} e'_t] = 0$	LP-IV$^{\perp}$ SVAR-IV SVAR-H	LP-IV$^{\perp}$ SVAR-H
Limited Lead-Lag Exogeneity but Contamination by Future Shocks		
$E[u^i_{t-j} z_t] \neq 0$ for some $j < 0$ but $E[u^i_{t-j} z_t^{\perp}] = 0$ and $E[u^i_{t-j} e'_t] = 0$	SVAR-IV	–

6. Monetary Policy Shocks (II)

Narrative Identification

Monetary Policy Shocks

Romer, Romer (2004)

- Romer and Romer (2000) presented evidence suggesting that the Fed had superior information when constructing inflation forecasts compared to the private sector
- Romer and Romer (2004) introduces a **new narrative measure** of MP shocks:
  1. Derive a series of 'intended federal funds rate changes' around FOMC meetings using data on actual changes and narrative accounts for the months without meetings
  2. Separate the endogenous response of policy to the economy from the exogenous shock by regressing the intended funds rate change on the current rate and on the (private) central bank's Greenbook forecasts of output growth, inflation and unemployment over the next two quarters

Monetary Policy Shocks

Romer, Romer (2004)

$$
\begin{aligned}
\Delta ffr_m = \phi_0 + \phi_i \widetilde{ffr}_m + \phi_{u_0} F_{cb,m} u_0 \\
+ \sum_{i=-1}^{2} \phi_{\pi,i} F_{cb,m} \Delta y_i + \sum_{i=-1}^{2} \varphi_{\Delta y,i} (F_{cb,m} \Delta y_i - F_{cb,m-1} \Delta y_i) \\
+ \sum_{i=-1}^{2} \phi_{\pi,i} F_{cb,m} \pi_i + \sum_{i=-1}^{2} \varphi_{\pi,i} (F_{cb,m} \pi_i - F_{cb,m-1} \pi_i) + z_m^{mp}
\end{aligned}
$$

- Changes associated with meeting m
- The i subscripts refer to the horizon of the forecast: 1 is the previous quarter, 0 is the current quarter, 1 and 2 are one and two quarters ahead

Monetary Policy Shocks

Cochrane, 2004 on Romer, Romer (2004)

To measure the effects of monetary policy on output it is enough that the shock is orthogonal to output forecasts.

The shock does not have to be orthogonal to price, exchange rate, or other forecasts.

It may be predictable from time t information; it does not have to be a shock to agent's or the Fed's entire information set.

Monetary Policy Shocks

Romer, Romer (2004)

Baseline regression:

$$\Delta y_t = a_0 + \sum_{k=1}^{11} a_k D_{kt} + \sum_{i=1}^{24} b_i \Delta y_{t-i} + \sum_{j=1}^{36} c_j z_{t-j}^{mp} + \varepsilon_t$$

- Autoregressive Distributed Lag (ADL) Model
- Single variable regression
- D_k are monthly dummies
- Additional controls for robustness

Monetary Policy Shocks

Romer, Romer (2004)

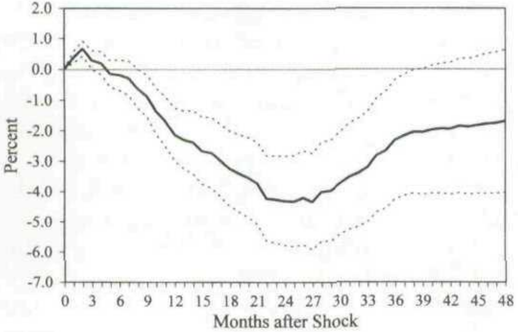


Figure: Output response to a MP shock, Romer, Romer (2004)

Monetary Policy Shocks

Romer, Romer (2004)

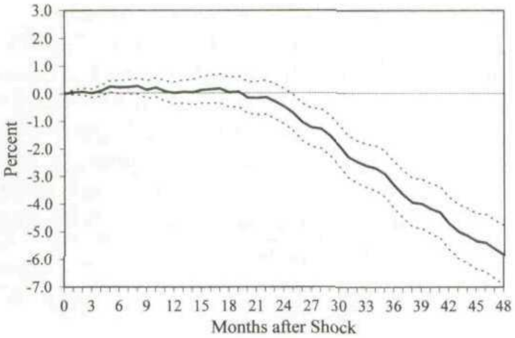


Figure: Price response to a MP shock, Romer, Romer (2004)

High-Frequency Identification

HFI: Intuition

- The price of an interest rate futures contract is a function of agents' expectations about future interest rates

$$\downarrow$$

Futures prices embed the expected policy path

- Reactions following central banks' announcements measure the unexpected component of policy

$$\downarrow$$

Monetary surprises measure innovation in monetary policy

Interest Rate Futures

Rudebusch (1989), Kuttner, (2001), Sack, (2004)

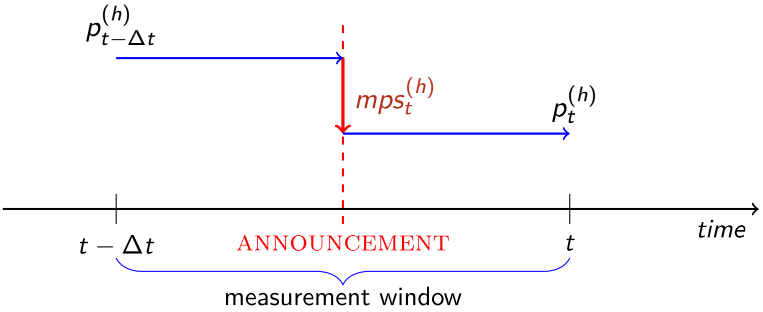
- Future contract that pays a function of i_{t+h}

$$p_t^{(h)} = F_t^M\left(i_{t+h}\right) + \zeta_t^{(h)}$$

- $p_t^{(h)}$: price of futures contract expiring at $t+h$
- $F_t^M\left(i_{t+h}\right)$ market forecast of interest rate expected at time $t+h$
- $\zeta_t^{(h)}$: risk premium

Monetary Surprises

Gürkaynak, Sack, Swanson (2005)



$$mps_t^{(h)} \equiv p_t^{(h)} - p_{t-\Delta t}^{(h)} = \underbrace{\left[F_t^M\left(i_{t+h}\right) - F_{t-\Delta t}^M\left(i_{t+h}\right)\right]}_{\text{expectation revision}} + \underbrace{\left[\zeta_t^{(h)} - \zeta_{t-\Delta t}^{(h)}\right]}_{\simeq 0}$$

A Typical Announcement Day

- Fed funds Futures Contract FF4

$$p_t^{(FF4)} = F_t^M \left(i_{t+3}\right) + \zeta_t^{(FF4)}$$

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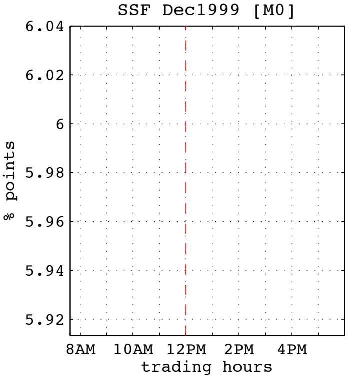

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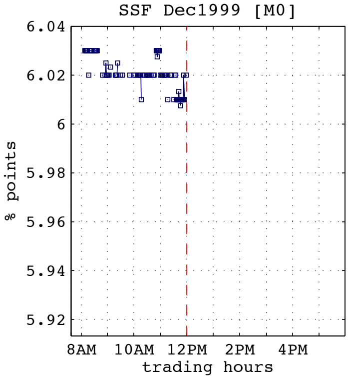

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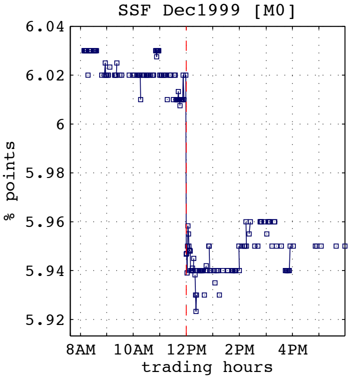

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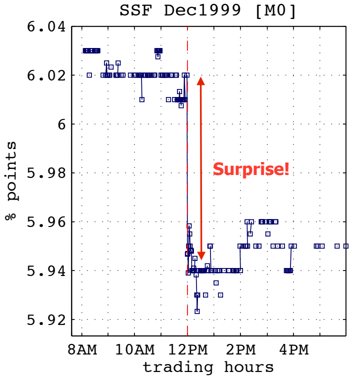


From Monetary Surprises to Monetary Policy Shocks

Gertler and Karadi (2015)

Assumptions:

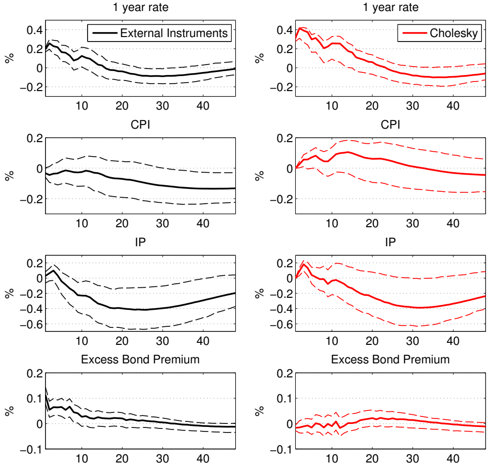
- The announcement is the only event in Δt
- The risk compensation $\zeta_t^{(h)}$ is unaffected by the monetary policy announcement
- Markets are fully rational
- Markets efficiently incorporate all available information as soon as it is released
- Markets have the same information set of the central bank

price updates $\Longleftrightarrow$ **monetary policy shock**

$$\boxed{mps_t = u_t^{mp} + \text{measurement error}}$$

Monetary Policy Shocks

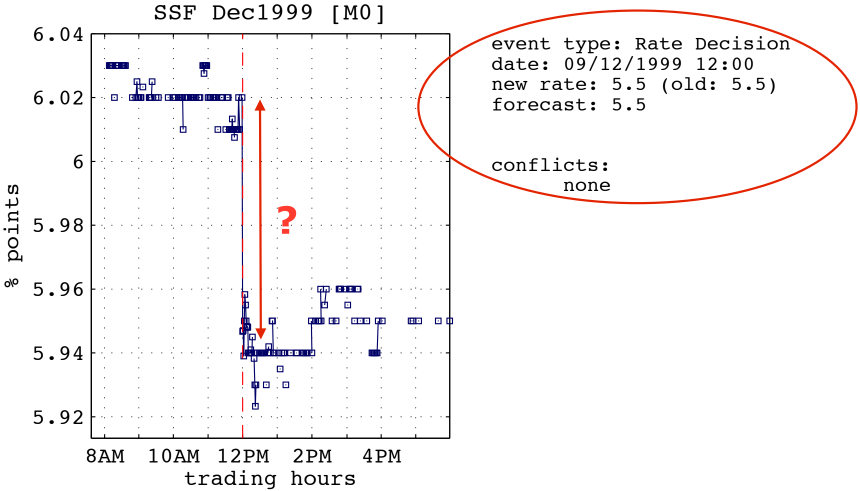
Gerthler, Karadi, 2015



First stage regression: F: 21.55 robust F: 17.64 R2: 7.76% Adjusted R2: 7.40%

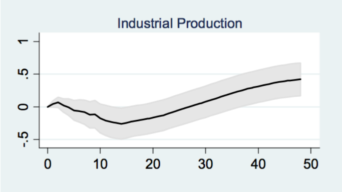
Fragility in the Empirical Evidence?

Puzzles in the Surprises?

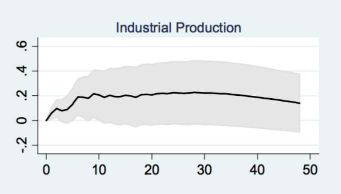


What are the Effects of MP shocks on Output?

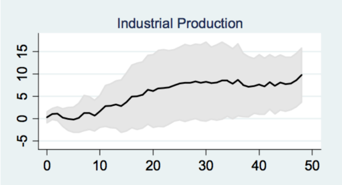
Ramey, 2015



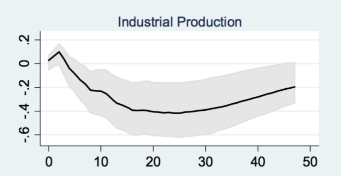
(a) hybrid VAR 69-07



(b) hybrid VAR 83-07



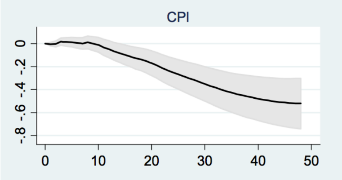
(c) GK Proxy LP 90-12



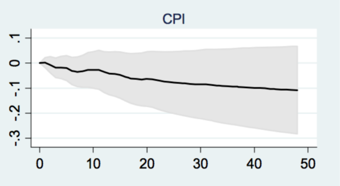
(d) GK Proxy SVAR 90-12

What are the Effects of MP shocks on Prices?

Ramey, 2015



(e) hybrid VAR 69-07



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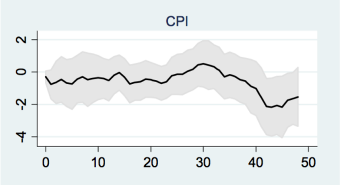

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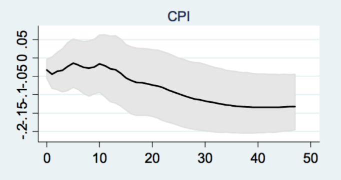

(h) GK Proxy SVAR 90-12

Signalling Effects & Monetary Policy Shocks

Monetary Policy and Information: Intuition

- **Interest rate hike** to informationally constrained agents
  - **MP shock**
    $\Longrightarrow$ lower output and inflation
  - **Endogenous reaction** to demand shocks
    $\Longrightarrow$ higher output and inflation

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- Agent's respond to to new information sluggishly

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- Agent's respond to to new information sluggishly

- Market surprises blend MP shocks with current and past macro shocks
  $\Longrightarrow$ **price and output puzzles**

Testing for Information Frictions

Greenbook Forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Inflation	2.538						
	(0.03)***						
Output		2.752					
		(0.02)***					
$h = -1$			2.024				
			(0.07)**				
$h = 0$				2.636			
				(0.02)***			
$h = 1$					2.436		
					(0.03)***		
$h = 2$						1.045	
						(0.40)	
All n & h							1.578
							(0.05)***
R^2	.036	.054	.027	.045	.040	.000	.058

Controlling for the Signalling Channel of Monetary Policy

Miranda-Agrippino, Ricco (2020)

1. At FOMC frequency $\Longrightarrow$ **'Signaling Channel'**

$$FF4_m = \alpha_0 + \sum_{j=-1}^{3} \theta_j F_t^{cb} x_{q+j} + \sum_{j=-1}^{2} \vartheta_j \left[F_t^{cb} x_{q+j} - F_{t-1}^{cb} x_{q+j} \right] + MPI_m$$

Controlling for the Signalling Channel of Monetary Policy

Miranda-Agrippino, Ricco (2020)

2. Monthly aggregation

$$\overline{MPI}_t = \sum_{m \in t} MPI_m$$

Controlling for the Signalling Channel of Monetary Policy

Miranda-Agrippino, Ricco (2020)

3. At monthly frequency $\Longrightarrow$ **Slow Absorption of Information**

$$\overline{MPI}_t = \phi_0 + \sum_{j=1}^{12} \phi_j \overline{MPI}_{t-j} + MPI_t$$

Controlling for the Signalling Channel of Monetary Policy

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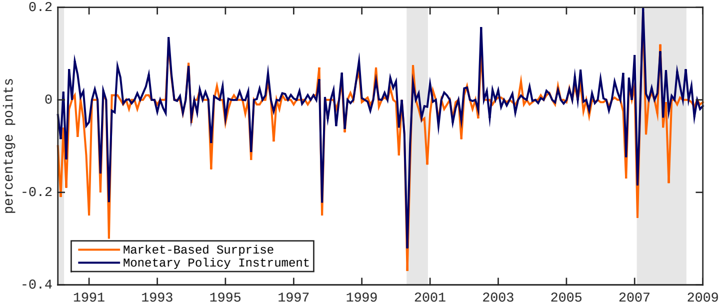
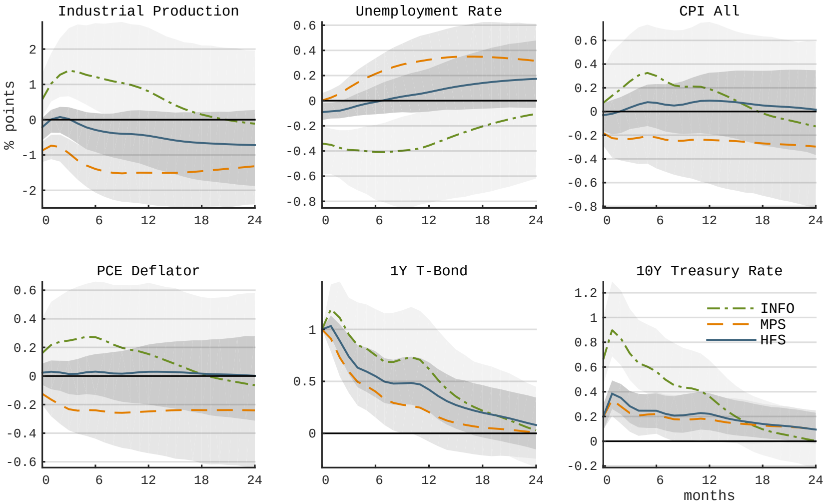


Information vs MP Shocks

Miranda-Agrippino, Ricco (2020)



Generalised IV identification

What if the shock is non-invertible?

- Leeper et al. (2013)'s RBC model with fiscal foresight
- Two iid shocks: technology, $u_{a,t}$, and tax $u_{\tau,t}$

$$
\begin{aligned}
a_t &= u_{a,t} \\
\tau_t &= u_{\tau,t-2},
\end{aligned}
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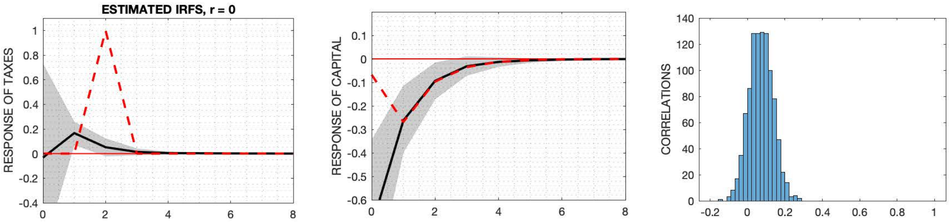
- SVAR in capital and taxes with IV on simulated data



IV identification in VARs

- External IV: Mertens and Ravn (2013) and Stock and Watson (2018)
  - Pros: flexible time span
  - Cons: partial invertibility needed
- Internal IV: Ramey (2011), Plagborg-Møller and Wolf (2021)
  - Pros: recoverability and invertibility not needed
  - Cons: same time span data and instrument

    (many additional parameters, potentially very large information set, lag order fixed by the VAR)

Innovations and shocks

- VAR residuals e_t are linear combinations of the current and lagged structural shocks u_t

$$e_t = A(L)y_t = A(L)B(L)u_t = Q(L)u_t \tag{6}$$

- Generally, the inverse map is not exact function of the e_t

$$u_t = P(u_t|\mathcal{H}) + s_t = D'(F)e_t + s_t \tag{7}$$

where P is the linear projection operator and
$\mathcal{H} = \overline{\text{span}}(e_{j,t-k}, j = 1, \ldots, n, k \in \mathbb{Z})$

Invertible & recoverable shocks

Invertibility

A shock is **invertible** if it is a contemporaneous linear combination of the VAR residuals

Recoverability

A shock is **recoverable** if it is a linear combination of the present and future values of the VAR residuals

A general representation result

General Representation

Any vector process Y_t with an SMA and VAR form can be represented as

$$\begin{aligned} Y_t = C(L)e_t &= C(L)Q^i u_t^i + C(L)Q^r(L)u_t^r + C(L)Q^n(L)u_t^n \\ &= C(L)\Sigma D^i u_t^i + C(L)\Sigma D^r(L)u_t^r + C(L)\Sigma D^n(L)u_t^n. \end{aligned} \tag{8}$$

where $C(L)$ the Wold representation coefficients and Σ is the covariance of e_t

- u_t^i the fundamental structural shocks
- u_t^r the recoverable (but nonfundamental) shocks
- u_t^n of the nonrecoverable ones
- $Q^h(L)u_t^h$, for $h = i, r, n$, is the projection of e_t onto u_{t-k}^h, with $k \geq 0$;
- $D^h(F)e_t$ is the projection of u_t^h onto e_{t+k}, with $k \geq 0$

A general representation result

General Representation

Moreover, the following properties hold:

(i) D^i and Q^i s.t $D^{f\prime}\Sigma D^i = Q^{f\prime}\Sigma^{-1}Q^i = I_{q_f}$, for q_f fundamental shocks;

(ii) $D^r(L)$ and $Q^r(L)$ s.t. $D^{r\prime}(F)\Sigma D^r(L) = Q^{r\prime}(F)\Sigma^{-1}Q^r(L) = I_{q_r}$, for q_r recoverable shocks

Instrument

The Instrument

$$\tilde{z}_t = \beta(L)\tilde{z}_{t-1} + \mu'(L)x_{t-1} + \alpha u_{it} + w_t,$$

where w_t is an error orthogonal to $u_{i,t}$, to z_{t-k}, x_{t-k}, $k > 0$, and to ε_{t+k}, $k \geq 0$

Instrument

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IV conditions

(i) $E(\tilde{z}_t, u_{it} | \tilde{z}_{t-k}, x_{t-k}, k > 0) = \alpha \neq 0$

(ii) $E(\tilde{z}_t, \varepsilon_{t+k} | u_{it}, \tilde{z}_{t-k}, x_{t-k}, k > 0) = 0 \quad \textit{for } k \geq 0$

Result #1: Invertible case

- Consider the projection equation

$$e_t = \psi z_t + e_t.$$

If u_{it} is fundamental, then its (absolute) IRF are

$$b_i(L) = \frac{C(L)\psi}{\sqrt{\psi' \Sigma^{-1} \psi}}.$$

- Consider the projection equation

$$z_t = \delta' e_t + e_t.$$

If u_{it} is fundamental, then

$$u_{it} = \frac{\delta' e_t}{\sqrt{\delta' \Sigma \delta}}.$$

Recoverability (but non-invertibility): intuition

- Residuals function of current and past structural shocks:

$$e_t = A(L)y_t = A(L)B(L)u_t = Q(L)u_t$$

- Under recoverability structural are functions of future residuals.

$$u_t = D'(F)e_t$$

- IRF

$$B(L) = C(L)Q(L)$$

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- $Q(L)$ obtained in a regression of the residuals on current and lagged instrument

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- Residuals function of current and past structural shocks:

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- Under recoverability structural are functions of future residuals.

$$u_t = D'(F)e_t$$

- IRF

$$B(L) = C(L)Q(L)$$

- $Q(L)$ obtained in a regression of the residuals on current and lagged instrument
- $D'(F)$ by regressing the instrument on current and future residuals

Result #2: Recoverable (but non-invertible) case

- Consider the projection equation

$$e_t = \psi(L) z_t + e_t.$$

If u_{it} is recoverable, its (absolute) impulse response functions are given by the equation

$$b_i(L) = \frac{C(L)\psi(L)}{\sqrt{\sum_{k=0}^{\infty} \psi_k' \Sigma^{-1} \psi_k}}.$$

- Let us consider the projection equation

$$z_t = \delta'(F) e_t + v_t.$$

$$u_{it} = \frac{\delta'(F) e_t}{\sqrt{\sum_{k=0}^{\infty} \delta_k' \Sigma \delta_k}}.$$

Result #3: Non-recoverable case

- Shock (by definition) cannot be retrieved.
- Relative impulse response functions can be obtained

$$b_i(L)\alpha = C(L)\psi(L)\sigma_z^2. \tag{9}$$

Testing for invertibility and recoverability

Invertibility:

- If invertible $\delta_k = 0$ for all positive k
- standard F-test for the joint significance of the coefficients of the leads in Eq. (10)
- test H_0 of fundamentalness vs H_1 nonfundamentalness
- If not invertibility, the degree of fundamentalness is

$$\hat{R}_f^2 = \hat{\delta}_0' \widehat{\Sigma} \hat{\delta}_0 / \sum_{k=0}^{r} \hat{\delta}_k' \widehat{\Sigma} \hat{\delta}_k.$$

Testing for invertibility and recoverability

Recoverability:

$$z_t = \delta'(F)e_t + v_t \tag{10}$$

- If recoverable $\hat{u}_{it} = \hat{\delta}(F)\hat{e}_t$ (Plagborg-Møller and Wolf, 2022)
- Ljung-Box Q-test to the estimated projection $\hat{\delta}(F)\hat{e}_t$
- H_0 is recoverability (serial uncorrelation) vs H_1 nonrecoverability (serial correlation)

IV Identification in practice

1. Regress $\tilde{z}_t$ onto its lags and a set of regressors x_t, to get z_t

$$\tilde{z}_t = \beta(L)\tilde{z}_{t-1} + \mu'(L)x_{t-1} + \alpha u_{it} + z_t \tag{11}$$

   If the F-test does not reject the null $H_0 : \beta(L) = 0$ & $\mu'(L) = 0$, step 1 can be skipped

2. Estimate a VAR(p) with OLS to obtain $\widehat{A}(L)$, $\widehat{C}(L) = \widehat{A}(L)^{-1}$, $\hat{e}_t$ and $\widehat{\Sigma}$

3. Regress $\hat{z}_t$ on the current value and the first r leads of the Wold residuals:

$$\hat{z}_t = \sum_{k=0}^{r} \hat{\delta}_k' \hat{e}_{t+k} + \hat{v}_t = \hat{\delta}(F)\hat{e}_t + \hat{v}_t$$

   Save the fitted value of the regression, $\hat{\eta}_t$

   Test for invertibility

IV Identification in practice

4. **Invertible shock**: Estimate δ and the unit-variance shock. Estimate

$$e_t = \psi' z_t + e_t$$

and estimate IRFs according to #1. Estimate the variance decomposition

4′. **Invertibility is rejected**: Test for recoverability

5. **Recoverable shock**: Estimate the unit-variance shock according. Estimate

$$e_t = \psi(L) z_t + e_t$$

and IRFs according to #2. Estimate the variance decomposition

IV Identification in practice

5′. **Nonrecoverable shock**:

- Expand information set of the VAR specification and repeat steps 2-4, or
- Estimate

$$e_t = \psi' z_t(L) + e_t$$

Estimate lower and upper bounds according and the corresponding variance contributions

IV Identification in practice