Lecture 4: Search and Matching

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1. https://sites.google.com/view/franck-malherbet/home/teaching/labor-economics

- In every developed country, jobs and manpower have movements of *considerable magnitude*.

- All developed economies are affected by unemployment in various ways.

- For a worker, the search for a job that fits her requirements and skills is a process that is costly and time consuming, and so does a firm in selecting individuals.

- Imperfections in the available information entail frictional unemployment which is characterized by the simultaneous presence of unemployed persons and vacant jobs.

- The level of unemployment depends on the processes of job destruction and creation.
- It also depends on institutional factors which influence the length of an unemployment spell and the hiring process of firms.
- From now on, we envisage the hiring process as a phenomenon of *matches* between employers and workers.
- The probability of finding a job (*a match between a firm and worker*) depends on the prevailing conditions on the labor market.
- These conditions are gathered in an aggregate index, the labor market tightness which is the ratio of the number of vacant jobs to the number of searching persons.

Beveridge curve

- Beveridge (1944) proposed to use the relationship between vacant jobs and the unemployment level to assess the extent of worker reallocation.
- The Beveridge Curve illustrates this linkage between the *unemployment rate*, u, and the *vacancy rate*, v (ratio of vacant jobs to labor force).
- It illustrates the **simultaneous presence** of unemployed persons and vacant jobs due to :
  - Mobility costs associated with *e.g.* location and skill,
  - Imperfect information.

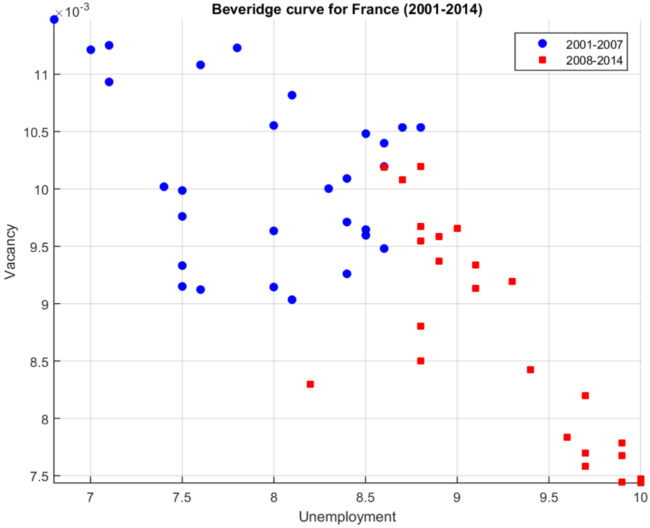


FIGURE 1 – Beveridge curve. Source : OECD

The Canonical Search and Matching model

- We develop a *simple model of the labor market* in which transaction costs explain the simultaneous existence of vacant jobs and unemployed workers.

- Main hypothesis :

  - Time is continuous. We focus on steady-states. [2]
  - There is a continuum of infinitely-lived workers and firms discounting time at a common rate $r > 0$.
  - Workers and firms are risk neutral.
  - Firms are small, *i.e.* have one job slot that is either vacant or filled.
  - Wages are not competitive, *i.e.* the workers are not paid their marginal productivity.

2. See e.g. Pissarides (2000) (chapter 1, section 1.7) for an out-of-steady-state analysis.

Matching function

- In practice, *job search procedures* are characterized by a large number of frictions :
  - mismatch between vacant jobs and skills of workers,
  - imperfect information about jobs and workers.
- The central idea is that trade in the labor market is a decentralized economic activity.
- It is :
  - uncoordinated
  - time consuming
  - costly

- The matching function is the key element of most macro models of the labor market.
- It builds on the idea of search frictions and can be seen as a way to endogenize the job arrival rate of job search models.
- The *matching function* goes straight to an *aggregate* level and does not take into account the diversity of individual actions. [3]
- It describes how the *flow of hires*, M, is linked to :
  - the number (stock) of vacant jobs, V,
  - the number of persons (stock) in search for work, S.

3. Put differently, we do not provide sound micro foundations for the matching function. See e.g. Cahuc et al. (2014) (chapter 9, section 3.1.1) for more details.

- A match is a meeting between a worker and a firm.

- The number of matches is a function of the amount of search on both sides of the market and is captured by the following aggregate function :

$$M = M(a \times V, s \times S)$$

where :

  - s, search effort of the worker,
  - a, advertising effort of the firm.

- Assuming no OTJS and normalizing search/advertising effort to one, the matching function rewrites :[4]

$$M = M(V, U)$$

where U stands for the number of unemployed workers.

4. See e.g. Pissarides (2000) (chapter 5) for a model with search intensity and job advertising or Gavazza, Mongey and Violante (2018), Aggregate recruiting intensity, *American Economic Review*.

Properties of the matching function

- The matching function summarizes the trading technology between these two sides of the market as a production function summarizes the production technology.

- It is increasing and concave in both of its arguments :

$$\frac{\partial M\left(V,U\right)}{\partial V}>0;\ \frac{\partial M\left(V,U\right)}{\partial U}>0;\ \frac{\partial^2 M\left(V,U\right)}{\partial V^2}<0;\ \frac{\partial^2 M\left(V,U\right)}{\partial U^2}<0$$

- In addition (essentiality) :

$$M=M(0,U)=M(V,0)=0$$

- It is generally assumed that the matching function is homogeneous of degree one (CRS).[5]

5. Not an innocuous assumption but supported by most of the empirical analyses, see e.g. Petrongolo and Pissarides (2001) for a survey.

Petrongolo and Pissarides (2001), Looking into the Black Box : A Survey of the Matching Function, *Journal of Economic Literature*.

Properties of the matching function (cont'd)

- Let us denote by $\theta \equiv \frac{V}{U}$, the labor market tightness.

- Making use of the CRS assumption, the probability of filling a vacancy during a small interval of time writes as :

$$h = \frac{M}{V} = \frac{M\left(V, U\right)}{V} = M\left(1, \frac{1}{\theta}\right) \equiv m\left(\theta\right)$$

  and the probability to find a job as :

$$f = \frac{M}{U} = \frac{V}{U}\frac{M\left(V, U\right)}{V} = \theta m\left(\theta\right)$$

- It is often assumed a function of the *Cobb-Douglas form* :

$$M = \kappa V^{\eta} U^{1-\eta}$$

  where $\kappa > 0$ is a scale parameter and $\eta \in (0, 1)$ is the elasticity of the matching function wrt the number of vacancies, V.

Properties of the matching function (cont'd)

- Using the Cobb-Douglas form, h and f rewrite as :

$$h(\theta) = \kappa\theta^{\eta-1} \text{ where } h'(\theta) < 0$$

$$f(\theta) = \kappa\theta^{\eta} \text{ where } f'(\theta) > 0$$

- It follows that the probability :
  - to fill a vacancy (*vacancy hiring rate*) is decreasing in θ,
  - to find a job (*job finding rate*) is increasing in θ.

- **Trading externalities** : the increase in the number of vacant jobs diminishes the rate at which vacant jobs are filled and increases the exit rate from unemployment.

- It is in the interest of unemployed workers for firms to create jobs, but in the interest of each firm for the number of vacancies to be as low as possible :
  - positive between-group externalities, *i.e.* spillover (*thick-market*) externality.
  - negative within-group externalities, *i.e.* congestion externality.

Empirical elements of the matching function

- Adopting the Cobb-Douglas form, the explanatory variables are the stocks of unemployed persons (U_t) and vacant jobs (V_t)

- Taking the log, the job finding rate rewrites :

$$\ln(f_t) = \eta \ln \theta_t + \phi_t$$

 where ϕ_t designates the logarithm of the efficiency parameter.

- Most often the estimation uses *simple OLS regression procedures* (see e.g. Petrongolo and Pissarides, 2001).

- Borowczyk-Martins et al. (2013) have noted however that those estimations are probably biased since the decision to post a vacancy is not independent of the efficiency of the matching process. [6]

6. Borowczyk-Martins, Jolivet, and Postel-Vinay (2013), Accounting for endogeneity in matching function estimation, *Review of Economic Dynamics*.

- Cahuc et al. (2016, 2020) estimate the parameter η to be approx. 0.4 for the *french labor market* over the period 2005-2010.[7]

TABLE 1 – Estimates of the parameters of the matching function

	(1) OLS	(2) IV
Dep. var.		Labor market tightness (log)
		First stage
Entries		$.63^{***}$
		(.05)
Entries (-1)		-1.31^{***}
		(.17)
Entries (-2)		-0.65^{***}
		(.12)
Dep. var		Job finding rate (log)
		Second stage
Labor market tightness (log)	$.38^{***}$	$.43^{***}$
	(.07)	(.15)
Date FE	Yes	Yes
R^2	0,33	
Nb. Observations	879	879

Source : Pole emploi and EMMO-DMMO. Note : Estimation of the parameter of the job matching function equation on 348 employment pools from 2005 to 2010.
* significant at 10 percent, ** significant at 5 percent, *** significant at 1 percent.

7. Benghalem et al. (2020), Taxation of Temporary Jobs : Good Intentions With Bad Outcomes ?, *The Economic Journal*.

Beveridge curve

- Let us denote by :
  - U, the number of unemployed workers,
  - L, the number of employed workers,
  - $N = U + L$, the labor force.
- The law of motion of the number of unemployed workers is given by :

$$\frac{\mathrm{d}U}{\mathrm{d}t} = qL - \theta m(\theta) U$$

  where q stands for the exogenous job separation rate.

- In a steady-state, one gets :

$$(q + \theta m(\theta)) U = qN \Longleftrightarrow (q + \theta m(\theta)) \underbrace{\frac{U}{N}}_{\text{unemp. rate}} = q$$

- The stationary value of the unemployment rate satisfies :

$$u = \frac{q}{q + \theta m(\theta)} \tag{1}$$

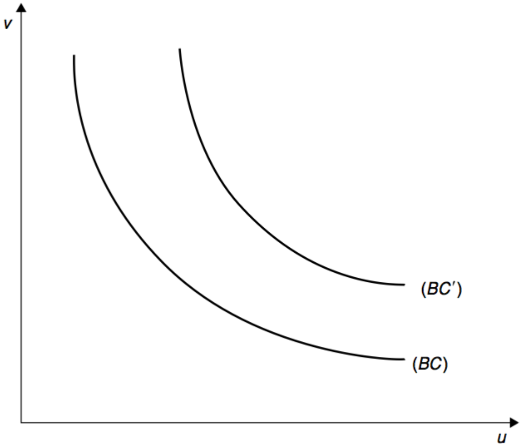


Figure 9.16
The Beveridge curve: the relation between the unemployment rate (u) and the vacancy rate (v).

- Equation (1) describes a relationship between the unemployment rate u and the vacancy rate v.

- It expresses the equilibrium of worker flows between employment and unemployment.

- This relationship yields the Beveridge curve, a downward-sloping and convex to the origin curve in the (u, v) or (u, θ) plane.

- The position of the Beveridge curve reflects the efficiency of the matching technology.

- Let us denote by :
  - Π_e, the expected profit of a filled job,
  - Π_v, the expected profit of a vacant job.

- In a *stationary environment*, the expected profit from a filled job satisfies :

$$\Pi_e = \frac{1}{1 + r\mathrm{d}t}\left[(y - w)\mathrm{d}t + q\mathrm{d}t\Pi_v + (1 - q\mathrm{d}t)\Pi_e\right]$$

  where w stands for the wage.

- This equation rewrites :

$$r\Pi_e = y - w + q(\Pi_v - \Pi_e) \tag{2}$$

- **Interpretation :**
  - A filled job generate a flow profit $y - w$ per unit of time and changes state at rate q.
  - The change of state yields a net return equal to $\Pi_v - \Pi_e$.

- The costs of a vacant job per unit of time are denoted by γ.

- These costs represent the expenses incurred in holding the position open and looking for an employee with the right skills to fill it.

- Since vacant jobs are filled at rate $m(\theta)$, the expected profit satisfies :

$$\Pi_v = \frac{1}{1 + r\mathrm{d}t} \left\{ -\gamma \mathrm{d}t + m(\theta)\mathrm{d}t\Pi_e + \left[1 - m(\theta)\mathrm{d}t\right] \Pi_v \right\}$$

- Or again,

$$r\Pi_v = -\gamma + m(\theta)(\Pi_e - \Pi_v) \tag{3}$$

- **Interpretation :**
  - A vacant job costs γ per unit of time and change state at rate $m(\theta)$.
  - The change of state yields a net return equal to $\Pi_e - \Pi_v$.

- As long as the expected profit from a vacant job remains positive, new entrepreneurs will enter the labor market and create jobs.

- In equilibrium all profit opportunities from new jobs are exploited, driving rents from vacant jobs to zero. Hence, the free entry condition implies :

$$\Pi_v = 0$$

- It follows, using (3), that :

$$\underbrace{\Pi_e}_{\text{Expected profit from a new job}} = \underbrace{\frac{\gamma}{m(\theta)}}_{\text{Expected cost of hiring a worker}}$$

- Combining with (2), one get the job creation (JC) condition :

$$\frac{\gamma}{m(\theta)} = \frac{y - w}{r + q} \tag{4}$$

- At free entry equilibrium, the average cost of a vacant job must be equal to the profit expected from a filled job.

- Differentiation wrt θ and w yields :

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}w}\right|_{JC} = \frac{(m(\theta))^2}{(r+q)\gamma m'(\theta)} < 0$$

- The job creation condition defines a decreasing relation between the wage and the labor market tightness, which is analogous to *labor demand* in the neoclassical theory of the firm.

- Let us denote by :
  - V_e, the expected utility of an employee,
  - V_u, the expected utility of an unemployed worker.

- When employed, a worker produces a quantity y, gets a real wage w and bears a risk of losing her job at rate q.

  The expected utility of an employee at stationary equilibrium satisfies :

$$rV_e = w + q(V_u - V_e) \tag{5}$$

- When unemployed, a worker gets z and finds jobs at rate $\theta m(\theta)$.

  The expected utility of an unemployed worker searching for a job satisfies :

$$rV_u = z + \theta m(\theta)(V_e - V_u) \tag{6}$$

Bargaining

- When a worker and a vacant job come together, the employer and the potential employee bargain over the wage.

- The wage bargaining outcomes is described by a surplus sharing rule.

- Let us define :
  - $\Pi_e - \Pi_v$, the firms' net surplus (rent),
  - $V_e - V_u$, the workers' net surplus (rent).

- Rent represents the difference between what individuals obtain through the contractual relationship and what the best opportunity outside the contract would bring them.

- The total surplus, Ω, is defined by the sum of the rents of a *job-worker match*, and satisfies :

$$\Omega = \Pi_e - \Pi_v + V_e - V_u$$

- The Nash (1950) solution is the most common solution to determine wages in the bulk of the literature on labor markets with frictions. [8]

- Why ? There is, a priori, **no particular rationale** (either empirical or theoretical) to restrict to the Nash solution.

- However several lines of arguments are put to the fore. The NBS :
  1. is flexible and tractable,
  2. has a natural strategic counterpart (see e.g. Osborne and Rubinstein, 1990), [9]
  3. ease comparison with the existing literature.

- The Nash solution has been challenged both on theoretical and empirical grounds.

- As emphasized by Thomson (1994), other solutions are important challengers to the Nash solution. [10]

8. Nash (1950), The Bargaining Problem, *Econometrica*.
9. Osborne and Rubinstein (1990), Bargaining and markets, Academic press.
10. Thomson (1994), Cooperative models of bargaining, Handbook of Game Theory.

Nash Bargaining

- The wage, w, derived from the (*generalized*) NBS maximizes the (*weighted*) expected present value of the job to the worker and to the employer, net of the value of searching for an alternative partner.

- Let us denote $\beta \in (0;1)$, the bargaining power of the worker.

- The wage, w, satisfies :

$$w = \arg\max \left(\Pi_e - \Pi_v\right)^{1-\beta} \left(V_e - V_u\right)^{\beta}$$

Appendix

- Taking the log and differentiating wrt w yields :

$$\frac{1-\beta}{\Pi_e - \Pi_v} = \frac{\beta}{V_e - V_u}$$

or

$$V_e - V_u = \beta\Omega \quad \text{and} \quad \Pi_e - \Pi_v = (1-\beta)\,\Omega$$

- Assuming free entry, using (2), (5) and the foc of the Nash solution, one gets :

$$(1-\beta)(w-rV_u)=\beta(y-w) \Leftrightarrow w=rV_u+\beta(y-rV_u)$$

- Workers receive their reservation wage rV_u and a fraction β of the net surplus that they create by accepting a job.

- It follows that :
  - If $\beta = 1$, the worker has all bargaining power then she reaps all of production y.
  - If $\beta = 0$, the employer has all the bargaining power then the worker is paid her reservation wage rV_u.
  - In the intermediate case, $(0 < \beta < 1)$, the wage is a *convex combination* of y and rV_u.

- Using $V_e - V_u = \beta\Omega$ and remarking that $\Pi_e = (1-\beta)\,\Omega = \frac{\gamma}{m(\theta)} \Longleftrightarrow \Omega = \frac{\gamma}{(1-\beta)m(\theta)}$, the expected utility of an unemployed worker rewrites :

$$rV_u = z + \theta m(\theta)\,\beta\Omega = z + \frac{\gamma\beta\theta}{1-\beta}$$

- The wage curve (WC) follows :

$$w = (1-\beta)\,rV_u + \beta y = (1-\beta)\,z + \beta\,(y + \gamma\theta) \tag{7}$$

where $\gamma\theta = \gamma\frac{v}{u}$ is the average hiring cost per unemployed worker.

- The wage equation defines an increasing function between the wage and the labor market tightness, which is analogous to the *labor supply* in Walrasian models.

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}w}\right|_{WC} = \frac{1}{\beta\gamma} > 0$$

Labor market equilibrium

- The labor market equilibrium is defined by a tuple-(w, θ) that solves the following two equations :

$$\begin{aligned} \frac{\gamma}{m(\theta)} &= \frac{y - w}{r + q} \\ w &= (1 - \beta) z + \beta (y + \gamma\theta) \end{aligned}$$

where γ, y, r, q, β and z are exogeneous parameters.

- The model is *block-recursive*, once θ and w are determined, the equilibrium unemployment rate follows :

$$u = \frac{q}{q + \theta m(\theta)}$$

- Most often the impact of exogeneous parameters on labor market equilibrium can be de deduced from shifts of the job creation condition (4) and the wage equation (7).

- Using (4) and (7), it can be useful to completely defines the equilibrium value of the labor market tightness :

$$\frac{\gamma}{m(\theta)} = \frac{(1-\beta)(y-z)}{r+q+\beta\theta m(\theta)} \tag{8}$$

- For instance, it is straightforward to remark from the equilibrium conditions that a rise in y increases :
  - the expected profit (push θ up)
  - the wage (push θ down)

- Total effect on θ is therefore **ambiguous** as there are two countervailing forces on wages and profits.

- Making use of (8), it is however easy to show that :[11]

$$\frac{d\theta}{dy} > 0$$

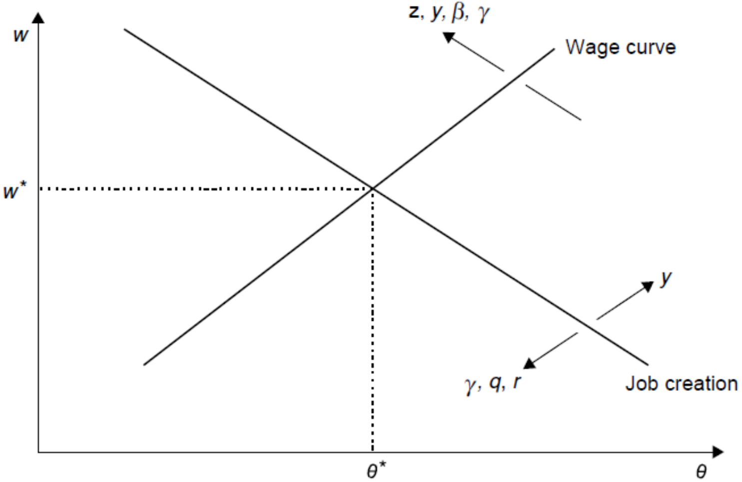
▸ Appendix

- Hence, a rise in productivity, y, increases the labor market tightness, θ. As the unemployment rate does not directly depend on y and is decreasing in θ, it decreases.

- Total differentiation with respect to the exogeneous parameters yields :

	γ	y	z	β	r	q
$\frac{d\theta}{d.}$	$-$	$+$	$-$	$-$	$-$	$-$

11. The derivation wrt remaining parameters are left as homework.


z, y, β, γ
Wage curve
w
w*
y
γ, q, r
Job creation
θ*
θ

- We assumed that firms holding a vacancy pay a flow cost γ and that free entry drives down the value of a vacant job to zero.

- Alternatively, consider the more realistic case where firms have to pay a sunk cost κ to enter the market. [12] Hence, firms enter the market until the expected value of a vacancy equals the entry cost :

$$\Pi_v = \kappa$$

- The labor market equilibrium is again defined by a tuple-(w, θ) and solves : [13]

$$\begin{aligned} \frac{r\kappa}{m(\theta)} &= \frac{y - w - r\kappa}{r + q} \\ w &= (1-\beta)z + \beta(y - r\kappa + \theta r\kappa) \end{aligned}$$

12. See e.g. Acemoglu (2001) for a similar hypothesis.
Acemoglu (2001), Good jobs versus bad jobs, *Journal of labor Economics*.

13. The demonstration is left as homework. Additionally, show that we can easily obtain an expression similar to (8).

Efficiency

- In the matching model, the allocation of labor resources is characterized by the presence of positive between-group externalities and negative within-group congestion effects.

- Every unemployed person would like to be the only person searching for a job and have as many vacant jobs as possible, while every employer would like to be the only one with vacant positions and to be facing a wide array of job seekers.

- There are *congestion effects* within each category and *positive spillovers* between the categories.

- In the decentralized equilibrium of the matching model with wage bargaining, individuals do not internalize these externalities.
- The decentralized equilibrium of the labor market is generally inefficient, except for a particular situation that verifies the so called Hosios-Diamond-Pissarides (HDP) condition.
- Let us assume, for the sake of simplicity, that $r \to 0$.[14]
- The planner's problem satisfies :[15]

$$\max_{\{\theta,u\}} \Gamma = y(1-u) + zu - \gamma v \quad \text{s.t.} \quad u = \frac{q}{q + \theta m(\theta)}$$

14. The general case is left as homework. See e.g. Pissarides (2000), chapter 8.

15. The planner's problem consists in finding the best allocation given the constraints on trading imposed by search and matching frictions, *i.e.* this is a second best allocation.

- Let ϑ denotes the elasticity of the matching function with respect to the unemployment rate.

- Assuming that the matching function is of the Cobb-Douglas form, the elasticity satisfies :[16]

$$\vartheta(\theta) = -\frac{\theta m'(\theta)}{m(\theta)} = -\frac{(\eta - 1)\,\theta\kappa\theta^{\eta-2}}{\kappa\theta^{\eta-1}} = 1 - \eta$$

- Using (1), the *planner's problem* rewrites :

$$\max_{\theta} \quad \Gamma = \left(y + \frac{q}{q + \theta m(\theta)} (z - y - \gamma\theta) \right)$$

16. Remark that with a Cobb-Douglas function, the elasticity is constant and does not depend on labor market tightness.

- The socially efficient allocation satisfies :

$$\frac{\gamma}{m(\theta)} = (1-\vartheta)\frac{y-z}{q+\vartheta\theta m(\theta)}$$

- In a decentralized equilibrium when $r \to 0$, we get :

$$\frac{\gamma}{m(\theta)} = (1-\beta)\frac{y-z}{q+\beta\theta m(\theta)}$$

- The decentralized equilibrium is socially efficient if :

$$\beta = \vartheta = 1-\eta$$

- This condition is known as the Hosios-Diamond-Pissarides (HDP) condition. [17]

17. Hosios (1990), On the efficiency of matching and related models of search and unemployment, *The Review of Economic Studies*.

- In general, since both firms and workers are causing congestion for each other, it is not possible to say whether equilibrium unemployment is above or below the socially efficient rate.

- Hence, it follows that :
  - if $\beta = \vartheta$, the unemployment rate is socially efficient.
  - if $\beta > \vartheta$, the unemployment rate is inefficiently high.
  - if $\beta < \vartheta$, the unemployment rate is inefficiently low.

- In the matching model with wage bargaining, the inefficiency of decentralized equilibrium comes from the absence of mechanisms giving agents an incentive to take into account the externalities linked to their decisions.

- These mechanisms can exist with other wage setting mechanisms, *e.g.* in competitive search models.

Competitive Search

- What is competitive search?
- In line with Wright et al. (2021), search is competitive if : [18]
  1. Search is directed,
  2. The direction is based on advertised prices.
- In practice, employers often *announce* the wage attached to their vacant jobs.
- Moen (1997) assumes that wages are no longer bargained over, but posted by employers. [19]

18. Wright, Kircher, Julien, and Guerrieri (2021), Directed search : A guided tour, *Journal of Economic Literature*.
19. Moen (1997), Competitive search equilibrium, *Journal of political Economy*.

- There are a large number of labor "pools" (indexed by i) where the mobility and information are perfect.

- In each sub-market (pool) :

  - The number of hires is determined by a matching function.
  - Employers post a hiring wage, w_i (*not renegotiable*).[20]

- Unemployed workers have perfect information on each sub-market.

- Hence, search can be directed toward their preferred sub-market.

20. See e.g Shi (2009) for a more complete (and subtle) model.
Shi (2009), Directed Search for Equilibrium Wage-Tenure Contracts, *Econometrica*,

- Entrepreneurs compete to attract workers into their respective pool in offering the same expected utility, V_u, to job seekers.

- They can do so in two ways :
  - opening few jobs against a high wage (*long queuing*),
  - opening many jobs against a low wage (*short queuing*).

- The expected utility of a person employed in labor pool i is :

$$rV_{ei} = w_i + q(V_u - V_{ei})$$

- The expected utility of a unemployed person is :

$$rV_u = z + \theta_i m(\theta_i)(V_{ei} - V_u)$$

- Eliminating V_{ei} between these last two equations gives :

$$\theta_i m(\theta_i) = (r+q)\frac{(rV_u - z)}{w_i - rV_u} \tag{9}$$

- The optimal strategy for the entrepreneurs consists of offering a wage so as to maximize the expected gain from vacant jobs.

- The expected gain Π_{vi} from an unfilled job, and from a filled one are :

$$r\Pi_{vi} = -\gamma + m(\theta_i)(\Pi_{ei} - \Pi_{vi})$$
$$r\Pi_{ei} = y - w_i + q\left(\Pi_{vi} - \Pi_{ei}\right)$$

- Eliminating Π_{ei} between these last two equations gives :

$$r\Pi_{vi} = \frac{-\gamma(r+q) + m(\theta_i)(y - w_i)}{r + q + m(\theta_i)} \tag{10}$$

- This equation gives the expected profit from a vacant job as a function of the wage, w_i, and the labor market tightness, θ_i.

- Entrepreneurs choose the wage in order to maximize the expected profit of a vacant job, such that :

$$w_i = \arg\max \Pi_{vi}$$

- Using the free entry condition, $\Pi_{vi} = 0$, together with the foc of the previous program yield :

$$w_i = rV_u + \vartheta(\theta_i)(y - rV_u)$$

Appendix

- The bargained wage in the *basic model* (*see infra*) yields :

$$w = rV_u + \beta(y - rV_u)$$

- Comparison of these two equations shows that this mode of wage setting entails at equilibrium the HDP condition and so ensures the efficiency of decentralized equilibrium.

- This example suggests that competition among firms can restore the efficiency of market equilibrium.

- Random or Directed (competitive) search ?

- Recent attempt to disentangle random versus directed search lead to mixed conclusions about their respective relevance.[21]

- Some workers bargain with prospective employers before accepting a job, others face a posted wage as a take-it-or-leave-it opportunity.

  - Hall and Krueger (2012) found a quite large share of workers bargaining over their wage almost equivalent to the share of posted wages.[22]

  - Marinescu and Wolthoff (2019) show that on the leading job board CareerBuilder.com, 80% of vacancies do not contain information about wages.[23]

21. See e.g. Engelhardt and Rupert (2017), Competitive versus random search with bargaining : An empirical comparison, *Labour Economics*.

22. Hall and Krueger (2012), Evidence on the Incidence of Wage Posting, Wage Bargaining, and On-the-Job Search, *American Economic Journal : Macroeconomics*.

23. Marinescu and Wolthoff (2019), Opening the Black Box of the Matching Function : the Power of Words, *Journal of Labor Economics*.

Other wage-setting mechanisms

- As emphasized by Mortensen and Pissarides (1999), the search equilibrium framework is useful in the study of the unemployment effects of alternative wage determination mechanisms.

- We have so far only focussed on two cases :
  - Nash bargaining
  - Competitive search equilibrium

- In the reminder of the lecture, we will extend the benchmark model to :
  - Specific cases
  - Kalai-Smorodinsky bargaining
  - Wage posting in the spirit of Burdett-Mortensen (1998)

- Beyond, the model can accommodate other mechanisms (not developed here) :
  - Unions' behavior (more in the next lecture)
  - Efficiency wages

Specific cases

- Let us first consider a case where the wage, w, is exogenous (*e.g.* existence of a completely binding minimum wage.[24])

- The model becomes dichotomous and we are only interested in firms' behavior.

- Equations (2) and (3) together with the FEC imply :

$$\frac{\gamma}{m(\theta)} = \frac{y - w}{r + q}$$

- The labor market tightness is uniquely pinned down by the job creation condition and we have that :

$$\frac{\mathrm{d}\theta}{\mathrm{d}w} = \frac{m(\theta)^2}{\gamma m'(\theta)(r+q)} < 0$$

24. Flinn (2010), The Minimum Wage and Labor Market Outcomes (MIT Press), provides a thorough introduction to the analysis of the effects of the minimum wage in search economies.

- Let us now consider a case where the bargaining power of the worker, β, is nil.[25]

- The model is again dichotomous and the firm reaps the entire surplus of the job-worker match such that (with the same notation as before) :

$$\cancel{\Omega = (1-\beta)\Pi_e} \longrightarrow \Omega = \Pi_e$$

- Workers are paid their reservation wage such that, $w = rV_u$, and using the FEC, the job creation condition rewrites $\frac{\gamma}{m(\theta)} = \Pi_e = \frac{y - rV_u}{r+q}$. Then as :

$$\cancel{rV_u = z + \theta m(\theta)\beta\Omega} \longrightarrow rV_u = z$$

It follows that :

$$\frac{\gamma}{m(\theta)} = \frac{y - z}{r + q}$$

- The labor market tightness is again uniquely pinned down by the job creation condition with $\frac{\gamma}{m(\theta)} = \frac{y-z}{r+q}$.

25. This may be the case with low-skilled workers who do not have good outside options.

Kalai-Smorodinsky Bargaining

- SaM models readily extend to other surplus splitting rules.[26]
- We consider the Kalai-Smorodinsky (KS) solution[27] and follow l'Haridon et al. (2013)[28] assuming for simplicity :
  - symmetric bargaining,
  - perfectly transferable (linear) utility.
- The symmetric KS solution is characterized by equal proportional concessions for both agents from their respective utopia points.
- Utopia points are defined by the maximum utility attainable subject to the constraint that no agent should receive less than her outside option :
  - From (2), if $w = y - r\Pi_v$, then $\Pi_e = \Pi_v$ and the instantaneous surplus of the worker is $y - r\Pi_v$.
  - From (5), if $w = rV_u$ then $V_e = V_u$ and the instantaneous surplus of the firm is $y - rV_u$,

26. See e.g. Petrosky-Nadeau and Wasmer (2017) (chapter 1 section 1.5) for further discussions on wages.
27. Kalai and Smorodinsky (1975), Other solutions to Nash's bargaining problem, Econometrica.
28. l'Haridon, Malherbet and Perez-Duarte (2013), Does bargaining matter in the small firms matching model ? Labour Economics.

- Let us denote by :
  - $\widehat{w}^f = rV_u$, the wage at utopia for the firm,
  - $\widehat{w}^e = y - r\Pi_v$, the wage at utopia for the worker.

- Firms' and workers' most optimistic expectation solves :

$$\begin{aligned} r\widehat{\Pi}_e &= y - \widehat{w}^f + q\left(\Pi_v - \widehat{\Pi}_e\right) = y - rV_u + q\left(\Pi_v - \widehat{\Pi}_e\right) \\ r\widehat{V}_e &= \widehat{w}^e + q\left(V_u - \widehat{V}_e\right) = y - r\Pi_v + q\left(V_u - \widehat{V}_e\right) \end{aligned}$$

- The KS solution satisfies :

$$\underbrace{\frac{\Pi_e - \Pi_v}{\widehat{\Pi}_e - \Pi_v}}_{\text{Relative utility gain from the firm}} = \underbrace{\frac{V_e - V_u}{\widehat{V}_e - V_u}}_{\text{Relative utility gain from the worker}}$$

Appendix

- The KS solution leads to an equalization of relative utility gains of the two players that holds for every point along the ray between (Π_v, V_u) and $(\widehat{\Pi}_e, \widehat{V}_e)$.

Does bargaining matter ?

- Assuming free entry and making use of Π_e, $\widehat{\Pi}_e$, V_e and $\widehat{V}_e$, the KS solution satisfies :

$$w = rV_u + \frac{1}{2}(y - rV_u)$$

- Remarking that the Nash solution with $\beta = \frac{1}{2}$ rewrites as $w = rV_u + \beta(y - rV_u) = rV_u + \frac{1}{2}(y - rV_u)$, the two solutions are identical.

- Hence when *utility is transferable* and *with wages flexibility*, the Nash and the Kalai-Smorodinsky converge to a single aggregate wage equation.

- In the absence of these two restrictions, this is not necessarily the case and the *labor market equilibrium is likely to be altered* :
  - Partially transferable utility,
  - Efficiency wages,
  - Wage floors.

Equilibrium Unemployment with Wage-Posting

- It is possible to nest the equilibrium search model of Burdett-Mortensen (1998) into the random search and matching model of Pissarides (2000). [29]

- Workers search (randomly) on and off the job.

- Total labor force is fixed and represented as a unit mass.

  Let us denote search inputs by :
  - u, the number of unemployed workers
  - $1 - u$, the number of employed workers
  - v, the number of vacant jobs

- The matching function equals in value the total flow of offers received by workers :

$$M(v, u, 1-u) = \lambda_u u + \lambda_e (1-u)$$

29. Mortensen (1998), Equilibrium Unemployment with Wage Posting : Burdett-Mortensen Meet Pissarides, Working Paper.

- Assume for simplicity that employed worker and unemployed workers are perfect substitutes and search at unit intensity :[30]

$$\lambda_e = \lambda_u = \lambda(v) = M(v, u, 1-u) = M(v, 1)$$

- Relations similar to those of lecture 3 are obtained except that the arrival rate of job offers now depends on v.

- We have that :

$$\begin{aligned} \frac{u}{1-u} &= \frac{q}{\lambda(v)} \\ G(w) &= \frac{qH(w)}{q+\lambda(v)\left[1-H(w)\right]} = \frac{H(w)}{1+\kappa(v)\left[1-H(w)\right]} \end{aligned}$$

where $\kappa(v) = \frac{\lambda(v)}{q}$.

- Let us define $\frac{M(v,1)}{v} = \frac{\lambda(v)}{v} = m(v)$ as the average rate at which vacancies are filled, *i.e.* the rate at which workers are contacted per vacancy.

30. Hence, only the number of searching workers matters in this context.

- The asset value of a vacant job solves (with the usual notations) :

$$r\Pi_v = \max_{w \geq x} \left\{ m(v) \left[u + (1-u) G(w) \right] \left(\Pi_e(w) - \Pi_v \right) - \gamma \right\} \tag{11}$$

  Hence, the worker contacted at rate, $m(v)$, accepts with :
    - prob. 1 among the pool, u of unemployed workers
    - prob. $G(w)$ among the pool, $1-u$ of employed workers

- The asset value of a filled job solves :

$$r\Pi_e(w) = y - w + \underbrace{q\left(\Pi_v - \Pi_e(w)\right)}_{\text{destruction}} + \underbrace{\lambda(v)\left[1 - H(w)\right]\left(\Pi_v - \Pi_e(w)\right)}_{\text{quit}}$$

  Hence, the job is destroyed at rate q and $\lambda(v)[1 - H(w)]$ is the rate at which a worker quits to a firm offering a better wage.

- Free entry drives down the value of a vacancy to zero, *i.e.* $\Pi_v = 0$.

- The asset value of a filled job rewrites :

$$\Pi_e(w) = \frac{y - w}{r + q + \lambda(v)\left[1 - H(w)\right]}$$

- Making use of the flows balance, FEC and Π_e, equation (11) rewrites as :

$$\frac{\gamma}{m(v)} = \max_{w \geq x} \left\{ \left(\frac{q}{q + \lambda(v)\left[1 - H(w)\right]} \right) \left(\underbrace{\frac{y - w}{r + q + \lambda(v)\left[1 - H(w)\right]}}_{\Pi_e(w)} \right) \right\}$$

- The expected cost of filling a vacancy (LHS) is equal to the expected profit (RHS) for every wage offer that maximizes the value of hiring a worker.

- A steady-state search equilibrium consists in :
  - the vacancy rate, v
  - the sampling distribution of wage offers, $H(w)$

- The lowest job offer is only accepted by unemployed, hence the equilibrium number of vacancies, v, solves :

$$\frac{\gamma}{m(v)} = \left(\frac{q}{q+\lambda(v)}\right)\left(\frac{y-x}{r+q+\lambda(v)}\right)$$

- Every wage in the support of equilibrium wage offer distribution must yield the same profit :

$$\frac{q(y-x)}{(q+\lambda(v))(r+q+\lambda(v))} = \frac{q(y-w)}{(q+\lambda(v)[1-H(w)])(r+q+\lambda(v)[1-H(w)])}$$

- This equation is a quadratic equation of the form :

$$\alpha_2 H^2(w) + \alpha_1 H(w) + \alpha_0 = 0$$

where :

$$\begin{aligned}
\alpha_2 &= \lambda^2(v) \\
\alpha_1 &= -\lambda(v)(r+2(q+\lambda(v)))H(w) \\
\alpha_0 &= (q+\lambda(v))(r+q+\lambda(v))\left(1-\frac{y-w}{y-x}\right)
\end{aligned}$$

- Solving for this equation, we get a closed-form expression for the wage distribution :

$$H(w) = \frac{r + 2(q + \lambda(v))}{2\lambda(v)} \left[1 - \sqrt{\frac{r^2 + 4(q + \lambda(v))(r + q + \lambda(v)) \left(\frac{y-w}{y-x}\right)}{(r + 2(q + \lambda(v)))^2}} \right]$$

- This solution limits to the Burdett-Mortensen (1998) equilibrium distribution :

$$\lim_{r \to 0} H(w) = \frac{q + \lambda(v)}{\lambda(v)} \left[1 - \sqrt{\left(\frac{y-w}{y-x}\right)} \right]$$

with the same properties and caveats, as detailed in lecture 3.

- Mortensen (1998, 2000) extends the model to match specific capital to accommodate for the typical hump shape of the wage distribution.

Conclusion

- In this lecture, we have studied the canonical search and matching model.
- The model can be enriched in many aspects :
  - Endogeneous job destruction,
  - Labor market institutions,
  - Product market regulations,
  - Macroeconomic turbulence,
  - ...
- In particular the model has been proved to be the workhorse to study labor market policies.

Appendix : Competitive search

- Differentiation of (10) wrt w_i yields :

$$
\begin{aligned}
-\frac{m(\theta_i)}{r+q+m(\theta_i)}+\frac{m'(\theta_i)(y-w_i)}{r+q+m(\theta_i)}\frac{\partial\theta_i}{\partial w_i}-\frac{m(\theta_i)(y-w_i)-\gamma(r+q)}{[r+q+m(\theta_i)]^2}m'(\theta_i)\frac{\partial\theta_i}{\partial w_i} &= 0\\
\left[m'(\theta_i)(y-w_i)\frac{\partial\theta_i}{\partial w_i}-m(\theta_i)\right](r+q+m(\theta_i))-m'(\theta_i)\frac{\partial\theta_i}{\partial w_i}\left[\underbrace{m(\theta_i)(y-w_i)-\gamma(r+q)}_{=0 \text{ if free entry}}\right] &= 0
\end{aligned}
$$

- Differentiation of (9) wrt to θ_i and w_i gives the derivative $\frac{\partial\theta_i}{\partial w_i}$:

$$
\frac{\partial\theta_i}{\partial w_i}=\frac{-\theta_i}{[1-\vartheta(\theta_i)](w_i-rV_u)} \; ; \; \vartheta(\theta_i)\equiv-\frac{\theta_i m'(\theta_i)}{m(\theta_i)}
$$

- Making use of the free entry condition $\Pi_{vi}=0$ such that $\frac{\gamma}{m(\theta_i)}=\frac{y-w_i}{r+q}$, and of $\frac{\partial\theta_i}{\partial w_i}$, one gets :

$$
\begin{aligned}
-\theta_i m'(\theta_i)\frac{y-w_i}{(1-\vartheta(\theta_i))(w_i-rV_u)}-m(\theta_i) &= 0\\
\vartheta(\theta_i)(y-w_i) &= (1-\vartheta(\theta_i))(w_i-rV_u)\\
w_i &= rV_u+\vartheta(\theta_i)(y-rV_u)
\end{aligned}
$$

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Appendix : Nash Solution (symmetric case)

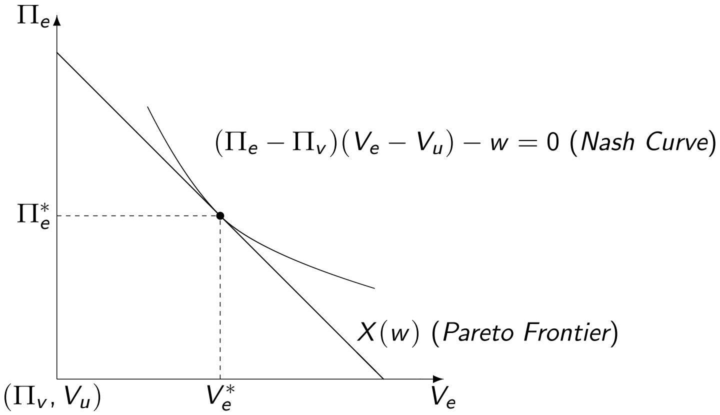

FIGURE 2 – Nash solution with transferable utility.

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Appendix : Kalai-Smorodinsky Solution (symmetric case)

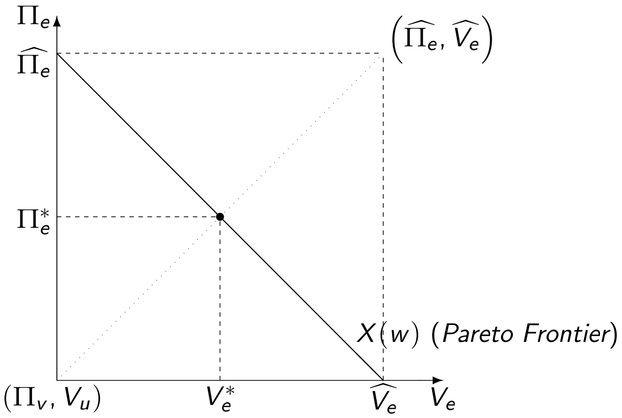

FIGURE 3 – Kalai-Smorodinsky solution with transferable utility.

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Appendix : Comparative statics

- Let us define $\phi = \phi(\theta, \gamma, \beta, y, z, r, q)$ such that :

$$\phi(\theta, \gamma, \beta, y, z, r, q) = \frac{\gamma}{m(\theta)} - \frac{(1-\beta)(y-z)}{r+q+\beta\theta m(\theta)}$$

- Using the implicit function theorem, one gets :

$$\frac{\mathrm{d}\theta}{\mathrm{d}p} = -\frac{\frac{\partial\phi}{\partial p}}{\frac{\partial\phi}{\partial\theta}} \text{ for } p = \{\gamma, \beta, y, z, r, q\}$$

- Differentiation wrt θ yields :

$$\frac{\partial\phi}{\partial\theta} = -\frac{\gamma m'(\theta)}{(m(\theta))^2} + \frac{(1-\beta)(y-z)\beta(1-\vartheta(\theta))}{(r+q+\beta\theta m(\theta))^2 m(\theta)} > 0$$

where $\vartheta(\theta) \equiv -\frac{\theta m'(\theta)}{m(\theta)} \in [0;1]$. It follows that $\frac{\mathrm{d}\theta}{\mathrm{d}p}$ is of the opposite sign of $\frac{\partial\phi}{\partial p}$.

- For instance, as $\frac{\partial\phi}{\partial y} = -\frac{1-\beta}{r+q+\beta\theta m(\theta)} < 0$, we have that $\frac{\mathrm{d}\theta}{\mathrm{d}y} > 0$. Hence, a rise in productivity increases the expected profit and the labor market tightness. As the unemployment rate does not directly depend on y and is decreasing in y, the unemployment rate decreases with productivity.

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