Labour demand

AE318 - Labour Economics

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Why do we care about labour demand?

- Keynesian views of unemployment attribute a large role to a lack of labour demand
- How do changes such as technology shocks, unionization, business cycle fluctuations, minimum wages affect labour demand (and wages)

Introduction

The theory of labour demand is deeply related to that of production factors.

The demand for labour must therefore depend not only on the cost of labour but also on the cost of the other factors.

- In the short run, volume of labour services is variable
- In the long run, there are possibilities of substituting capital K for labour L
- This is precisely the purpose of the theory of labour demand: to study the behavior of firms when there are more than two factors of production given the trade-offs between the number of its employees and number of hours worked

Short-Run Labour Demand

Long-Run Labour Demand

The Static Model of Labour Demand

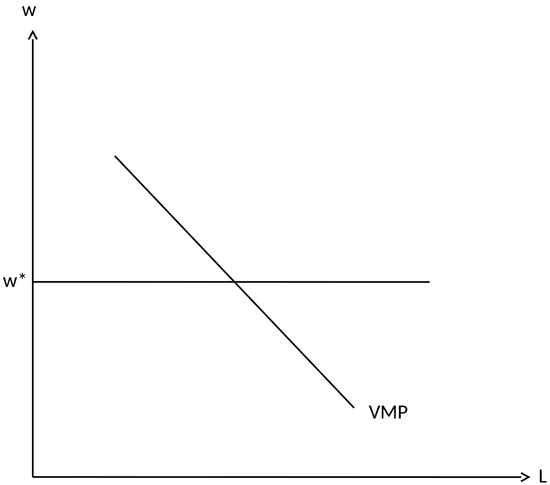
- The static theory of labour demand assumes no adjustment costs in the amount of labour employed.
- We distinguish between short-run and long-run labour demand:
  - Short-run: capital (e.g. machinery and factories) does not change.
  - Long-run: all factors of production (including capital) may change.
- In the short-run the demand for labour depends on the wage and the market power (in the product market) of the firm.

Perfect competition

- In the short run, capital is assumed to be fixed.
- With perfect competition in the product market the firm cannot affect the price of output P
- Production function: $Y = F(L)$
- F is a strictly increasing ($F' > 0$) and concave ($F'' < 0$) function
- Firm's profit: $\Pi(L) = PY - wL$
  - Substituting $F(L)$ for Y
    $$\Pi(L) = PF(L) - wL$$
  - Profit maximization gives the following FOC:
    $$\Rightarrow PF'(L) = w$$

    value marginal product (VMP) = wage

Perfect competition



Market power

- With market power in the product market the firm's choice of output affects product prices
- Inverse demand function $P = P(Y)$
- Its elasticity is defined as:

$$\eta_Y^P \doteq YP'(Y)/P(Y)$$

  - with $\eta_Y^P \in (-1, 0)$

Market power

- Firm's profit becomes:

$$\Pi(L) = P[Y]Y - wL = P[F(L)]F(L) - wL$$

- FOC:

$$\Pi'(L) = F'(L)[P(F(L)) + P'(F(L))F(L)] - w = 0$$

- Rewriting:

$$P(F(L))F'(L)(1 + \eta_Y^P) = w$$

- If a firm has market power in the product market
  - Marginal productivity $=$ real wage $\times$ markup $\nu = (1 + \eta_Y^P)^{-1}$
  - The firm pays workers less than their value marginal product

Short-run labour demand

- Cost function: $C(Y) = wL = wF^{-1}(Y)$
  - Its derivative: $C'(Y) = w/F'(Y)$
  - So that the FOC leads to $P(Y) = \nu C'(Y)$
  - Price: markup on the marginal cost
- Differentiating $P(F(L))F'(L) = \nu w$ wrt to L and w:

$$\frac{\partial L}{\partial w} = \frac{\nu}{F'^2 P' + PF''} < 0$$

- An increase in w will: increase the price of the good, decrease labour demand and product output

Short-Run Labour Demand

Long-Run Labour Demand

Long-Run Labour Demand

- In the long run, capital K becomes a flexible factor
- The analysis proceeds in 2 stages:
  1. Level of production taken as given and we look for the optimal combinations of K and L by which that level can be reached
  2. Look for the level of output that will maximize the firm's profits

Stage 1: Cost minimization

- The optimal combination of inputs is obtained by minimizing the cost linked to production level Y

$$\min_{K,L} wL + rK \text{ s.t. } F(K,L) \geq Y$$

- The Lagrangian of the problem is:

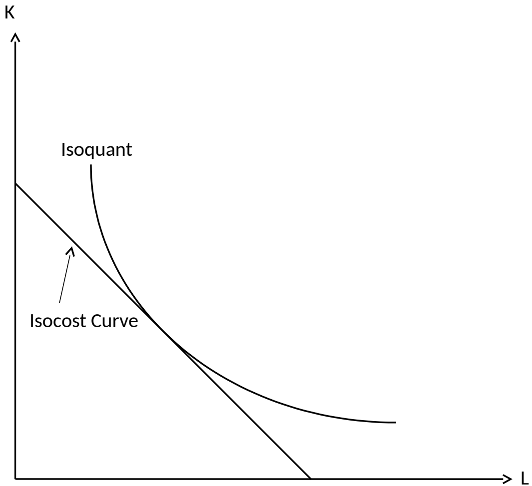
$$\mathcal{L} = wL + rK - \lambda[Y - F(K,L)]$$

- FOC:

$$\frac{\partial \mathcal{L}}{\partial L} = w + \lambda F_L = 0$$
$$\frac{\partial \mathcal{L}}{\partial K} = r + \lambda F_K = 0$$

- from (1) and (2) we get: $\frac{w}{r} = \frac{F_L}{F_K}$ The price ratio is equal to the technical rate of substitution

Stage 1: Cost minimization



Stage 1: Solution and cost function

- Solutions: conditional factor demand $\tilde{K}(w, r, Y)$ and $\tilde{L}(w, r, Y)$
- Cost function $C(w, r, Y) = w\tilde{L} + r\tilde{K}$, with the following properties:
  1. Non-decreasing w.r.t. w,r, and Y
  2. Concave in w and r: $C_{ww} < 0$ and $C_{rr} < 0$
  3. Homogeneous of degree 1 in input prices, of degree $1/\theta$ in Y (when the production function is homogeneous of degree θ):

$$C(tw, tr, Y) = tC(w, r, Y) \text{ for } t > 0$$
$$C(w, r, Y) = C(w, r, 1)Y^{\frac{1}{\theta}}$$

  4. Shephard's lemma: $\tilde{L} = C_w(w, r, Y)$ and $\tilde{K} = C_r(w, r, Y)$

Elasticity of substitution between capital and labour

Elasticity of substitution between capital and labour

- A crucial parameter of interest is the elasticity of substitution between K and L, i.e. holding output Y constant:

$$\sigma = \left. \frac{\text{\% change in K/L}}{\text{\% change in w/r}} \right)_{Y=\text{constant}} = \frac{w/r}{\tilde{K}/\tilde{L}} \frac{\partial(\tilde{K}/\tilde{L})}{\partial(w/r)}$$

- This elasticity measures how firms change the mix of inputs to respond to a change in relative prices of these inputs, maintaining the level of output
- If $\sigma \to \infty$ the two factors become perfect substitutes
- If $\sigma = 0$ the factors cannot substitute for each other
- The elasticity also takes the following form: $\sigma = \frac{CC_{wr}}{C_w C_r}$

Example: CES Production Function

- Assume the CES production function:

$$Y = [(\alpha_L L)^{\frac{\sigma-1}{\sigma}} + (\alpha_K K)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

- Cost-minimization leads to FOC:

$$\frac{\partial \mathcal{L}}{\partial L} = w + \lambda \frac{\sigma-1}{\sigma}[(\alpha_L L)^{\frac{\sigma-1}{\sigma}} + (\alpha_K K)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} \alpha_L^{\frac{\sigma-1}{\sigma}} \frac{\sigma}{\sigma-1} L^{\frac{-1}{\sigma}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial K} = r + \lambda \frac{\sigma-1}{\sigma}[(\alpha_L L)^{\frac{\sigma-1}{\sigma}} + (\alpha_K K)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} \alpha_K^{\frac{\sigma-1}{\sigma}} \frac{\sigma}{\sigma-1} K^{\frac{-1}{\sigma}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y - [(\alpha_L L)^{\frac{\sigma-1}{\sigma}} + (\alpha_K K)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} = 0$$

Example CES Production Function (2)

- From FOC (1) and (2) we obtain:

$$\frac{K}{L} = \left(\frac{w}{r}\right)^{\sigma} \left(\frac{\alpha_L}{\alpha_K}\right)^{1-\sigma}$$

- Parameter σ is the elasticity of substitution between the two inputs.

More than two inputs

- The program can be generalized to n inputs (X^i of price W^i)

$$\min_{(X^1,\dots X^n)} \sum_{i=1}^{n} W^i X^i \text{ s.t. } F(X^1,\dots X^n) \geq Y$$

- The conditional factor demands are defined by the equations:

$$\frac{F_i(\tilde{X}^1,\dots \tilde{X}^n)}{F_j(\tilde{X}^1,\dots \tilde{X}^n)} = \frac{W^i}{W^j}, \forall i,j = 1\dots n$$

- The cost function is defined accordingly: $C(W^1,\dots W^n, Y)$
- From Shephard's lemma:

$$\frac{\partial \tilde{X}^i}{\partial W^j} = C_{ij}$$

P-substitute and p-complement

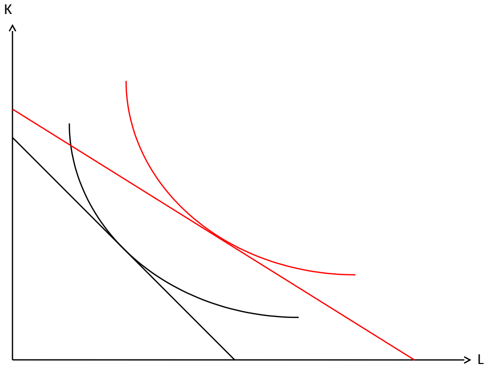
- If $\partial \tilde{X}^i / \partial W^j > 0$, factors i and j are called p-substitutes; in the opposite case, p-complements
- Two factors are p-substitutes if, to attain a given output, the demand of one factor increases when the price of the other factor increases
- If there are only two factors, they are necessarily p-substitutes

Stage 2: Profit Maximization

- The producer can also choose the level of production
- The unconditional factor demands for K and L are then different from the conditional demands
- First look at an example:
  - What happens if the price of labour falls from w_0 to w_1?

Stage 2: Profit Maximization

- The profit maximizing firm will adjust its output when w falls
- A fall in w lowers the marginal cost of production
- This fall in marginal cost encourages the firm to expand



Stage 2: Profit Maximization

- Maximize profit function written as a function of Y and factor costs:
  - Program: $\max_Y \Pi(w, r, Y) = P(Y)Y - C(w, r, Y)$
  - FOC: $\frac{\partial \Pi}{\partial Y} = P'Y + P - C_Y = 0 \Rightarrow P = \nu C_Y$
  - This relation define the optimal level of output Y^* as a function of w, r
- Maximize profit as a direct function of factors K, L:
  - Program: $\max_{K,L} \Pi = P(F(K, L))F(K, L) - wL - rK$
  - FOC: $F_L(K, L) = \nu \frac{w}{P}$ and $F_K(K, L) = \nu \frac{r}{P}$
- These equations define *unconditional* factor demands $K^*,\ L^*$

Conditional and unconditional demands

- Conditional demands $\tilde{K}, \tilde{L}$ depend on w, r, Y
- Unconditional demands K^*, L^* depend on w, r only
- At the level of output Y^*, conditional and unconditional demands coincide

Labour demand elasticity

- How does an increase in wages impact the labour demand?
  - Substitution effect: at fixed level of output, firms will use less labour and more capital
  - Scale effect: because production costs increase, the level of output will be lower
- The following relation links the different elasticities:

$$\eta_w^L = \bar{\eta}_w^L + \bar{\eta}_Y^L \eta_w^Y$$

  - η_w^L: elasticity of unconditional labour demand wrt wage
  - $\bar{\eta}_w^L$: elasticity of conditional labour demand wrt wage
  - $\bar{\eta}_Y^L$: elasticity of conditional labour demand wrt output
  - η_w^Y: elasticity of optimal output Y^* wrt wage

Gross substitutes and gross complements

- The cross-elasticity can be obtained similarly:

$$\eta_r^L = \bar{\eta}_r^L + \bar{\eta}_Y^L \eta_r^Y$$

- While $\bar{\eta}_r^L$ is always positive, the sign of the scale effect is ambiguous
- When $\eta_r^L > 0$ labor and capital are gross substitutes; otherwise, they are gross complements