CHAPTER 5

Job Search

In this chapter we will:

- See what unemployed workers do to find a job
- Study how the duration of unemployment depends on the characteristics of unemployed workers
- Study how unemployment insurance influences the duration of unemployment
- Learn how economists empirically evaluate this influence
- Examine how the contribution of Lalive, van Ours, and Zweimüller (2006) assesses the impact of modifications in the Austrian unemployment insurance system on the duration of unemployment (The main results of this contribution can be replicated with data and programs available at www.labor-economics.org.)
- Review the effects of job search help and checking on job search effort
- See how the equilibrium search model explains why identical workers can be paid differently
- Learn why wages rise, on average, as workers gain experience and why large firms pay higher wages than small firms to identical workers

INTRODUCTION

The economic theory of labor supply pays no attention to the time and cost of looking for work. The consumption of "leisure"—even when this term is extended to cover home production—remains the sole alternative to waged work, and by definition an agent who utilizes the total amount of time at his disposal in the form of leisure is described as a *nonparticipant*. So from this perspective there is no place for the unemployed person, even though her principal activity amounts to looking for work. Such a description of the labor market implicitly assumes a structure of *perfect information*. It supposes that each agent knows all the particulars about all the jobs on offer and that he merely has to decide the number of hours—potentially as low as zero—that he wants to devote to work, given the (supposedly) single and universally known wage prevailing in the labor market. There is no need to *look* for a job that would suit him. Such a hypothesis is no doubt too simplistic, to the extent that, as we document below, many unemployed workers do devote considerable effort to looking for work without getting satisfactory job offers. So, we must examine the consequences of *imperfect information*. This is precisely the purpose of job search theory: to study the behavior of an individual who has imperfect information about jobs and wages.

In the job market, the imperfection of the available information occurs in the form of a number of possible wages that an agent might be able to command. Hence the job seeker surveys the labor market so as to find the highest wage being paid for the services she can supply. This procedure is no different from that adopted by a person looking for an apartment (at the best possible rent) or a loan (at the best possible rate of interest). It was Stigler (1961, 1962) who first highlighted this common process in all markets where information is imperfect. The modern theory of the job search arose in the 1970s with the formalizations of McCall (1970) and Mortensen (1970).

The first section of this chapter portrays the activity of job search, using surveys that describe the amount of time that persons both employed and unemployed spend looking for work each day. It lays particular stress on the influence of economic incentives on the amount of time devoted to job search.

Section 2 of this chapter lays out the basic job search model, in which an agent keeps looking as long as she entertains the hope of improving her welfare by continuing to search. This model is useful to understand how the duration of the search depends on individual preferences and the overall characteristics of the environment in which it takes place. The theory of job search is not in conflict with the theory of labor supply. By giving a prominent role to imperfect information, job search theory adds the category "unemployed" to those of "employed" and "nonparticipant." In this way it sheds supplementary light on the decision to participate in the labor market, which no longer takes the form of a choice between work and nonparticipation; rather, the choice now lies in knowing whether it is *worthwhile* to *look* for work. In other words, to hold a paid job you must first have decided to look for one. A good synthesis of this theory can be found in Mortensen (1986) and Mortensen and Pissarides (1999).

Section 3 presents the empirical analysis of a reform of the Austrian unemployment insurance system carried out by Lalive et al. (2006). It details the econometric strategies utilized to identify and evaluate the effects of reforms to a system of unemployment insurance on the duration of unemployment and the rate of return to employment. This section also presents the econometric techniques adopted to estimate the models of duration and offers a synthesis of the principal empirical results in the domain of job search.

In the first three sections of this chapter, the distribution of wages is a given parameter, which is not explained. We will see in section 4 that, *when the distribution of wages is rendered endogenous*, the search model allows us to go further and to explain why identical workers can be paid differently, contrary to the model of perfect competition studied in chapter 3, which assumes that wage differences reflect only differences in talent and the hardship of tasks. We start to see in section 4 that many empirical studies do suggest that wage differences do not solely reflect differences of talent and the hardship of tasks. It is apparent that persons who have the luck to be hired by larger or more productive firms have higher wages than persons with identical characteristics who have not had the same luck. These differences in remuneration are enduring and represent an important portion, on the order of one quarter, of wage differences as a whole. The existence of job search costs or "frictions" can explain this phenomenon. In the presence of job search costs, the competition that would allow wage earners to be remunerated at their marginal productivity cannot fully play out. Search costs cause "rents" to materialize: the wage earner incurs a loss if she loses her job when it is costly to find another; the employer also incurs a loss when a worker quits if it is costly to hire another. To avoid this loss, the employer is ready to give wage earners a remuneration higher than what they could get by looking for another job. For her part, the worker is ready to accept a wage lower than what she could extract from another employer if job search is costly. Hence the costs of job hunting can exert influence on wage formation. From that standpoint, we will analyze the behavior of employers in the context of the job search model. For a long time the theory of job search developed within the framework of partial equilibrium, which left it unable to explain the formation of the wage distribution that confronts job seekers. To make it complete, the behavior of employers has been introduced so as to arrive at a description of labor market equilibrium. By attributing well-defined strategic behavior to firms, these "equilibrium search models" are able to portray the process of wage formation as endogenous and to explain why identical workers can be paid differently. We see that in reality it is essential to take into account not just job search costs and the search activity associated with them but also on-the-job search in order to explain the empirical properties of wage distribution.

1 WHAT DO JOB SEEKERS DO?

Discussion of the situation of unemployed persons often devolves into mere caricature. Some take the view that the unemployed can always find work if they really want to, so there is no point in supporting them financially while they do so. At the opposite extreme, the unemployed are viewed as victims who deserve the most generous indemnification possible. It is possible to rise above these caricatural stances thanks to surveys that tell us precisely how the unemployed react to economic incentives. Such information is very useful in making unemployment insurance function more effectively.

Job search is linked to the work available: it is an activity aimed at the goal of earning remuneration. But the returns to job search are generally different from the returns to wage-earning activity. If you are unemployed, the income you can derive from an hour of job search will certainly be less than what you could obtain from an hour of work, if you had a job. From an hour of work you derive a wage, whereas an hour of searching for work gives you the chance of obtaining a job interview, or in the best case, of being hired. This variation in return implies that time devoted to job search is highly likely to be shorter than the time devoted to waged work, which is indeed the case, as we will see. We will show further that time devoted to job search, just like time devoted to work, is sensitive to financial incentives.

1.1 How Job Seekers Spend Their Time

Detailed surveys of how persons spend their time can be mined to shed valuable light on the behaviors of the unemployed and wage earners (Krueger and Mueller, 2010, 2011, 2012). Table 5.1, based on the American Time Use Survey (ATUS), presents comparative information about how wage earners and unemployed persons in the United States spend their time. The ATUS is a nationally representative sample drawn from households that have completed their final interview for the Current Population Survey (CPS). Individuals are queried in detail on how they used their time the day before the day of the survey. Unemployed workers are individuals who declared that they did not work in the previous week, that they actively looked for work in the previous four weeks, and that they were available to start work. Employed workers are all those who declared that they had a waged job. The sample is restricted to people age 20 to 54 years. Job search activities typically include calling or visiting a labor office, reading and replying to job advertisements, and job interviews.

Table 5.1 reveals that the unemployed spend on average 32 minutes per day looking for work, a duration that is far less than the time wage earners spend at work, which amounts to 325 minutes. This difference may flow from differences in observed characteristics, such as age, educational qualification, gender, and differences in unobserved ones, such as psychological state, between the unemployed and wage earners. It may also flow from responses to incentives. The labor supply model suggests that two effects may influence the amount of time spent searching for work. First, given that an hour of job search returns less than an hour of waged work, the *substitution effect* ought to result in less time being devoted to job search by an unemployed person than a waged employee devotes to working. Second, since the income of an unemployed person is less than that of a wage earner, the *income effect* ought to result in more time being spent on job search. Table 5.1 suggests that the substitution effect is largely dominant. Thus the unemployed devote more time to domestic production, shopping, and taking care of other members of their household than wage earners do. Sleep, leisure, sports, and socializing also bulk large in their use of their time.

It should be stressed that the time devoted to job search by the unemployed reported in table 5.1 is an average that comprises a high proportion of persons whose use of their time indicates that they did not look for a job at all the day before the survey. In fact, just 20% of unemployed persons engaged in job search activity the day before

Table 5.1
Average minutes per day by activity and employment status in the United States in 2003–2006.

	Employed	Unemployed
Sleep	496	555
Personal care and eating	110	97
Home production, shopping, care of others	158	254
Leisure, travel, sports, and socializing	320	442
Work	325	10
Job search	1	32

Source: Krueger and Mueller (2012, table 3, p. 773) and personal computations.

the survey. The average daily search duration of unemployed persons who did actually hunt for work the day before they were queried is thus 160 minutes.

Table 5.1 also shows that those earning a wage devote on average no more than a minute per day to job search. The fact is, only 0.7% of wage earners search for (another) job, and those who do so spend about 14 minutes a day on the task. From this perspective, the situation of wage earners is much like that of inactive persons, meaning those who declare that they do not have a job and are not searching for one: 0.7% of inactive persons state that they took steps to search for work the day before the survey and did so for an average duration similar to wage earners who searched.

The preceding data apply to the United States, but Krueger and Mueller (2010, 2012) have reported analogous observations for Canada and European countries.

1.2 How Economic Incentives Affect the Time Dedicated to Job Search

Research on job search activity shows that unemployed persons react to economic incentives. Thus Krueger and Mueller (2010) find that workers who expect to be recalled by their previous employer search substantially less than the average unemployed worker, and that across the 50 U.S. states and D.C. the time spent looking for a job is inversely correlated to the level of unemployment benefits, with an elasticity between −1.6 and −2.2. Analysis of a more specific survey of unemployed persons receiving benefits in New Jersey leads, however, to an elasticity of around −0.3, clearly weaker in absolute terms (Krueger and Mueller, 2011). Krueger and Mueller (2010) also find that job seekers who likely have less access to financial resources (e.g., because they do not have a working spouse) tend to respond more to unemployment insurance (UI) benefits than do those with greater financial wherewithal.

As a general rule, more generous unemployment benefits paid for a longer period diminish the amount of time devoted to job search. This phenomenon is illustrated in figure 5.1, which reports the amount of time spent searching by persons eligible for unemployment insurance, and those not eligible, as a function of the duration of their spell of unemployment in the United States over the period 2003–2006.

This figure shows that job search by unemployed persons benefiting from unemployment insurance intensifies as week 26 (the point at which benefits will come to an end) approaches. The increase in the amount of time devoted to the search is considerable, going from less than 20 minutes to more than 70 minutes between week 15 and week 26, and falling back to 20 minutes toward week 34. This observation, which is not replicated in the case of noneligible unemployed persons, strongly suggests that financial support during unemployment influences the amount of time devoted to job search. It is verisimilar to suppose that unemployed persons receiving benefits intensify their job search as their period of eligibility draws to a close in order to avoid the drop in income set to occur at that time if they have not found work.

But figure 5.1 also reveals that the amount of time devoted to job search by those not receiving unemployment benefits is, for the most part, inferior to that of the eligible unemployed, which runs counter to the supposition that the more generous the unemployment benefit, the less time will be spent on job search. Krueger and Mueller (2010) show that this gap persists even for persons with the same *observable* characteristics (age, gender, education, marital status, number of children). It might, however, arise

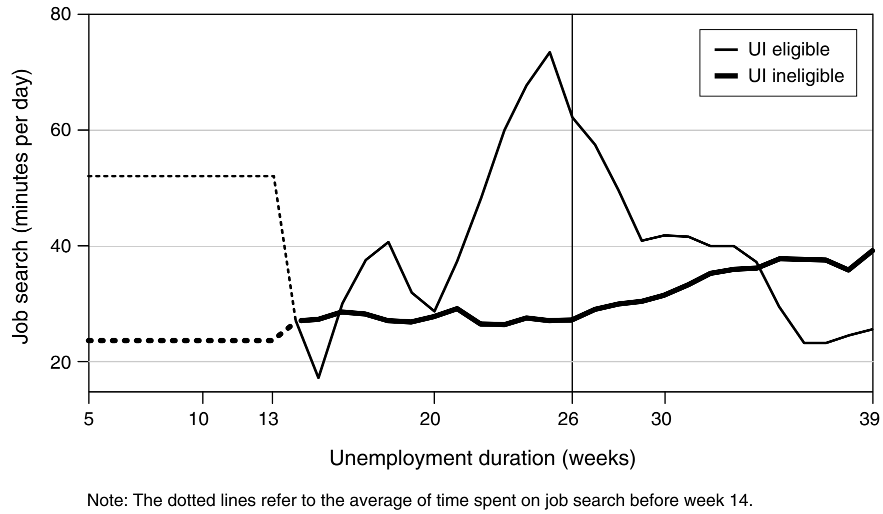

Figure 5.1
Job search by unemployment duration in the United States over the period 2003–2006.

Source: Krueger and Mueller (2010, figure 3, p. 305).

from *unobservable* characteristics like personal motivation, self-esteem, or any other characteristic capable of influencing both eligibility for unemployment insurance and the intensity of job search. It is in fact highly likely that psychologically fragile persons who experience difficulty in career planning will have had shorter spells of past employment, which bar them from access to unemployment benefits, and will likewise be less motivated to hunt for work, irrespective of the level of unemployment benefit. In this context, the difference between the two groups in the intensity of job search flows not from the generosity of unemployment insurance but rather from differences in the unobserved characteristics of the two populations. Hence the observation that the duration of job search differs between the eligible and noneligible unemployed does not warrant the conclusion that financial support during unemployment has a causal impact on the amount of time devoted to job search. In section 3.1 of this chapter we will examine in greater detail the empirical strategies used to try to identify such a causal impact.

1.3 Methods of Job Search: An Internet Revolution?

On the basis of the National Longitudinal Survey of Youth (NLSY97) conducted during 2008–2009, Kuhn and Mansour (2011) have documented with great precision the search methods of American job seekers aged between 24 and 28. Table 5.2 presents their principal results. The data used make it possible to know, for each search method, the proportion of unemployed persons using the Internet and the proportion of unemployed using *offline* methods. Clearly these two channels are not exclusive.

We observe that, on average, an unemployed person makes use of 1.58 of the 9 methods of *offline* research classified as active, the most frequent being "contacted friends or relatives," followed by "contacted employer directly." The proportion is 1.44

TABLE 5.2
Search methods of unemployed workers.

Method	Share of workers using offline methods	Share of workers using online methods
Active search method		
Contacted employer directly	0.36	0.29
Contacted public employment agency	0.19	0.19
Contacted private employment agency	0.07	0.08
Contacted friends or relatives	0.44	0.11
Contacted school/university employment center	0.05	0.06
Sent out résumés or filled out applications	0.24	0.48
Checked unions or professional registers	0.03	0.03
Placed or answered ads	0.16	0.17
Other active methods	0.04	0.03
Total active search methods	**1.58**	**1.44**
Passive search methods		
Looked at ads	0.30	0.32
Attended job training programs or courses	0.06	0.03
Other passive methods	0.02	0.02
Total passive search method	**0.38**	**0.37**

Source: Kuhn and Mansour (2011, table 2, p. 22).

out of 9 for *online* job search, the most frequent being "sent out resumes or filled out applications," followed by "contacted employer directly." It is interesting to note that job search on the Internet is much more "formal" than offline search. The "contacted friends or relatives" approach is adopted by only 11% of unemployed persons via Internet, whereas it is adopted by 44% of offline job seekers. Combining the online and offline methods, a typical job seeker uses 3.02 of the nine active methods. Table 5.2 also shows that the use of passive search methods is identical online and offline, the passive method most frequently resorted to being "looked at ads" in both cases.

Table 5.2 testifies to the importance the Internet has assumed in job search. It might lead us to suppose that the Internet has increased the efficiency of job search, probably by supplying more information, more rapidly, to job seekers and employers. Kuhn and Skuterud (2004), based on data analogous to those of Kuhn and Mansour (2011), but for the period 1998–2000, arrive at no such conclusion. Against all expectation, they found that for identical observable characteristics, unemployed workers who look for work online have longer unemployment durations than non-Internet searchers. With data relative to the same population but pertaining to 2008–2009, hence markedly more recent, Kuhn and Mansour (2011) find, in contrast, that for identical observable characteristics, unemployed workers who look for work online have an average duration of unemployment 25% shorter than non-Internet searchers.

Kuhn and Mansour (2011) put forward two reasons for this spectacular trend reversal. The first is the improved quality of the majority of Internet job search sites, both public and private. The second is the enormous growth in penetration by the Internet: over the 10 years that separate the two studies, the proportion of young unemployed who looked for work online went from 24.2% to 74.4%. By connecting many more workers with many more employers in very short timespans and at very low cost, the Internet has in all likelihood effected a large reduction in labor market frictions. Research on this subject is still too sparse, however, for us to accept such a conclusion unreservedly.

We now describe job search behavior more precisely, with the help of a model that has proved its utility for understanding and evaluating with precision the effects of the various parameters of unemployment insurance systems.

2 BASIC JOB SEARCH THEORY

Job search theory arises initially out of a basic model—called today the partial model—describing the behavior of a person looking for work in a situation of imperfect information. This model furnishes precise conclusions about the effects of a change in the environment or in economic policy. The basic model is grounded, however, on oversimple hypotheses, and these we must abandon in order to describe the reality of the search process better. For one thing, in this model all the unemployed have access, in exogenous fashion, to unemployment insurance benefits, they are not allowed to select the intensity of their search, and they cannot look for (another) job once they are employed. Finally, the basic model is situated in a stationary environment. We first lay out the basic model, then analyze the changes that emerge as we abandon these four hypotheses.

2.1 The Basic Model

In the job search model, the optimal strategy of a person looking for work consists simply of choosing a *reservation wage* that represents the lowest remuneration he will accept. The amount chosen depends on all the parameters that go to make up the economic environment, in particular the benefits paid to those who are unemployed and the arrival rate of job offers. Hence most often it is enough to know how the reservation wage varies in order to discern the effects of economic policy on the duration of unemployment. As well, when it is linked to the labor supply model presented in chapter 1, the job search model makes it possible to shed light on the choice of nonparticipation, unemployment, or work.

2.1.1 The Search Process and the Reservation Wage

The basic job search model aims to describe the behavior of an unemployed person who dedicates all of his efforts to looking for a job, when the conditions in which this search takes place do not vary over time. The dynamic aspect of the model makes it possible to define the optimal job search strategy. The model is explicitly dynamic but is situated in a stationary environment.

The Discounted Expected Utility of an Employee

The main hypothesis of the job search model is that the job seeker does not know exactly what wage each job pays. So by looking, he can expect to improve his prospects of earning. We epitomize these imperfections in the available information by postulating that the job seeker knows only the cumulative distribution of the possible wages. We further assume that this distribution is the same at each date and that successive wage offers are independent draws from this distribution. This stationarity assumption means that, at any time, a person looking for work faces the same structure of information. We use $H(.)$ to denote the cumulative distribution function of all possible wages.

A job offer comes down to the proposal of a constant real wage w, which the worker will receive on each date as long as he remains with the firm that makes the offer. If we assume that the agent is risk-neutral and if, for the sake of simplicity, we leave out of account the disutility of work, his *instantaneous* utility then simply equals w. This means that over a short interval of time, $\mathrm{d}t$ in length,[1] the agent attains a level of instantaneous satisfaction equal to $w\mathrm{d}t$. Let us further assume that over each short interval of time $\mathrm{d}t$, any job whatsoever can disappear at the rate $q\mathrm{d}t$, where $q > 0$ is a constant exogenous parameter. Over each short interval of time $\mathrm{d}t$, a waged worker thus loses his job at the rate $q\mathrm{d}t$. Let us assume that the real instantaneous rate of interest r is constant and exogenous. A single dollar invested in the financial market on date t brings in $1 + r\mathrm{d}t$ dollars in $t+\mathrm{d}t$. The discounted value of a dollar at date t that will be available at date $t+\mathrm{d}t$ is thus equal to $1/(1 + r\mathrm{d}t)$. The term $1/(1 + r\mathrm{d}t)$ thus represents the discount factor over each short interval of time $\mathrm{d}t$. In a stationary state, the discounted expected utility V_e of an employed person receiving wage w satisfies the following relation:

$$V_e = \frac{1}{1 + r\mathrm{d}t}\left[w\mathrm{d}t + (1 - q\mathrm{d}t)V_e + q\mathrm{d}tV_u\right] \tag{5.1}$$

This relation indicates that the discounted expected utility stemming from being hired is equal to the discounted sum of the flow of income $w\mathrm{d}t$ over the interval of time $\mathrm{d}t$, and the discounted expected future income. With probability $(1 - q\mathrm{d}t)$ this future income does coincide with the expected utility V_e associated with continued employment, and with complementary probability $q\mathrm{d}t$ it conforms instead to V_u, the discounted expected utility of an unemployed person. Multiplying the two sides of relation (5.1) by $1 + r\mathrm{d}t$ and rearranging the terms of this expression, we arrive at:

$$rV_e = w + q(V_u - V_e) \tag{5.2}$$

This equation is easy to interpret.[2] It shows that, at every moment, a job entails discounted expected flow of income rV_e equal to wage w, to which is added average income

[1] The unfolding of time may be described in continuous or discrete manner; we have chosen the former because it is analytically more simple and has been adopted almost universally by all published work in this field.

[2] Mathematical appendix D at the end of the book supplies a rigorous proof of formulas analogous to equation (5.2) and shows that they effectively correspond to the stationary state of a model where a particular event (here, the loss of work) follows a Poisson process.

$q(V_u - V_e)$ deriving from any possible change in the employee's status. This average income is in fact a loss resulting from the wage worker's having quit his job.

Equality (5.2) allows us to express the discounted expected utility of an employee receiving wage w—which we henceforth denote $V_e(w)$—in the following manner:

$$V_e(w) - V_u = \frac{w - rV_u}{r + q} \tag{5.3}$$

It is thus apparent that the difference between the expected utility of an employee and that of an unemployed person expands with the wage accepted and shrinks with the discounted expected utility of the unemployed person.

The Optimal Search Strategy

To simplify the exposition we will assume that a job seeker can only meet a single employer on any date (see Mortensen, 1986, for the possibility of multiple offers). The employer offers the job seeker the constant wage w over the duration of her employment, which she is free to accept or refuse. The optimal job search strategy is then as follows:

1. If the job seeker receives no offer on date t, she continues looking. This behavior results from the stationarity of intertemporal utility V_u.

2. If the job seeker receives a wage offer w, she accepts if $V_e(w) > V_u$. If not, she continues looking.

Since a job seeker's expected utility V_u does not depend on a particular wage offer w, relation (5.3) shows that $V_e(w)$ is an increasing linear function of the wage offered. This relation also shows that phase 2 of the search strategy amounts to the adoption of a "stopping rule" that dictates accepting wage w if and only if it is superior to a threshold-value x defined by:

$$x = rV_u \tag{5.4}$$

The acceptance of an offer exactly equal to x procures for the job seeker the same level of utility that she gets by remaining unemployed; in other words, $V_e(x) = V_u$. As in the theory of labor supply laid out in chapter 1, wage x continues to be called the *reservation wage*, but we will see in section 2.1.3 that it means something tangibly different.

Thus the job search model shows that the optimal strategy consists of continuing to hunt for a job as long as incoming job offers entail wages below the reservation wage. The optimality of this strategy, which is known as sequential search, was demonstrated by McCall (1970). Other strategies are conceivable. Stigler (1962), for example, proposes a nonsequential strategy that consists of deciding, at the moment unemployment commences, to review a set number of job offers. This strategy is not optimal, as it may lead to continuing to search for a job even after receiving an offer greater than the reservation wage.

The Discounted Expected Utility of a Job Seeker

The existence of the stopping rule makes it possible to deduce numerous characteristics of the search process from those of the reservation wage. To make the factors that determine the reservation wage explicit, we need to specify more precisely the discounted expected utility V_u of a job seeker. Accordingly, we will designate by λ the arrival rate of job offers. This rate encapsulates the difficulties encountered while looking for a job. It reflects the general state of the labor market, but it also depends on the personal characteristics of the job seeker—age and educational qualifications, for example—and the effort he puts into the search. In the basic model, we assume that this rate λ is a constant exogenous quantity. Moreover, the search for a job entails costs at every turn. Some are financial, like the cost of getting about, buying specialized magazines, and sending out applications. But it is equally necessary to include the opportunity cost of the search, in other words, the value of a period of time that could have been devoted to other activities. All these costs will be summed up, at each date, by a single scalar $c > 0$. There are also gains associated with periods of looking for work. These comprise unemployment benefits and also perhaps the consumption of domestic production and leisure. If, for each date, we express the sum of these gains by the scalar $b > 0$, the *net* instantaneous income from looking for work, denoted z, is then equal to $(b - c)$.

At any moment the status of a job seeker may change with rate λ. If he does actually receive an offer, he will not accept unless the wage that goes with it is more than his reservation wage x. The discounted utility V_λ expected upon receiving an offer of employment is thus equal to:

$$V_\lambda = \int_0^x V_u \mathrm{d}H(w) + \int_x^{+\infty} V_e(w)\mathrm{d}H(w)$$

Conversely, if the job seeker receives no offers, he keeps looking, which procures for him a discounted expected utility equal to V_u. Now, during a short interval of time $\mathrm{d}t$ in length, a job seeker gains $z\mathrm{d}t$ and has a probability $\lambda\mathrm{d}t$ of receiving a job offer. In the stationary state, his expected utility thus satisfies:

$$V_u = \frac{1}{1 + r\mathrm{d}t}\left[z\mathrm{d}t + \lambda\mathrm{d}tV_\lambda + (1 - \lambda\mathrm{d}t)V_u\right]$$

If we multiply the two sides of this equality by $1 + r\mathrm{d}t$ and rearrange terms, we find that a job seeker's discounted expected utility is defined by the following trade-off equation:

$$rV_u = z + \lambda \int_x^{+\infty} \left[V_e(w) - V_u\right] \mathrm{d}H(w) \tag{5.5}$$

Like relation (5.2) defining an employee's discounted expected utility, this equation has to be interpreted by examining the various ways the assets V_u of an unemployed person may be invested. In the "financial" market these assets will bring in rV_u at any moment, while if "invested" in the labor market they will procure income z augmented by the value $\lambda(V_\lambda - V_u)$ of the average gain linked to the change of status of a person who is looking for work.

Reservation Wage, Hazard Rate, and Average Duration of Unemployment

With the help of relations (5.3) and (5.4), which define respectively the intertemporal utility $V_e(w)$ of an employee and the reservation wage x as a function of the discounted expected gain V_u of an unemployed person, we easily arrive at the following equation, which implicitly characterizes the reservation wage as a function of the parameters of the model:

$$x = z + \frac{\lambda}{r+q}\int_x^{+\infty}(w-x)\mathrm{d}H(w) \tag{5.6}$$

We can show (1) that there is only one optimal value for this reservation wage, and (2) that it maximizes the intertemporal utility of a job seeker. For that, we need merely observe that relation (5.5) defines V_u as a function of x and verify that the derivative of this function is null for the value of x given by (5.6). Equation (5.6) shows that the reservation wage is equal to the net income from the job search plus the discounted expected value of what the job search can yield above the reservation wage. This way of characterizing the reservation wage is instructive, for it brings out clearly the optimality of the search strategy adopted by the job seeker.

The values of two other important variables flow from knowing the reservation wage. These are the “hazard rate,” or the exit rate from unemployment, and the average duration of unemployment. Since a job seeker becomes employed when (1) she receives a wage offer—which occurs at rate λ—and (2) the offer is at least equal to her reservation wage—which occurs with probability $[1 - H(x)]$—the exit rate from unemployment takes the value $\lambda[1 - H(x)]$ at any moment. When the number of job seekers is large, this rate merges with the hazard rate. The average duration of unemployment, denoted T_u, is then given by:

$$T_u = \frac{1}{\lambda[1-H(x)]} \tag{5.7}$$

The interpretation of this last relation is very intuitive: it means that if a job seeker has one chance in ten of becoming employed in any week, she will on average remain unemployed for ten weeks.[3] Relation (5.7) also shows that the average duration of unemployment is an increasing function of the reservation wage: when a person who is looking for work raises the level of the wage she is demanding, on average it prolongs the duration of the search.

2.1.2 Comparative Statics of the Basic Model

The comparative statics properties of the job search model are very easily obtained if we write relation (5.6), which defines the reservation wage, in the following form:

$$\Phi(x,z,r,\lambda,q) = 0 \quad \text{with} \quad \Phi(x,z,r,\lambda,q) \equiv x - z - \frac{\lambda}{r+q}\int_x^{+\infty}(w-x)\mathrm{d}H(w) \tag{5.8}$$

We can easily verify that the partial derivatives of the function Φ possess the following properties:

$$\Phi_x > 0, \quad \Phi_z < 0, \quad \Phi_r > 0, \quad \Phi_\lambda < 0 \quad \text{and} \quad \Phi_q > 0$$

[3] Mathematical appendix D at the end of the book shows that if a random variable follows a Poisson process of parameter a, then the mathematical expectation of this variable is equal to $1/a$.

As relation (5.8) implies $\partial x/\partial i = -\Phi_i/\Phi_x$, $i = z, r, \lambda, q$, we immediately obtain the direction of the variations in the reservation wage as a function of the parameters of the model, or:

$$\frac{\partial x}{\partial z} > 0, \quad \frac{\partial x}{\partial \lambda} > 0, \quad \frac{\partial x}{\partial r} < 0 \quad \text{and} \quad \frac{\partial x}{\partial q} < 0 \tag{5.9}$$

With the help of relation (5.7), we deduce from this the main comparative statics properties of the average duration of unemployment. The result is:

$$\frac{\partial T_u}{\partial z} > 0, \quad \frac{\partial T_u}{\partial r} < 0 \quad \text{and} \quad \frac{\partial T_u}{\partial q} < 0$$

The rise in the reservation wage and the average duration of unemployment that follow from a rise in the net income z from looking for work, constitute an important result of this theory. This means, all other things being equal, that an increase in unemployment benefits should have the effect of lengthening the duration of unemployment. This result is highly intuitive: it simply makes sense that a job seeker receiving higher compensation will be more demanding in terms of the wage he hopes to get, and that that on average will lengthen the amount of time he spends looking. This strong prediction of the theory of job search has often been contested (see Atkinson and Micklewright, 1991, for a detailed critical analysis of it). On the theoretical level, however, it is unassailable, since the person looking for work does in fact receive benefit payments from the unemployment insurance system (which is the case in the basic model just presented). Let us suppose, for example, that unemployment benefits rise from 0% to 100% of the current average wage. It is hard to believe that a change of that magnitude in the size of the payments will have no positive influence on the average duration of unemployment. But leaving aside this exaggerated example, the extent of the influence is a priori unknown. Moreover, a very large percentage of those looking for work receive no unemployment benefits. We will see that, for them, an increase in unemployment benefits is highly likely to have an inverse effect on their reservation wage (a point rigorously established in section 2.2.1 of this chapter, which deals with the eligibility effect). Given these circumstances, we have to turn to empirical studies to get an idea of the sign and the order of magnitude of the unemployment benefits elasticity of the average duration of unemployment. We will see below that in general this elasticity is slight when the amount of unemployment benefits takes a "reasonable" magnitude.

The other implications of the model are also easy to grasp. A rise in r is characteristic of a job seeker who places less value on the future than another. A person of this type has a lower reservation wage and on average spends less time looking for work. When the job loss rate $q$increases, the current demands of job seekers diminish, since the gap between the expected utility of an employee and that of a job seeker shrinks, which reduces the average duration of unemployment. Another interpretation of this relation is that when jobs are of shorter duration, workers are less demanding because they know they will have other opportunities in the future. On the other hand, relation (5.7) shows that an increase in λ, the arrival rate of wage offers, has an ambiguous effect on the amount of time devoted to looking for a job. In this case, job seekers revise their reservation wage upward, which entails a lowering of the term $[1 - H(x)]$ representing the probability of accepting an offer. The direction of consequent change in the rate of exit from unemployment $\lambda[(1-H(x)]$ and the average duration of unemployment $T_u = 1/\lambda[(1-H(x)]$ is then unknown. It should be noted, however, that if the frequency with which job offers arrive has little effect on the reservation wage, the average duration of unemployment decreases with this frequency. Empirical studies do seem to indicate as much (see section 3.2.2).

2.1.3 The Choice of Nonparticipation, Job Seeking, or Employment

Decisions to participate in the labor market are envisaged one way under the theory of labor supply and another way under the theory of job search. The theory of labor supply comprises only two possible states: either one is a participant or one is not. The theory of job search just outlined assumes that workers do participate in the labor market and are thus faced only with the choice between unemployment and employment. It is possible, though, to contemplate a hybrid model that takes into account three possible states: nonparticipation, job seeking, and employment.

The Reservation Wage and Alternative Income

In the theory of labor supply, participation in the labor market depends on a comparison between the current wage w and the reservation wage w_A defined by relation (1.3) in chapter 1. In this theory, decisions to participate can be summarized in the following manner:

$$\begin{cases} w > w_A \implies \text{employee} \\ w \leq w_A \implies \text{nonparticipant} \end{cases} \tag{5.10}$$

The theory of job search defines the reservation wage x as the wage at which the job seeker is indifferent between accepting a job and continuing to look. It depends on the overall characteristics of the labor market, which we will designate by Ω. According to equation (5.6) defining x, these characteristics include the distribution $H(.)$ of possible wages, the net income z associated with the job search, the job offers arrival rate λ, the interest rate r, and the job destruction rate q. Thus in symbolic terms we may write $\Omega = \Omega(H, z, q, \lambda, r)$ and $x = x(\Omega)$. The choice between participation and nonparticipation is based on a comparison between the expected utility of a job seeker V_u and that of a nonparticipant V_I. If the latter receives a constant income R_I at each date, her expected utility is defined by the equality $rV_I = R_I$. This can easily be compared to that of a job seeker, which is such that $rV_u = x$. An agent decides to participate in the labor market if and only if $V_I \leq V_u$, which translates into the inequality $x(\Omega) \geq R_I$. It is apparent that the decision to participate in the labor market is made by comparing the reservation wage to the "alternative income" R_I that a nonparticipant is capable of obtaining at any moment. Individual decisions hence take the following form:

$$\begin{cases} x(\Omega) \geq R_I \implies \text{participant} \\ x(\Omega) \leq R_I \implies \end{cases} \tag{5.11}$$

Moreover, when a participant receives a wage offer w, she accepts if it exceeds her reservation wage. In other words, the decisions of a participant come down to:

$$\begin{cases} w > x(\Omega) \implies \text{employee} \\ x(\Omega) \geq w > R_I \implies \text{unemployed} \end{cases} \tag{5.12}$$

The theory of job search suggests that the rate of participation depends on the set Ω of all the factors affecting the labor market. For example, some studies reveal that a rise in unemployment insurance benefits (an increase of z) is often accompanied by a rise in the participation rate, which itself takes the form of a rise in the unemployment rate (see Moorthy, 1989). In the same way, an increase in the unemployment rate, by lessening the probability of exiting from unemployment, tends to diminish the reservation wage and thus the participation rate. This relationship augments the procyclical character of the participation rates deduced from the labor supply model, in which the lowering of wages in bad economic times gives individuals incentive to withdraw from the labor market.

Discouraged Workers

The theory of job search only takes account of the wage prevailing in the marketplace through the distribution of its possible values. Hence, among nonparticipants, it is difficult to distinguish those who don't want to work at the "current" wage from those who would accept a job for that amount of remuneration but who give up looking because of the costs incurred by doing so and the time they would have to wait before being hired. These nonparticipants are called *discouraged workers*. If we assimilate the average of possible wages $\mathbb{E}(w) = \int_0^{+\infty} w\mathrm{d}H(w)$ to the "current" wage, we can conclude that individuals for whom $x(\Omega) \leq R_I \leq \mathbb{E}(w)$ form the category of discouraged workers. More generally, the "discouraged worker effect" is cited whenever change in certain characteristics of the economic environment implies a lowered participation rate. For example, if job offers arrive with reduced frequency, the reservation wage $x(\Omega)$ falls, and consequently the participation rate falls too (since the latter is by definition the percentage of the population for whom the relation $x(\Omega) \geq R_I$ is satisfied).

Numerous studies allow us to obtain an estimate of the number of discouraged workers. It suffices to identify, among the individuals who claim to be looking for work, those who have not made efforts that count as really "significant" (see OECD, 1994, volume 1 for a precise definition of this adjective). Table 5.3 shows that their number is not negligible.

The Frontier Between Nonparticipation and Job Seeking

The existence of discouraged workers suggests that the frontier between nonparticipation and participation in the labor force is difficult to draw. When does the intensity of

Table 5.3
Discouraged workers and job seekers in 2011 (as a percentage of the labor force).

Country	Discouraged workers	Job seekers
Denmark	0.15	7.6
Spain	1.33	21.6
France	0.12	9.3
Germany	0.14	5.9
United States	0.65	8.9
Japan	1.04	4.5

Source: OECD Labor Force Statistics.

the effort made by an individual to find a job qualify him as an active job seeker? The varying definitions of unemployment supply different, and perforce arbitrary, answers to this question. Measurements of unemployment derive from investigations in which, to be considered unemployed, you have to have been without work (during the period in question), have taken steps to look for work, and be ready to start work (in principle) immediately. But these three conditions, in particular the second pertaining to the process of looking, can have different meanings. Thus in the United States individuals who employ passive methods (like looking in the want ads) are classed as nonparticipants, while numerous OECD countries consider job seekers employing both passive and active methods as unemployed (see U.S. Bureau of Labor Statistics, www.bls.gov/cps/cps_htgm.htm).

A number of factors point to the conclusion that the distinction between nonparticipation and unemployment often turns out to be arbitrary. Re-interview programs carried out in the United States with individuals already interviewed the week before reveal that, especially for individuals situated close to the frontier of nonparticipation, the answers given (regarding the same period of reference) can be quite different (Abowd and Zellner, 1985). Some people are hard to classify, and their answers are highly sensitive to the way the interviews are conducted. Jones and Riddell (1999) show that individuals classed as nonparticipants by surveys of the labor force in Canada are anything but uniform in their behavior. These authors distinguish four categories of individuals: the employed, the unemployed, individuals marginally attached to labor market participation, and nonparticipants. Individuals marginally attached to labor market participation, traditionally considered nonparticipants, say that they are not looking for a job but would like to work. They represent 25% to 30% of the volume of unemployment over the period studied by Jones and Riddell. The matrix of transition between different states is presented in table 5.4. It is apparent that individuals marginally attached to labor market participation behave differently on average than nonparticipants, since they have a much higher probability of returning to full participation. The rates at which individuals on the margin of participation do return to employment are closer to those of the unemployed than to those of genuine nonparticipants. Jones and Riddell also emphasize that the category of individuals marginally attached to participation is extremely heterogeneous. Consequently, within the overall group of those who say they would like to work but are not looking for a job, Jones and Riddell distinguish persons who are “waiting” for a job—because they are “waiting to be recalled by their former

Table 5.4

The transition matrix between different states in the labor market. Monthly rates for the year 1992 in Canada (standard errors are in parentheses).

From ↓ To ⟶	Employed	Unemployed	Nonparticipant + marginally attached
Unemployed	0.112 (0.004)	0.708 (0.005)	0.180 (0.005)
Marginally attached	0.098 (0.005)	0.171 (0.007)	0.731 (0.008)
Nonparticipant	0.026 (0.001)	0.030 (0.001)	0.944 (0.002)

Source: Jones and Riddell (1999, table 1).

employer," or "have found a job but haven't been hired yet," or "are waiting for an answer from an employer"—and discouraged persons who "believe there are no jobs matching their qualifications available in their region." It is apparent that those who are waiting for a job have a rate of return to employment higher than that of the unemployed (equal to 0.200), whereas discouraged workers show behavior closer to that of genuine nonparticipants (their rate of return equals 0.044).

These examples show that taking job search behavior into consideration renders the distinction between labor market participation and nonparticipation ambiguous. In consequence, assessments of unemployment and of the labor force are necessarily arbitrary, and it is generally useful to supplement them with other indicators in order to get a clear picture of the state of the labor markets. In this regard, the employment rate—equal to the ratio between the number of jobs and the population of working age, generally taken to be all those between 15 and 64 years of age—is a supplementary indicator frequently used to gauge what is happening in the labor markets.

2.2 Extensions of the Basic Model

The results obtained using the basic model are numerous. They have however been obtained using hypotheses that are sometimes very restrictive. To expand on what the basic model has to tell us, we first examine the consequences of the conditions of eligibility for unemployment insurance benefits. We then look at the changes we must make when an individual is able to look for a job while he or she is already working. After that we make the assumption that agents can decide how much effort to put into their job search. And finally we study the consequences of the fact that unemployment insurance benefits are not stationary.

2.2.1 Eligibility and Unemployment

In most countries, those who work in exchange for wages have to pay premiums into an unemployment insurance system that allows a wage earner to receive compensation if she loses her job. When these conditions are met, we say that the worker is *eligible* for unemployment insurance benefits. But many people, in particular new entrants into the labor market and those who have been unemployed for a long time, are not eligible for such benefits. For them, finding a job also means becoming eligible, or becoming eligible again. This entails that the reservation wage of those who are not eligible *sinks* when the benefits paid to the unemployed *who do meet the eligibility requirements* rise.

Two Types of Job Seeker

To make this intuition perfectly explicit, we will assume in what follows that there are two types of job seekers: those who are eligible for unemployment insurance benefits and those who are not. This circumstance can be formalized quite simply by assuming on one hand that the instantaneous income of the former always amounts to z, while that of the latter has the value $z_n < z$, and on the other that an individual becomes and remains eligible if she has been employed at least once. In this context, z represents the benefits paid by the unemployment insurance system, while z_n is determined by the welfare system, which generally pays out smaller amounts.

The situation of the eligible job seeker is identical to that of the basic model, and her reservation wage, always denoted by x, continues to be defined by equation (5.6). But the behavior of a noneligible job seeker is not so simple because her expected utility, denoted V_{un}, depends on that of an eligible job seeker, which continues to be denoted V_u. When a noneligible job seeker accepts a job offering an instantaneous wage w, her expected utility $V_e(w)$ satisfies the following equation:

$$rV_e(w) = w + q\,[V_u - V_e(w)] \tag{5.13}$$

It should be noted that it is the expected utility V_u of an *eligible* job seeker that appears in this expression, for it is assumed, for the sake of simplicity, that unemployment insurance benefits are paid whenever an agent has been employed at least once. For given V_u, relation (5.13) indicates that $V_e(w)$ increases with w, and that the reservation wage of a noneligible job seeker, denoted x_n, satisfies the equality $V_e(x_n) = V_{un}$. Since we always have $x = rV_u$, equation (5.13) allows us to express the expected utility of a noneligible job seeker as a function of the two reservation wages, x and x_n. The result is:

$$rV_{un} = \frac{rx_n + qx}{r + q} \tag{5.14}$$

Assuming that the frequency with which a noneligible job seeker receives job offers is always equal to λ, her expected utility is defined by the following equation, which is analogous to relation (5.5) in the basic model:

$$rV_{un} = z_n + \lambda \int_{x_n}^{+\infty} [V_e(w) - V_{un}]\, \mathrm{d}H(w) \tag{5.15}$$

The Reservation Wage of Noneligible Job Seekers

Observing, from (5.13), that $rV_e(w) = (rw + qx)/(r + q)$, and utilizing expression (5.14) of V_{un}, we arrive, thanks to (5.15) and after several simple calculations, at a relation that implicitly defines the reservation wage x_n of a noneligible person as a function of that of an eligible person. It is written:

$$rx_n = (r + q)z_n - qx + \lambda \int_{x_n}^{+\infty} (w - x_n)\mathrm{d}H(w)$$

It is easy to verify that this relation implies a negative linkage between x_n and x. Since x increases with the instantaneous income z of eligible job seekers, the reservation wage x_n of *noneligible* job seekers is a *decreasing* function of z. This outcome is explainable as follows: a noneligible job seeker knows that by accepting an offer of work, he risks becoming unemployed again in the future at rate q. But in that case, he also knows that he will henceforth be eligible for unemployment benefits $z > z_n$. A rise in z therefore increases the loss occasioned by refusing a job offer, which gives him incentive to lower his reservation wage. On the other hand, we may note that an increase in welfare payments z_n exerts upward pressure on the reservation wage of noneligible job seekers. This implies that a rise in unemployment benefits has an ambiguous impact on unemployment because it increases the unemployment spell of eligible job seekers but it decreases the unemployment spell of those who are not eligible.

2.2.2 On-the-Job Search

As a general rule, an individual who has a job is still able to carry out a search for another one. For the sake of simplicity, we will assume that the costs of job search are negligible for a worker who is employed. The advantage of this hypothesis is that we do not have to make a distinction between employees who have a low wage and are looking for another job and those who are receiving a high wage and therefore are not looking, since the cost of doing so would be too high compared to their earnings prospects. If the costs of searching for a job are null for an employed worker, she always has an interest in looking for another job, and accepts the first offer that exceeds her present wage.

The Behavior of Agents

Let us assume that an employed person receives job offers with a frequency of λ_e, and that she risks losing her job, at any time, with an exogenous constant probability of q. The discounted utility $V_e(w)$ expected by a wage earner whose current remuneration comes to w then has three components. The first corresponds to the instantaneous income w deriving from her waged labor, the second is the average discounted expected gain $q[V_u - V_e(w)]$ due to job loss, and the third is the discounted expected earnings $\lambda_e \int_w^{+\infty} [V_e(\xi) - V_e(w)]\mathrm{d}H(\xi)$ consequent upon a change of employer (which occurs for every wage offer that exceeds the present wage w). Finally, $V_e(w)$ is defined by the following equation:[4]

$$rV_e(w) = w + q[V_u - V_e(w)] + \lambda_e \int_w^{+\infty} [V_e(\xi) - V_e(w)]\,\mathrm{d}H(\xi) \tag{5.16}$$

Deriving this relation with respect to w, we get:

$$V_e'(w) = \frac{1}{r + q + \lambda_e[1 - H(w)]} \tag{5.17}$$

In this way we easily verify that the discounted expected utility $V_e(w)$ of an employee increases with wage w; hence the optimal search strategy for a job seeker is characterized by a reservation wage x such that $V_e(x) = V_u$. Assuming that the arrival rate of job offers is equal to λ_u for a job seeker, and again designating her instantaneous gain by z, her discounted expected utility V_u continues to be defined by equation (5.5), so that:

$$rV_u = z + \lambda_u \int_x^{+\infty} [V_e(\xi) - V_u]\,\mathrm{d}H(\xi) \tag{5.18}$$

Making $w = x$ in (5.16) and comparing (5.18), we immediately get:

$$x = z + (\lambda_u - \lambda_e) \int_x^{+\infty} [V_e(\xi) - V_u]\,\mathrm{d}H(\xi) \tag{5.19}$$

[4] The reader who is not yet sufficiently familiar with this type of equation will benefit from working with a small interval of time $[t, t+\mathrm{d}t]$. In the stationary state, we thus have:

$$(1 + r\mathrm{d}t)V_e(w) = w\mathrm{d}t + q\mathrm{d}tV_u + (1 - q\mathrm{d}t)\left[\lambda_e\mathrm{d}t \int_w^{+\infty} V_e(\xi)\mathrm{d}H(\xi) + \lambda_e\mathrm{d}tV_e(w)H(w) + (1 - \lambda_e\mathrm{d}t)V_e(w)\right]$$

By rearranging a few terms and making $\mathrm{d}t \to 0$ in this formula, we come back to equation (5.16).

Compared to the basic model, this equation indicates that a job seeker must henceforth weight the discounted expected utility of the job search $\int_x^{+\infty}[V_e(\xi)-V_u]\mathrm{d}H(\xi)$ by the *difference* $\lambda_e-\lambda_u$ of the rates with which job offers arrive.

Properties of the Reservation Wage

We will see further in this chapter (section 4.2) that the possibility of moving from one job to another plays an essential role in the elaboration of *equilibrium* search models, that is, models in which the cumulative distribution function $H(.)$ is endogenous. In this regard, it is useful to determine precisely the expression of $V_e(\xi)-V_u$ appearing in (5.19) so as to bring out the dependence between the reservation wage x and the function $H(.)$. By applying the formula of integration by parts[5] to the right-hand side of (5.19), we arrive at:

$$x=z+(\lambda_u-\lambda_e)\left[\left[-\bar{H}(\xi)\left[V_e(\xi)-V_u\right]\right]_x^{\infty}+\int_x^{+\infty}\bar{H}(\xi)V_e'(\xi)\mathrm{d}\xi\right]\quad \text{with}\quad \bar{H}(\xi)\equiv 1-H(\xi)$$

As we still have $V_e(w)-V_u=\int_x^w V_e'(\xi)\mathrm{d}\xi$, utilizing (5.17) and assuming that $\lim_{\xi\to\infty}\bar{H}(\xi)\left[V_e(\xi)-V_u\right]=0$, we finally have:

$$x=z+(\lambda_u-\lambda_e)\int_x^{+\infty}\frac{\bar{H}(\xi)}{r+q+\lambda_e\bar{H}(\xi)}\mathrm{d}\xi \tag{5.20}$$

This equation implicitly defines the reservation wage as a function of the parameters λ_u, λ_e and the cumulative distribution function $H(.)$. When $\lambda_e=0$, that is, when there is no on-the-job search, we come back to the reservation wage of the basic model. Vice versa, if $\lambda_e>0$, the job seeker takes account of the possibilities of future income associated with continuing to look for a job while employed. Adopting this stance has the effect of lowering the reservation wage. If $\lambda_e=\lambda_u$, the reservation wage is equal to the net income z of the job seeker, for a worker then has as many chances of receiving an acceptable offer while employed as he does while unemployed. It is also interesting to note that if $\lambda_e>\lambda_u$, the reservation wage falls *below* z. In this configuration of the parameters, an employee has more chances of obtaining an acceptable offer than a job seeker. The latter thus has an incentive to accept "bad" jobs, which nevertheless afford him better prospects than his present situation of being unemployed. The bulk of the estimations show however that the inequality $\lambda_u\geq\lambda_e$ is the most probable. For example, using data from the Netherlands, van den Berg and Ridder (1998) find that λ_u differs very little from λ_e, while Bontemps (1998) and Kiefer and Neumann (1993) estimate, using French and American data, that λ_u is respectively 10 times and 5 times higher than λ_e. This likely comes about because unemployed job seekers devote more effort to looking for work than employed job seekers do. Be that as it may, taking into account on-the-job search ($\lambda_e>0$) has the effect of diminishing the size of the reservation wage in comparison to the one that emerges from the basic model ($\lambda_e=0$).

[5] This formula reads $\int u\mathrm{d}v=uv-\int v\mathrm{d}u$, where u and v are two functions. Here, we posit: $u=V_e(\xi)-V_u$, $\mathrm{d}u=V_e'(\xi)\mathrm{d}\xi$, $\mathrm{d}v=h(\xi)\mathrm{d}\xi$, and $v=-\bar{H}(\xi)$.

2.2.3 Choosing How Hard to Look

The hypothesis that both the arrival rate of job offers and the costs of the job search do not vary is unsatisfactory, since it does not allow us to take into account the fact that a job seeker may make sedulous efforts that increase the costs of the job search but at the same time increase her chances of receiving job offers. This relation is well documented by the empirical research cited in the first section of this chapter, which shows that persons who dedicate more time to looking for work exit more rapidly from unemployment.

Optimal Effort

Let us designate the intensity of the job search by the scalar e, which can be interpreted as the time and/or the intensity of the effort devoted to search. The notion that more job offers should result from greater effort devoted to search amounts to postulating that the rate at which offers arrive increases with e. For the sake of simplicity and without loss of generality, we postulate a linear relation $\lambda = \alpha e$. The parameter $\alpha > 0$ we interpret as an indicator of the state of the labor market, independent of individual efforts. This parameter is a function of, among other things, the number of vacant jobs, the number of job seekers, and objective characteristics like age, sex, and educational level. We will denote by $\phi(e)$ the cost arising from the search effort e, with $\phi' > 0$ and $\phi'' > 0$. So henceforth the instantaneous utility of a job seeker will be written $[z - \phi(e)]$. For ease of exposition, we also assume that there is no on-the-job search and for that matter the opposite assumption would change the outcome very little (see Mortensen, 1986). Thus we can follow exactly the line of reasoning worked out in the basic model in section 2.1, positing in the first stage that the amount of effort e is given.

The reservation wage x is always implicitly defined by the equation (5.6), which will henceforth be written:

$$x = z - \phi(e) + \frac{\alpha e}{r+q}\int_{x}^{+\infty}(w-x)\mathrm{d}H(w) \tag{5.21}$$

This relation gives the value of the reservation wage associated with a given amount of effort e. Now the optimal value of effort ought, by definition, to maximize the intertemporal utility V_u of a job seeker. Since $V_u = x/r$, this value is reached by differentiating relation (5.21) with respect to e and looking for the value of e for which $\partial x/\partial e = 0$. The result is:

$$\phi'(e) = \alpha \int_{x}^{+\infty} \frac{w-x}{r+q}\mathrm{d}H(w) \tag{5.22}$$

The convexity of function $\phi(.)$ guarantees that the amount of effort defined by this relation is indeed a maximum. This equation states that it is optimal to equalize the marginal cost of effort to its marginal return. The latter is equal to α (the increased probability of getting a job offer) times the expected gains associated with a job offer.

With the help of (5.21), we further obtain:

$$x = z + e\phi'(e) - \phi(e) \tag{5.23}$$

The Properties of Optimal Effort

In what follows, it will be helpful to view equations (5.22) and (5.23) as forming a system determining in an implicit manner the reservation wage and the optimal effort, respectively written $x(\alpha, z)$ and $e(\alpha, z)$. By differentiating relation (5.23) with respect to α, it is easy to show that $\partial x(\alpha, z)/\partial \alpha$ and $\partial e(\alpha, z)/\partial \alpha$ are of the same sign.[6] With the help of this property, differentiating equation (5.22) with respect to α implies:[7]

$$\frac{\partial x(\alpha, z)}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial e(\alpha, z)}{\partial \alpha} > 0$$

We knew already that an improvement in the state of the labor market causes the reservation wage to rise—see (5.9)—and it is apparent that it also increases the intensity of the job search. In other words, when the economy is going well, or when it is easier to find a job, it pays a job seeker to look harder, which also allows him to raise his wage demands. Conversely, when the economy slows, a job seeker both lowers his reservation wage and reduces his search efforts (see also van den Berg and van Ours, 1994).

Differentiating relation (5.22) with respect to z, we deduce that $\partial x(\alpha, z)/\partial z$ and $\partial e(\alpha, z)/\partial z$ are of opposed signs. Using this result, differentiating relation (5.23) with respect to z further implies:

$$\frac{\partial x(\alpha, z)}{\partial z} > 0 \quad \text{and} \quad \frac{\partial e(\alpha, z)}{\partial z} < 0$$

Thus, as in the basic model, a rise in the income of a job seeker raises the reservation wage—see further (5.9)—but we also observe that such a rise tends to reduce the search effort. This results from the fact that an increase in z increases the intertemporal utility of the job seeker. He can thus reduce the amount of effort he puts into searching, because the marginal gain from intensified effort sinks below the level of marginal disutility that it provokes. Therefore, increases in z unambiguously increase unemployment spells, equal to $1/\alpha e[1 - H(x)]$. Finally, it should be noted that a *simultaneous* lowering of α and z has an ambiguous effect on optimal effort. It can indeed happen that certain categories of persons (the long-term unemployed in particular) find themselves facing a reduced number of job offers and a reduction in their unemployment benefits.

2.2.4 JOB SEARCH AND WEALTH

To this point we have neglected risk aversion and the possibility of saving or taking on debt. In actuality, individuals generally have an aversion to risk that may lead them to save when they are in work so as to lessen the impact of the drop in income that would result should they become unemployed. They may also borrow, if borrowing is possible, when they are unemployed. Such behavior modifies the expectation of utility during a

[6] Differentiating (5.23) with respect to α gives $\frac{\partial x}{\partial \alpha} = \frac{\partial e}{\partial \alpha} e\phi''(e)$.

[7] Differentiating equation (5.22) with respect to α yields:

$$\frac{\partial e}{\partial \alpha}\phi''(e) + \alpha \frac{\partial x}{\partial \alpha}\frac{1 - H(x)}{r + q} = \int_x^{+\infty} \frac{w - x}{r + q} \mathrm{d}H(w) > 0$$

spell of unemployment, since unemployed persons have an interest in dipping into their savings, and even in taking on debt, and so reducing their wealth. These modifications in the behavior of the unemployed are analyzed by Danforth (1979), Lentz and Tranaes (2005), Chetty (2008), and Lammers (2012).

To simplify, we approach this problem with a static model where the unemployed person disposes of a given initial wealth, denoted a. The utility of a job seeker is written $v(c) - \phi(e)$, where $v(c)$ is an increasing concave function corresponding to the utility of consumption c and $\phi(e)$ is an increasing convex function that represents the disutility of the search effort e. The probability of receiving an offer is equal to αe. We assume that the utility of an agent remunerated at wage w is written $v(a+w) - \psi$, where ψ designates the disutility of work. If this agent does not find work, her utility is simply equal to $v(a+z)$ where z designates the income obtained from sources other than work. At the beginning of the period, the unemployed person effects a search effort e that allows her to receive wage offers picked from a distribution $H(.)$. At the close of the period, the unemployed person works if she has received and accepted an offer and consumes all her wealth and income. In this setting, an unemployed person chooses her reservation wage x and her search effort e so as to maximize her expected utility, or:

$$\max_{(e,x)} -\phi(e) + (1-\alpha e)\, v(a+z) + \alpha e \left\{ \int_x^{+\infty} [v\,(a+w) - \psi]\, \mathrm{d}H(w) + v(a+z)H(x) \right\}$$

The first-order conditions of this problem define the optimal effort and the reservation wage:

$$\phi'(e) = \alpha \int_x^{\infty} [v\,(a+w) - \psi - v(a+z)]\, \mathrm{d}H(w) \tag{5.24}$$

$$v(a+x) = v(a+z) + \psi \tag{5.25}$$

We see that her reservation wage depends on her wealth and that it is greater than z. Differentiating the equation (5.25) with respect to a, we get:

$$\frac{\mathrm{d}x}{\mathrm{d}a} = \frac{v'(a+z) - v'(a+x)}{v'(a+x)} > 0$$

An increase in wealth thus raises the reservation wage. Deriving and differentiating equation (5.24) yields:

$$\phi''(e)\frac{\mathrm{d}e}{\mathrm{d}a} = \alpha \int_x^{+\infty} \left[v'\,(a+w) - v'(a+z)\right] \mathrm{d}H(w)$$

The utility function v being concave, $v'\,(a+w) < v'(a+z)$ when $w > z$. Since for that matter the effort function is convex, we have $\phi''(e) > 0$, which implies that search effort decreases with wealth a. This result illustrates the intuitive view that wealthier persons have less incentive to look for work, since the marginal utility of their consumption is less.

Starting from the observation that wealth undergoes diminution with the duration of unemployment, this model suggests that the reservation wage ought to fall during a spell of unemployment while the search effort ought to rise. Note, however, that the results of this static model are not general. Lentz and Tranaes (2005) show, in a dynamic setting, that the sign of the impact of wealth on the reservation wage and on search effort is a priori undetermined. It depends on the form of the utility function, the form of the search cost function, and the existence of liquidity constraints.

2.2.5 The Effect of Benefit Sanctions

In general, the unemployed must meet obligations in order to receive unemployment benefits. Primarily, they must take verifiable steps to look for work, and their payments may be temporarily or permanently suspended if they fail to meet their obligations or refuse a job that is offered to them. The fulfillment of these obligations is monitored with widely varying strictness across OECD countries (figure 5.2). In addition to such steps, unemployed persons may be subjected to a requirement of availability for work during participation in an active program or to demands for occupational or geographical mobility. Sanctions can also be imposed in case of refusal to participate in an active labor market program such as training or intensive placement services. The strictness of the sanctions imposed in cases of noncompliance has been summarized by Venn (2012) in a synthetic index. Figure 5.3 shows that even though sanctions are applied in principle in most countries, there are large differences in their degree of strictness.

The model of job search with endogenous effort is well adapted to analyzing the consequences of sanctions (see Lalive et al., 2005; Abbring et al., 2005; Boone and van Ours, 2006; Boone et al., 2007, 2009). The possibility of being sanctioned may be formalized with the assumption that the probability of being sanctioned decreases with search effort. The sanction consists of a *permanent* reduction in unemployment benefit

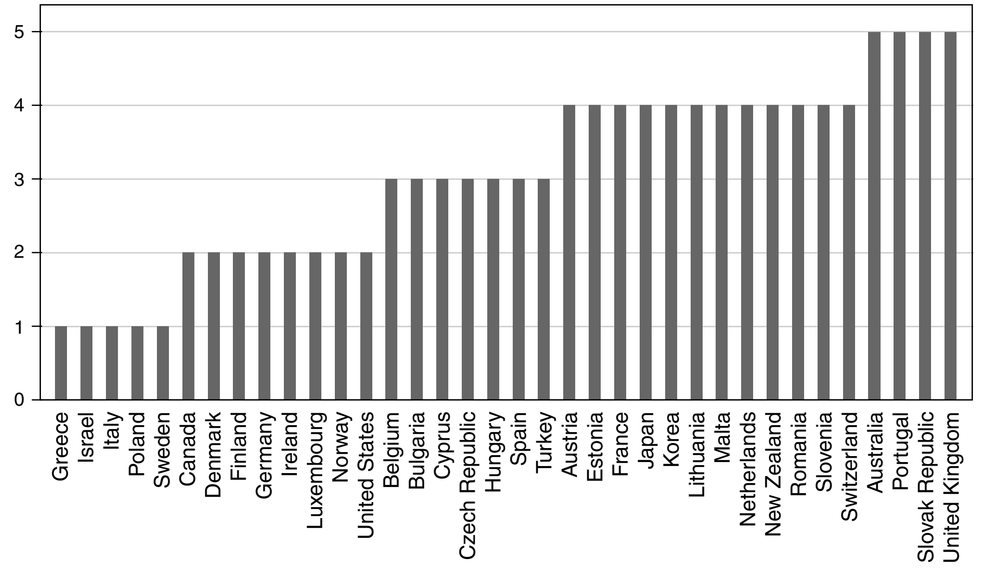

Figure 5.2
Strictness of job search monitoring, scored from 1 (least strict) to 5 (most strict).

Source: Venn (2012, figure 4, p. 18).

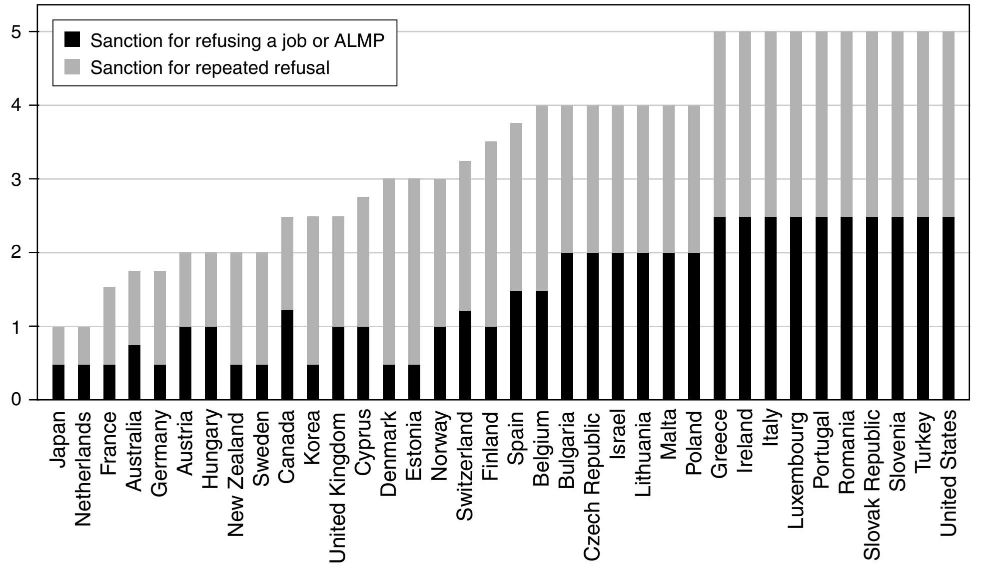

Figure 5.3
Strictness of sanctions, scored from 1 (least strict) to 5 (most strict).

Source: Venn (2012).

by an amount s. This reduction occurs at a rate of $\sigma(e)$, where σ is a decreasing and convex function of search effort e. The search effort made by a job seeker not subject to sanction we denote by e_u and that of a sanctioned job seeker e_s. The corresponding probability to receive a job offer is respectively αe_u and αe_s. To simplify the analysis, we will assume that all jobs have the same wage w and that the rate of job destruction is null, such that the value of a job, V_e, is simply equal to w/r. So that there may exist unemployed persons not subject to sanctions at stationary equilibrium, we will assume that all agents have an instantaneous death rate equal to δ and that there is a number of entrants equal to the number of the deceased at each instant. All entrants onto the labor market start out as unsanctioned unemployed. We denote by N the size, constant and exogenous, of the active population. U and S designate respectively the number of unsanctioned unemployed at stationary equilibrium and the number of sanctioned unemployed. At stationary equilibrium, the equality of the flows entering and exiting unsanctioned and sanctioned unemployment entails:

$$\delta N = \delta U + \sigma(e_u)U + \alpha e_u U \quad \text{and} \quad \sigma(e_u)U = \delta S + \alpha e_s S$$

whence:

$$U = \frac{\delta}{\delta + \alpha e_u + \sigma(e_u)} N \quad \text{and} \quad S = \frac{\sigma(e_u)}{\delta + \alpha e_s} \cdot \frac{\delta}{\delta + \alpha e_u + \sigma(e_u)} N$$

The last two equations yield the values of U and S as a function of search efforts e_u and e_s. From that we deduce the number of jobs held, denoted L, with the identity $L = N - U - S$.

In order to know the search efforts of agents, we have to specify their behavior. If ρ designates the discount rate, every agent is in fact subject to an effective discount rate $r = \rho + \delta$, given that she might die at any instant with a probability δ. Under these conditions, at stationary equilibrium, the expected utility of an unsanctioned unemployed person, denoted V_u, and that of a sanctioned unemployed person, denoted V_s, verify the two following equations:

$$rV_u = z - \phi(e_u) + \alpha e_u (V_e - V_u) + \sigma(e_u)(V_s - V_u) \tag{5.26}$$

$$rV_s = z - s - \phi(e_s) + \alpha e_s (V_e - V_s) \tag{5.27}$$

The unemployed choose levels of search effort that maximize their respective expected utilities. The first-order conditions are then written as follows:

$$\phi'(e_u) = \alpha (V_e - V_u) + \sigma'(e_u)(V_s - V_u) \tag{5.28}$$

$$\phi'(e_s) = \alpha (V_e - V_s) \tag{5.29}$$

The convexity of functions ϕ and σ defining the cost of search effort and the probability of being sanctioned ensures that the second-order conditions are satisfied.

These equations allow us to study the impact of the extent s of the sanction on search efforts. Since $V_e = w/r$ is independent of s, and taking into account the first-order condition (5.29), the derivation of (5.27) with respect to s gives $(r + \alpha e_s)\frac{\mathrm{d}V_s}{\mathrm{d}s} = -1$, and consequently we have $\frac{\mathrm{d}V_s}{\mathrm{d}s} < 0$. The function ϕ being convex, equation (5.29) thus shows that the search effort of the sanctioned unemployed increases with the sanction. This is what is called the ex-post effect of the sanction: when the sanction is applied, search effort increases on account of the reduction of unemployment benefit. But the sanction also has an ex-ante effect: the threat of its imposition modifies the behavior of the unemployed who are as yet unsanctioned. Thus, taking into account the first-order condition (5.28), the derivation of (5.26) with respect to s yields $\frac{\mathrm{d}V_u}{\mathrm{d}s} = \frac{\sigma(e_u)}{r+\sigma(e_u)+\alpha e_u}\frac{\mathrm{d}V_s}{\mathrm{d}s} < 0$. With the help of this last relation, the derivation of the first-order condition (5.28) entails, after several simple calculations:

$$\left[\phi''(e_u) + \sigma''(e_u)(V_u - V_s)\right]\frac{\mathrm{d}e_u}{\mathrm{d}s} = \frac{(r + e_u)\sigma'(e_u) - \alpha\sigma(e_u)}{\sigma(e_u)}\frac{\mathrm{d}V_u}{\mathrm{d}s}$$

The function σ being decreasing, we thus have $\frac{\mathrm{d}e_u}{\mathrm{d}s} > 0$. Note that the search effort of the unsanctioned unemployed increases with the amount of the sanction. The unemployed not (yet) subject to sanction increase their search effort to avoid the sanction, the more so to the degree the sanction is severe. In this perspective, it is interesting to compare the search effort of the sanctioned and unsanctioned unemployed. Rewriting the term $(V_e - V_s)$ in the form $(V_e - V_u) + (V_u - V_s)$ in equation (5.29), we find that:

$$\phi'(e_u) = \phi'(e_s) - \left[\alpha + \sigma'(e_u)\right](V_u - V_s)$$

As $V_u > V_s$, we observe that the search effort of the unsanctioned unemployed exceeds that of the sanctioned unemployed if (and only if) $\sigma'(e_u) < -\alpha$. Hence, when the probability of being sanctioned rises sufficiently in response to reduced search effort, the threat of being sanctioned pushes the search effort of the unsanctioned unemployed higher than that of the sanctioned unemployed. Boone et al. (2009) observe this phenomenon in an experimental setting. The threat of the sanction may be more effective than the application of the sanction. This result is important, for it suggests that a well-managed system of sanctions may be an effective tool for giving the unemployed an incentive to search hard for a job while paying them a generous unemployment benefit (Boone et al., 2007). This problem will be analyzed in greater detail in chapter 14.

2.2.6 Nonstationary Environment

The hypothesis that a job seeker's environment is stationary does not apply in a number of cases. Financial constraints increase the longer unemployment lasts, job offers most often grow scarcer, and net income from the search falls off, since as a general rule unemployment insurance systems mandate a reduction, or even a termination, in the payment of benefits at the end of a certain period. In what follows, we focus only on this last cause of nonstationarity; van den Berg (1990) presents a model taking into account a number of causes of nonstationarity. More precisely, we assume that the net instantaneous income of a job seeker diminishes (in the broad sense) over time. We will thus have $z(t) \leq z(t')$ for all $t \geq t'$.

In this nonstationary environment, the discounted expected utility of a person entering unemployment, or $V_u(0)$, is no longer necessarily equal to the discounted expected utility $V_u(t)$ of a person who has already been unemployed for a period $t > 0$. We do however continue to assume that a job offer is a proposal of a constant wage which an employee will receive as long as he remains with the firm that makes the proposal. Thus, the discounted expected utility $V_e(w)$ of a person paid a constant wage w is stationary. Assuming for simplicity that there is no on-the-job search, it is defined by the following equation:

$$rV_e(w) = w + q\left[V_u(0) - V_e(w)\right] \tag{5.30}$$

The optimal job search strategy still consists of refusing all proposals that offer an expected utility less than that of an unemployed person and accepting all others. Since, following relation (5.30), $V_e(w)$ is an increasing function of w, the optimal strategy comes down to choosing, at every moment, a reservation wage such that only offers that exceed it will be accepted. Let us denote by $x(t)$ the reservation wage of a person whose duration of unemployment is equal to t; this wage is then characterized by the equality $V_e[x(t)] = V_u(t)$. Since the function $V_e(.)$ is increasing, the reservation wage $x(t)$ varies in the same direction as the discounted expected utility $V_u(t)$. Now intuition suggests that $V_u(t)$ ought to decrease with the duration t of unemployment, inasmuch as the resources $z(t)$ of a job seeker diminish with this duration. To see this clearly, we may focus on a short interval of time $[t, t+\mathrm{d}t]$ and make explicit the trade-off equation giving the value of $V_u(t)$. If λ continues to designate the rate at which job offers arrive, we then have:

$$V_u(t) = \max_s \frac{z(t)\mathrm{d}t + \lambda \mathrm{d}t\left[\int_s^{+\infty} V_e(w)\mathrm{d}H(w) + V_u(t+\mathrm{d}t)H(s)\right] + (1-\lambda \mathrm{d}t)V_u(t+\mathrm{d}t)}{1 + r\mathrm{d}t} \tag{5.31}$$

In the maximization problem appearing in this equation, the discounted expected utility $V_u(t + \mathrm{d}t)$ at date $(t + \mathrm{d}t)$ has to be considered as given, for on that date the job seeker decides on a *new* reservation wage independently of the choice made at date t. The optimal reservation wage is then obtained by setting to zero the derivative with respect to s of the term between brackets in the expression (5.31) of $V_u(t)$. After several simple calculations, we arrive at $V_e[x(t)] = V_u(t{+}\mathrm{d}t)$, which corresponds exactly to the characterization $V_e[x(t)] = V_u(t)$ of the reservation wage $x(t)$ when $\mathrm{d}t \to 0$.[8]

Since the net income $z(t)$ of an unemployed person decreases over time, equation (5.31) shows that $V_u(t) \leq V_u(t')$ necessarily obtains for every $t \geq t'$. Since his reservation wage and discounted expected utility vary in the same direction, we can deduce that $x(t) \leq x(t')$ for every $t \geq t'$. Hence reservation wages fall with time spent searching for a job when unemployment insurance benefits are regressive. This result implies that the rate of leaving unemployment, or $\lambda[1 - H(x(t))]$, increases with the duration t of the unemployment spell—a conclusion confirmed by a number of observations, in particular concerning the behavior of certain categories of job seekers as the period of their entitlement to unemployment insurance benefits draws to a close (see section 4.1.2). On the other hand, the long-term unemployed have, in general, a smaller probability of exiting from unemployment than do the short-term unemployed. This phenomenon can be explained by the fact that job offers arrive less frequently the longer one is unemployed, either because the productive abilities of the individual decline or simply because employers take the view that too long a period of unemployment sends a bad "signal." In these circumstances, the fact that one's reservation wage has fallen may be offset, or more than offset, by the declining arrival rate of job offers. The rate of exit from unemployment is then no longer obliged to decrease with the duration of the job search.

The foregoing analysis can easily be applied to the case of a change in the length of time unemployment insurance benefits are paid.[9] For example, if this period is shortened, that means that the intertemporal resources of the job seeker shrink, and that diminishes both his discounted expected utility and reservation wage. Thus, for a period of unemployment of the *same length*, and for the *same amount* of benefits, a shortened period of entitlement to benefits leads to a lowering of the reservation wage and consequently a reduction in the average duration of unemployment.

3 EMPIRICAL ASPECTS OF JOB SEARCH

The job search model contains a number of predictions that it is important to test and quantify in order to have at our disposal the kind of information public policy makers require.

[8] One can check as well that the second derivative with respect to s of the term between brackets is negative when this equality is satisfied. So what we have is indeed a maximum.

[9] By way of illustration, the interested reader can characterize the reservation wages associated with a system of unemployment insurance benefit such that $z(t) = z_0$ for $0 \leq t \leq T$, and $z(t) = z < z_0$ for $t > T$, where z, z_0, and T are constant exogenous parameters. A reduction in the length of time over which benefits are paid is similar to a lowering of T.

In this perspective, the economist is faced with the classic problem of detecting causal relations. A demonstrated correlation between the generosity of unemployment insurance benefit and the duration of unemployment is not a sufficient basis for inferring a cause-and-effect sequence. Such an observed correlation might flow from a combination of two effects. The first is the actual impact on behavior of the insurance mechanism, precisely the causal phenomenon we wish to isolate. The second is the effect flowing from differences among job seekers, such as age, previous labor market experience, place of residence, date of registration as unemployed, individual motivation, life goals, and so on. Given that certain characteristics are not observed by the economist (motivation for example), it is illegitimate, even if observable characteristics are controlled for, to conclude from a correlation between the generosity of unemployment benefit and the duration of unemployment that the one caused the other.

This is a point of primordial importance. Empirical research that merely presents correlations without any convincing strategy for isolating a causal relation—what is called an "identification strategy"—must be regarded as descriptive in nature and inadequate to ground an inference of causality. Yet such research is not without value, for it may suggest approaches that can help us, where necessary, to pinpoint causal relations.

Hence empirical research on the impact of unemployment insurance on the behavior of the unemployed is implicitly focused on isolating relations of cause and effect. We will present this research with the aid of the contribution of Lalive et al. (2006), which assesses the impact of modifications in the Austrian unemployment insurance system on the duration of unemployment. We start with the identification strategy chosen by these authors, setting it in the context of the range of strategies adopted in this domain. We then proceed to the estimation properly speaking, and the empirical results. The main results of this contribution can be replicated with data and programs available at www.labor-economics.org.

3.1 The Identification Strategy

3.1.1 Controlled Experiments

Ideally, the evaluation of the impact of a measure forming part of an unemployment insurance system ought to rest on controlled experiments in which its beneficiaries would be drawn at random from the potentially eligible population, resulting in the division of the eligible population into a "test group" (or "treated group") of beneficiaries and a "control group" of nonbeneficiaries. Such a procedure offers a guarantee that the two groups are comparable and have the same characteristics on average, since they have been selected at random. It also offers a guarantee that any performance differential between the two groups is indeed attributable to the measure being tested, on condition that the control group is not indirectly affected by the measure. For example, a measure to help with job search may affect individuals who are not its beneficiaries if they are in competition with those who are. In this case, the employment outlook for the control group is negatively affected by the measure, and failure to take this effect into account may lead to an overly positive evaluation of the efficiency of the help with job search given to the treatment group (Cahuc and Le Barbanchon, 2010; Gautier et al., 2012). It is also possible that members of the control group may alter their behavior out of disappointment at not having been included among the beneficiaries. Controlled experiments focused on job search help and the monitoring of the activity of job seekers have been carried out in the United States (see chapter 14, section 4.2.1) and in Europe (see van den Berg and van der Klaauw, 2006; Micklewright and Nagy, 2010). But such experiments are hard to set up, which is why most studies of the effects of unemployment insurance rely on "natural experiments." Such experiments take advantage of policy changes or external shocks that exert varying effects on groups of persons having characteristics as unvarying as possible. This is the approach taken by Lalive et al. (2006), which we will now present.

3.1.2 DIFFERENCE-IN-DIFFERENCES

Lalive et al. (2006) identify the causal effect of benefit duration on the willingness of individuals to accept jobs using a policy change that took place in Austria on 1 August 1989. The replacement ratio, equal to the ratio of unemployment benefits over the previous wage, was increased by about 15% for workers earning below a certain threshold, whereas for workers above this threshold the replacement ratio remained unchanged. The potential benefit duration was increased, depending on age and experience: for workers younger than age 40 and/or for workers with little previous work experience, potential benefit duration remained unchanged; for workers with high levels of previous work experience, potential benefit duration increased, respectively, from 30 to 39 weeks for the age group 40 to 49; and from 30 to 52 weeks for workers aged 50 and older.

Accordingly, the policy change affected various unemployed workers differently, as shown in table 5.5. A first group, denoted eRR, experienced an increase in the replacement ratio. A second group, denoted ePBD, experienced an extension of potential benefit duration. A third group, denoted ePBD-RR, experienced both a higher replacement ratio and a longer potential benefit duration. The control group experienced no change in the policy parameters.

To assess the impact of changes to financial incentives on transition rates out of unemployment, Lalive et al. use longitudinal individual data from two sources: the Austrian social security database, which contains detailed information on individuals' employment, unemployment, and earnings history since the year 1972 and some information on the employer, like region and industry affiliation; and the Austrian unem-

TABLE 5.5
Changes in the replacement ratio (RR) and in potential benefit duration (PBD) on 1 August 1989 in Austria.

		Age			
		Younger than 40 work experience		40 and older work experience	
		Low	High	Low	High
Monthly income	$\leq$ 12,610 Austrian shillings	eRR	eRR	eRR	ePBD-RR
	$>$ 12,610 Austrian shillings	Control	Control	Control	ePBD

Note: Work experience "low" refers to less than 6 years of experience out of the previous 10 years and less than 9 years of experience out of the previous 15 years. Work experience "high" refers to more than 6 years of experience out of the previous 10 years and more than 9 years of experience out of the previous 15 years. ePBD: eligible for increase in potential benefit duration; eRR: eligible for increase in replacement ratio; ePDB-RR: eligible for increase in potential benefit duration and in replacement ratio.
Source: Lalive et al. (2006, table 2, p. 1018).

ployment register, which supplies information on the relevant socioeconomic characteristics. From these data Lalive et al. extract a sample that contains all unemployment entrants for the period between 1 August 1987 (two years before the policy change) and 31 July 1991 (two years after the policy change). They concentrate on job seekers in the age bracket 35 to 54, who have worked at least 52 weeks within the last two years before entering into unemployment and who reside in regions that were never eligible for a special regional extended benefit program. They end up with 225,821 unemployment spells.

The median duration of unemployment is 12 weeks. More than 85% of spells end in a job, while 14% of spells end in a non-job exit destination (long-term sickness, pension, unknown). Since spells are observed until May 1999, only 1% of unemployment entrants in the period between 1 August 1987 and 31 July 1991 are still looking for a job in May 1999. These spells are *censored*. We speak of "left censoring" when the (unknown) date of the start of the unemployment spell falls prior to the date on which the survey commences, and "right censoring" when an individual is still looking for work on the date when the survey stops. The survey simply reveals that when a spell is censored, the actual duration of unemployment is at least equal to the reported duration of this censored spell in the survey. The data exploited by Lalive et al. are of very high quality, since the proportion of censored spells is very small. But that is not always the case, and we will see below how to deal with censored data.

In this setting it becomes possible to evaluate the impact of the changes that came into effect on 1 August 1989, thanks to the difference-in-differences technique, which consists of comparing the respective trajectories of the average performances of the treated and untreated groups. Let $\bar{Y}_B^T$ be the average duration of unemployment for a treated group before the date of the reform (B for before) and $\bar{Y}_A^T$ its average duration after the date of the reform (A for after). Let $\bar{Y}_B^C$ be the average duration of unemployment for the control group before the reform and $\bar{Y}_A^C$ its average duration after the reform. The difference-in-differences estimator, denoted $\tilde{\Delta}_{DD}$, is defined by:

$$\tilde{\Delta}_{DD} = \left(\bar{Y}_A^T - \bar{Y}_B^T\right) - \left(\bar{Y}_A^C - \bar{Y}_B^C\right) \tag{5.32}$$

Thus the difference-in-differences estimator is equal to the difference between the before-after estimator of the treated group $\left(\bar{Y}_A^T - \bar{Y}_B^T\right)$, and the before-after estimator of the control group $\left(\bar{Y}_A^C - \bar{Y}_B^C\right)$. Plainly it is not possible to observe what would have happened if the reform had not taken place. Hence the validity of this estimator of the average effect of the treatment rests on the hypothesis that the difference in the duration of unemployment between the treatment group and the control group would have remained constant in the absence of the reform (see chapter 14, section 3.3.1 for a more formal presentation). This is the *common trend assumption*. For it to be credible, the researcher must strive to ensure that the observable characteristics of individuals in both groups are as alike as possible. So the goal will be to set bounds to the groups described in table 5.5 by selecting persons with age ranges and income levels as close to one another as possible. But the number of observations available frequently imposes limits. For this reason, in a first iteration, Lalive et al. take into account individuals aged 35 to 54 to present their main results, which puts at their disposal 225,821 spells of unemployment. This is a fairly wide bracket, which is why in subsequent iterations they limit themselves to narrower brackets, at the cost of fewer observations against which to test the robustness of their results.

It is also necessary to check that the previous patterns of unemployment duration of the two groups are parallel and that their composition remains stable over time, in order to be certain that the results observed do not arise out of a change in the composition of the groups. Finally, it is important to verify that the reform was not anticipated; otherwise the behaviors of the individuals in the treatment group might have altered prior to the date of the reform.

The identification strategy chosen by Lalive et al. fits well with their natural experiment and the data available to them. It is a strategy frequently employed in labor economics. But its precondition is a change in economic policy that affects comparable groups differently. Such events are not always readily to hand. But other strategies are possible. Blundell and Costa Dias (2009) offer an overview of these strategies for the whole area of public policy.

3.2 Estimation

First we show that the difference-in-differences estimator does allow us to evaluate the impact of the unemployment insurance reform on the average duration of unemployment. Then we will see how the consequences of the reform may be studied in greater depth by examining its impact not just on the average but also on the distribution of unemployment durations, with the help of survival and hazard functions, estimations of which yield rates of exit from unemployment sorted by the duration of the spell of unemployment.

3.2.1 Unemployment Duration

It is simple to make a first pass at estimating the impact of the reform of the Austrian unemployment insurance system that took place in August 1989 using the difference-in-differences estimator defined by equation (5.32). The unemployment durations of persons who entered unemployment before 1 August 1989 serve to calculate average durations prior to the reform, since such persons were not its beneficiaries. The durations of persons who entered unemployment after 1 August 1989 serve to calculate average durations subsequent to the reform. Since these data are available only for the two years prior to the reform, durations greater than 104 weeks (or 2 years) are left out. Let t_u denote the realized duration of unemployment measured in weeks. Unemployment duration is defined as $t_u^{104} \equiv \min(t_u, 104)$. This definition discards unemployment durations greater than 104 weeks. As the latter concern no more than 1.65% of the sample, we may assume that the results obtained in this way furnish a good first approximation. They are presented in table 5.6. Column 3 shows that the duration of unemployment rises for all groups after August 1989. Column 4, however, which presents the difference-in-differences estimator, reveals that the average duration of unemployment rose more for the groups that benefited from more generous unemployment insurance after that date.

These results, which are coherent with the predictions of the job search model, permit us an initial overview of the impact of the reform. They allow us to calculate that an increase of 1% in the replacement ratio leads to a lengthening of the duration

TABLE 5.6
Average unemployment duration in first 104 weeks (measured in weeks).

	Before August 1989	After August 1989	Change (after-before)	Diff-in-diff (compared to control)
ePBD group	16.25 (.08)	18.67 (.09)	2.42 (.12)	1.13 (.18)
N	48,294	51,110		
eRR group	17.79 (.12)	20.*m*03 (.16)	2.24 (.20)	.96 (.24)
N	17,160	15,310		
ePBD-RR group	19.01 (.17)	23.55 (.24)	4.53 (.20)	3.25 (.24)
N	11,992	9,182		
Control group	15.24 (.08)	16.52 (.09)	1.29 (.13)	
N	33,815	38,958		

Note: Standard errors in parentheses. N: number of unemployment spells in the group. Diff-in-diff: difference-in-differences; RR: replacement rate; PBD: potential benefit duration; ePBD: eligible for increase in potential benefit duration; eRR: eligible for increase in benefit RR; ePBD-RR: eligible for both.

Source: Lalive et al. (2006, table 4, p. 1020).

of unemployment of 0.3%, in other words, an elasticity of 0.3.[10] They also allow us to show that the elasticity of the duration of unemployment to the potential duration of the payment of benefit is of the order of 0.17.[11] These results fall within the ranges obtained by many other research efforts in this domain.

It is worth noting, however, that the difference-in-differences estimates of table 5.6 are based on rather different groups. Unbiased estimates will be obtained only if there are no group-specific trends in unemployment durations. Lalive et al. (2006) provide a variety of robustness tests, including a focus on more narrowly defined groups and on groups that are just below or just above the eligibility threshold.

Postulating an upper boundary of 104 weeks on individual unemployment durations leads to a perceptible reduction in the average unemployment duration: under this postulate, the average duration amounts to 16.84 months (with a standard deviation of 0.037), whereas without this postulate, it amounts to 18.55 months (with a standard deviation of 0.067). Nor for that matter does this postulate provide any solution to the problem of censored data. It is possible, though, to take better account of the censored data and also to go beyond mere average durations by estimating the impact of the

[10] The unemployment rate of group eRR is in fact 20.03 after the reform. The last column of table 5.6 indicates that it would have been weaker by 0.96 point in the absence of a rise in the replacement ratio, or $20.03 - 0.96 = 19.07$. Hence the augmentation of the replacement ratio raised the unemployment rate by $0.96/19.07 = 5\%$. Since the replacement ratio increases by around 15%, the elasticity of unemployment duration with respect to the replacement ratio is equal to $5\%/15\% = 0.3$.

[11] Persons 50 and older compose 16% of the sample. The potential duration of benefit payments thus grew by $(0.16)(22/30) + (0.84)(930) = 37\%$. Table 5.6 indicates that the increase in the duration of benefit caused the duration of unemployment to lengthen by $(1.13)/(18.67 - 1.13) = 6.4\%$. Consequently the elasticity of the duration of unemployment to the potential duration of benefit payment is $6.4/37 = 0.17$.

reform on the rates of exit from unemployment as a function of the amount of time spent unemployed. This is the procedure adopted by Lalive et al. in the following part of their article. It consists of estimating hazard and survival functions.

3.2.2 Hazard Function and Survival Function

Let us illustrate the "hazard function," which is a basic concept of duration models. In what follows, we will denote the continuous random variable representing the duration of unemployment by T. Like every random variable, the duration of an individual's unemployment spell is characterized by knowledge of its cumulative distribution function denoted $F(t)$, or its probability density $f(t) = F'(t)$. Recall that the cumulative distribution function is defined by $F(t) = \Pr\{T < t\}$ and so represents the probability that the unemployment spell lasts less than t units of time. Theoretical job search models are capable of producing a certain number of predictions about this function, but they most naturally lead to characterizations of the hazard function. The latter represents, for an individual, the instantaneous conditional probability of exiting from unemployment when she has been unemployed for at least a period of length t. For example, in the model in section 2.2.6, in which unemployment insurance benefits are not stationary, the hazard function is equal to $\lambda[1 - H(x(t))]$, where $x(t)$ designates the reservation wage after an unemployment spell equal in length to t. More generally, designating the hazard function by $\varphi(.)$ and knowing that the individual has been unemployed for at least a period of length t, the conditional probability $\varphi(t)\mathrm{d}t$ that the duration of unemployment is located within the small interval of time $[t, t+\mathrm{d}t]$ is defined by $\varphi(t)\mathrm{d}t = \Pr\{t \leq T < t + \mathrm{d}t \mid T \geq t\}$. Applying the definition of conditional probabilities[12] gives us:

$$\varphi(t)\mathrm{d}t = \frac{\Pr\{t \leq T < t + \mathrm{d}t\}}{\Pr\{T \geq t\}} = \frac{f(t)\mathrm{d}t}{1 - F(t)}$$

The hazard function is thus characterized by the equality:

$$\varphi(t) = \frac{f(t)}{\overline{F}(t)}, \quad \text{with} \quad \overline{F}(t) \equiv 1 - F(t) \tag{5.33}$$

In this expression there appears the *survival function* $\overline{F}(t)$, representing the probability that the unemployment spell lasts at least a period of length t. Obviously, there is a relation between the survival function and the expected duration of unemployment: $\mathbb{E}(T) = \int_0^{T_u} tf(t)\mathrm{d}t$, where T is defined on the support $[0, T_u]$. Integrating this equation by part, we get:[13]

$$\mathbb{E}(T) = \int_0^{T_u} \overline{F}(t)\mathrm{d}t$$

[12] Given two events A and B, this definition is written:

$$\Pr\{A \mid B\} = \frac{\Pr\{A \cap B\}}{\Pr\{B\}}$$

With $A = \{t \leq T < t + \mathrm{d}t\}$ and $B = \{T \geq t\}$, we find the formula given in the text.

[13] We use the formula $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$, and posit $u = t$, $\mathrm{d}u = \mathrm{d}t$, $\mathrm{d}v = f(t)\,\mathrm{d}t$, $v = -[1 - F(t)]$.

so that the expected duration of the spell of unemployment is equal to the integral of the survival function.

It is also useful to link the survival function to the integral $\Phi(t)$ of the hazard function. This integral, also called the "integrated hazard," is defined by $\Phi(t) = \int_0^t \varphi(\xi)\mathrm{d}\xi$. Relation (5.33) can also be written $\varphi(t) = -\partial[\ln \overline{F}(t)]/\partial t$; integrating this equality, we find:

$$\Phi(t) = -\ln \overline{F}(t) \tag{5.34}$$

The integrated hazard is thus equal to the opposite of the logarithm of the survival function.

In practice it is important to know if the duration of the phenomenon under study, in this case the duration of an unemployment spell, increases, diminishes, or remains constant with time already spent unemployed. The hazard function allows us to characterize this notion of "duration dependence" very easily. If $\varphi'(t) > 0$, the probability of exiting from unemployment increases with the amount of time t already spent unemployed, and we refer to "positive duration dependence." Conversely, if $\varphi'(t) < 0$, the probability of exiting from unemployment diminishes with the amount of time t already passed in this state, and we then refer to "negative duration dependence." The model presented in section 2.2.6, for example, in which unemployment benefits tail off as the time spent looking for a job lengthens, exhibits positive duration dependence. It should be noted that the hazard function is not necessarily monotonic: it may increase for certain values of t and diminish for others. The hazard function may equally be independent of the length of an unemployment spell, as is the case in the basic job search model in section 2.1, where the exit rate from unemployment $\lambda[1 - H(x)]$ is a constant.

3.2.3 Nonparametric Estimation

It is possible to estimate the survival function by adopting what is called a nonparametric approach, which makes no hypothesis about the form of the distribution of the durations of unemployment spells. This approach is very useful in the first stage of data exploitation. Following Lalive et al. (2006), we most often adopt the Kaplan-Meier (1958) estimator of the survival function. Table 5.7 presents an extract from the data concerning the 225,821 spells of unemployment that began between 1 August 1987 and 31 July 1991. Column 1 assigns an identifier to each spell of unemployment. Column 2 gives its duration, expressed in weeks. The durations have been ranked in ascending order. The shortest duration is equal to .0712128, which corresponds to half a day (6 hours, assuming that the week comprises 7 days of 12 hours each). There are 17 unemployment spells with a duration of half a day. The variable in column 3 takes the value 1 if the spell is censored, and zero if not. We observe that unemployment spell 189540 is censored and that it matches an individual 36.2 years old at the date he entered into unemployment, as shown in column 4.

Let K designate the number of different durations of unemployment inventoried in the sample of the $n = 225{,}821$ observations and let us rank these durations in ascending order, $\tau_1 < \tau_2 < \ldots < \tau_K$. Let us denote n_i the number of unemployment spells the duration of which is at least equal to τ_i. If there is no censoring, the Kaplan-Meier estimator

TABLE 5.7
Extraction from the data set of Lalive et al. (2006).

id	dur	uncc	age
1	0.712128	0	49.99863
2	0.712128	0	49.99863
...	...	...	...
189540	25.78669	1	36.21355
...	...	...	...

Note: id: identification number of unemployment spells; dur: duration of the unemployment spell; uncc equals 1 if the spell is censored and equals zero otherwise; age: age of the individual at the beginning of the unemployment spell.

of the survival function, $\bar{F}(\tau_i)$, in other words the probability that the duration of unemployment is at least equal to τ_i, is simply equal to the proportion of persons whose unemployment duration is greater than τ_i, or $\hat{S}(\tau_i) = n_i/n$. But this estimator requires modification when there are censored observations. Such is the case here, since some persons remained unemployed in May 1999. Let d_j be the number of unemployment spells with a duration equal to τ_j and let c_j be the number of censored spells lying between τ_j and τ_{j+1}. We may then define the number of spells of unemployment with a duration at least equal to τ_i by $n_i = \sum_{j=i}^{K}(d_j + c_j)$. In this case, an estimation of the hazard function, $\hat{\varphi}(\tau_i)$, which here corresponds to the probability that the unemployment spell has a duration of exactly τ_i, is given by:

$$\hat{\varphi}(\tau_i) = \frac{\text{Number of spells with duration equal to } \tau_i}{\text{Number of spells with duration at least equal to } \tau_i} = \frac{d_i}{n_i}$$

The quantity

$$1 - \hat{\varphi}(\tau_i) = \frac{\text{Number of spells with duration greater than } \tau_i}{\text{Number of spells with duration at least equal to } \tau_i} = \frac{n_i - d_i}{n_i}$$

will then represent an estimation of the probability that a spell of unemployment has a duration greater than τ_i. Now, if one's spell of unemployment has lasted longer than τ_i, one has to have been unemployed for all the durations $\tau_j,\ j \leq i$. The Kaplan-Meier estimator of the survival function—in other words, an estimation of the probability that the duration of unemployment is at least equal to τ_i—may then be defined as follows:

$$\hat{S}(\tau_i) = \prod_{j<i}[1 - \hat{\varphi}(\tau_j)] = \prod_{j<i}\frac{n_j - d_j}{n_j}$$

The Kaplan-Meier estimators of the survival function and the hazard function are programmed into the standard software used in econometrics. They prove highly useful in describing the form of the survival and hazard functions of different groups.

Figure 5.4 compares the Kaplan-Meier estimators of the survival functions before and after August 1989 of persons more than 40 years old whose wage prior to their entry into unemployment was greater than 12,610 Austrian shillings. Members of this group benefited from the reform of August 1989 (see table 5.5), which raised the potential duration of unemployment benefit from 30 to 39 weeks for those younger than 50, and from 30 to 52 weeks for those 50 and older. Each curve in figure 5.4 portrays an estimation of the probability of still being unemployed as a function of the number of weeks already spent being unemployed. We observe that the two survival functions diverge after 15 weeks. The gap widens until the 40-week point, then narrows and becomes constant from 65 weeks on. These results are coherent with the predictions of the job search model, which indicate that the unemployed raise their reservation wage and reduce their search effort when the unemployment insurance system becomes more generous. Under these conditions, the probability of remaining unemployed increases.

The Kaplan-Meier estimator of the hazard function, shown in figure 5.5, allows us to visualize the rates of exit from unemployment during spells of unemployment. This figure shows a significant peak in the exit rate at week 30 for unemployment spells that began before the reform, and that two significant peaks appear at weeks 39 and 52 for spells that began after the reform. Such a pattern of movement in the dates of the peak rates of exit from unemployment suggests that the reform did indeed have a causal impact on exit rates from unemployment. This can be seen with greater precision in figure 5.6, which represents the difference-in-differences of the rates of exit from unemployment as between this group and the control group. At week 30, we observe a drop in the exit rate from unemployment for the test group relative to the control group. The peaks in the differences between the exit rates shift toward week 39 and especially toward week 52. We also note that the difference-in-differences are large for the spell situated before week 60 and subsequently become quite small, the reason being that the reform did not alter the parameters of the benefits scheme for spells of unemployment that exceed 52 weeks.

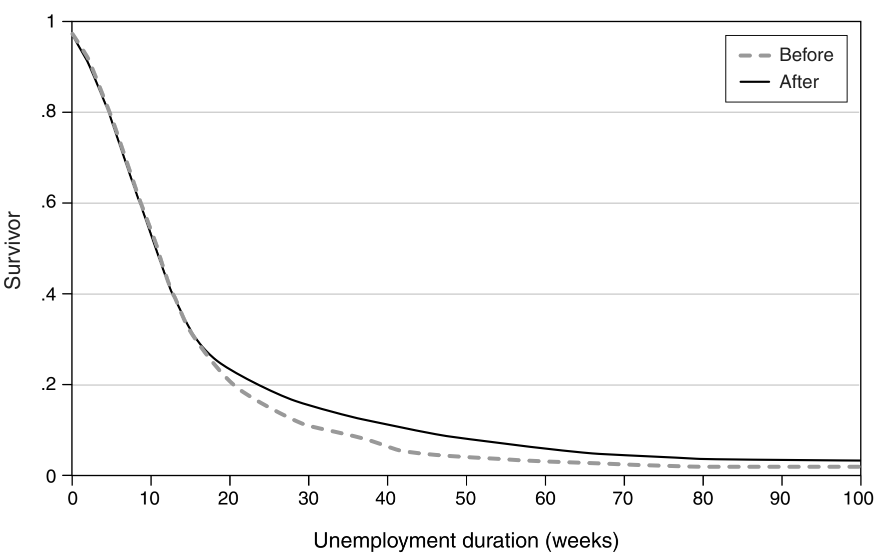

Figure 5.4
Kaplan-Meier survivor functions before and after the reform for the group of individuals potentially eligible to the extension of potential duration of benefits.

Source: Data from Lalive et al. (2006).

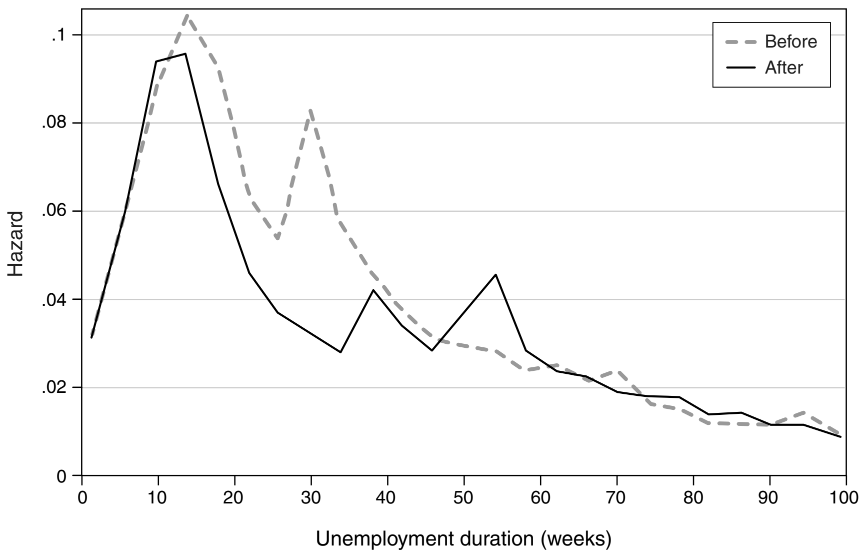

Figure 5.5
Kaplan-Meier hazard functions before and after the reform for the group of individuals potentially eligible to the extension of potential duration of benefits. Hazard functions are smoothed on 4-week windows.

Source: Data from Lalive et al. (2006).

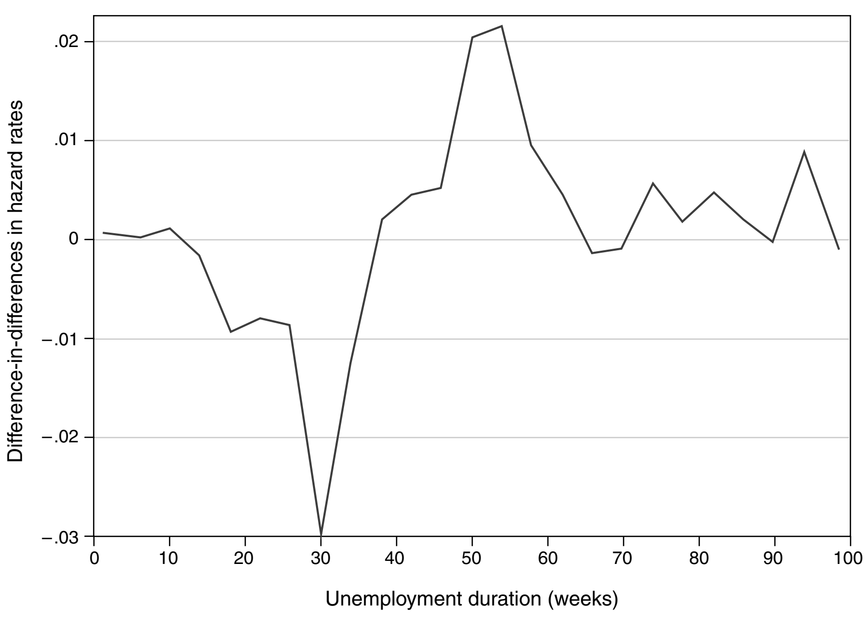

Figure 5.6
Difference-in-differences in Kaplan-Meier hazard functions for the group of individuals potentially eligible to the extension of potential duration of benefits and the control group. Hazard functions are smoothed on 4-week windows.

Source: Data from Lalive et al. (2006).

The nonparametric estimation of the survival and hazard functions constitutes an important stage in the process of evaluation. It allows us to observe the alterations in the average rates of exit from unemployment for the treated groups and the control group. Still, it does not allow us to estimate the impact of the unemployment insurance modifications conditional upon an array of explanatory variables. Of course, it is possible to calculate the survival and hazard functions for groups of either gender, different age, and different educational level. But in order to detect the behavior of the unemployed conditional upon their individual characteristics, it is necessary to estimate a model that expresses the survival and hazard functions as a function of these characteristics. This step is highly instructive, given that the characteristics of the persons constituting the treatment and control groups differ. To accomplish it, Lalive et al. (2006) adopt (as most research in this domain does) a parametric approach: they make hypotheses about the form of the distribution of the probabilities of unemployment duration. It then becomes possible to estimate the rate of exit from unemployment taking into account individual heterogeneity. This approach is called "the reduced form approach" because it simply relies on the predictions of some predefined models. This approach is different from the structural approach, in which each study estimates parameters of a specific model explicitly derived from the theory. For a good example of the structural model, consult Wolpin (1987), Devine and Kiefer (1991, chapter 5), and the survey of Eckstein and van den Berg (2007).

3.2.4 Parametric Estimation

To estimate the impact of financial incentives on the rate of exit from unemployment while taking into account the observable heterogeneity of individual characteristics, Lalive et al. (2006) use a model with proportional hazard.

The Likelihood Function

In the proportional hazard model, we assume that the vector $\boldsymbol{\theta}$ of the explanatory variables is composed of two subsets, $\boldsymbol{\theta_0}$ and $\boldsymbol{\theta_x}$, and that the hazard function takes the following form:[14]

$$\varphi(t, \mathbf{x}, \boldsymbol{\theta}) = \varphi_0(t, \boldsymbol{\theta_0})\rho(\mathbf{x}, \boldsymbol{\theta_x}) \tag{5.35}$$

Function φ_0 is called the "baseline hazard" because it is identical for all individuals, so that $\boldsymbol{\theta_0}$ is a vector of parameters to be estimated, independent of individuals' characteristics. Most often we utilize a well-specified function, for example, the Weibull distribution or, as Lalive et al. do, a piecewise constant function of elapsed duration.[15]

Relation (5.35) shows that, in the proportional hazard model, the effect of the vector $\mathbf{x}$ of individual characteristics is to multiply the baseline hazard by the scale

[14] This hypothesis amounts to assuming that the cumulative distribution function of the random variable T takes the expression $F(t) = 1 - e^{-\rho(\mathbf{x}, \boldsymbol{\theta_x}) \int_0^t \varphi_0(\tau, \boldsymbol{\theta_0}) d\tau}$.

[15] Note that for proportional hazard models, it is possible to proceed to a semiparametric estimation by specifying the scale factor a priori while not imposing any particular form for the baseline hazard (in that case, we must utilize the empirical distribution of the unemployment durations). This so-called partial-likelihood approach was suggested by Cox (1975); one may consult Kiefer (1988, IV-C) for a good introduction to it.

factor $\rho(\mathbf{x},\boldsymbol{\theta}_x)$ independent of the duration t of unemployment. A specification frequently used, as Lalive et al. do, for the scale factor is $\rho(\mathbf{x},\boldsymbol{\theta}_x)=\exp(\mathbf{x}\boldsymbol{\theta}_x)$, which has the advantage of being positive and supplying a simple interpretation of the components of the vector $\boldsymbol{\theta}_x$. If we denote by x_k the k^{th} component of vector $\mathbf{x}$ of individual characteristics, relation (5.35) defining the hazard function shows that $(\partial \ln \varphi/\partial x_k)=\theta_{xk}$, where θ_{xk} designates the k^{th} component of vector $\boldsymbol{\theta}_x$. If we have been careful to specify the explanatory variables in terms of logarithms, vector $\boldsymbol{\theta}_x$ then represents the vector of the elasticities of the hazard function, that is, the elasticities of the conditional probability of exiting unemployment with respect to the explanatory variables.[16]

The estimators of vectors $\boldsymbol{\theta}_x$ and $\boldsymbol{\theta}_0$ are obtained by maximizing the likelihood function of the sample with respect to the components of vectors $\boldsymbol{\theta}_x$ and $\boldsymbol{\theta}_0$. If there is no censoring, the likelihood function of the sample is written $\prod_{i=1}^{n} f(t_i,\mathbf{x},\boldsymbol{\theta})$, where f denotes the probability density of unemployment durations and t_i designates the length of observation i. This likelihood can be computed from the hazard function and the survival function using equation (5.33). The likelihood is merely $\prod_{i=1}^{n}\varphi(t_i,\mathbf{x},\boldsymbol{\theta})\bar{F}(t_i,\mathbf{x},\boldsymbol{\theta})$. But in reality some spells are censored. If observation t_i is censored, the survey simply reveals that the duration of unemployment T_i is *at least* equal to t_i. The contribution of this observation to the likelihood of the sample is then equal to $\Pr\{T_i\geq t_i\}\equiv\overline{F}(t_i|\mathbf{x},\boldsymbol{\theta})$. Let us define the dummy variable k_i by $k_i=1$ if the observation is not censored, and by $k_i=0$ if it is. Then the likelihood function becomes[17] $\prod_{i=1}^{n}[\varphi(t_i,\mathbf{x},\boldsymbol{\theta})]^{k_i}\bar{F}(t_i,\mathbf{x},\boldsymbol{\theta})$. It is possible to express this likelihood function solely with the help of the hazard function $\varphi(t,\mathbf{x},\boldsymbol{\theta})$ and its integral, the integrated hazard $\Phi(t,\mathbf{x},\boldsymbol{\theta})$. Relations (5.33) and (5.34) thus give $\ln f(t_i,\mathbf{x},\boldsymbol{\theta})=\ln\varphi(t_i,\mathbf{x},\boldsymbol{\theta})-\ln\overline{F}(t_i,\mathbf{x},\boldsymbol{\theta})$ with $\Phi(t_i,\mathbf{x},\boldsymbol{\theta})=-\ln\overline{F}(t_i,\mathbf{x},\boldsymbol{\theta})$, and the likelihood of the sample becomes, in logarithmic form, which is generally used to proceed to maximization:

$$L(\boldsymbol{\theta})=\sum_{i=1}^{n}k_i\ln\varphi(t_i,\mathbf{x},\boldsymbol{\theta})-\sum_{i=1}^{n}\Phi(t_i,\mathbf{x},\boldsymbol{\theta}) \tag{5.36}$$

In practice the estimator $\widehat{\boldsymbol{\theta}}$ of vector $\boldsymbol{\theta}$ of the parameters corresponds to the value of $\boldsymbol{\theta}$ that maximizes this log-likelihood function. This maximization most often gives no analytical solution, and it is necessary to fall back on numerical methods. Maximum likelihood-based methods are now so common that standard econometric software packages have routines for many of these methods.

In the hazard proportional model, it is easy to check that if we assume that the scale factor takes the form $\exp(\mathbf{x}_i\boldsymbol{\theta}_\mathbf{x})$, the log-likelihood function is:

$$L(\boldsymbol{\theta})=\sum_{i=1}^{n}k_i\left[(\mathbf{x}_i\boldsymbol{\theta}_\mathbf{x})+\ln\varphi_0(t_i,\boldsymbol{\theta}_\mathbf{0})\right]-\sum_{i=1}^{n}\Phi_0(t_i,\boldsymbol{\theta}_\mathbf{0})\exp(\mathbf{x}_i\boldsymbol{\theta}_\mathbf{x})$$

In this expression, function Φ_0 represents the integrated hazard of the baseline hazard φ_0.

[16] Recall that the elasticity of function $f(\mathbf{x}):\mathbb{R}^n\to\mathbb{R}$, with respect to x_i is $(x_i/f(\mathbf{x}))\,(\partial f(\mathbf{x})/\partial x_i)=\partial\ln(f(\mathbf{x}))/\partial\ln(x_i)$.

[17] This expression of the likelihood function assumes that the censoring mechanism is independent of the duration T_i of unemployment.

The Estimation of Parameters

The set of individual characteristics $\mathbf{x}$ accounted for by Lalive et al. comprises age, marital status, gender, education, log (previous monthly income), recall status, blue collar (versus white collar), seasonal industry, manufacturing industry, time spent unemployed, tenure, and quarter of inflow into unemployment. Their baseline hazard φ_0 is a piecewise constant function of elapsed duration defined as follows:

$$\varphi_0(t_i, \boldsymbol{\theta_0}) = \exp\left(\sum_{j=0}^{14} \theta_j \mathbb{I}\left(4j < t_i \leq 4(j+1)\right) + \theta_{15}\mathbb{I}(t_i > 60)\right)$$

where $\mathbb{I}(x)$ is an indicator function equal to 1 if condition x is verified, and zero if not. This expression of the baseline hazard incorporates the assumption that the hazard rate shifts in every four-week interval until week 60 and, because there are very few transitions beyond week 60, the last time interval covers the entire remaining duration of the spell as of week 60. Parameters θ_j allow us to detect the evolution of the hazard according to the duration of the spell of unemployment, and thus furnish a measure of the "duration dependence" of the process of exiting unemployment and hence serve to pinpoint the impact of the reform of August 1989.

Parameters θ_j can be estimated with the help of a specification of the difference-in-differences type. Let us limit ourselves to considering only the impact of the prolongation of the potential duration of unemployment benefit. Eligibility for the increase in potential benefit duration is denoted by a function *ePBD* equal to 1 if individuals belong to the group *ePBD* defined in table 5.5 and to zero otherwise, that is, $ePBD = I(ePBD = 1)$. Second, we introduce the function $A89$ equal to 1 if the unemployment spell of length t_i began after 1 August 1989 and to zero otherwise for all individuals. Thus, the interaction term $ePBD \times A89$ indicates that an individual satisfying all eligibility criteria for the increase in potential benefit duration has entered the period when this policy change has been enacted. The duration dependence of the hazard rate is defined as follows:

$$\begin{aligned} \theta_j &= \beta_{0j} + \beta_{1j} ePBD + \beta_{2j} A89 + \delta_j ePBD \times A89 \\ j &= 0, \ldots, 15 \end{aligned}$$

Assuming at this stage that the sample includes only individuals belonging to the control group and to the *ePBD* group, parameters β_{1j} capture ex ante differences between the *ePBD* group and the control group, parameters β_{2j} capture changes to duration dependence occurring over time for all individuals, and parameters δ_j measure the change in the duration dependence of the hazard rate due to changes in the potential benefit duration in August 1989.

Lalive et al. (2006) use this type of specification for all groups defined in table 5.5. They estimate the parameters β and δ proper to each group by maximizing the likelihood function of the sample. With the parameters estimated, it becomes possible to simulate the impact of the reform on the hazard and survival functions. Figure 5.7 represents the simulation of the impact of the reform on the hazard rate of group *ePBD*, which benefited from the prolongation of the potential duration of unemployment benefit from 30 to 39 weeks. Comparison with the control group reveals that this prolongation shows

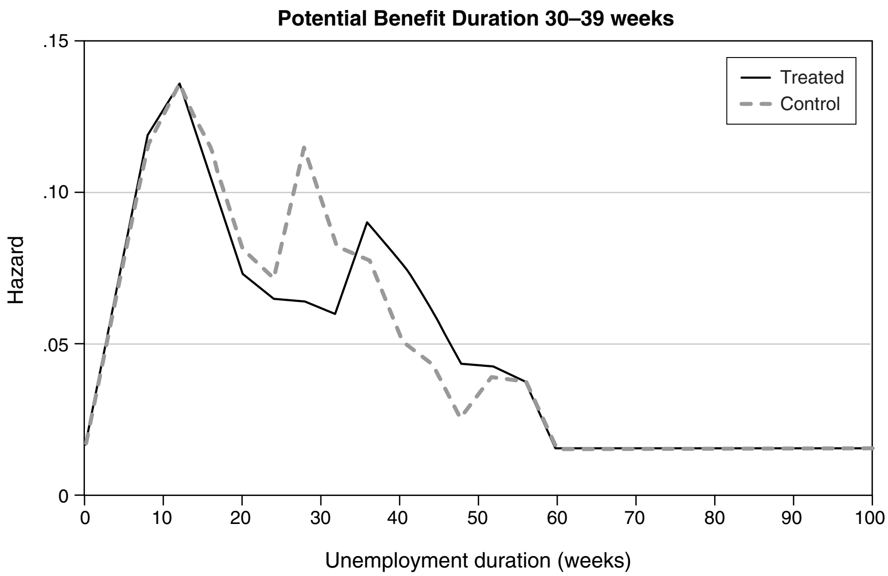

FIGURE 5.7
Estimated average treated and control hazard rates for the group who benefited from the extension of the potential benefit duration.

Source: Lalive et al. (2006, figure 5, p. 1026).

up in a shift of the peak point for exit from unemployment from week 30 to week 39. Taking individual heterogeneity into account thus confirms the result obtained by the nonparametric Kaplan-Meier estimation (see figure 5.5).

Parametric estimation makes it possible to perceive the impact of modification of the parameters of unemployment insurance as a function of observable individual characteristics. For example, it turns out that older unemployed persons react more strongly to these modifications. This might result from the greater difficulty they experience in reinserting themselves into the labor force or from their nearness to retirement age, which might make their search effort and reservation wage more sensitive to unemployment insurance (Hairault et al., 2010).

3.2.5 UNOBSERVED HETEROGENEITY

Explanatory variables such as sex, educational attainment, and past experience allow us to control to a degree the heterogeneity among individuals. But unobserved heterogeneity always remains: for example, personal motivation is generally not observed. The omission of some variables, or specification errors in the impact of the exogenous ones, are formally much like unobserved heterogeneity. Failure to take this type of heterogeneity into account leads to bias in the estimation of time dependency.

To see this clearly, consider an example in which there is a fraction p of the population which has a constant hazard function γ_1 and a fraction $(1-p)$ which has a constant hazard function γ_2. The hazard function of the whole sample is equal to:

$$\varphi(t) = \frac{p\gamma_1 e^{-\gamma_1 t} + (1-p)\gamma_2 e^{-\gamma_2 t}}{pe^{-\gamma_1 t} + (1-p)e^{-\gamma_2 t}}$$

It is easy to verify that $\varphi'(t) < 0$. Consequently, the omission of unobserved heterogeneity can falsely introduce a negative duration dependence, since in reality the individual probability of finding a job is independent of the amount of time spent unemployed.

The presence of unobserved heterogeneity can also lead to a selection bias prejudicial to the estimation of hazard functions. Such is the case, for example, in Lalive et al. (2006), if the unemployment insurance reform exerts different effects on the behavior of persons whose *unobservable* characteristics are different. Let us suppose, for example, that members of group *ePBD*—beneficiaries of the increased potential duration of unemployment insurance—have on average observable and unobservable characteristics identical to those of the control group at the moment of the onset of the reform, 1 August 1989. Let us also suppose that those persons with the most motivation to look for work are less sensitive to the increased potential duration of unemployment insurance and that this motivation is unobservable. In this setting, the unobservable characteristics of the group that benefited from the increased potential duration of unemployment insurance will play a different role than the unobservable characteristics of the control group during spells of unemployment. If these (plausible) hypotheses are verified, then the ratio between hazard rates at any given instant will simultaneously reflect the causal effect of the potential duration of benefit payments and a selection bias on unobservable characteristics.

To get around these difficulties we may assume that the probability density of the dependent variable is written (leaving out vectors $\mathbf{x}$ and $\boldsymbol{\theta}$ for the sake of simplicity) $f(t, v)$, where v is a random variable of density $p(.)$ marking the unobserved heterogeneity among agents. For example, in the proportional hazard model, it is possible to introduce this form of heterogeneity by assuming that the hazard function takes the form $\varphi(t, \mathbf{x}, \boldsymbol{\theta}) = \rho(\mathbf{x}, \boldsymbol{\theta}_x)\varphi_0(t, \boldsymbol{\theta}_0)v$. We thus obtain the *mixed proportional hazard model* studied in detail by Lancaster (1979), van den Berg (2001), and Abbring and van den Berg (2003). The probability density function $p(.)$ of the random variable v is unknown and must therefore be estimated. In practice it can be assumed to follow a Gamma distribution, with mean normalized to 1 and variance equal to $1/\sigma$. This adds a single parameter to estimate in the likelihood. A discrete law (v_k, p_k) is also often used, with $p_k = \Pr\{v = v_k\}$ for $k = 1, .., K$, and we estimate the vector $(v_1, .., v_K; p_1, .., p_K)$ along with all the other parameters of the model. Lalive et al. (2006) did robustness checks of their estimates with a mixed proportional hazard model allowing for a discrete distribution of unobserved heterogeneity with two mass points. Although they find evidence of the presence of these two mass points, the estimated effects of a change in the replacement ratio and in the potential benefits duration are not affected by taking unobserved heterogeneity into account.

3.3 Main Results on the Determinants of Unemployment Duration

The study of Lalive et al. (2006) allows readers to grasp the procedure generally followed in estimating the impact of changes to the unemployment insurance system on the duration of unemployment. But it gives no more than a glimpse of the large body of results obtained in this domain and reported in a burgeoning literature. The average duration of spells of unemployment proves to be strongly linked to the replacement rate and potential benefit duration. There exists, however, a strong heterogeneity in the duration of spells of unemployment in relation to the individual characteristics of job seekers. We present here a synthesis of the main results.

3.3.1 The Effects of Potential Benefit Duration and Replacement Rate

In line with the predictions of the job search model, empirical studies find that potential benefit duration and the replacement rate exert significant effects on the duration of unemployment. They also find that the quality of the jobs that are found may be affected by these two parameters, but here the results are less striking.

The survey of Tatsiramos and van Ours (2012), which selects studies with relevant identification strategies, shows that the magnitude of the effects of unemployment benefits differs for different countries and different types of policy changes but that the effects themselves differ less. Their survey warrants us to accept the two following orders of magnitude: first, the elasticity of the duration of unemployment with respect to the replacement ratio varies between 0.4 and 1.6; and second, an increase of a week in the potential duration of benefit payments leads to an increase in the duration of unemployment ranging between 0.1 and 0.4 of a week.

Numerous studies, following the contribution of Meyer (1990), highlight a significant discontinuity in the exit rate from unemployment in the period immediately preceding the exhaustion of entitlement to unemployment insurance benefits, as displayed in Figure 5.8. The size of the spike is influenced by the characteristics of workers and by the institutional environment. This is illustrated by the studies of Dormont et al. (2001) on French data. They show as well that the exit rate from unemployment to employment rises more at the end of the entitlement period for better-qualified job seekers. Figure 5.8 clearly illustrates this phenomenon. It traces the exit rate from unemployment for individuals whose benefits drop significantly in the 14th month of unemployment. At that

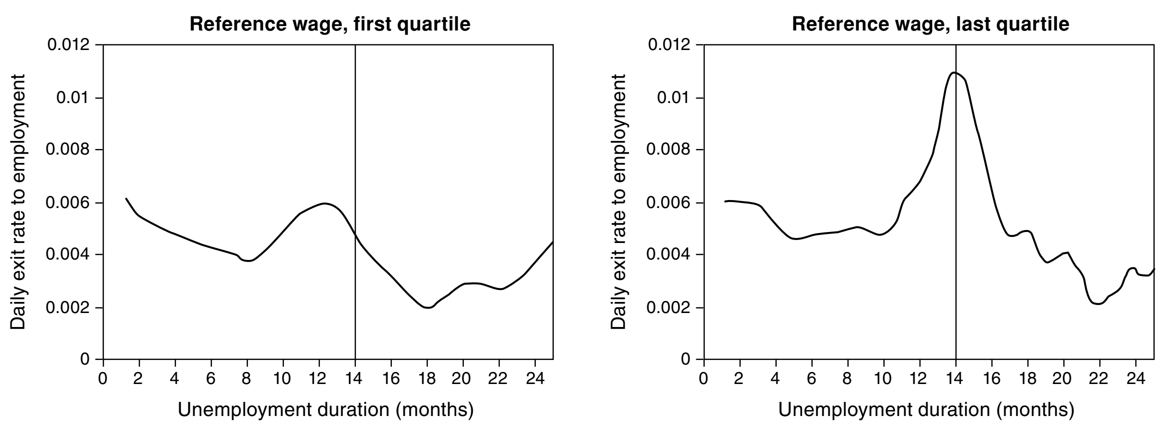

Figure 5.8

Exit rate from unemployment into employment and the end of entitlement to benefits. Period: 1986–1992. Population: individuals aged 25 and older. The reference wage corresponds to the average wage for the 12 months immediately preceding job loss.

Source: Dormont et al. (2001).

time, benefits pass from a magnitude of 57% to 75% of the *previous* wage to a fixed sum corresponding to roughly 60% of the *minimum* monthly wage. Figure 5.8 shows that the probability of exit rises significantly as the 14th month approaches. Further, this effect is much more marked for job seekers who previously earned high wages. Two causes contribute to this phenomenon. First, better-qualified workers, ones earning higher wages, are also those who can find jobs more easily and behave in a more opportunistic manner. Second, the fall in income in the 14th month is weaker to the extent that the reference wage was low to begin with. The question of the relative importance of these two causes remains open.

The studies of Dormont et al. (2001) on French data limit themselves to exits from unemployment into employment. But Card et al. (2007b) have pointed out that the peak observed is markedly more pronounced if all types of exit from unemployment are taken into account. The fact is, a large number of such exits are transitions into other states such as inactivity or training. Card et al. find, on Austrian data, that the rate of exit from unemployment is multiplied by 2.4 around the maximum duration of benefit, compared to the rate observed at the onset of an unemployment spell. The rate of return to employment, in contrast, is multiplied by only 1.15. In the case of seniors, the results of Hunt (1995) suggest that a substantial portion of the transitions observed at the exhaustion of benefit entitlement amount to exits from the labor market altogether. More generally, Card et al. (2007b) stress that the magnitude of any spike in the reemployment rate depends on institutional factors and labor market conditions that include the availability of post-exhaustion benefits (Pellizzari, 2006), the participation of UI recipients in the informal sector, and the incentives for firms to cycle workers through temporary unemployment.

3.3.2 Post-unemployment Outcomes

The job search model predicts that an increase in the amount of unemployment benefit or in its potential duration ought, by raising the reservation wage, to entail an improvement in the quality of the jobs accepted by the unemployed. The empirical results are very mixed. The first research published on this topic by Burgess and Kingston (1976) and Ehrenberg and Oaxaca (1976) estimated that the generosity of unemployment insurance had a positive impact on the wages of jobs accepted upon exit from unemployment. Belzil (2001) for Canada and Centeno (2004) for the United States find, on one hand, that the jobs accepted at the close of the potential duration of unemployment insurance are more unstable and on the other that a higher replacement ratio leads to more stable jobs. These predictions fit the job search model: as the end of the potential duration of benefit approaches, the lowering of their reservation wage induces the unemployed to accept lower-quality, more unstable jobs. An increase in the amount of unemployment benefit has the contrary effect.

More recent studies, using difference-in-differences or regression discontinuity approaches, which provide more reliable identification, find much more mixed results. Card et al. (2007b) detect no impact of the generosity of unemployment insurance on the wages or the stability of jobs in Austria. Van Ours and Vodopivec (2006) arrive at the same conclusion for Slovenia. Centeno and Novo (2009) detect very weak effects on wages in Portugal.

Hence, in the present state of knowledge, the impact of the generosity of unemployment insurance on the quality of jobs remains an open question.

3.3.3 The Effects of Wealth and Liquidity Constraints

The theoretical model elaborated in section 2.2.4 above showed that a rise in the total wealth of an unemployed person ought to have a disincentive effect on job search by raising her reservation wage and reducing her search effort. From this it follows that a rise in the wealth of an unemployed person may cause spells of unemployment to be more prolonged.

Using a portion of the Austrian data already exploited by Lalive et al. (2006), which we described above, Card et al. (2007a) examine the effect of an entitlement to severance pay (which augments the wealth of the unemployed person) on the intensity of job search. To be entitled to severance pay, one must have worked for at least 36 months. Card et al. use a regression discontinuity design to estimate the effects of severance pay, essentially comparing the search behavior of people who were laid off just before and just after the 36-month cutoff for severance pay eligibility. They verify that the firing decisions of firms are not linked to the payment of severance (the regulations for firing employees are very strict in Austria, and the law forbids any strategic behavior in the timing of such separations). In this case, one may reasonably suppose that there is indeed a random selection around the eligibility threshold. Card et al. find that the rate of return to employment (during the normal period of entitlement to unemployment insurance, which means the first 20 weeks of being unemployed) is between 8% and 12% lower for those who are just eligible to receive a severance payment than for those who are just ineligible.

The contribution of Lammers (2012) brings the behavior of unemployed persons following a change in their wealth into sharper focus. It relies on a Dutch panel that contains detailed information on individual wealth and income, subjective reservation wages, and proxies for search effort. Lammers finds that the wealth of an unemployed person has a positive effect on her reservation wage but has no significant effect on her search effort. So the prolongation of a spell of unemployment would be the consequence of her rejecting more of the job offers she receives rather than her receiving fewer offers because her effort has slackened.

Recent empirical studies, therefore, arrive at strongly convergent results regarding the effects of wealth and liquidity constraints. But they are not yet sufficiently numerous to yield definitive conclusions (Tatsiramos and van Ours, 2012).

3.3.4 The Reservation Wage

The survey of Krueger and Mueller (2011), mentioned above in connection with the use of their time by the unemployed, also supplies interesting indications concerning reservation wages. More than 6,000 unemployment insurance recipients in New Jersey were regularly queried each week throughout autumn 2009 and into spring 2010. Their answers when asked what the lowest wage is they would be willing to accept if they received a job offer are treated as their reservation wages. Table 5.8 reports the average ratio of the reservation wage to the pre-unemployment wage.

We observe that the average values of the reservation wage are very close to the previous wage, whatever the duration of the unemployment spell. But these average values conceal a steep variability. In Krueger and Mueller's survey the 25th percentile reservation wage ratio is 0.70, the median is 0.91, and the 75th percentile is 1.17. We further observe that in 6.7% of cases the reservation wage is below the unemployment benefit.

Table 5.8
Reservation wage ratio by duration of unemployment.

All durations	< 5 weeks	5–9 weeks	10–14	15–19	20–24	25–49	> 50
0.99	1.04	1.02	1.01	1.00	1.06	0.95	0.94

Source: Krueger and Mueller (2011, table 4.1).

Table 5.9
Elasticities of the reservation wages with respect to the income of unemployed persons.

Authors	Data	Elasticities
Lynch (1983)	UK (youth)	0.08 – 0.11
Holzer (1986)	US (youth)	0.018 – 0.049
van den Berg (1990)	Netherlands (30–55 years)	0.04 – 0.09

Source: Devine and Kiefer (1991, table 4.2, p. 75).

An initial series of studies attempted to make direct estimates of relations like equation (5.6) giving the value of the reservation wage in the basic model. To that end, as with the recent work of Krueger and Mueller (2011), they relied on data from surveys in which unemployed persons were asked to answer more or less directly the question, "What for you is the lowest acceptable wage?" Table 5.9 gives the magnitudes of the elasticity of the reservation wage with respect to the income of an unemployed person for three studies that use this type of data.[18] It results that, as the basic model predicts, this elasticity is positive. Its magnitude is however very slight.

3.3.5 Helping and Monitoring the Unemployed

The majority of OECD countries have adopted measures aimed at increasing the efficiency of the job search by those receiving unemployment insurance benefits. In the United States, Denmark, the Netherlands, and the United Kingdom, starting in the 1980s, these measures combine help in looking for a job with sanctions, generally consisting of a reduction in benefit, when the rules imposed by the body administering unemployment insurance are not adhered to (Venn, 2012). More precisely, we can distinguish three types of instruments that are generally used in combination: programs giving individual counseling to job seekers, stronger measures to check that eligibility conditions have been met, and stronger measures to check that suitable efforts to find a job are being made. Studies of experimental and nonexperimental programs usually find that the surveillance and counseling programs may have a significant effect on unemployment exit rates among those who need help. They also exert pressure on a percentage of the eligible unemployed who are not experiencing any real difficulty in finding work. The impact of these programs on unemployment duration and wages is reviewed in detail in chapter 14, section 4.

[18] These are averages of estimates for Lynch (1983) and Holzer (1986). The study by van den Berg (1990) estimates the value of reservation wage elasticity at the onset of a spell of unemployment in relation to the future income of an unemployed person.

4 SEARCH FRICTIONS AND WAGE DIFFERENTIALS

In this section we extend the basic job search model studied above to render *endogenous* the dispersion of wages for individuals endowed with identical productive abilities and preferences. This perspective is important inasmuch as it allows us to understand how individuals with identical productive abilities and preferences and with identical jobs can receive different wages. We begin by providing empirical facts about wage differentials which suggest that workers of identical productivity are paid differently. Then, we will see how the equilibrium search model can explain this phenomenon.

4.1 Empirical Facts About Wage Differentials

In a perfectly competitive labor market, with given productive abilities and working conditions, the wage of any individual ought to be independent of the firm or industry in which she is employed. If one industry or firm pays better than others, perfect mobility of workers ought to lead to a flow of labor supply toward that firm or industry and a consequent drop in remuneration. But the existence of persistent wage differentials among industries and firms is a stark, and abundantly documented, fact. Slichter (1950) had already established that this was the case for American workers between 1923 and 1946.

4.1.1 Interindustry Wage Differentials

Let w_{it} be the hourly wage of an individual i at date t; let $\mathbf{x}_{it}$ be the vector of her personal characteristics and those of her job at the same date; let $J(i,t)$ define worker i's industry at date t and let $d_{ijt} = \mathbb{I}[J(i,t) = j]$ be variable indicators equal to 1 if worker i works in industry j at date t and to zero otherwise. Interindustry wage differences are generally highlighted by estimating an equation of the form:

$$\ln w_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \sum_{j=1}^{J} \gamma_j d_{ijt} + \varepsilon_{it} \tag{5.37}$$

In this equation, ε_{it} designates a residual with zero mean. The coordinates of vector $\boldsymbol{\beta}$ and the γ_j are parameters to be estimated. Assuming that the residuals ε_{it} are independent of the d_{ijt}, the ordinary least squares estimators of the γ_j indicate the impact of the industries on wages. Note that the hypothesis of independence between the ε_{it} and the d_{ijt} is strong, since it means that the allocation of workers among sectors is independent of the error term, which includes unobserved heterogeneity. In other words, we assume that the allocation of workers among sectors is independent of their efficiency, something not observed by the econometrician.

Under these hypotheses, if the γ_j coefficients are significantly different from 0, we must conclude either that there are omitted variables or that there are interindustry wage differentials. Traditionally, the estimation of this type of equation gives coefficients γ_j that are significantly different from 0. The first column of table 5.10 presents the results obtained by Goux and Maurin (1999) for France in 1990–1995, with a breakdown into

39 industries. According to these estimations, the standard gap due to industry is of the order of 8% to 9%. We also observe that the industries that pay the least (agriculture, food retail, and hotel, bar, and restaurant) offer wages 15% lower on average than the rest of the economy. The industries that pay the most (petroleum, mining, chemicals) have wages that are on average 15% above those in the rest of the economy. Goux and Maurin note as well that these wage differences persist over time.

4.1.2 The Importance of Unobserved Worker Ability Differences

The results presented in the first column of table 5.10 are similar to those obtained for France and the United States by Dickens and Katz (1987), Krueger and Summers (1988), Katz and Summers (1989), and Abowd et al. (1999). At first glance, these results suggest that the labor markets are far from being perfectly competitive. Their interpretation is a delicate matter, however. It is quite possible that wage differentials are caused by an unobserved heterogeneity of workers. If those with the greatest productive abilities (unobserved) are concentrated in the same industries, those industries must pay higher wages. In that case, the model explaining wages is not described by equation (5.37), but by:

$$\ln w_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \sum_{j=1}^{J} \gamma_j d_{ijt} + \alpha_i + \varepsilon_{it} \tag{5.38}$$

where α_i designates the unobserved time-invariant characteristics of individual i. If data for several periods are available, it is possible to eliminate this term by estimating equation (5.38) in differences. Such estimations, carried out on U.S. and French data, and covering a sufficiently large number of industries (Murphy and Topel, 1987; Abowd et al., 1999; Goux and Maurin, 1999), find that interindustry wage differentials are to

Table 5.10

Estimates of interindustry wage differences in France, 1990–1995. Hourly wages. Field: men, wage earners. Selected industries.

Industry	Model without fixed individual effect	Model with fixed individual effect
Agriculture	−.101 (.07)	−.017 (.016)
Mining (coal)	.139 (.020)	.058 (.056)
Petroleum	.210 (.018)	.049 (.027)
Electricity	.108 (.007)	.058 (.019)
Chemical	.163 (.009)	.016 (.019)
Food retail	−.112 (.007)	−.043 (.014)
Hotels, bars, and restaurants	−.175 (.006)	−.008 (.012)

Note: The figures in parentheses are standard errors. Aside from industry, the variables taken into account are experience in the labor market, job seniority, place of residence, education, nationality (French or foreign), and profession. How to read the table: according to the model without fixed individual effect, a wage earner in agriculture receives, on average, a wage 10.1% lower than in the reference sector (public utilities).

Source: Goux and Maurin (1999, table 3).

a very large extent explained by the characteristics of workers. The second column of table 5.10, taken from Goux and Maurin (1999), gives a striking illustration of this. It shows that the contribution of industry to wage setting is much smaller, and often not significantly different from 0, at the threshold of 5%. Further, Goux and Maurin point out that there is a very weak correlation, less than 0.25, between the coefficients estimated by the models with and without fixed individual effect. Finally, they find that the wage variations incurred by an individual who changes industries do not exceed 2% to 3%.

These results point to the conclusion that interindustry wage differences are essentially explained by individual effects. They are very different from the results obtained by Krueger and Summers (1988) and Gibbons and Katz (1992), who, with a smaller number of industries (around 20, rather than around 40), show that industry makes a significant contribution to wage formation, after controlling for fixed individual effects. But if they adopt a breakdown similar to that of Krueger and Summers (1988) and Gibbons and Katz (1992), Goux and Maurin (1999) arrive at conclusions close to theirs for French data. That being so, the results that tend to prove the importance of the industry component obtained by Krueger and Summers (1988) and Gibbons and Katz (1992) are most likely the result of an aggregation bias caused by using too few industries.

4.1.3 Industry Effect and Firm Effect

Although the impact of industry on wage formation appears very slight, it is quite possible that wages are heterogeneous among firms in the same industry. An approach like the one just described must be used to study this problem, but with identification of the firms, and not just the industries, in which individuals work. This is the aim of Abowd et al. (1999), who have estimated the importance of firm-fixed effects with equation (5.38), where the "j" now represents indexes of firms rather than sectors. The estimation of this equation by ordinary least squares permits an evaluation of the contribution of firm-fixed effects to wage dispersion under the hypothesis that the mobility of wage earners among firms is independent of the residue ε_{it}. It is important to note that the identification of firm-fixed effects rests on the observation of wage earners who change firms. It is therefore a requirement to observe a sufficient number of entries into and exits from each firm in order to be able to identify these fixed effects with precision. Thus the panel must have a duration sufficiently protracted to satisfy this condition.

Use of this method shows that firm-fixed effects explain a significant part of wage distribution. Table 5.11 shows that in France firm-fixed effect explains 17% of the variation in the logarithm of wages, while individual-fixed effect explains 29%. In the

Table 5.11
Components, in percentage, of variations of (log of) real annual wages in France and in the United States.

	France (1976–1999)	United States (1990–1999)
Experience and experience squared	17	5
Person effect	29	24
Firm effect	23	24
Residual	31	47

Source: Abowd et al. (2003).

United States, firm- and individual-fixed effects each explain 24% of wage distribution. Abowd et al. (2013) confirmed these results using more recent data and improved estimation techniques.

Studies that take this approach show unambiguously that the impact of the firm is greater than that of the industry. Goux and Maurin (1999), for example, assess the average of the difference in wages paid to an identical worker employed at two different firms in France at 20% to 30%, whereas it does not exceed 2% to 3% for a change of industry. They show, moreover, that these differences are positively correlated with the size and the capital/labor ratio of firms. The correlations with productivity and profitability are much smaller and much less significant.

All in all, these studies show that individuals with identical time-invariant characteristics are paid differently when they work in different firms. Once again, the interpretation of these results is a delicate matter. It is possible that wage differences among firms are the result of unobserved differences linked to working conditions on a perfectly competitive labor market. The compensating differential theory of wages (see chapter 3, section 2) indicates that a wage reflects not just productive ability but also the content of the tasks an employee must carry out at her workplace: more dangerous, more unstable, and more laborious jobs are offset by higher wages. As these characteristics of jobs are generally poorly measured, it remains possible that the unobserved heterogeneity of jobs does explain wage differences among firms, according to a perfectly competitive logic.

The dispersion of wages may also be the consequence of market imperfections. It may for instance be the consequence of coalitions of employees and employers deciding wages and working conditions jointly, through negotiation. We study this question in chapter 7, dedicated to collective bargaining. In this chapter, we examine the consequences of limited mobility. When there are barriers to mobility, monopsony situations (those in which a single employer confronts a large number of suppliers of labor) can arise (see chapter 12, section 2.2.1). Fundamentally, monopsony is one of the textbook cases in which mobility costs work to the disadvantage of wage earners. If these costs did not exist, the monopsony would be powerless vis-à-vis employees who could quit their jobs at any time. But the converse is just as valid: the costs of hiring, firing, and training are obstacles to mobility of jobs that can be exploited by employees in such a way as to capture a share of the rent. No matter what their source, mobility costs and the rent-sharing that attends them generate wage differentials that are unrelated to productivity differentials and that hinder the efficiency of the competition mechanism. From this viewpoint, job search models generalize the monopsony model: they take into account the costs of mobility, called "friction costs," for employers and for workers. We will see that by making possible a deeper understanding of wage formation, these models have made a profound advance in our conception of how the labor market functions.

4.2 The Equilibrium Search Model

The basic job search model focuses solely on the behavior of job seekers and takes the distribution of wages as given. This approach leaves the setting of wages unexplained. Equilibrium search models have as their goal the explanation of how wages are set through the attribution of well-defined strategic behavior to firms. Labor market equilibrium is therefore characterized by an *endogenous* distribution of wages.

4.2.1 Diamond's Critique

In the job search models presented above, the cumulative distribution function $H(.)$ of wage offers is exogenous. This hypothesis must be abandoned if we wish to understand how wages are determined. To achieve that, we must make explicit the behavior of employers. This poses a conceptual difficulty known as *Diamond's critique*. Diamond (1971) was the first to emphasize that if the reactions of employers are introduced into the basic job search model, the outcome is necessarily a labor market equilibrium in which the distribution of wages is concentrated at a single point. To better understand this result, let us assume that the economy is composed of a large number of identical suppliers of labor and a large number of firms, likewise identical, and let us suppose that workers sequentially receive job offers (i.e., cannot recall former offers if declined) and accept any offers with a wage that is above their reservation wage, as shown above in section 2.1. Since the workers accept *without distinction* all proposals that equal or exceed the reservation wage, the firms gain no advantage by offering wages that exceed it (because as a general rule the profit per capita diminishes with the cost of labor). At equilibrium, the distribution of wages is thus concentrated at value x of the reservation wage. The definition of the reservation wage, which was obtained above in equation (5.6), is:

$$x = z + \frac{\lambda_u}{r+q}\int_x^{+\infty}(w-x)\mathrm{d}H(w) \tag{5.39}$$

where λ_u denotes the arrival rate of job offers and q the job destruction rate, implies that the distribution of wages is thus concentrated at a value equal to the instantaneous gain z of the unemployed workers. This result arises essentially out of the hypothesis that workers never (voluntarily) leave their employers and from the fact that employers have no power over the reservation wage level of workers. Indeed, x does not depend on the wage offered by any firm in particular but on the distribution H on the market. Hence firms have no incentive to post wages superior to the minimum acceptable z, for doing so allows them neither to attract nor to retain more workers. At first sight, Diamond's critique appears to deprive the basic job search model of all its relevance, since within this model we cannot explain why the distribution of wages does not degenerate to a single point.

In reaction to the critiques directed at the basic job search model, *equilibrium* search models have been elaborated, in which the distribution of wages becomes an endogenous variable dependent on, among other things, the wage strategies of employers. An initial approach consists of extending the basic model—often termed the partial model for clarity—by introducing heterogeneity among the workers and assuming that firms post non-renegotiable wages that cannot be contingent on the reservation wages (Albrecht and Axell, 1984). Under certain conditions, labor market equilibrium is compatible with a nondegenerated distribution of wages that coincides with that of the reservation wages of different categories. This means that an individual can encounter a distribution of wages because he is surrounded by individuals with different reservation wages.

4.2.2 The Mean-Min Wage Ratio

The solution to Diamond's critique based on different reservation wages is not totally satisfactory, inasmuch as Hornstein et al. (2011) have shown that the basic search model cannot generate large wage differentials for identical individuals for plausible values of preference parameters and of labor flows. More precisely, they argue that the basic search model predicts that the ratio between the mean wage and the reservation wage is very small for plausible parameters values. In order to show this, let ρ denote the replacement ratio, equal to the ratio between the instantaneous gains of unemployed workers and the average wage, so that $z = \rho\overline{w}$, where $\overline{w} = \int_x^\infty \frac{w}{1-H(x)} \mathrm{d}H(w)$ stands for the mean wage. Then equation (5.39) can be written:

$$x = z + \frac{\lambda_u^*}{r+q}(\overline{w} - x)$$

where $\lambda_u^* = \lambda_u [1 - H(x)]$ is the job finding rate. This equation yields a relation between the minimum observed wage x and the mean wage, which can be written under the form of the mean-min wage ratio:

$$Mm \equiv \frac{\overline{w}}{x} = \frac{\frac{\lambda_u^*}{r+q} + 1}{\frac{\lambda_u^*}{r+q} + \rho} \tag{5.40}$$

which is a measure of the dispersion of wages. The distribution of wages is degenerated to a single mass point if this ratio is equal to 1. And the bigger this ratio, the wider the wage dispersion. Hornstein et al. (2007) have estimated on U.S. data that the mean-min ratio of the frictional distribution of wages, the distribution of wages for identical workers, belongs to the interval $[1.5, 2]$.

An interesting feature of the mean-min wage ratio defined in equation (5.40) is that it does not depend *directly* on the wage offer distribution H. Accordingly, one does not need to have direct information about this distribution to know its mean-min ratio. The mean-min wage ratio is merely a function of four parameters: the job finding rate, the job separation rate, the discount rate, and the replacement rate. Hornstein et al. (2007) have shown that the mean-min wage ratio is very close to 1 for plausible values of these parameters. Setting the period to one month and calibrating the model on the U.S. economy over the period 1991–2007 implies a monthly job finding rate of 0.43, a monthly job separation rate of 0.03, and a replacement ratio ρ equal to 0.4. Assuming that the annual discount rate is equal to 5%, which implies a monthly discount rate $r = 0.0041$, the mean-min wage ratio would amount to 1.04. Hence the predicted mean-min wage ratio is very close to 1, meaning that the baseline job search model predicts that there is very little wage dispersion.

It is easy to see that this result arises out of the fact that the term $\lambda_u^*/(r+q)$ that appears in the numerator and in the denominator of the expression of the mean-min wage ratio is much bigger than 1 to the extent that the job finding rate is much bigger than the job separation rate; it is about 10 times bigger on most labor markets. This can be seen more clearly if we express the mean-min wage ratio as a function of the unemployment rate, denoted u. Normalizing the size of the labor force to 1, equality between entries into and exits out of unemployment, respectively equal to $q(1-u)$ and $\lambda_u^* u$, implies that $u = q/(q + \lambda_u^*)$. Now the mean-min wage ratio can be written:

$$Mm = \frac{\frac{q}{r+q}\frac{1-u}{u} + 1}{\frac{q}{r+q}\frac{1-u}{u} + \rho} \simeq \frac{\frac{1-u}{u} + 1}{\frac{1-u}{u} + \rho}$$

where the approximation is justified by the fact that the discount rate r is small with respect to the job separation rate, about 10 times smaller. In this expression of the mean-min ratio, we see that the term $(1-u)/u$ goes from 19 to 9 when the unemployment rate goes from 5% to 10%, driving the mean-min wage ratio up from 1.03 to 1.06. Therefore, on average, the mean-min wage ratio predicted by the baseline job search model is in the neighborhood of 1.05, between 10 times and 20 times smaller than the actual mean-min ratio. Hornstein et al. (2011) show that the mean-min wage ratio remains small under a large set of changes in the assumptions of the model, such as the introduction of risk aversion, of compensating wage differential, of return to experience, and of endogenous search effort.

Hornstein et al. (2011) convincingly show that the basic job search model does predict a narrow dispersion of wages. However, they also show that the on-the-job search models, initiated by Burdett and Mortensen (1998), can explain larger wage dispersion. On-the-job search implies that there is competition among employers to attract workers, which alters the way wages are determined. The model that follows sheds light on how this works.

4.2.3 WORKER TURNOVER AND WAGE DISPERSION

We consider an economy composed of a continuum of firms and a continuum of workers. For simplicity, these two continuums are assumed to be of unitary mass. This hypothesis allows us to account simply for the fact that there exists a large given number of firms and workers. The job search behavior of suppliers of labor is identical to that in the model with on-the-job searching studied above, in section 2.2.2. In particular, q always designates the instantaneous job destruction rate, and the parameters λ_u and λ_e represent respectively the arrival rate of job offers for an unemployed job seeker and for one who has a job. The reservation wage of the former, always denoted x, is then given by relation [as shown above in equation (5.20)]:

$$x = z + (\lambda_u - \lambda_e)\int_x^{\infty} \frac{\bar{H}(\xi)}{r + q + \lambda_e \bar{H}(\xi)} \mathrm{d}\xi \quad \text{with} \quad \bar{H}(\xi) \equiv 1 - H(\xi) \tag{5.41}$$

This equation implicitly defines the reservation wage as a function of the parameters λ_u, λ_e, and the cumulative distribution function H. When $\lambda_e = 0$, that is, when there is no on-the-job search, we come back to the reservation wage of the baseline model. Vice versa, if $\lambda_e > 0$, the job seeker takes account of the possibilities of future income associated with continuing to look for a job while employed. Now and henceforth, contrary to the model of section 2.2.2, the cumulative distribution function $H(.)$ is an endogenous variable.

Let us designate by $G(w)$ the cumulative distribution function of wages among employees, that is, the share of employees paid a wage lower than w. It is important to remark that $G(w)$ is different from $H(w)$, which is also called the *sampling* distribution, the cumulative distribution function of wage offers that job seekers receive. We can get a relation between these two cumulative distribution functions using the equality between exits and entries in jobs that pay wages lower than w. Let us denote by u the unemployment rate. The entries into jobs that pay less than w are composed of unemployed job seekers who have received a wage offer inferior to w. Now at each date an unemployed job seeker receives offers at rate λ_u, and these offers are lower than w with a probability $H(w)$. Entries of unemployed job seekers into jobs offering a wage lower than w then amount to $\lambda_u uH(w)$. These comprise all the entries into these jobs because we only consider the net number of entries; we do not count the entries and exits into and out of jobs that pay less than w. As regards exits, employment in the jobs paying less than w is equal to $(1-u)G(w)$. These jobs are destroyed at rate q, and employees get job offers at rate λ_e that they accept only if the wage offered is above w, which occurs with probability $1 - H(w) = \bar{H}(w)$. Therefore, at stationary equilibrium, the equality of the flows of entries and exits is given by the following equation:

$$\lambda_u uH(w) = (1-u)G(w)\left[\lambda_e \bar{H}(w) + q\right] \quad \textbf{(5.42)}$$

At stationary equilibrium the value of the unemployment rate results directly from the equality between the flows of workers entering into and exiting from unemployment. The former amounts to $q(1-u)$ and the latter is equal to $\lambda_u[1 - H(x)]u = \lambda_u u$ because the reservation wage x is the lower bound of the sampling distribution to the extent that it cannot be optimal to make wage offers that are always rejected. The stationary unemployment rate is then given by:

$$u = \frac{q}{\lambda_u + q} \quad \textbf{(5.43)}$$

Using equations (5.42) and (5.43), we get:

$$G(w) = \frac{H(w)}{1 + \kappa \bar{H}(w)}, \quad \text{where } \kappa = \frac{\lambda_e}{q} \quad \textbf{(5.44)}$$

This equation implies that $G(w) < H(w)$, when $\lambda_e > 0$. In other words, the probability of occupying a job with a wage less than w is lower than the probability of receiving a job offer with a wage less than w. The distribution of wages among employees thus dominates the distribution of wages offered. This stems simply from the fact that wage earners holding a job accept only wages higher than their current wage. Wage earners holding a job thus obtain higher and higher wages every time they change jobs. Moreover, equation (5.44) shows that the gap between the sampling distribution and the distribution of wages among employees grows with parameter κ, which represents the average number of job offers received in the interval between two job-destroying shocks. Parameter κ is an inverse measure of search frictions, which are small when there are many job offers between two consecutive job-destroying shocks.

Jolivet et al. (2006) have compared the distribution of wages of persons holding jobs with the wages at which unemployed persons are hired in 10 European countries and in the United States. They find that the distribution of the hiring wages of the unemployed is systematically dominated by that of the wages of persons who hold jobs. Figure 5.9 illustrates this phenomenon: it shows that the cumulative distribution functions of the hiring wages of the unemployed are systematically situated below those of the wages of job holders. Of course, the difference between these distributions may be influenced by factors other than job search. In particular, employers may have an interest in offering contracts in which the wage rises with seniority in order to raise the performance of their employees. The lower wages of entrants are then explained by reasons of incentivation, which we will study in chapter 6. It remains the case that the job search model with on-the-job search furnishes a highly convincing explanation of an empirical phenomenon that is well identified.

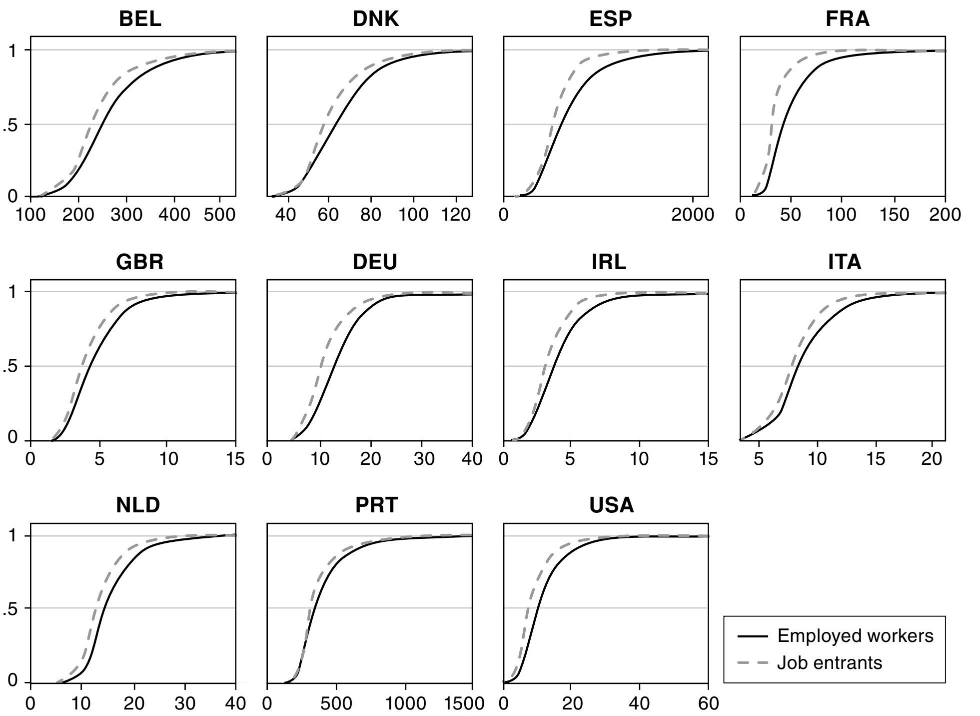

Figure 5.9
Cumulative distribution functions of hourly wages (in local currency) in 10 European countries (1994–1997) and in the United States (1993–1996).

Source: Jolivet et al. (2006).

4.2.4 Wage-Posting Models

To explain wage distribution, it is necessary to specify the manner in which wages are set. Let us begin by presenting the wage-posting model of Burdett and Mortensen (1998), where every firm unilaterally proposes an identical wage for all the workers it hires. In this model, workers of the same productivity who hold jobs of the same degree of difficulty obtain different wages.

The Behavior of Firms

Burdett and Mortensen (1998) assume that firms compete by posting wages to attract workers. At every instant, the probability of encountering a worker is identical for all firms. Each firm decides unilaterally on the *constant* wage that will be paid to its employees. It is thus assumed that the workers at one firm all receive the same wage. It is also assumed that at each moment a worker is capable of producing, if she is employed, a constant exogenous quantity y of goods. If there are $\ell(w)$ workers in a firm that pays wage w, the instantaneous profit of this firm works out to $(y-w)\ell(w)$. For simplicity, we assume that the real rate of interest r is close to 0 (an approximation we can justify by noting that in practice r is clearly smaller than the rates λ_u, λ_e, and q). Under this hypothesis, each firm sets its wage in such a way as to maximize its stationary instantaneous profit $(y-w)\ell(w)$, with the wages being paid in the other firms being taken as given (so what we have is a noncooperative equilibrium of the Cournot-Nash type). Let us first note that each firm must necessarily propose a wage w higher than the reservation wage x, or $w \geq x$, so as to be able to attract the unemployed at least. The optimal wage is then defined by the equality:

$$\frac{\ell'(w)}{\ell(w)} = \frac{1}{y - w}, \quad w \geq x \tag{5.45}$$

The employment function $\ell(w)$ is obtained using relation (5.45), which characterizes the optimal behavior of a firm. As this relation is true for all wages belonging to the support of H, it can be considered a first-order differential equation in $\ell(w)$. Quantity $\ell'(w)/\ell(w)$ representing the derivative of $\ln \ell(w)$, and the integral of $(y-w)^{-1}$ being equal to $-\ln(y-w)$, this equation is written $\int \ln \ell(w) \mathrm{d}w = -\int \ln(y-w)\mathrm{d}w$, or $\ln \ell(w) = -\ln(y-w) + a$, and then $\ell(w) = \exp(a)/(y-w)$, where a is a constant that does not depend on w. The intuitive view proves correct: employment does indeed increase with wages. We observe as well that the wage is lower than the marginal productivity of work, as in the monopsony model, and that the profits $(y-w)\ell(w)$ of the different firms are all equal to $\exp(a)$ at equilibrium. In other words, there exists a distribution of wages such that at equilibrium firms can realize the same level of profit with low wages and a small workforce, or with high wages and a large workforce. Consequently, firms that pay low wages face a relatively low hiring rate and a relatively high quit rate, which results, at stationary equilibrium, in a small workforce.

The Equilibrium Wage Distribution

The wage-setting policy of firms allows us to obtain a relation between the wage and employment. We can obtain another relation between employment and wages, deriving from worker flows, from equation (5.44). There are $\ell(w)$ employees in each firm that pays wage w and there are $H'(w)$ firms that pay wage w. Therefore, the total mass of employees paid wage w amounts to $H'(w)\ell(w)$. By definition, the mass of employees paid wage w is also equal to $G'(w)(1-u)$. Thus:

$$G'(w)(1-u) = \ell(w)H'(w) \tag{5.46}$$

Using this equality with equation (5.44) yields:[19]

$$\frac{\ell'(w)}{\ell(w)} = \frac{2\kappa H'(w)}{1 + \kappa \bar{H}(w)} \tag{5.47}$$

[19]Deriving equation (5.44) with respect to w yields:

$$G'(w) = \frac{H'(w)\left[1 + \kappa G(w)\right]}{1 + \kappa \bar{H}(w)} \tag{i}$$

Substituting this expression of $G'(w)$ into (5.46) gives:

$$\left[1 + \kappa G(w)\right](1-u) = \ell(w)\left[1 + \kappa \bar{H}(w)\right]$$

The derivation of the logarithm of this equation with respect to w gives:

$$\frac{\kappa G'(w)}{1 + \kappa G(w)} = \frac{\ell'(w)}{\ell(w)} - \frac{\kappa H'(w)}{1 + \kappa \bar{H}(w)}$$

This equation, together with equation (i), implies equation (5.47).

This differential equation implicitly defines the relation between functions $\ell(.)$ and $H(.)$ compatible with equilibrium of flows on the labor market. Comparison of equations (5.45) and (5.47) reveals that distributions of wage offers compatible with both equilibrium of flows on the labor market and strategic behavior by firms in setting wages necessarily satisfy, for any value of w, relation:

$$2(y-w)H'(w) + H(w) = \frac{1+\kappa}{\kappa} \tag{5.48}$$

This equality, which holds for all w, is interpretable as a first-order differential equation in $H(w)$. If A designates any constant, then the general solution of this differential equation is written:[20]

$$H(w) = A\sqrt{y-w} + \frac{1+\kappa}{\kappa}$$

The constant A is obtained using the fact that firms have no interest in offering a wage smaller than the reservation wage x of unemployed job seekers. Thus it is certain that $H(x) = 0$. Utilizing this property, we find that the *unique* possible equilibrium wage distribution is expressed by:

$$H(w) = \frac{1+\kappa}{\kappa}\left[1 - \sqrt{\frac{y-w}{y-x}}\right] \tag{5.49}$$

The upper bound of the distribution of wages, denoted w_{sup}, satisfies $H(w_{\text{sup}}) = 1$. It is defined as a function of the reservation wage by the formula:

$$w_{\text{sup}} = y - (y-x)\left(\frac{1}{1+\kappa}\right)^2 \tag{5.50}$$

If the reservation wage is less than the instantaneous production of a worker y (which is a necessary condition of the existence of equilibrium), we can verify that the upper bound $\overline{w}$ of wages is likewise smaller than individual production y. Taking into account (5.49), the equilibrium wage distribution takes the form:

$$H'(w) = \frac{1+\kappa}{2\kappa}\frac{1}{\sqrt{(y-x)(y-w)}} \tag{5.51}$$

The equilibrium density of the sampling distribution $H'(.)$ of this model turns out to *increase* as the level of wages rises. This result is a consequence of both the property that all agents are homogeneous and the firms' strategy of simply proposing an invariable wage. Under these conditions, a firm that raises its wage w increases its volume of employment to the detriment of employment in the other firms. This movement leads to an increasing relation between the wage and the size of the firms.

[20] Recall that the general solution of a linear differential equation is obtained by adding a particular solution to the general solution of the *homogeneous* equation. The latter is written $H'(w)/H(w) = -1/2(y-w)$; it is integrated exactly like equation (5.45), which gives us $H(w) = A\sqrt{y-w}$, where A is an arbitrary constant. We get a particular solution of equation (5.48); by making $H' = 0$ in this equation, we immediately find $H(w) = (q+\lambda_e)/\lambda_e$, and from that the general solution of equation (5.48).

All the relations giving the equilibrium values of the endogenous variables of the model depend on the reservation wage x, which is itself an endogenous variable. Now the reservation wage is always defined by equation (5.41) of the partial model, on condition of positing $r = 0$. Taking account of expression (5.49) of the equilibrium wage distribution, it is possible to obtain an explicit analytic form of this wage. After several calculations, we arrive at:

$$x = \frac{z(q+\lambda_e)^2 + (\lambda_u - \lambda_e)\lambda_e y}{(q+\lambda_e)^2 + (\lambda_u - \lambda_e)\lambda_e} \tag{5.52}$$

If there is no possibility of on-the-job search, or $\lambda_e = 0$, we have $x = z$ and, following (5.50), $w_{\text{sup}} = z$. We thus come back to the paradox pointed out by Diamond (1971)—the only possible equilibrium in the partial job search model occurs when the distribution of wages is entirely concentrated at the level of the instantaneous gain z of an unemployed job seeker. When $\lambda_u \to +\infty$, there is no friction in the labor market and the workers obtain the totality of product. The wage is thus uniform and equal to the value of production ($x = w_{\text{sup}} = y$). Searching while working is thus pointless ($\lambda_e = 0$). These characteristics describe a perfectly competitive equilibrium where there is no unemployment ($u = 0$) and where the wage equals the marginal productivity of labor.

The Empirical Implications of the Wage-Posting Model

The version of the wage-posting model we have just presented yields a certain number of pertinent empirical predictions. First, in the equilibrium search model, the wage of an individual employee rises when she moves from one job to another. Although that is not in practice the only reason for individual pay to rise, this phenomenon is in fact observed in the majority of transitions of this type (see for example Topel and Ward, 1992). Moreover, in this model the wage is positively correlated with the size of the firm, which fits well with observations that tell us that even after controlling for the heterogeneity of workers and firms, bigger firms pay higher wages than do smaller ones (Abowd et al., 1999).

Second, wages rise, on average, as workers gain experience. Assuming that new entrants begin as job seekers, the wage at which they are hired is a minimum corresponding to the reservation wage x. After that, their wage rises every time they change firms. More senior employees, who have on average had the most job offers, thus enjoy the highest wages. This prediction of the equilibrium search model agrees with the observation that a worker's wage increases with the time she has spent in the labor market (Abowd et al., 1999).

Third, the lower bound of the equilibrium wage distribution being equal to the reservation wage, an unemployed job seeker accepts all the offers he receives. This conclusion fits very well with that of empirical studies, which do in fact find that the probability of accepting an offer is close to 1.[21]

Fourth, the model with on-the-job search predicts more wage dispersion than the baseline job search model. In particular, the predicted mean-min wage ratio is larger.

[21] See Devine (1988), van den Berg (1990), and Wolpin (1987).

When there is on-the-job search, the mean-min wage ratio can be approximated by the expression (assuming as above that $r \to 0$):[22]

$$Mm \simeq \frac{\frac{\frac{1-u}{u}-\kappa}{1+\kappa}+1}{\frac{\frac{1-u}{u}-\kappa}{1+\kappa}+\rho}$$

This formula shows that the mean-min wage ratio increases with κ, which means that the mean-min ratio increases when there is more competition among firms. Jolivet et al. (2006) provide estimates of parameter κ for 10 European countries and for the United States. They find that κ is between 0.27 (for Portugal) and 2.03 (for France). Assuming that the unemployment rate equals 10% and that $\rho = 0.4$, the mean-min ratio is equal to about 1.2 when κ equals 2. Therefore, the on-the-job search model with wage posting can predict a value of the mean-min wage ratio that is significantly larger than the baseline job search model but that is still below the empirical mean-min wage ratio.

The search equilibrium model does present one major flaw: the density of wage distribution—see (5.51)—is an *increasing* function of the wage. This prediction turns out to conflict with all observations, which reveal that this density is increasing at first, then decreasing, with a maximum generally not too far from the lower bound, as shown by figure 5.10.

To remedy this flaw, one solution lies in introducing heterogeneity among agents. Assume that upon receiving a job offer, workers draw the productivity of the firm from which the offer comes from an exogenous distribution denoted $\Gamma(y)$, where y stands for the time-invariant productivity of each firm. Let us denote $H(w)$ the corresponding sampling distribution of wage offers. From equations (5.43), (5.44), and (5.46) we get:

$$\ell(w) = \frac{\lambda_u(1+\kappa)}{(\lambda_u+q)\left[1+\kappa\bar{H}(w)\right]^2} \qquad \textbf{(5.53)}$$

In equilibrium, the sampling distribution of wages and firm types must be identical, $H[w(y)] = \Gamma(y)$, and the firms with the smallest productivity offer workers their reservation wage and hire workers only from the unemployment pool. This distribution is an equilibrium if each firm offers a wage $w(y)$ that maximizes its steady-state profit flow, $\pi(y, w) = (y - w)\ell(w)$, which implies that $\mathrm{d}\pi[y, w(y)]/\mathrm{d}y = \delta\pi[y, w(y)]/\partial y = \ell[w(y)] > 0$. Or, utilizing equation (5.53):[23]

$$\frac{\mathrm{d}\pi[y, w(y)]}{\mathrm{d}y} = \frac{\lambda_u(1+\kappa)}{(\lambda_u+q)\left[1+\kappa\bar{\Gamma}(y)\right]^2}$$

If we assume that there is free entry into the market for goods, the profit of the firm with the weakest productivity is zero, or $\pi(y_{\inf}, x) = 0$, where x designates the reservation wage and $y_{\inf}$ represents the lower bound of the productivity distribution. This condition entails that the firm with the weakest productivity, equal to $y_{\inf}$, hires job seekers by

[22] See Hornstein et al. (2011).

[23] Note that individual firm decisions have no impact on $H[w(y)]$.

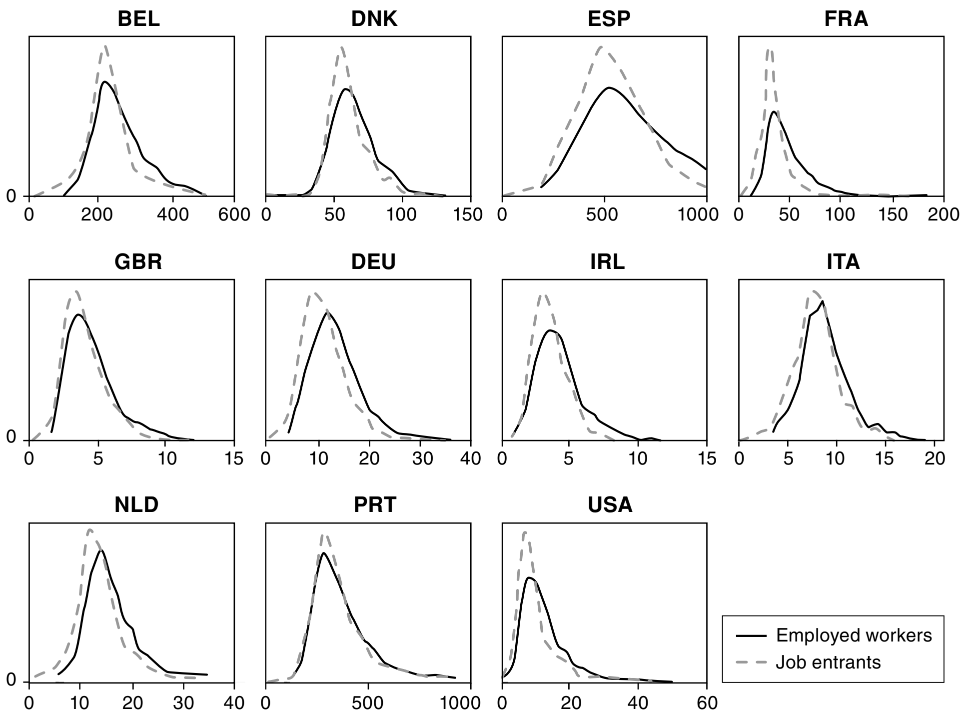

FIGURE 5.10
Density functions of hourly wages (in national currency) in 10 European countries (1994–1997) and in the United States (1993–1996).

Source: Jolivet et al. (2006).

offering them a reservation wage equal to their productivity, or $x = y_{\text{inf}}$. The integration of the above equation then entails:

$$\pi\left[y, w(y)\right] = \int_x^y \frac{\lambda_u(1+\kappa)}{(\lambda_u + q)\left[1 + \kappa\bar{\Gamma}(\xi)\right]^2} \mathrm{d}\xi$$

and thus, using the definition of profit, $\pi(y, w) = (y - w)\ell(w)$ and equation (5.53):

$$w(y) = y - \left[1 + \kappa\bar{\Gamma}(y)\right]^2 \int_x^y \frac{\mathrm{d}\xi}{\left[1 + \kappa\bar{\Gamma}(\xi)\right]^2} \tag{5.54}$$

This equation shows that, as in the previous model where jobs all have the same productivity, wages are lower than productivity. It also shows that wages rise with productivity whenever persons already in work receive offers, which corresponds to the case in which $\kappa = \lambda_e/q$ is strictly positive. For in the case where the rate at which job holders receive offers is zero, all wages are equal to the reservation wage x, itself equal to the instantaneous gain of job seekers, z.

Several contributions have estimated this model by inferring the distribution of the productivities of firms on the basis of the distribution of wages and transitions between jobs and unemployment (see Bontemps et al. 2000; Postel-Vinay and Robin, 2006; Eckstein and van den Berg, 2007). Bontemps et al. (2000), for example, estimate the cross-sectional distribution of wages, G, on French data, by a nonparametric method. Availing themselves of this distribution, and noting that the hazard rate of an episode of employment is equal to $q\left[1+\kappa \bar{H}(w)\right]$, with $\bar{H}(w)=(1+\kappa)G(w)/[1+\kappa G(w)]$, they can then estimate parameter κ by maximizing the likelihood of hazard rates on the labor market.[24] Finally, knowledge of the distribution H, of κ, and of wages w makes it possible to estimate the inverse wage function, meaning the distribution of productivities as a function of wages, which, in line with equation (5.54), takes the expression:

$$y(w)=w+\left[1+\kappa \bar{H}(w)\right]^2 \int_x^w \frac{\mathrm{d}\xi}{\left[1+\kappa \bar{H}(\xi)\right]^2}$$

Comparison of the distribution of productivities thus obtained with that directly estimated on the basis of firm data shows that the wage-posting model predicts a distribution of the productivities of firms with an excessively high proportion of high-productivity firms (the "tail" of the density function is too thick). This result comes from the fact that the wage-posting model artificially limits competition among firms, since they are barred from making counteroffers. This hypothesis, which limits the market power of wage earners, entails that in order to explain high wages productivities must be higher than those observed in the data.

4.2.5 Sequential Auctions and Bargaining

The wage-posting model assumes a limited degree of competition among firms, to the extent that an employer whose employee receives an offer higher than his current wage cannot, by hypothesis, make a counteroffer. Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006) have set forth models of sequential auction and bargaining where firms can make such counteroffers. We will see that these models make it possible to explain the distribution of wages better than models of wage posting do.

To make the equilibrium job search model more empirically pertinent, Postel-Vinay and Robin (2002) elaborate a model in which a firm can make a counteroffer when its worker receives an offer from a competing firm. In this setting, wage earners have more market power than in the wage-posting model because they can drive employers into direct competition. Postel-Vinay and Robin assume that firms make take-it-or-leave-it offers to workers. The productivities of firms and the situation of each worker are observed by all.

Hence, when a firm encounters a job seeker, the firm offers him his reservation wage, which he accepts, as in the wage-posting model. When two firms are in contact with the same worker, they simultaneously make him take-it-or-leave-it offers. Such is the case when a job holder paid at wage w in a firm of productivity y (with $w \leq y$) receives

[24] The maximum likelihood method within the framework of duration models is presented in section 3.2.

an offer from a firm of productivity y'. A range of outcomes are possible, depending on the amount of wage w and the productivities y and y'.

1. If $y < y'$ the competing firm, of productivity y', offers the lowest wage possible that allows it to attract the worker, while the firm of productivity y offers the highest wage possible that is compatible with nonnegative profits, to try to retain the worker. The firm of productivity y thus offers a wage equal to y. At equilibrium the worker is hired by the firm with the highest productivity, where she obtains a wage that makes her indifferent between staying with the type y firm at wage y or receiving wage $\omega(y, y')$ in the type y' firm. If we denote by $V_e(w, y)$ the expected discounted utility of a worker paid at wage w in a type y firm, the wage $\omega(y, y')$ obtained in the new type y' firm verifies $V_e[\omega(y, y'), y'] = V_e(y, y)$.

2. When $y' \leq y$, two possibilities must be considered, depending on whether the firm currently employing the worker is compelled to make a counteroffer in order to retain her.
   a. If $V_e(w, y) > V_e(y', y')$, the firm of productivity y' can offer at most wage y', but the expected utility of a worker who accepted that offer would be inferior to what she would get by staying with her current employer. So she stays with the productivity y firm and keeps wage w.
   b. If $V_e(w, y) < V_e(y', y')$, the productivity y firm can retain its employee by offering her a wage $\omega(y', y) > w$ such that $V_e[\omega(y, y'), y] = V_e(y', y')$. Thus the worker obtains a wage rise without changing firms.

Thus, in the sequential auction model, the offers of competing employers permit wage earners to obtain wage rises without changing firms. Furthermore, in this model workers can change firms while accepting drops in their wage. A worker paid at wage w can move from a productivity y firm to a productivity $y' > y$ firm where the wage is $\omega(y, y') < w$ since the higher productivity of the new firm may allow her to obtain larger wage rises in the future. With this in mind, the sequential auction model permits us to explain why a nonnegligible percentage of wage earners (from 20% to 35% depending on the country in the data of Jolivet et al., 2006) do change jobs with no interval of unemployment, but they do so at a reduced wage.

Cahuc et al. (2006) have introduced bargaining into the sequential auction model by making the assumption that workers can counter the offers of firms with propositions of their own. Bargaining gives workers the possibility to obtain higher wages than they can in the base model of sequential auction.[25] Estimation of the model of Cahuc et al. (2006) on French data for the period 1993–2000 indicates that low-skilled workers have very weak bargaining power, not significantly differing from zero, whereas more highly skilled workers have positive bargaining power.

[25] Bargaining models are presented in chapter 7.

By giving more market power to wage earners than the wage-posting model gives, sequential auction models with bargaining allow us to better replicate the empirical distribution of wages than the wage-posting model. These models also generate a wider dispersion of wages than the wage-posting model, to the extent that wages are heterogeneous within each firm. Papp (2013) has shown that the model of Cahuc et al. (2006) predicts values of the mean-min wage ratio that are compatible with empirically derived values, which lie between 1.5 and 2. Overall then, on-the-job search models with sequential auction and bargaining have the capacity to produce good empirical predictions.

An operational description of the labor market would also require that parameters λ_u, λ_e, and q describing a worker's transitions between different possible states be made endogenous. In particular, the job offers arrival rates depend on the number of vacant jobs and the number of job seekers—quantities that derive from the behavior of firms and the way in which wages are set. The job destruction rate q is in all likelihood influenced by variations in productivity and by the way wages are set. The matching models, which we develop in chapter 9, partly fill these gaps.

5 SUMMARY AND CONCLUSION

- Job search theory assumes that individuals know only the distribution of wages existing in the economy and that they must search in order to encounter employers who will make them definite wage offers. The optimal strategy for a job seeker consists of accepting any wage offer higher than his or her *reservation wage*. The latter depends on the set of parameters affecting the labor market, in particular the job destruction rate, the arrival rate of job offers, and unemployment insurance benefits.
- To get unemployment insurance benefits, one must in general have worked previously and contributed to an unemployment insurance fund for a specified period. One is then eligible for unemployment insurance benefits. A rise in the level of benefits increases the duration of unemployment for eligible job seekers but diminishes that of ineligible job seekers.
- Empirical studies of the determinants of the exit rates from unemployment generally utilize duration models, which explain the amount of time passed in a certain state—for example the length of unemployment spells—as a function of institutional data and the characteristics of a sample of individuals followed over a certain period. The estimation of these models poses problems linked in particular to the specification of the functions defining the exit rates from unemployment and the existence of censored data.
- Empirical studies show that the reservation wage and the average length of an unemployment spell are sensitive to the amount of unemployment insurance benefits. The elasticity of the duration of unemployment with respect to the replacement ratio varies between 0.4 and 1.6. An increase in the potential duration of benefits increases the effective duration of an unemployment spell by around 20% of the increase in the potential duration. Much of the effect exerted by an increase in the period of benefit payment is due to easing of liquidity constraints.

- Unemployment insurance influences the arrival rates of job offers by its effect upon the intensity and efficiency of the job search carried out by job seekers receiving benefits.
- Providing that the data are disaggregated to a sufficient degree, unobserved individual heterogeneity explains the core of interindustry wage differences. Yet the share of wage differences explained by the heterogeneity of firms remains substantial. In France, for example, the average of the differences in wage paid to an identical worker employed in two different firms lies in the 20% to 30% range but does not exceed the 2% to 3% range from one industry to another.
- Because it integrates the strategic behavior of firms, the equilibrium search model is characterized by an endogenous distribution of wages. It offers the advantage of explaining the wage-setting process and thus making possible the analysis of the overall effects of economic policy. The equilibrium search model also highlights the importance of taking into account on-the-job search with sequential auctions and bargaining when it comes to explaining the wage distribution. This model explains why the wage of an individual employee can increase or decrease when she moves from one job to another, why wages rise, on average, as workers gain experience, and why large firms pay higher wages than small firms to identical workers.

6 RELATED TOPICS IN THE BOOK

- Chapter 1, section 1: The reservation wage and the choice between consumption and leisure
- Chapter 3, section 2: Compensating wage differentials and the hedonic theory of wages
- Chapter 8, section 2: Theories of discrimination
- Chapter 9, section 3: The matching model
- Chapter 10, section 1: Technological progress and unemployment
- Chapter 12, section 2.2.2: Minimum wage in labor market with frictions
- Chapter 13, section 1: Unemployment insurance
- Chapter 14, section 2.1: Manpower placement services
- Chapter 14, section 3: Evaluation of labor market policies
- Chapter 14, section 4.2: Job search assistance and monitoring

7 FURTHER READINGS

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Lalive, R., van Ours, J., & Zweimüller, J. (2006). How changes in financial incentives affect the duration of unemployment. *Review of Economic Studies*, *73*, 1009–1038.