Monetary Economics

Chapter 1: The Basic New Keynesian Model

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Objective of the chapter

- The chapter presents the **basic NK model** and derives its implications regarding the role of expectations in the transmission of MP.
- As explained in the General Introduction, the motivation for considering this model is threefold:
  1. like RBC models, it is a (DS)GE model, so that
     - it is not subject – or little sensitive – to Lucas' (1976) critique,
     - it provides a welfare criterion to assess the desirability of policies,
  2. unlike RBC models, it provides an active role for MP, due to
     - the **inefficiency** of economic fluctuations,
     - the **non-neutrality** of MP in the short term,
  3. it is simple.
- This model will be used in Chapters 2, 3, and 6.

From the standard RBC model to the basic NK model

- As also explained in the General Introduction, the basic NK model corresponds to the standard RBC model in which
  - there is no endogenous capital accumulation (for simplicity),
  - there is **monopolistic competition** in the goods market, so that firms are price-makers (not price-takers),
  - there is **price stickiness** in the goods market, so that
    - economic fluctuations are inefficient,
    - MP is non-neutral in the short term (due to its effects on real money balances and the short-term real interest rate).
- In particular, it is a GE model, so that its equilibrium conditions are
  - the first-order conditions of the private agents' optimization problems,
  - the constraints of these problems,
  - market-clearing conditions.

Which version of the basic NK model? I

- There are different versions of the basic NK model, depending on
  1. the nature of shocks,
  2. the degree of steady-state inefficiency,
  3. the nature of price stickiness,
  4. the role of money,
  5. the nature of labor.

- **Shocks** $\equiv$ stochastic exogenous variables (normalized to have a zero mean).

- **Steady state** $\equiv$ equilibrium in the absence of shocks.
  Typically endogenous variables are constant over time

Which version of the basic NK model? II

1. For simplicity, I consider only **two shocks**:
   - technology shocks,
   - shocks to the elasticity of substitution between differentiated goods.

$\hookrightarrow$ Other shocks could be considered, whose normative implications would be qualitatively similar to those of technology shocks:

- consumption-utility shocks,
- labor-disutility shocks,
- government-expenditures shocks,
- distortive-tax shocks.

Which version of the basic NK model? III

2. I assume that there is a fiscal authority that offers an employment subsidy (financed by lump-sum taxes), so as to partially or completely offset the monopolistic-competition distortion. Monop-competiton > distortions = not at steady state
ex: too little labour employed, too little goods produced

   ↪ This assumption enables me to consider alternative degrees of **steady-state inefficiency**. Emp. subsidy = different degrees of inefficiency :
   +subsd = - inefficiency as firms employ more / produce more

3. I consider **price stickiness**
   - **à la Calvo (1983)** (at each date, some randomly chosen firms are allowed to change their prices),
   - not à la Rotemberg (1982) (at each date, each firm faces a resource cost in changing its price, which is quadratic in the price-change size).

   ↪ When the steady-state distortion is small, these two alternative ways of modeling price stickiness have qualitatively similar positive and normative implications.

Which version of the basic NK model? IV

4. I first consider a **cashless model**, in which money implicitly serves only as a unit of account.

   $\hookrightarrow$ I show at the end of the chapter how to introduce money explicitly into this model.

5. I assume that there is a **single kind of labor** as in Galí (2015, Chapter 3), not several as in Woodford (2003, Chapter 4).

   $\hookrightarrow$ These two alternative assumptions have qualitatively similar positive and normative implications.

Time and agents

- Time is discrete, indexed by t, and the horizon is infinite.
- There are **four kinds of agents**:
  - a large number of households,
  - a large number of firms,
  - a single monetary authority,
  - a single fiscal authority. Providing emp. subsidy to mitigate steady-state inefficiency = degree of inef.
- All of them are infinitely lived.
- All households are identical, so that there is a representative household (RH).
- The RH is the only agent to have a utility function, so that the basic NK model is a **representative-agent model**. Natural goal of MP maker = maximise Utilise of RH

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Markets

- A continuum of **monopolistically** competitive **goods** markets:
  - demand from households,
  - supply from firms.
- A **perfectly** competitive **labor** market:
  - demand from firms,
  - supply from households.
- A **perfectly** competitive **one-period-bond** market:
  - demand from households,
  - supply from households.

Outline of the chapter

- Introduction
- Households' behavior
- Firms' behavior
- Equilibrium conditions
- Role of the private sector (PS)'s expectations
- MP instrument

  Interest rate or supply of money best instrument ?

RH's intertemporal utility

- RH's **intertemporal utility** function at date 0 is

$$\mathcal{U}_0 \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U\left(C_t, N_t\right) \right\}, \text{ with } C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}},$$

where $\beta \in]0,1[$ is the discount factor, U is continuous and twice differentiable, and, for each date t,

  - C_t is the consumption index,
  - $C_t(i)$ is the quantity of good i consumed by RH,
  - $\varepsilon_t > 1$ is the elasticity of substitution between differentiated goods,
  - N_t is the number of hours worked by RH,
  - $U_{c,t} > 0$, $U_{cc,t} < 0$, $U_{n,t} < 0$, $U_{nn,t} < 0$.

RH's optimization problem in words

- RH chooses how much
  - of each good to consume,
  - labor to supply,
  - bonds to hold,

  in order to maximize
  - her intertemporal utility function,

  subject to
  - her intertemporal budget constraint,

  taking as given
  - the price of each good,
  - the wage,
  - the price of bonds

  (given the market structure and the large number of households).

RH's optimization problem formalized

$$\underset{\substack{[C_t(i)]_{i\in[0,1],t\in\mathbb{N}},\\ (N_t)_{t\in\mathbb{N}},(B_t)_{t\in\mathbb{N}}}}{Max} \quad \mathbb{E}_0\left\{\sum_{t=0}^{+\infty}\beta^t U\left(C_t,N_t\right)\right\}$$

subject to

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon_t-1}{\varepsilon_t}}di\right]^{\frac{\varepsilon_t}{\varepsilon_t-1}} \quad \text{and}$$

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + W_tN_t + T_t \quad \text{for} \quad t\in\mathbb{N},$$

taking as given

$$B_{-1},\ \left[P_t(i)\right]_{0\leq i\leq 1,t\in\mathbb{N}},\ \left(Q_t\right)_{t\in\mathbb{N}},\ \left(W_t\right)_{t\in\mathbb{N}}, \text{ and } \left(T_t\right)_{t\in\mathbb{N}}.$$

Notations and resolution

- For each date t,
  - $P_t(i)$ is the price of good i,
  - Q_t is the price of one-period nominal bonds (paying one unit of money at maturity),
  - B_t is the quantity of one-period nominal bonds held by RH,
  - W_t is the nominal wage,
  - T_t is a lump-sum component of income.

- RH's optimization problem can be solved in **two steps**:
  1. for any given consumption index C_t, characterize RH's choice of the distribution of consumption across goods $\left[C_t(i)\right]_{0 \leq i \leq 1}$,
  2. characterize RH's choice of the consumption index C_t and the number of hours worked N_t.

Distribution of consumption across goods I

- Dual optimization problem:

$$\underset{[C_t(i)]_{0\leq i\leq 1}}{Max}\left[\int_0^1 C_t(i)^{\frac{\varepsilon_t-1}{\varepsilon_t}}di\right]^{\frac{\varepsilon_t}{\varepsilon_t-1}}$$

subject to $\int_0^1 P_t(i)C_t(i)di = Z_t$.

- Lagrangian:

$$L=\left[\int_0^1 C_t(i)^{\frac{\varepsilon_t-1}{\varepsilon_t}}di\right]^{\frac{\varepsilon_t}{\varepsilon_t-1}}-\lambda\left[\int_0^1 P_t(i)C_t(i)di-Z_t\right].$$

- First-order conditions (FOCs): $C_t(i)^{\frac{-1}{\varepsilon_t}}C_t^{\frac{1}{\varepsilon_t}}=\lambda P_t(i)$ for all $i\in[0,1]$.

Distribution of consumption across goods II

- Using these FOCs to replace $C_t(i)$ in the definition of C_t gives $\lambda = P_t^{-1}$ where
$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon_t} di \right]^{\frac{1}{1-\varepsilon_t}}$$
is the aggregate price index.

- Replacing λ by P_t^{-1} in the FOCs gives the **demand schedule**
$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon_t} C_t \quad \text{for all} \quad i \in [0,1].$$

- The limit case $\varepsilon_t \longrightarrow +\infty$ corresponds to perfect competition.

Consumption index and number of hours worked I

- Replacing $C_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon_t}}$ by $C_t(i) P_t(i) C_t^{\frac{-1}{\varepsilon_t}} P_t^{-1}$ in the definition of C_t gives

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t.$$

- Therefore, the second step of RH's optimization problem can be rewritten as

$$\underset{(C_t)_{t\in\mathbb{N}},(N_t)_{t\in\mathbb{N}},(B_t)_{t\in\mathbb{N}}}{Max} \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U\left(C_t, N_t\right) \right\}$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \quad \text{for} \quad t \in \mathbb{N},$$

taking as given

$$B_{-1},\ \left(P_t\right)_{t\in\mathbb{N}},\ \left(Q_t\right)_{t\in\mathbb{N}},\ \left(W_t\right)_{t\in\mathbb{N}}, \text{ and } \left(T_t\right)_{t\in\mathbb{N}}.$$

Consumption index and number of hours worked II

- The FOCs of this problem are, for all $t \in \mathbb{N}$,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \quad \text{(labor-consumption trade-off condition),}$$

$$Q_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \quad \text{(Euler equation).}$$

- Interpretation: at the optimal plan, it must be the case that
  - $U_{c,t} dC_t + U_{n,t} dN_t = 0$ for any pair (dC_t, dN_t) satisfying the budget constraint $P_t dC_t = W_t dN_t$,
  - $U_{c,t} dC_t + \beta \mathbb{E}_t \{U_{c,t+1} dC_{t+1}\} = 0$ for any pair (dC_t, dC_{t+1}) satisfying the budget constraint $P_{t+1} dC_{t+1} = -\frac{P_t}{Q_t} dC_t$.

Consumption index and number of hours worked III

- Specific functional-form assumption for the period utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where $\sigma > 0$ and $\varphi > 0$.

- The previous FOCs then become

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \quad \text{(labor-consumption trade-off condition)},$$

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad \text{(Euler equation)}.$$

Consumption index and number of hours worked IV

- The labor-consumption trade-off condition can be rewritten in log-linear form as
$$w_t - p_t = \sigma c_t + \varphi n_t,$$
where, for any variable Z_t, $z_t \equiv \log Z_t$.

- A log-linear approximation of the Euler equation around a steady state with a zero inflation rate and a constant consumption level is
$$c_t = \mathbb{E}_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - \bar{i}\right),$$
where $i_t \equiv -\log Q_t$ is the log of the gross yield on one-period nominal bonds (referred to as the short-term **nominal interest rate**), $\bar{i} \equiv -\log \beta$, and $\pi_t \equiv p_t - p_{t-1}$ is the **inflation rate**.

Production function and price rigidity

- There is a continuum of firms indexed by $i \in [0,1]$, each firm producing a differentiated good.
- All firms use the same technology, represented by the **production function**

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where $\alpha \in]0,1[$ and A_t is a stochastic exogenous factor.

- As in Calvo (1983), at each date, each firm may reset its price only with probability $1-\theta$ (independent of the time elapsed since the last adjustment), where $\theta \in [0,1]$, so that
  - at each date, a measure $1-\theta$ of firms reset their prices,
  - at each date, a measure θ of firms keep their prices unchanged,
  - the average duration of a price is $(1-\theta)^{-1}$,
  - θ is a natural index of price stickiness.

Aggregate price level dynamics

- At each date t, all firms resetting their prices will choose the same price noted P_t^* because they face the same problem.

- Therefore, $P_t = \left[\theta \left(P_{t-1}\right)^{1-\varepsilon_t} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon_t}\right]^{\frac{1}{1-\varepsilon_t}}$.

- Dividing by P_{t-1}, one gets $\Pi_t^{1-\varepsilon_t} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon_t}$, where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

- Log-linearization around a steady state with $\Pi_t = 1$ yields

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}).$$

Firms' optimization problem

- A firm re-optimizing at date t will choose the price P_t^* that maximizes the current market value of the profits generated while this price remains effective:

Proba that firm is stuck with Pt* at time t+k

$$\underset{P_t^*}{Max} \sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k} \left[P_t^* Y_{t+k|t} - \Psi_{t+k}\left(Y_{t+k|t}\right) \right] \right\},$$

Expected profits in euro at date t

Euros at date t !

In Euros at date t+k

where

- $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor for nominal payoffs between t and $t+k$,
- $Y_{t+k|t}$ is output at $t+k$ for a firm that last reset its price at t,
- $\Psi_t(.)$ is the nominal cost function at t,

subject to $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_{t+k}} C_{t+k}$ for $k \in \mathbb{N}$,

Market clearing conditions =>
Demand faced = Demand schedule of RH
= nb of goods sold

taking $(C_{t+k})_{k\in\mathbb{N}}$ and $(P_{t+k})_{k\in\mathbb{N}}$ as given.

Aggregates taken as given, single firm is atomistic

FOC of this problem

- The FOC of this problem (henceforth "firms' FOC") is

$$\sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\varepsilon_{t+k} - 1\right) \left(P_t^* - \mathcal{M}_{t+k} \psi_{t+k|t} \right) \right\} = 0,$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the nominal marginal cost at $t+k$ for a firm that last reset its price at t, and $\mathcal{M}_{t+k} \equiv \frac{\varepsilon_{t+k}}{\varepsilon_{t+k}-1}$.

- Under flexible prices, this FOC collapses to $P_t^* = \mathcal{M}_t \psi_{t|t}$, so that $\mathcal{M}_t$ is the **"desired" (or frictionless) markup**.

- Dividing by P_{t-1}, one gets

$$\sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\varepsilon_{t+k} - 1\right) \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M}_{t+k} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0,$$

growth inflation rate t-1 -> t+k

where $\Pi_{t-1,t+k} \equiv \frac{P_{t+k}}{P_{t-1}}$ and $MC_{t+k|t} \equiv \frac{\psi_{t+k|t}}{P_{t+k}}$ is the real marginal cost at $t+k$ for a firm whose price was last set at t.

Zero-inflation-rate steady state

- At the **zero-inflation-rate steady state** (ZIRSS),
  - P_t^* and P_t are equal to each other and constant over time,
  - therefore, all firms produce the same quantity of output,
  - this quantity is constant over time, as the model features no deterministic trend,
  - therefore, $\frac{P_t^*}{P_{t-1}} = 1$, $\Pi_{t-1,t+k} = 1$, $\mathcal{M}_{t+k} = \mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$, $Q_{t,t+k} = \beta^k$, and we note $Y_{t+k|t} = Y$ and $MC_{t+k|t} = MC$,
  - firms' FOC then implies $MC = \frac{1}{\mathcal{M}}$. (FOC of previous slide is general so in particular holds at the SS)

Assume the economy fluctuates around the ZIRSS for the first order linearization to be valid

Log-linearization of firms' FOC

- Log-linearization of firms' FOC around the ZIRSS yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\left\{\mu_{t+k} + mc_{t+k|t} + (p_{t+k} - p_{t-1})\right\},$$

where $\mu_{t+k} \equiv \log \mathcal{M}_{t+k}$.

- This equation can be rewritten as

real marginal cost (in log term)

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\left\{\mu_{t+k} + mc_{t+k|t} + p_{t+k}\right\}.$$

Takes a weighted average of ideal prices for next periods !
Takes a weighted average of the best price for each next period
bc it knows it can be stuck

log of desired markup

log price level

sum of the two is log of nominal marginal cost !

In level : desired markup*nominal marginal cost

- Hence, firms resetting their prices choose a price that corresponds to a weighted average of their current and expected future desired markups over their nominal marginal costs.

Setting theta = 0 (perfectly flexible prices) we only have k=0 in the sum that is not 0
Thus pt* = desired markup*nominal marginal cost which we knew

Market-clearing conditions

- Market clearing in the **goods markets** requires, for all i and t,

$$Y_t(i) = C_t(i).$$

- Therefore, $Y_t = C_t$, where $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} di\right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$.

- Market clearing in the **labor market** requires, for all t,

$$N_t = \int_0^1 N_t(i) di.$$

Aggregate production function

- Using the market-clearing conditions, the production function, and the demand schedule, one gets

$$N_t = \int_0^1 \left[\frac{Y_t(i)}{A_t}\right]^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{\frac{-\varepsilon_t}{1-\alpha}} di,$$

and therefore the **aggregate production function**

$$y_t = (1-\alpha)n_t + a_t - d_t,$$

where $d_t \equiv (1-\alpha)\log\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{\frac{-\varepsilon_t}{1-\alpha}} di$ is a measure of price (and, hence, output) dispersion across firms.

Average real marginal cost

- In the neighborhood of the ZIRSS, d_t is equal to zero up to a first-order approximation, so that, at the first order,

$$y_t = (1-\alpha)n_t + a_t$$

(the proof is postponed to Chapter 2).

- Noting mc_t the **average real marginal cost** at t, mpn_t the average marginal product of labor at t, and τ the constant employment subsidy, and using $y_t = (1-\alpha)n_t + a_t$, one gets

$$\begin{aligned} mc_t &= \log(1-\tau) + (w_t - p_t) - mpn_t \\ &= \log(1-\tau) + (w_t - p_t) - (a_t - \alpha n_t) - \log(1-\alpha) \\ &= \log(1-\tau) + (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha). \end{aligned}$$

Real marginal cost

- For any firm-level variable z, let $z_{t+k|t}$ denote the value of z at $t+k$ for a firm that last reset its price at t.

- Using the demand schedule and the goods-market clearing condition, one gets, at the first order, the **real marginal cost**

$$\begin{aligned}
mc_{t+k|t} &= \log(1-\tau) + (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \\
&= \log(1-\tau) + (w_{t+k} - p_{t+k}) - \frac{a_{t+k} - \alpha y_{t+k|t}}{1-\alpha} - \log(1-\alpha) \\
&= mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\
&= mc_{t+k} - \frac{\alpha\varepsilon_{t+k}}{1-\alpha}(p_t^* - p_{t+k}) = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}).
\end{aligned}$$

- Under constant returns to scale ($\alpha = 0$), one has $mc_{t+k|t} = mc_{t+k}$: the real marginal cost is independent of the output level and, hence, is common across firms.

Rewriting firms' FOC

- Using the previous result, firms' FOC can be rewritten as $p_t^* - p_{t-1}$

$$\begin{aligned} &= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\left\{\Theta\left(\mu_{t+k}+mc_{t+k}\right)+\left(p_{t+k}-p_{t-1}\right)\right\} \\ &= (1-\beta\theta)\Theta\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\left\{\mu_{t+k}+mc_{t+k}\right\}+\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\left\{\pi_{t+k}\right\} \\ &= \beta\theta\mathbb{E}_t\left\{p_{t+1}^*-p_t\right\}+(1-\beta\theta)\Theta\left(\mu_t+mc_t\right)+\pi_t, \end{aligned}$$

  where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$.

- Using the aggregate price level dynamics equation, one then gets

$$\pi_t = \beta\mathbb{E}_t\left\{\pi_{t+1}\right\}+\chi\left(\mu_t+mc_t\right),$$

  where $\chi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$.

Natural level of output

- Independently of the nature of price setting, the average real marginal cost can be rewritten, at the first order, as

$$\begin{aligned} mc_t &= \log(1-\tau) + (w_t - p_t) - mpn_t \\ &= \log(1-\tau) + (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \log(1-\tau) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha), \end{aligned}$$

  using the labor-consumption trade-off condition, the goods-market-clearing condition, and the (approximate) aggregate production function.

- Now, firms' FOC implies that, under flexible prices, $mc_t = -\mu_t$.

- Therefore, the **natural level of output**, defined as the equilibrium level of output under flexible prices and noted y_t^n, is such that

$$-\mu_t = \log(1-\tau) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha).$$

Output gap and New Keynesian Phillips curve

- Therefore, the natural level of output is

$$y_t^n = \frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}\left[\log\left(\frac{1-\alpha}{1-\tau}\right)+\frac{1+\varphi}{1-\alpha}a_t-\mu_t\right].$$

- The natural level of output does not depend on i_t, i.e. **MP is neutral under flexible prices**.

- Subtracting the two equations on the previous slide, one gets $mc_t + \mu_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\widetilde{y}_t$, where $\widetilde{y}_t \equiv y_t - y_t^n$ is called the **output gap**.

- Rewriting firms' FOC, one gets the **New Keynesian Phillips curve** (NKPC)

$$\pi_t = \beta \mathbb{E}_t\left\{\pi_{t+1}\right\} + \kappa \widetilde{y}_t,$$

where $\kappa \equiv \chi\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

Interpretation of the NKPC

- The NKPC is **forward-looking** because, when a firm resets its price, it knows that it will not be able to change this price for some (random) time.
- Therefore, the current inflation rate depends on
  - the current situation (term $\kappa \widetilde{y}_t$),
  - the expected future situation (term $\beta \mathbb{E}_t \{\pi_{t+1}\}$).
- The slope κ of the Phillips curve is decreasing in θ and β: the stickier the prices or the higher the discount factor, the less prices react to the current situation.
- As prices become flexible ($\theta \longrightarrow 0$), the NKPC becomes $y_t = y_t^n$.

IS equation

- Using the goods-market-clearing condition and the definition of the output gap, one can rewrite the Euler equation as the **IS equation**

$$\widetilde{y}_t = \mathbb{E}_t \{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \{\pi_{t+1}\} - r_t^n\right),$$

where

$$\begin{aligned} r_t^n &\equiv \bar{i} + \sigma \mathbb{E}_t\{\Delta y_{t+1}^n\} \\ &= \bar{i} + \frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}\left(\frac{1+\varphi}{1-\alpha}\mathbb{E}_t\{\Delta a_{t+1}\} - \mathbb{E}_t\{\Delta \mu_{t+1}\}\right) \end{aligned}$$

is the **natural rate of interest** (unique equilibrium value of the ex ante short-term real interest rate $i_t - \mathbb{E}_t\{\pi_{t+1}\}$ consistent with the output level being constantly equal to its natural level).

Set of equilibrium conditions

- Given $(a_t, \mu_t, i_t)_{t\in\mathbb{N}}$, $(\widetilde{y}_t, \pi_t)_{t\in\mathbb{N}}$ is determined by
  - the IS equation $\widetilde{y}_t = \mathbb{E}_t\left\{\widetilde{y}_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - r_t^n\right)$,
  - the NKPC $\pi_t = \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + \kappa\widetilde{y}_t$,

  for $t \in \mathbb{N}$, which implies that **MP is not neutral** (unless $\theta \longrightarrow 0$).

- Given $(a_t, \mu_t, i_t, \widetilde{y}_t, \pi_t)_{t\in\mathbb{N}}$, $(y_t, c_t, n_t, w_t - p_t)_{t\in\mathbb{N}}$ is determined by
  - the definition of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$,
  - the goods-market-clearing condition $c_t = y_t$,
  - the aggregate production function $y_t = (1-\alpha)n_t + a_t$,
  - the labor-consumption trade-off condition $w_t - p_t = \sigma c_t + \varphi n_t$,

  for $t \in \mathbb{N}$.

Role of PS's expectations I

- Provided that $\lim_{k \longrightarrow +\infty} \mathbb{E}_t \{\widetilde{y}_{t+k}\} = 0$, iterating the IS equation forward yields

$$\widetilde{y}_t = -\frac{1}{\sigma}\mathbb{E}_t \left\{\sum_{k=0}^{+\infty} \left(r_{t+k} - r_{t+k}^n\right)\right\},$$

  where $r_t \equiv i_t - \mathbb{E}_t \{\pi_{t+1}\}$ is the ex ante **short-term** real interest rate.

- Using a no-arbitrage condition, one can interpret $\mathbb{E}_t \left\{\sum_{k=0}^{+\infty} r_{t+k}\right\}$ as the ex ante **long-term** real interest rate.

- Therefore, the output gap depends on PS's expectations of the future path of the short-term interest rate (through the long-term interest rate).

Role of PS's expectations II

- Provided that $\lim_{k \longrightarrow +\infty} \mathbb{E}_t \{\pi_{t+k}\} = 0$, iterating the NKPC forward yields

$$\pi_t = \kappa \mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \beta^k \widetilde{y}_{t+k} \right\}.$$

- Therefore, the inflation rate also depends on PS's expectations of the future path of the short-term interest rate.

- So MP affects the economy not only through changes in the current short-term interest rate, but also through changes in **PS's expectations** of the future path of the short-term interest rate.

Role of PS's expectations III

- As an illustration, assume for simplicity that CB controls directly r_t and follows the rule $r_t = \gamma r_{t-1} + \xi_t$, where $\gamma \in]0,1[$ and ξ_t is an i.i.d. MP shock occurring at date t.

- Then $\mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} r_{t+k} \right\} = \frac{r_t}{(1-\gamma)}$, $\widetilde{y}_t = \frac{-r_t}{\sigma(1-\gamma)} + f\left[\left(\mathbb{E}_t \{ r_{t+k}^n \} \right)_{k \in \mathbb{N}} \right]$ and $\pi_t = \frac{-\kappa r_t}{\sigma(1-\gamma)(1-\beta\gamma)} + g\left[\left(\mathbb{E}_t \{ r_{t+k}^n \} \right)_{k \in \mathbb{N}} \right]$, where f and g are linear functions.

- Therefore, the more persistent the short-term interest rate (the closer to 1 is γ), the larger the effect of the MP shock on the long-term interest rate, the output gap, and the inflation rate.

In Woodford's (2003) words I

"Successful monetary policy is not so much a matter of effective control of overnight interest rates as it is of shaping market expectations of the way in which interest rates, inflation, and income are likely to evolve over the coming year and later. (...) [O]ptimizing models imply that private sector behavior should be forward-looking; hence expectations about future market conditions should be important determinants of current behavior. It follows that, insofar as it is possible for the central bank to affect expectations, this should be an important tool of stabilization policy. (...) Not only do expectations about policy matter, but, at least under current conditions, very little else matters.

In Woodford's (2003) words II

[T]he current level of the overnight interest rates as such is of negligible importance for economic decisionmaking. The effectiveness of changes in central-bank targets for overnight rates in affecting spending decisions (and hence ultimately pricing and employment decisions) is wholly dependent upon the impact of such actions upon other financial-market prices, such as long-term interest rates, equity prices, and exchange rates. These are plausibly linked, through arbitrage relations, to the short-term interest rates most directly affected by central-bank actions. But it is the expected future path of short-terms rates over coming months and even years that should matter for the determination of these other asset prices, rather than the current level of short-term rates by itself. Thus the ability of central banks to influence expenditure, and hence pricing, decisions is critically dependent upon their ability to influence market expectations regarding the future path of overnight interest rates, and not merely their current level."

In Bernanke's (2004b) words I

"Informal discussions of monetary policy sometimes refer to the Fed as 'setting interest rates.' In fact, the FOMC does not set interest rates in general; rather, the Committee 'sets' one specific interest rate, the federal funds rate. The federal funds rate, the interest rate at which commercial banks borrow and lend to each other on a short-term basis (usually overnight) is not important in itself. Only a tiny fraction of aggregate borrowing and lending is done at that rate. From a macroeconomic perspective, longer-term interest rates–such as home mortgage rates, corporate bond rates, and the rates on Treasury notes and bonds–are far more significant than the funds rate, because those rates are the most relevant to the spending and investment decisions made by households and businesses. These longer-term rates are determined not by the Fed but by participants in deep and sophisticated global financial markets.

In Bernanke's (2004b) words II

Although the FOMC cannot directly determine long-term interest rates, it can exert significant influence over those rates through its control of current and future values of the federal funds rate. The crucial link between the federal funds rate and longer-term interest rates is the formation of private-sector expectations about future monetary policy actions. Loosely speaking, long-term interest rates embody the expectations of financial-market participants about the likely future path of short-term rates, which in turn are closely tied to expectations about the federal funds rate. Thus, to influence long-term interest rates, such as thirty-year mortgage rates or the yields on corporate bonds, the FOMC must influence private-sector expectations about future values of the federal funds rate. The Committee can do this by its communication policies, by establishing certain patterns of behavior, or both."

In European central bankers' words

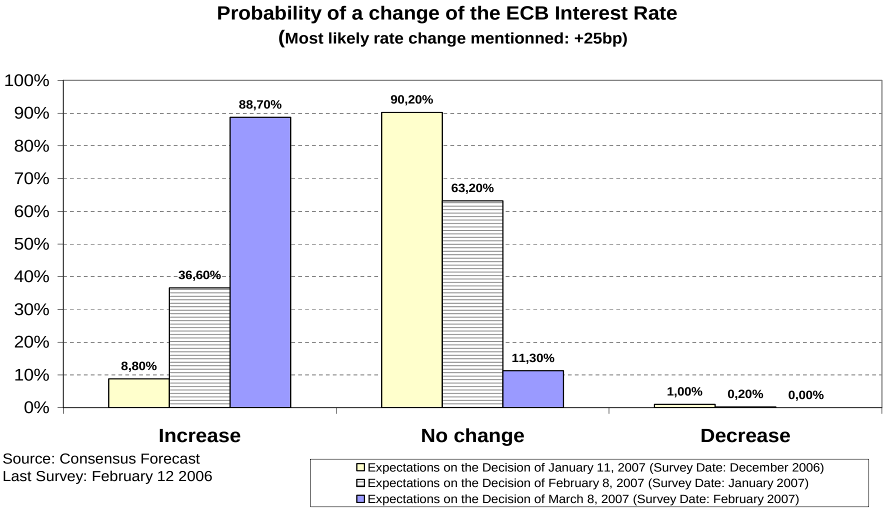
González-Páramo (2007): *"expectations play an important role in the transmission of monetary policy. Consider, for instance, the term structure of interest rates. Central banks have a direct influence only on short-term interest rates through their monetary policy instruments – typically an overnight call rate. However, consumption and investment decisions, and thus medium-term price developments, are to a large extent influenced by longer-term interest rates, which in turn depend on private sector expectations regarding future central bank decisions and future inflation."*

Trichet (2008): *"Through their actions, central banks can directly control very short-term interest rates. However, given that for consumption and investment decisions the longer-term interest rates are more relevant, the whole yield curve is relevant for the effectiveness of monetary policy. Medium- and long-term interest rates largely depend on private expectations regarding future central bank decisions."*

Very short-term predictability

- CBs monitor markets' expectations of their next policy-rate decision.
- Sometimes, they communicate about this decision to ensure that markets' expectations are aligned with their intentions.
- For instance, the ECB has used the code words "vigilance" and "strong vigilance" in its communication between 2005 and 2011 to signal a likely policy-rate hike two months ahead and one month ahead respectively (presumably as a trade-off between commitment and predictability).

Expectations of the next monetary-policy decision

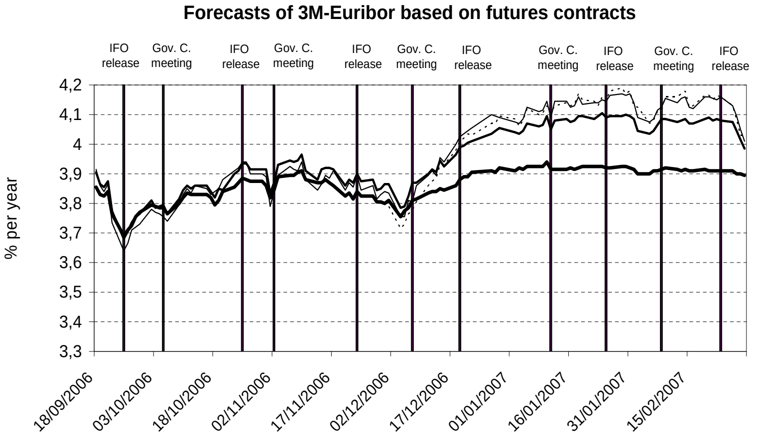


Short- to medium-term predictability

- However, as we have just seen, what matters for the economy is much less the expectation of next month's policy rate than the expectation of the whole future trajectory of the policy rate.

- This is why CBs also monitor markets' expectations of their subsequent policy-rate decisions, and sometimes communicate about these decisions to ensure that markets' expectations are aligned with their intentions.

Expectations of up to one-year-ahead policy-rate decisions



Source: BSME.
Production: POMONE.
Last update: 02/03/2007.

In Bernanke's (2004a) words

"The fact that market expectations of future settings of the federal funds rate are at least as important as the current value of the funds rate in determining key interest rates such as bond and mortgage rates suggests a potentially important role for central bank communication: If effective communication can help financial markets develop more accurate expectations of the likely future course of the funds rate, policy will be more effective (...).

It is worth emphasizing that the predictability of monetary policy actions has both short-run and long-run aspects. A central bank may, through various means, improve the market's ability to anticipate its next policy move. Improving short-term predictability is not unimportant, because it may reduce risk premiums in asset markets and influence shorter-term yields. But signaling the likely action at the next meeting is not sufficient for effective policymaking. Because the values of long-term assets are affected by the whole trajectory of expected short-term rates, it is even more vital that information relevant to estimating that trajectory be communicated."

Two possible MP instruments I

- CBs have the monopoly of issuing bank notes and bank reserves.
- Therefore, they control the monetary base (i.e. the supply of money).
- In the basic NK model, there are two alternative MP instruments: the **supply of money** and the **short-term nominal interest rate**.
- One of the simplest way to introduce money in this model is to make real money balances enter the utility function in a separable way.
- In this case, the corresponding FOC leads to the following log-linearized money-demand equation: $m_t^d - p_t = \frac{\sigma y_t - i_t}{\nu}$, where $\nu \equiv -\frac{U_{mm}M}{U_m P} > 0$.
- The money-market-clearing condition then gives $m_t^s - p_t = \frac{\sigma y_t - i_t}{\nu}$.

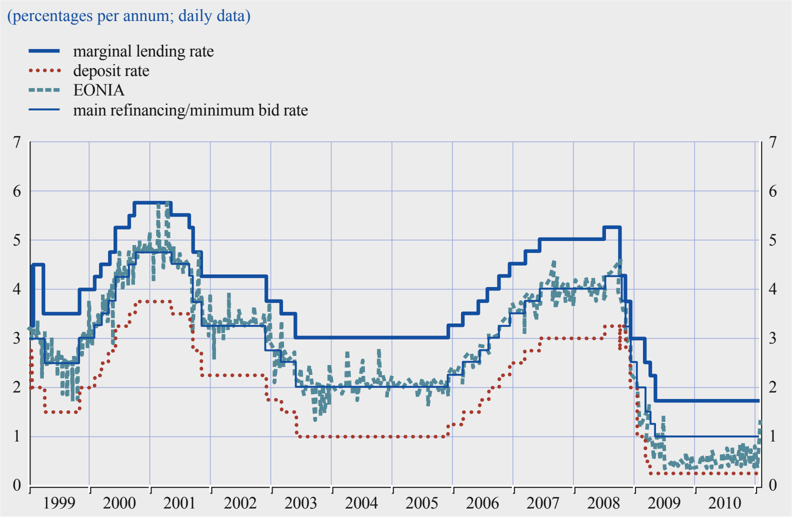
Two possible MP instruments II

- Money then plays a residual role, as it appears only in latter equation.
- If the instrument is m^s, then CB chooses m^s, and the price on the money market (interest rate i) adjusts so that $m^d(i) = m^s$.
- If the instrument is i, then CB chooses i and adjusts m^s so that $m^s = m^d(i)$.
- In the presence of money-demand shocks (i.e. shocks added to the money-demand equation) that CB does not observe in real time, i may be preferable to m^s as an instrument.
- This is because unlike m^s, i isolates the real variables from these shocks.

Two possible MP instruments III

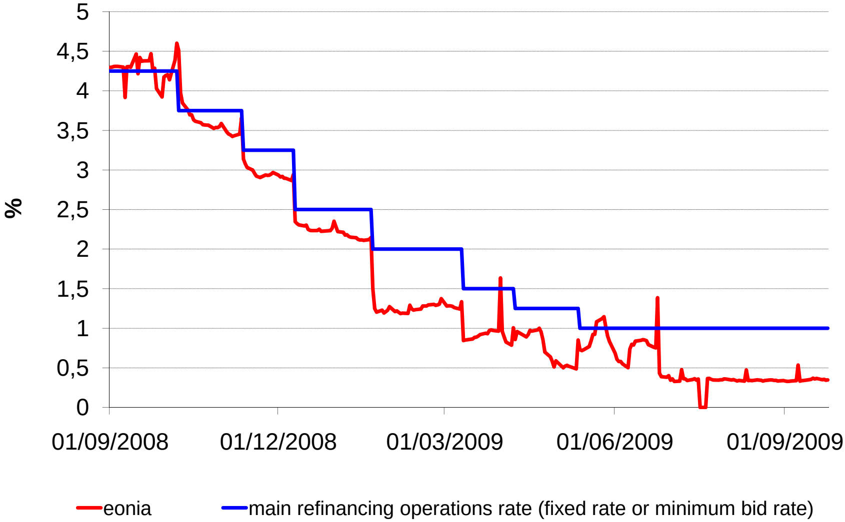
- In practice, before the 2008-2009 crisis (i.e. for conventional MP), the MP instrument was i:
  - a MP committee chose a value for i without considering the adjustment in m^s necessary for i to reach this value,
  - a specialized staff of the CB adjusted m^s for i to reach this value.
- The basic NK model is consistent with this practice.
- However, the MP instrument has not necessarily been i since the 2008-2009 crisis, as unconventional MP (quantitative or credit easing) has, on some occasions and in some places, pushed m^s higher than the value consistent with the market interest rate being close to the policy rate chosen.

ECB policy rates and EONIA



Source: ECB (2011).

ECB MRO rate and EONIA



Targeted and effective Fed Funds rates