# Labour supply
## AE318 - Labour Economics

Roland Rathelot

Classical theory of labour supply
   Static model of labour supply
   Policy: EITC in the US
   Dynamic model of labour supply

New empirical evidence on labour supply

Neoclassical vs. behavioural models of labour supply?

Labour rationing

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# The worker utility

Each worker chooses its level of consumption and leisure to maximizes its utility

- max $U(C, L)$
    - $C$: consumption
    - $L$: leisure hours
- Try plotting reasonable gradients, connect your plot to assumptions about first derivatives of $U$
- Define iso-utility curves as: $\forall \bar{U}, \{(C, L) \text{ s.t } . U(C, L) = \bar{U}\}$
- From equation $U(C, L) = \bar{U}$, define implicit function $C_{\bar{U}}(L)$
- Differentiating the equation: $U_C dC + U_L dL = 0$ so that:

$$C'_{\bar{U}}(L) = -\frac{U_L}{U_C}$$

# The worker problem

Each worker chooses its level of consumption and leisure to maximizes its utility
under a budget constraint

- max $U(C, L)$
  - $C$: consumption
  - $L$: leisure hours
- such that $pC = wH + Y$
  - $p$: unit price of consumption good
  - $w$: wage
  - $Y$: non-labour income
  - $H$: working hours
- Rewrite budget constraint as: $C + wL = Y + wT$
  - $T = L + H$ is the total time available (168 hours/week)
  - price $p$ is normalised to 1

# Resolution

▶ The Lagrangian of this program is:
$$\mathcal{L}(C, L, \lambda) = U(C, L) - \lambda[C + wL - Y - wT]$$

▶ First order conditions:

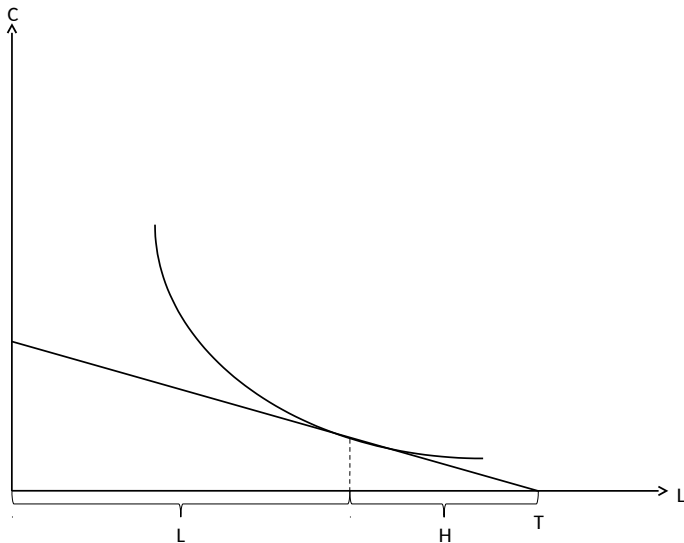$$\frac{\partial \mathcal{L}}{\partial C} = U_C - \lambda = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial L} = U_L - \lambda w = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = C + wL - Y - wT = 0 \tag{3}$$

▶ From (1) and (2): the equilibrium $(C^*, L^*)$ is such that the marginal rate of substitution equals the wage

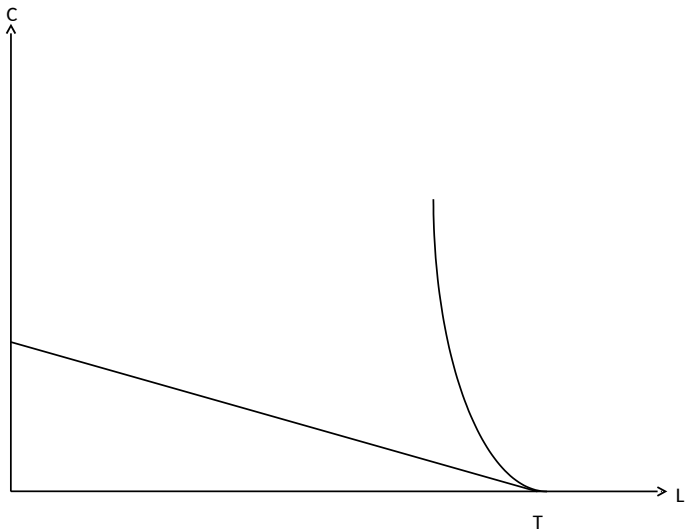$$\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w$$

# Optimal Labour Supply Decision

# Reservation wage

- ▶ The reservation wage is the lowest wage rate that leaves an individual indifferent between working and not working
- ▶ Marginal rate of substitution (slope of indifference curve) = Slope of budget constraint ($w$) at 0 working hours

# A change in unearned income $Y$

- ▶ How does an increase in unearned income $Y$ will change leisure hours $L$?
- ▶ Depends on the properties of the utility function
- ▶ With usual properties, an increase in unearned income $Y$ unambiguously increases leisure hours $L$ and reduced working hours: pure *income effect*

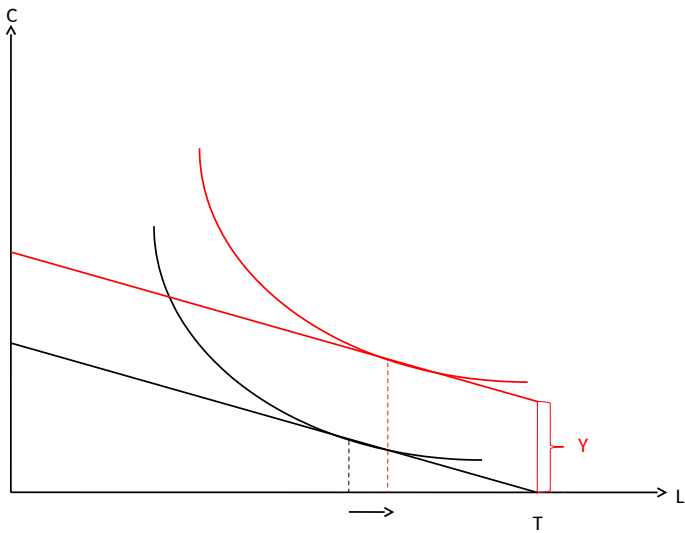# A change in unearned income $Y$: Details

- Denote: $R = Y + wT$
- We want to compute $\partial L/\partial R$ ($w$ being fixed)
- Start from the equilibrium equation, inserting the constraint:

$$U_L(R - wL, L) = wU_C(R - wL, L)$$

- Differentiate it wrt $L$ and $R$ and re-arrange the terms:

$$\frac{dL}{dR} = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

- If $U_{CC}, U_{LL} < 0$ and $U_{CL} > 0$, no ambiguity: $\frac{dL}{dR} > 0$

# An increase in the wage rate $w$

▶ An increase in the wage $w$ has an ambiguous effect on labour ~~supply.~~
  1. *Income Effect*: consume more leisure, reduce labour supply.
  2. *Substitution Effect*: leisure becomes more expensive (relative to consumption), which increases labour supply.

# A change in the wage rate $w$: Details

▶ The objective is to compute $\partial L/\partial w$ (holding $Y$ constant)

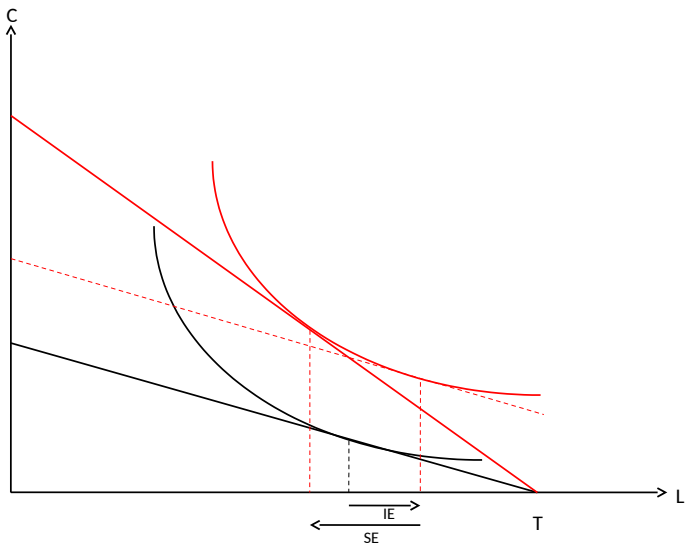▶ Start from the equilibrium equation, inserting the constraint:

$$U_L(Y + w(T - L), L) = wU_C(Y + w(T - L), L)$$

▶ Differentiate it wrt $L$ and $w$ and re-arrange the terms:

$$\frac{dL}{dw} = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}(T - L) + \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

▶ An income effect ($> 0$) and a substitution effect ($< 0$)

$$\frac{dL}{dw} = \frac{dL}{dR}(T - L) + \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

# Marshallian (non-compensated) labour supply

▶ Problem parameterized in $w$ and $Y$:

$$\max_{C,L} U(C, L) \text{ s.t. } C + wL \leq Y + wT$$

▶ Solutions: $L^*(w, Y)$ and $C^*(w, Y)$

▶ Differentiating FOC by $L$, $w$ and $Y$, partial derivatives of $L^*$:

$$L_1^* = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}(T - L) + \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

$$L_2^* = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

▶ Marshallian elasticity of labour supply to wages:

$$\eta_M \doteq \frac{w}{T - L}\frac{\partial(T - L^*)}{\partial w} = -\frac{wL_1^*}{T - L}$$

# Hicksian (compensated) labour supply

- Dual problem parameterized in $w$ and $\bar{U}$:

$$\min_{C,L} C + wL \text{ s.t. } U(C,L) \geq \bar{U}$$

- Solutions: $\hat{L}(w, \bar{U})$ and $\hat{C}(w, \bar{U})$
- Hicksian elasticity of labour supply to wages:

$$\eta_H \doteq \frac{w}{T-L} \frac{\partial(T - \hat{L})}{\partial w} = -\frac{w\hat{L}_1}{T-L}$$

# Hicks and Marshall: two good buddies

▶ Define $y(w, \bar{U}) \doteq \hat{C}(w, \bar{U}) - w[T - \hat{L}(w, \bar{U})]$

$$L^*(w, y(w, \bar{U})) = \hat{L}(w, \bar{U})$$

▶ Deriving the equation by $w$: $L_1^* + y_1 L_2^* = \hat{L}_1$

▶ Shephard's lemma: $y_1 = -(T - \hat{L})$

$$L_1^* - (T - \hat{L})L_2^* = \hat{L}_1 \qquad (4)$$

▶ So that

$$\hat{L}_1 = \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}} < 0 \text{ and } \eta_H > 0$$

▶ Multiplying (4) by $w/(T - \hat{L})$ and rearranging:

$$\eta_M = \eta_H - wL_2^*$$

# The individual labour supply curve

- Labour supply curve: labour supply as a function of wage
- The slope of the labour supply curve depends on the relative magnitudes of income and substitution effects
- Below the reservation wage the individual will not want to work
- At wages slightly above the reservation wage the labour supply curve must be positively sloped (SE > IE)
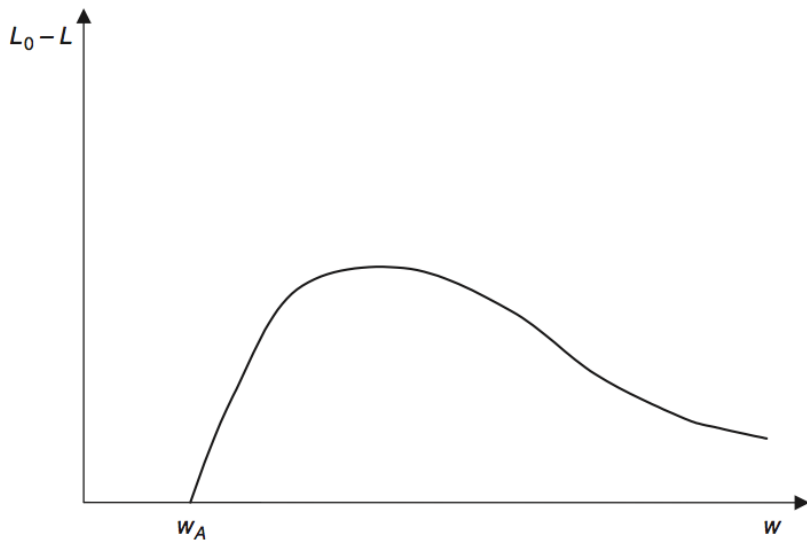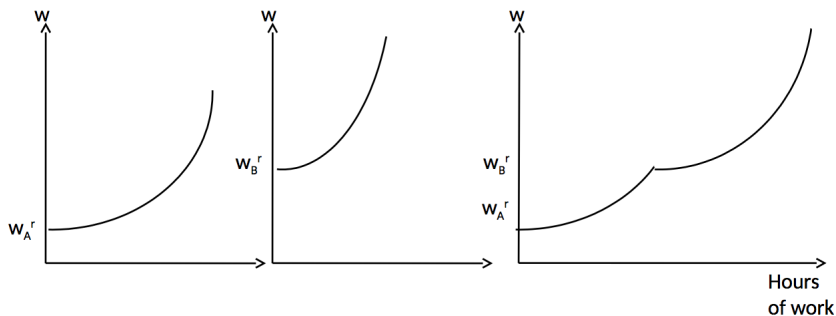- It may become backward bending for high wage levels (IE > SE)

**FIGURE 1.13**
The individual labor supply.

# Market labour supply curve

▶ For a given wage, the aggregate labour supply is equal to the sum of the hours supplied by each worker

- ▶ Let's just plot wages and number of hours worked across individuals in the population
- ▶ What is possibly wrong with this?

# Goldberg (2016)

Kwacha Gonna Do? Experimental Evidence about Labor Supply in Rural Malawi, *American Economic Journal: Applied Economics*, 2016



Jessica Goldberg
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# Kwacha Gonna Do?

- ▶ What is wage elasticity of employment in rural Malawi?
- ▶ RCT:
  - ▶ Job offers for workfare programme (4 hours of agricultural work on a day)
  - ▶ Daily wage randomly assigned at the village-week level
  - ▶ Wage offers: between 10th and 90th percentiles (USD 0.3-1)
- ▶ Data:
  - ▶ Administrative data from the randomisation and workfare programme
  - ▶ Baseline survey: socio-demographics
  - ▶ 3 follow-up (week 4, 8, 12): reasons for working or not, recall questions
  - ▶ 529 adults in 10 villages
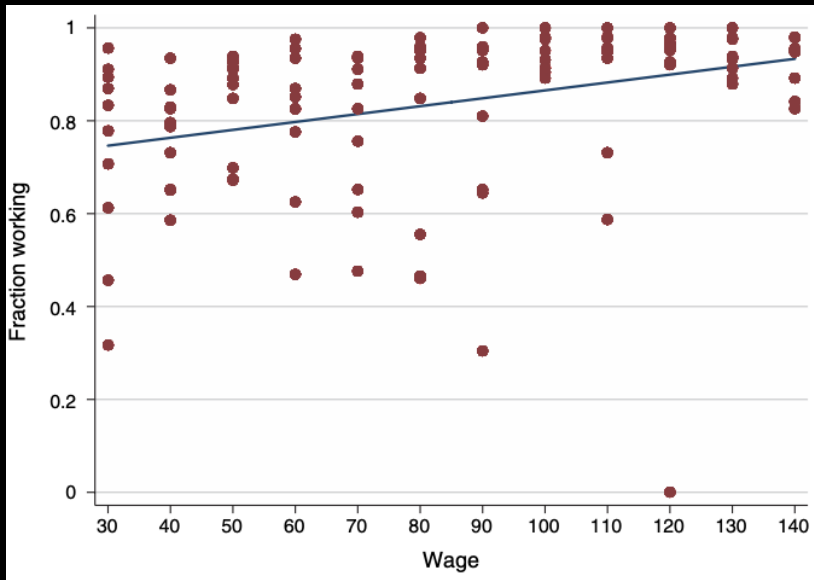
# Kwacha Gonna Do?

- Results:
  - Elasticity of .15 (lower bound of the literature)
  - Little heterogeneity

# Cesarini, Lindqvist, Notowidigdo and Östling (2017)

The Effect of Wealth on Individual and Household Labor Supply: Evidence from Swedish Lotteries, *American Economic Review*, 2017



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