Closing standard SOE models

Closing standard models

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Lecture 4b

Aim of this lecture

- Is the model we studied stationary? Why do we care?
- What are unit roots?
- How to simulate a SOE RBC model?
- Discuss the role of external debt accumulation.

References

- Schmitt-Grohé, S. and M. Uribe (2003), "Closing small open economy models", *Journal of International Economics*, 61, 163-185.
- Martin Uribe, Stephanie Schmitt-Grohe (2017), *Open Economy Macroeconomics,* Princeton University Press.

- More lecture notes on complete markets in open economy? (Let me know if you are interested)

Unit roots

- Consumption and external debt are characterized by a random walk behaviour (see previous lectures). Intrinsic feature of the SOE model.
- In an univariate model, unit roots imply $\rho = 1$. Thint at an AR(1) process with $\rho = 1 \longrightarrow$ random walk!
- A new ss following each shock: multiple ss!
- Careful, a model with unit roots can verify Blanchard Khan conditions for saddle path stability!
- Why should we care? Nice saddle path in the previous chapter!
- ..but..no second moments..the variance of a random walk $\longrightarrow \infty$. How can we compute RBC statistics to match data?

How to "fix" the standard SOE model

- The literature proposes several "tricks" to stationarize the model.We will review the most popular ones.
- Remember that the aim is to stationarize the model.
- But you should of course choose the trick that makes more sense in your model (knowledge of your country).

The standard model

- The standard framework we have studied with a stochastic setting: productivity shocks (temporary but persistent, AR(1), $0 < \rho < 1$)
- capital and adjustment costs
- Markets could be complete or incomplete.

Complete markets

- Agents are insured agains any state of the nature because they have access to a full portfolio (complete) of assets delivering 1 unit of consumption good in each state of the nature.
- Not our focus: not consistent with financial frictions, overborrowing, balance of payments crisis etc etc.
- The CAM is not featured by unit roots: no endogenous random walk problem.
- It eliminates the random walk component of consumption. It fixes the marginal utility of consumption of the soe to the one of the other country (exogenous).
- The ca is just a residual in the model that can be computed by finding the change in the complete asset portfolio.

Incomplete markets

- Interest rates are risk-free returns on securities.
- What's the steady state? Is it unique?
- The steady state depends on initial conditions and shocks $\longrightarrow$ transient shocks have permanent effects on the state of the economy.
- Equilibrium dynamics posseses a random walk component $\longrightarrow$ the variance of consumption is infinite!
- Stationarity inducing methods: SGU (2003); Cole and Obstfeld, Corsetti and Pesenti; Ghironi.

Endogenous discount factor

$$E_0 \sum_{t=0}^{\infty} \theta_t U\left(C_t, N_t\right)$$

$$\begin{aligned}
\theta_0 &= 1 && (1)\\
\theta_{t+1} &= \beta\left(C_t, N_t\right)\theta_t && (2)\\
t &\geq 0; \beta_c < 0; \beta_N > 0 &&\\
k_{t+1} &= \left(1-\delta\right)k_t + I_t && (3)
\end{aligned}$$

$$D_t = D_{t-1}\left(1 + r_{t-1}\right) + C_t + I_t - Y_t + \Phi\left(k_{t+1} - k_t\right)$$

$$\lim \frac{D_{t+j}}{\Pi_{s=0}^{j}\left(1 + r_s\right)} \leq 0$$

FOCs

Lagrangian at $t = 0$

$$
\begin{aligned}
L_0 \quad = \quad & E_0 \sum_{t=0}^{\infty} \{\theta_t U(C_t, N_t) \\
& -\theta_t \lambda_t (-D_t + D_{t-1}(1 + r_{t-1}) + C_t + k_{t+1} - (1-\delta) k_t - Y_t + \Phi (\\
& -\eta_t (\beta(C_t, N_t)\theta_t - \theta_{t+1})\}
\end{aligned}
$$

$$
\begin{aligned}
C_t &: \lambda_t = U_c - \eta_t \beta_c & (4) \\
D_t &: \lambda_t = \beta(C_t, N_t) E_t \lambda_{t+1} (1 + r_t) & (5) \\
N_t &: U_{N,t} = \eta_t \beta_N - \lambda_t A_t F_{N,t} & (6)
\end{aligned}
$$

$$
\begin{aligned}
\theta_{t+1} &: \eta_t = -E_t U(C_{t+1}, N_{t+1}) + E_t \eta_{t+1} \beta(C_{t+1}, N_{t+1}) & (7) \\
k_{t+1} &: 0 = \beta(C_t, N_t) E_t \lambda_{t+1} [A_{t+1} F_k(k_{t+1}, N_{t+1}) & (8) \\
& + (1-\delta) + \Phi'(k_{t+2} - k_{t+1})] - \lambda_t [1 + \Phi'(k_{t+1} - k_t)]
\end{aligned}
$$

Steady state

- Let $\beta(C_t, N_t) = \left(1 + C_t - \frac{N_t^{\omega}}{\omega}\right)^{-\psi_1}$
- Capital, output and labor are uniquely pinned down and independent from external debt.
- Consumption and debt are pinned down thanks to ψ_1
- More functional forms: $U(C_t, N_t) = \frac{\left(C_t - \frac{N_t^{\omega}}{\omega}\right)^{1-\gamma} - 1}{1-\gamma}$; $Y_t = A_t k_t^{\alpha} N_t^{1-\alpha}$; $\Phi = \frac{\phi}{2}\left(k_{t+1} - k_t\right)^2$

Steady state

Try to pin down the steady state. Hints:

1. Use (4)+(6) to find: $(U_c - \eta_t \beta_c) A_t F_{N,t} = \eta_t \beta_N - U_{N,t} \rightarrow A_t F_{N,t} = N_t^{\omega - 1}$
2. Use (8) in ss to obtain: $[F_k + (1-\delta)] = \frac{1}{\beta} \rightarrow \frac{k}{N} = \left(\frac{\alpha}{r+\delta}\right)^{1/(1-\alpha)} \Rightarrow$
3. $1+2 \rightarrow N = \left[(1-\alpha)\left(\frac{\alpha}{r+\delta}\right)^{\alpha/(1-\alpha)}\right]^{1/(\omega-1)}$ Labor depends on parameters only.
4. $\rightarrow$SS capital labor ratio: $\left[(1-\alpha)\left(\frac{k}{N}\right)^{\alpha}\right]^{1/(\omega-1)} = N$; SS output from production function

Steady state

- Consumption: From (5): $1 = \beta(C_t, N_t)(1 + r_t)$
- Moreover: $\beta(C_t, N_t) = \left(1 + C - \frac{N^\omega}{\omega}\right)^{-\psi_1}$

  thus:$1 = \left(1 + C - \frac{N^\omega}{\omega}\right)^{-\psi_1}(1 + r_t)$. You pin down C.
- Trade balance now:$C = Y - TB - I$;
- $tb = 1 - \frac{I}{Y} - \frac{\left[\frac{1}{(1+r)}\right]^{-1/\psi_1} + \frac{N^\omega}{\omega} - 1}{Y}$ is uniquely pinned down. It depends on the value of ψ_1.

Model's performance

Variable	Canadian Data			Model		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1	3.1	0.61	1
c	2.5	0.7	0.59	2.3	0.7	0.94
i	9.8	0.31	0.64	9.1	0.07	0.66
h	2	0.54	0.8	2.1	0.61	1
$\frac{tb}{y}$	1.9	0.66	-0.13	1.5	0.33	-0.012
$\frac{ca}{y}$				1.5	0.3	0.026

Note. Empirical moments are taken from Mendoza (1991). Standard deviations are measured in percentage points.

Calibration

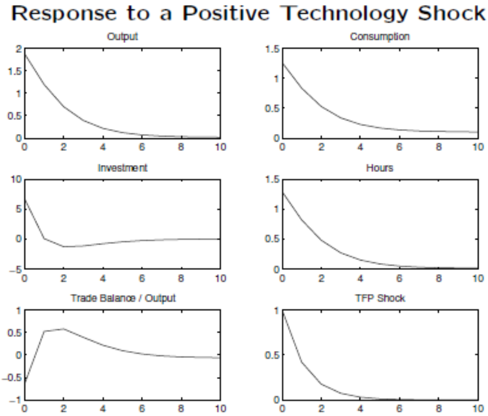
Model 1: Calibration

γ	ω	ψ_1	α	ϕ	r	δ	ρ	σ_ϵ
2	1.455	0.11	0.32	0.028	0.04	0.1	0.42	0.0129

SGU, (2003)

IRFs

Response to a 1% productivity shock



Source: Schmitt-Grohé and Uribe (JIE, 2003)

External discount factor (EDF)

Same as before but the discount factor funcion changes in the following way:

$$\begin{aligned} \theta_{t+1} &= \beta\left(\bar{c}_t, \bar{n}_t\right)\theta_t \\ t &\geq 0 \end{aligned}$$

$$C_t : U_{c,t} = \lambda_t$$

externalities are not internalized.

$$D_t : \lambda_t = \beta\left(\bar{c}_t, \bar{n}_t\right) E_t \lambda_{t+1}\left(1 + r_t\right)$$

$$N_t : U_{N,t} = -\lambda_t A_t F_{N,t}$$

External discount factor (EDF)

$$
\begin{aligned}
k_{t+1} \quad &: \quad \beta\left(\bar{c}_t, \bar{n}_t\right) E_t \lambda_{t+1}\left[A_{t+1} F_k\left(k_{t+1}, N_{t+1}\right)+(1-\delta)+\Phi'\left(k_{t+2}-k_{t+
\right.\\
&= \quad \lambda_t\left[1+\Phi'\left(k_{t+1}-k_t\right)\right]
\end{aligned}
$$

Notice that in equilibrium:

$$
\begin{aligned}
C_t &= \bar{c}_t \\
N_t &= \bar{n}_t
\end{aligned}
$$

External debt-elastic interest rate (EDEIR)

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

BC does not change:

$$D_t = D_{t-1}\left(1 + r_{t-1}\right) + C_t + I_t - Y_t + \Phi\left(k_{t+1} - k_t\right)$$

the discount rate is exogenous and equal to β.Stationarity is insured by the following assumption:

$$r_t = r^* + p\left(\bar{d}_t\right) \tag{9}$$

Functional forms

$$p\left(\bar{d}_t\right) = \psi_2\left(e^{d_t - \bar{d}} - 1\right)$$

Calibration:

Model 2: Calibration of parameters not shared with Model 1

β	$\bar{d}$	ψ_2
0.96	0.7442	0.000742

SGU, (2003)

Steady state

$$r = r^* + p\left(\bar{d}\right)$$

from Euler eq:

$$D_t : \lambda_t = \beta \lambda_{t+1}\left(1 + r_t\right) \longrightarrow$$

you always need to impose in ss: $\beta = \frac{1}{1+r} = \frac{1}{1+r^*} \longrightarrow$ substituting for:$r = r^* + \psi_2\left(e^{d-\bar{d}} - 1\right)$, we obtain:

$$1 = \beta\left(1 + r^* + \psi_2\left(e^{d-\bar{d}} - 1\right)\right)$$

thus, in ss, $d = \bar{d}$!!, debt is pinned down uniquely by the parameter $\bar{d}$!!

Portfolio adjustment costs

Same utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

no interest rate premia, discount rate is exogenous, β.
Stationarity is insured by portfolio adjustment costs: $\frac{\psi_3}{2}(D_t - \bar{D})^2$

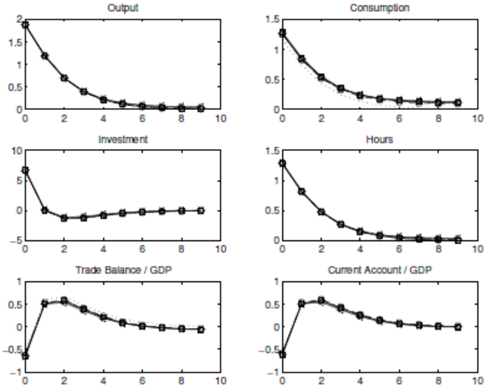
$$D_t = D_{t-1}(1 + r_{t-1}) + C_t + I_t - Y_t + \Phi(k_{t+1} - k_t) + \frac{\psi_3}{2}(D_t - \bar{D})^2$$

In, the Euler equation becomes:

$$[1 - \psi_3(D - \bar{D})] = \beta(1 + r)$$

Moreover:$\beta = \frac{1}{1+r} \rightarrow D = \bar{D}$!!, debt is pinned down uniquely by the parameter $\bar{D}$.

Performance



To resume:

Small open economy models with incomplete markets generally present unit roots $\rightarrow$debt and consumption are not pinned down uniquely. Stationarity inducing methods: allow to pin down steady-state debt/consumption.

- **Endogenous discount factor**. They allow to pin dow consumption, and thus the trade balance and debt (see above).
- **EDF endogenous discount factor** allow to pin down consumption and labor (which are equal to the population average)$\rightarrow$ pin down uniquely debt.
- **External debt elastic interest rate (EDEIR).** It allows to pin down uniquely debt (via eq 9 in ss)
- **Portofolio adjustment costs**. Allow to pin down uniquely debt (via adj. costs).