Labour supply

AE318 - Labour Economics

Roland Rathelot

Classical theory of labour supply

- Static model of labour supply
- Policy: EITC in the US
- Dynamic model of labour supply

New empirical evidence on labour supply

Neoclassical vs. behavioural models of labour supply?

Labour rationing

Classical theory of labour supply

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Labour rationing

The worker utility

Each worker chooses its level of consumption and leisure to maximizes its utility

- max $U(C, L)$
  - C: consumption
  - L: leisure hours
- Try plotting reasonable gradients, connect your plot to assumptions about first derivatives of U
- Define iso-utility curves as: $\forall \bar{U}, \{(C, L) \text{ s.t } .U(C, L) = \bar{U}\}$
- From equation $U(C, L) = \bar{U}$, define implicit function $C_{\bar{U}}(L)$
- Differentiating the equation: $U_C dC + U_L dL = 0$ so that:

$$C'_{\bar{U}}(L) = -\frac{U_L}{U_C}$$

The worker problem

Each worker chooses its level of consumption and leisure to maximizes its utility under a budget constraint

- max $U(C, L)$
  - C: consumption
  - L: leisure hours
- such that $pC = wH + Y$
  - p: unit price of consumption good
  - w: wage
  - Y: non-labour income
  - H: working hours
- Rewrite budget constraint as: $C + wL = Y + wT$
  - $T = L + H$ is the total time available (168 hours/week)
  - price p is normalised to 1

Resolution

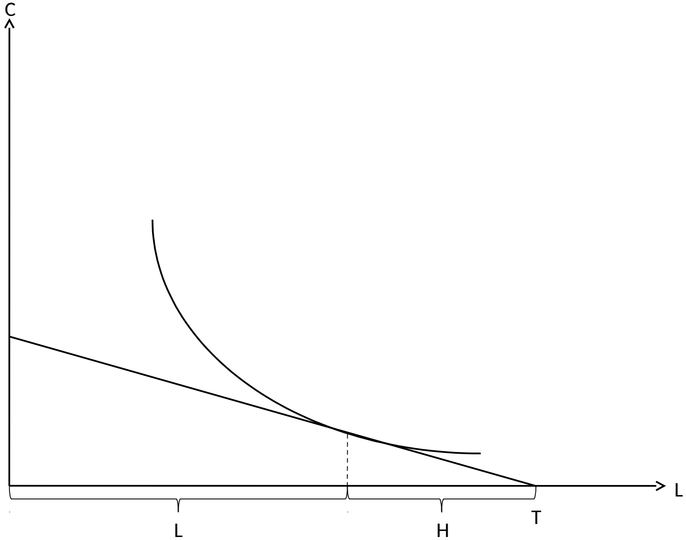
- The Lagrangian of this program is:
  $\mathcal{L}(C, L, \lambda) = U(C, L) - \lambda[C + wL - Y - wT]$
- First order conditions:

$$\frac{\partial \mathcal{L}}{\partial C} = U_C - \lambda = 0 \tag{1}$$
$$\frac{\partial \mathcal{L}}{\partial L} = U_L - \lambda w = 0 \tag{2}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = C + wL - Y - wT = 0 \tag{3}$$

- From (1) and (2): the equilibrium (C^*, L^*) is such that the marginal rate of substitution equals the wage

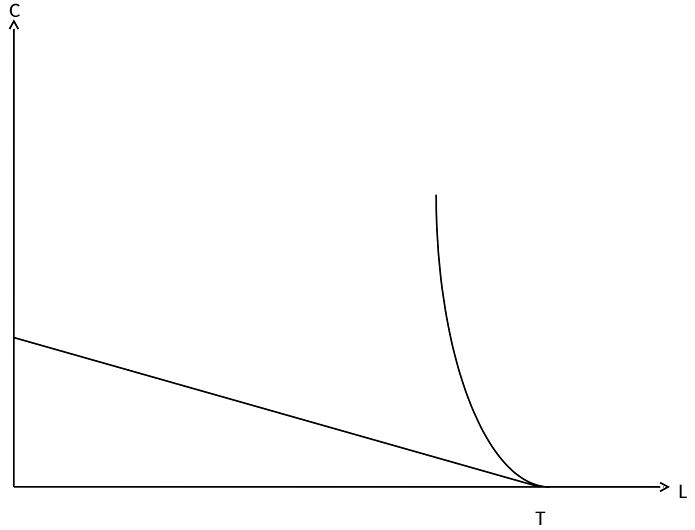
$$\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w$$

Optimal Labour Supply Decision



Reservation wage

- The reservation wage is the lowest wage rate that leaves an individual indifferent between working and not working
- Marginal rate of substitution (slope of indifference curve) $=$ Slope of budget constraint (w) at 0 working hours


C
L
T

A change in unearned income Y

- How does an increase in unearned income Y will change leisure hours L?
- Depends on the properties of the utility function
- With usual properties, an increase in unearned income Y unambiguously increases leisure hours L and reduced working hours: pure *income effect*

A change in unearned income Y: Details

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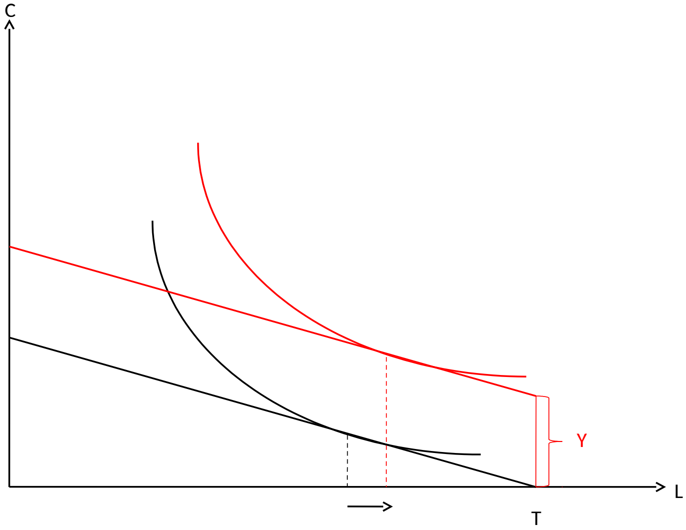
- Denote: $R = Y + wT$
- We want to compute $\partial L / \partial R$ (w being fixed)
- Start from the equilibrium equation, inserting the constraint:

$$U_L(R - wL, L) = wU_C(R - wL, L)$$

- Differentiate it wrt L and R and re-arrange the terms:

$$\frac{dL}{dR} = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

- If $U_{CC}, U_{LL} < 0$ and $U_{CL} > 0$, no ambiguity: $\frac{dL}{dR} > 0$


C
Y
L
T

An increase in the wage rate w

- An increase in the wage w has an ambiguous effect on labour supply.
  1. *Income Effect*: consume more leisure, reduce labour supply.
  2. *Substitution Effect*: leisure becomes more expensive (relative to consumption), which increases labour supply.

A change in the wage rate w: Details

- The objective is to compute $\partial L/\partial w$ (holding Y constant)
- Start from the equilibrium equation, inserting the constraint:

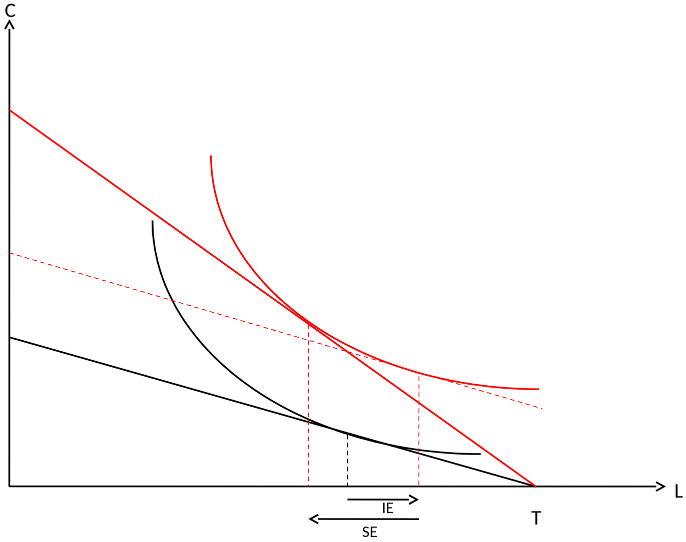
$$U_L(Y + w(T - L), L) = wU_C(Y + w(T - L), L)$$

- Differentiate it wrt L and w and re-arrange the terms:

$$\frac{dL}{dw} = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}(T - L) + \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

- An income effect (> 0) and a substitution effect (< 0)

$$\frac{dL}{dw} = \frac{dL}{dR}(T - L) + \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$


C
L
IE
SE
T

Marshallian (non-compensated) labour supply

- Problem parameterized in w and Y:

$$\max_{C,L} U(C,L) \text{ s.t. } C + wL \leq Y + wT$$

- Solutions: $L^*(w,Y)$ and $C^*(w,Y)$
- Differentiating FOC by L, w and Y, partial derivatives of L^*:

$$L_1^* = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}(T - L) + \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$
$$L_2^* = \frac{U_{CL} - wU_{CC}}{2wU_{CL} - U_{LL} - w^2 U_{CC}}$$

- Marshallian elasticity of labour supply to wages:

$$\eta_M \doteq \frac{w}{T-L}\frac{\partial(T-L^*)}{\partial w} = -\frac{wL_1^*}{T-L}$$

Hicksian (compensated) labour supply

- Dual problem parameterized in w and $\bar{U}$:

$$\min_{C,L} C + wL \text{ s.t. } U(C, L) \geq \bar{U}$$

- Solutions: $\hat{L}(w, \bar{U})$ and $\hat{C}(w, \bar{U})$
- Hicksian elasticity of labour supply to wages:

$$\eta_H \doteq \frac{w}{T-L}\frac{\partial(T-\hat{L})}{\partial w} = -\frac{w\hat{L}_1}{T-L}$$

Hicks and Marshall: two good buddies

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- Define $y(w, \bar{U}) \doteq \hat{C}(w, \bar{U}) - w[T - \hat{L}(w, \bar{U})]$

$$L^*(w, y(w, \bar{U})) = \hat{L}(w, \bar{U})$$

- Deriving the equation by w: $L_1^* + y_1 L_2^* = \hat{L}_1$
- Shephard's lemma: $y_1 = -(T - \hat{L})$

$$L_1^* - (T - \hat{L})L_2^* = \hat{L}_1 \tag{4}$$

- So that

$$\hat{L}_1 = \frac{-U_C}{2wU_{CL} - U_{LL} - w^2 U_{CC}} < 0 \text{ and } \eta_H > 0$$

- Multiplying (4) by $w/(T - \hat{L})$ and rearranging:

$$\boxed{\eta_M = \eta_H - wL_2^*}$$

The individual labour supply curve

- Labour supply curve: labour supply as a function of wage
- The slope of the labour supply curve depends on the relative magnitudes of income and substitution effects
- Below the reservation wage the individual will not want to work
- At wages slightly above the reservation wage the labour supply curve must be positively sloped (SE $>$ IE)
- It may become backward bending for high wage levels (IE $>$ SE)

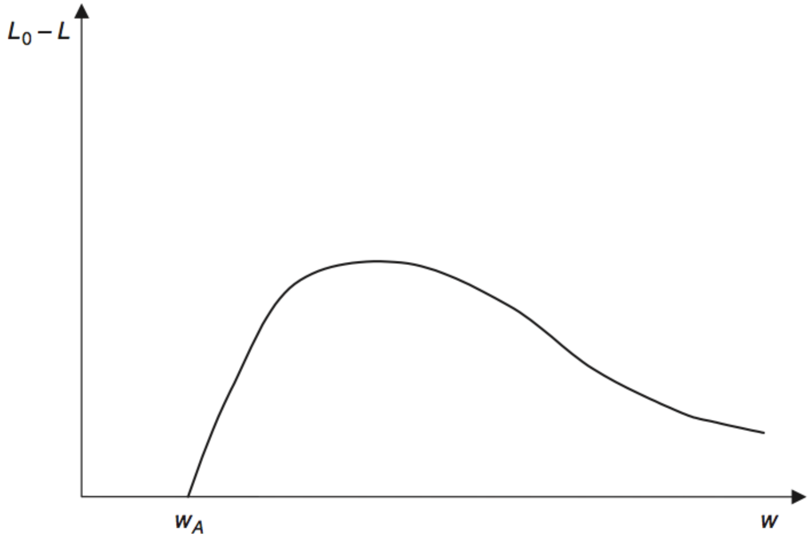
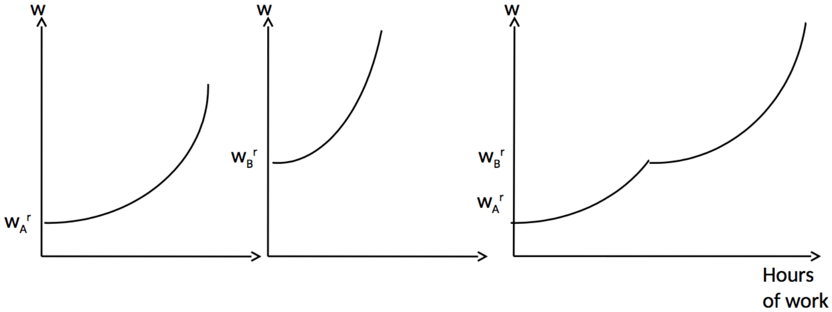


Figure 1.13
The individual labor supply.

Market labour supply curve

- For a given wage, the aggregate labour supply is equal to the sum of the hours supplied by each worker



Estimating the labour supply curve

- Let's just plot wages and number of hours worked across individuals in the population
- What is possibly wrong with this?

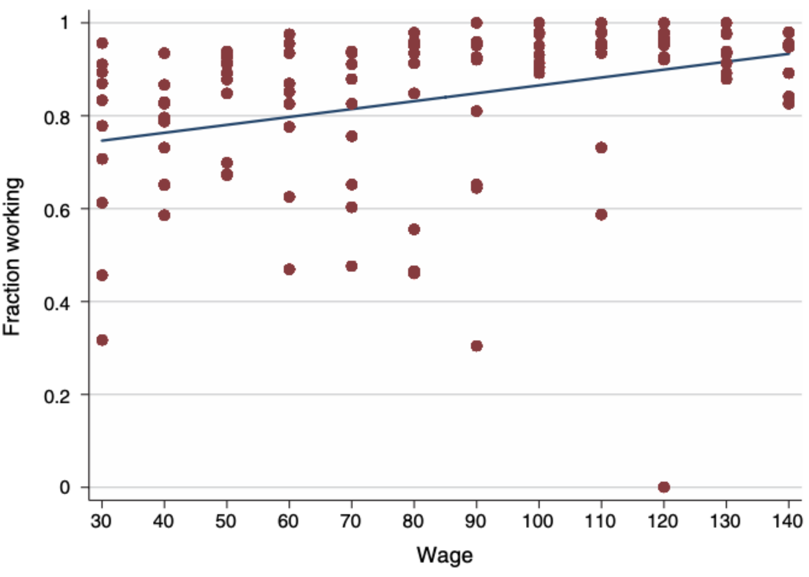
Goldberg (2016)

Kwacha Gonna Do? Experimental Evidence about Labor Supply in Rural Malawi, *American Economic Journal: Applied Economics*, 2016

Jessica Goldberg
Maryland

Kwacha Gonna Do?

- What is wage elasticity of employment in rural Malawi?
- RCT:
  - Job offers for workfare programme (4 hours of agricultural work on a day)
  - Daily wage randomly assigned at the village-week level
  - Wage offers: between 10th and 90th percentiles (USD 0.3-1)
- Data:
  - Administrative data from the randomisation and workfare programme
  - Baseline survey: socio-demographics
  - 3 follow-up (week 4, 8, 12): reasons for working or not, recall questions
  - 529 adults in 10 villages



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Kwacha Gonna Do?

- Results:
  - Elasticity of .15 (lower bound of the literature)
  - Little heterogeneity

Cesarini, Lindqvist, Notowidigdo and Östling (2017)

The Effect of Wealth on Individual and Household Labor Supply: Evidence from Swedish Lotteries, *American Economic Review*, 2017

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