Lecture 2: Job Search (Part II)

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1. https://sites.google.com/view/franck-malherbet/home/teaching/labor-economics

- The basic job search model allows to illustrate the search behavior of an individual in a simple way.
- This simplicity comes at a cost since some hypotheses may be deemed too restrictive.
- The model can however easily accommodate **richer (more realistic) ingredients**.

In this lecture, we will :

1. Examine the consequences of the conditions of eligibility for unemployment insurance benefits.
2. Observe the implications of allowing an individual to seek a job while he is already employed.
3. Suppose that agents can decide how much effort to put into their job search.
4. Analyze how active labor market policies affect job search behavior.
5. Extend the basic framework to a non stationary environment.
6. Present some empirical aspects of job search.

- In most countries, not all job-seekers are eligible to UI, *i.e.* only those who have sufficiently contributed to UI finance will receive benefits when they become unemployed. [2]

Percentages of unemployed persons qualifying for unemployment insurance benefits in 1995.

Austria	66
Belgium	81
Denmark	66
Finland	73
France	45
Germany	70
Greece	9
Ireland	67
Italy	7
Netherlands	50
Portugal	27
Spain	24
Sweden	70

Source: Manning (1998, table 1, p. 144).

- In practice, about one third of the unemployed are not covered by unemployment insurance in OECD countries ($\approx$ 50% in the U.S.)

2. In France for instance, you are eligible, if you have been employed for at least 6 months in the last 24 months. This condition can be met with one or more contracts with different employers.

The model

- In most countries waged workers have to pay a premium to an unemployment insurance system (UIS). [3]
- Two types of job seekers :
  1. eligible workers
  2. non-eligible workers
- Eligible workers to unemployment benefits are those who receive compensation from the UIS following a job loss.
- However, *new entrants*, those with too short employment spells and those who have been *unemployed for a long time* are not (anymore) eligible for such benefits.
- In this case, the reservation wage of those who are not eligible decreases when the benefits paid to the unemployed who do meet the eligibility requirement increase.

3. This is implicit in the model presented below.

- Instantaneous income of the unemployed always amounts to z (benefits paid by the UIS).
- Individuals are eligible once they have been employed **at least once**.[4]
- For non-eligible workers, welfare system provides $z_n < z$.
- Let us denote by :
  - V_u, the expected utility of an eligible unemployed worker getting z,
  - V_{un}, the expected utility of a non-eligible unemployed worker getting z_n,
  - $V_e(w)$, the expected utility of a worker getting w.
- Then, we get :

$$rV_e(w) = w + q\left[V_u - V_e(w)\right] \tag{1}$$

4. For models with eligible and non eligible workers, see e.g. :
Ortega and Rioux (2010), On the extent of re-entitlement effects in unemployment compensation, *Labour Economics*.
Andersen, Kristoffersen, and Svarer (2018), Benefit reentitlement conditions in unemployment insurance schemes, *Labour Economics*.

- We now have two reservation wages to determine : x and x_n.

- We still have $x = rV_u$, while x_n need to be determined, and results from $V_{un} = V_e(x_n)$. Thus, equation (1) implies :

$$\begin{aligned}(r+q)\,V_e(x_n) &= x_n + qV_u \\ (r+q)\,V_e(x_n) &= x_n + q\frac{x}{r} \\ (r+q)\,V_{un} &= x_n + q\frac{x}{r}\end{aligned}$$

or

$$rV_{un} = \frac{rx_n + qx}{r+q} \tag{2}$$

- The expected utility of a non-eligible job seeker is a *weighted average* of the two reservation wages.

- Assuming that eligible and ineligible job seekers receive offers at the same rate λ , we have :

$$rV_{un} = z_n + \lambda \int_{x_n}^{+\infty} [V_e(w) - V_{un}] \, \mathrm{d}H(w)$$

which together with (2) implies :

$$rx_n = z_n(r+q) - qx + \lambda \int_{x_n}^{+\infty} (w - x_n) \mathrm{d}H(w) \tag{3}$$

- This relation implies a negative link between x_n and x.
- The reservation wage of a non-eligible worker x_n is decreasing in z and increasing in z_n.[5]

$$\frac{\mathrm{d}x_n}{\mathrm{d}z_n} > 0 \quad \text{and} \quad \frac{\mathrm{d}x_n}{\mathrm{d}z} < 0$$

5. Recall from chapter 1 that $\frac{\mathrm{d}x}{\mathrm{d}z} > 0$.

- **Interpretation :**
  - A non-eligible job seeker knows that by accepting an offer, she risks becoming unemployed again in the future at rate q.
  - However, in this case, she will henceforth be eligible for unemployment benefits $z > z_n$.
  - Hence, a non-eligible worker has an incentive to reduce her reservation wage x_n in order to become eligible.
  - This is the entitlement or eligibility effect stressed by Mortensen (1977). [6]

6. See :
Mortensen (1977), Unemployment insurance and job search decisions, *ILR Review*.
Devine and Kiefer (1991), Empirical Labor Economics : The search approach, Oxford University Press.

On-The-Job-Search (OTJS)

- In practice, people often move from one job to another without ever becoming unemployed : there is on-the-job search (OTJS). Appendix
- We assume that the cost of job search is negligible for a worker who is employed. [7]
- We hence do not have to make a distinction between employees who have a low wage and are looking for another job and those who are receiving a high wage.
- If the cost of searching for a job is null for an employed worker, she always has an interest in looking for another job that would pay more.

7. See e.g. Pissarides (2000) for a search (and matching) model where searching on the job is costly.

The model

- An employed person receives job offers with a frequency of λ_e, and she risks losing her job, at any time, with an exogenous constant probability $q > 0$.
- The discounted expected utility of a wage earner **currently paid** w rewrites :

$$rV_e(w) = w + q\left[V_u - V_e(w)\right] + \underbrace{\lambda_e \int_w^{+\infty} \left[V_e(w') - V_e(w)\right] \mathrm{d}H(w')}_{\text{expected gain from finding } w' \geq w} \tag{4}$$

- The derivative with respect to w yields :

$$V_e'(w) = \frac{1}{r + q + \lambda_e[1 - H(w)]} \tag{5}$$

- Remark that $V_e(w)$ is increasing in w.

- The optimal search strategy implies :

$$V_e(x) = V_u \tag{6}$$

- Assuming the arrival rate of job offers is equal to λ_u for a job seeker, her discounted expected utility V_u satisfies :

$$rV_u = z + \lambda_u \int_x^{+\infty} \left[V_e(w') - V_u\right] \mathrm{d}H(w') \tag{7}$$

- Making $w = x$ in (4), yields, together with (7) :

$$x = z + (\lambda_u - \lambda_e) \int_x^{+\infty} \left[V_e(w') - V_u\right] \mathrm{d}H(w')$$

- Compared to the basic model, this equation indicates that a job seeker must weight the discounted expected utility of the job search $\int_x^{+\infty} [V_e(w') - V_u] \mathrm{d}H(w')$ by the *difference* $(\lambda_u - \lambda_e)$ of the rates with which job offers arrive.

- From the reservation wage (integrating by parts) :

$$x = z + (\lambda_u - \lambda_e)\left[\left[-\bar{H}(\nu)\left[V_e(\nu) - V_u\right]\right]_x^{\infty} + \int_x^{+\infty} \bar{H}(\nu) V_e'(\nu) \mathrm{d}\nu\right]$$

  where $\bar{H}(\nu) \equiv 1 - H(\nu)$.

- Remarking that $\lim_{\nu \to \infty} \bar{H}(\nu)\left[V_e(\nu) - V_u\right] = 0$ and making use of (5), we get :

$$x = z + (\lambda_u - \lambda_e) \int_x^{+\infty} \frac{\bar{H}(\nu)}{r + q + \lambda_e \bar{H}(\nu)} \mathrm{d}\nu$$

- This equation implicitly defines the reservation wage as a function of the parameters λ_u, λ_e and the CDF H.

- Thus the possibility of OTJS affects job seekers' behaviour in the following way :

$$x \underset{<}{\overset{>}{-}} z \text{ if } \lambda_u \underset{<}{\overset{>}{-}} \lambda_e \text{ as } \int_x^{+\infty} \frac{\bar{H}(\nu)}{r + q + \lambda_e \bar{H}(\nu)} \mathrm{d}\nu > 0$$

- For instance, if $\lambda_e > \lambda_u$ then $x < z$:
  - The job seeker will accept a wage lower than z because she expects her labour market prospects to improve once employed.
  - Put differently, the loss of instantaneous income will be more than offset by the potential capital gains once employed.
  - Getting a job is a stepping stone to better paid jobs !

Four cases occur naturally :

1. $\lambda_e = 0$, there is no otjs and we are back to basic model where $x > z$.
2. If $\lambda_e > 0$, the job seeker takes account of the future wage improvement while searching on the job :
   - 2a. If $\lambda_e < \lambda_u$, we have the case where λ_e lowers the reservation wage and where $x > z$.
   - 2b. If $\lambda_e = \lambda_u$, it follows that $x = z$.
   - 2c. If $\lambda_e > \lambda_u$, we have the case where λ_e lowers the reservation wage and where $x < z$.
3. In all cases, OTJS decreases the reservation wage and can be seen as a stepping stone towards better-paid jobs.

- Many empirical studies show that the inequality $\lambda_u \geq \lambda_e$ is the most probable.
  - Van den Berg and Ridder (1998) find that λ_u differs very little from λ_e using data from the Netherlands. [8]
  - Bontemps (1998) and Kiefer and Neumann (1993) find that $\lambda_u >> \lambda_e$ using French and US data respectively.
  - Robin (2011), using US data finds that $\lambda_e = 0.12 \times \lambda_u$. [9]
- This likely occurs because unemployed job-seekers devote more effort looking for a job than employed job-seekers (see *e.g.* Krueger and Mueller, 2010).
- Results are not consensual however. Faberman et al. (2022) challenge the view that $\lambda_u \geq \lambda_e$ and argue instead that employed workers are much more efficient at finding new jobs than unemployed. [10]

8. Van den Berg and Ridder (1998), An empirical equilibrium search model of the labor market, *Econometrica*.
9. Robin (2011), On the dynamics of unemployment and wages distributions, *Econometrica*.
10. Faberman, Mueller, Sahin and Topa (2022), Job Search Behavior among the Employed and Non-Employed, *Econometrica*.

What is the extent of OTJS ?

In the US and in a typical month in 2013, 2% to 2.5% of people over age 16 who were employed in one month had moved to a different job by the next month.

Source : Bosler and Petrosky-Nadeau (2016). Survey of Income and Program Participation (SIPP). Gray bars represent NBER recession dates. Line breaks show periods with missing data.

Search intensity

- The hypothesis that both the arrival rate of job offers and the costs of the job search do not vary is unsatisfactory.
- A job seeker may make efforts increasing both the costs of job search and the probability of receiving an offer.
- To simplify, we assume there is no on-the-job-search. [11] We denote by e the intensity of job search such that :

$$\lambda = \lambda(e) = \alpha e, \quad \lambda'(e) > 0 \tag{8}$$

where α is an indicator of the state of the labor market *independent* of individual efforts.

- Effort is costly :

$$c = c(e), \quad c'(e) > 0, \quad c''(e) > 0 \tag{9}$$

11. See e.g. Bloemen (2004) for a model with endogeneous search intensity and on-the-job search or Faberman et al. (2022). Bloemen (2004), Job Search, Search Intensity, and Labor Market Transitions, *Journal of Human Resources*.

- The reservation wage x is now defined as :

$$x = b - c(e) + \frac{\alpha e}{r+q} \int_x^\infty (w - x) \, \mathrm{d}H(w) \tag{10}$$

- Optimal search effort results from :

$$\max_e \quad rV_u = b - c(e) + \frac{\alpha e}{r+q} \int_x^\infty (w - x) \, \mathrm{d}H(w)$$

which leads to

$$\underbrace{c'(e)}_{\text{marginal cost}} = \underbrace{\frac{\alpha}{r+q} \int_x^\infty (w - x) \, \mathrm{d}H(w)}_{\text{marginal benefit}} \tag{11}$$

- It is possible to get rid of the integral in (10) by reinserting (11) in (10), we get :

$$x = b + ec'(e) - c(e) \tag{12}$$

- Making use of (11) and (12), we have the following properties :

$$\frac{\partial x}{\partial \alpha} > 0 \text{ and } \frac{\partial e}{\partial \alpha} > 0$$

$$\frac{\partial x}{\partial b} > 0 \text{ and } \frac{\partial e}{\partial b} < 0$$

- An improvement in the state of the labor market α causes the reservation wage to rise and increases the intensity of the job search.
- A rise in the income of a job seeker b raises the reservation wage but reduces the search effort.
- Finally, it should be noted that a *simultaneous changes* of α and b has an ambiguous effect on optimal search effort.

Solving the job search model with endogeneous search effort

- Sketch of the procedure :
  - Set parameters (r, α, q, b)
  - Choose the sampling distribution, $H(w)$, and set related parameters, e.g. assume a log-normal distribution :

$$H(w) \sim \log \mathcal{N}(\mu, \sigma)$$

  - Assume search effort is quadratic :

$$c(e) = \frac{1}{2} c_0 e^2, \quad c_0 > 0$$

  - Find (x, e) as the solution of :

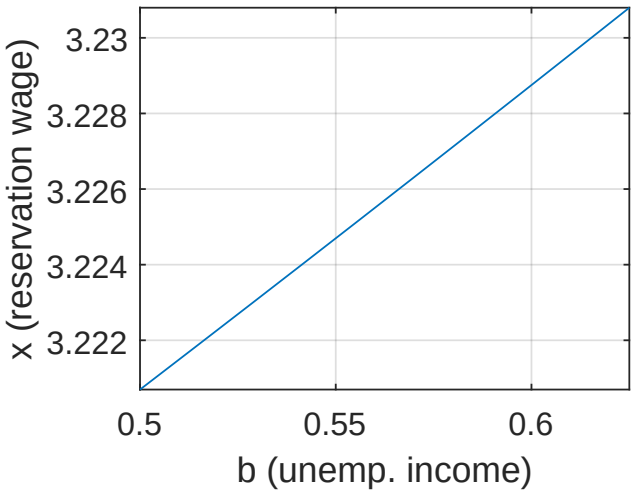
$$c'(e) - \frac{\alpha}{r+q} \int_x^{+\infty} (w - x) \, \mathrm{d}H(w) = 0$$

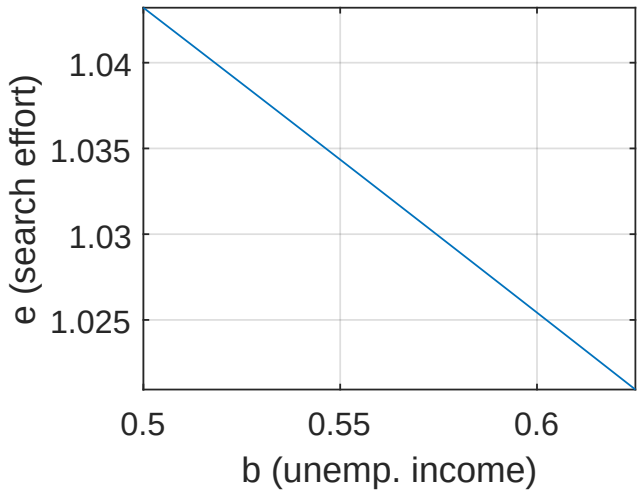
$$x - b - e.c'(e) + c(e) = 0$$

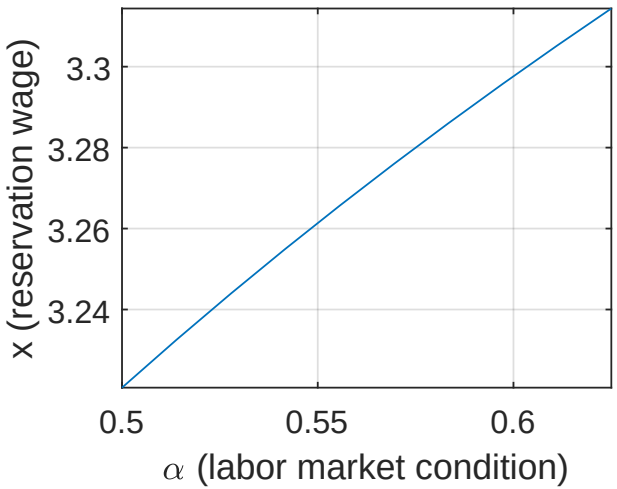
  - Derive model's outcomes : moments and comp. statics

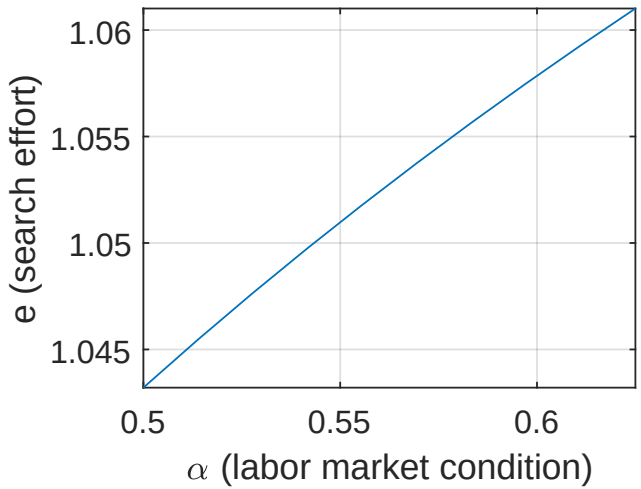
- MATLAB script : Simu_JobSearch_effort.m

Solving the job search model with endogeneous search effort









The effect of benefit sanctions (ALMP)

- All unemployment insurance systems are accompanied with sanctions that cut benefits if lack of job search. Is this useful ?

- In general, the unemployed must meet obligations to receive unemployment benefits :
  - They must take verifiable steps to look for work.
  - They must accept a job that is offered to them, if not their payments may be decreased or suspended.
  - All unemployed receiving benefits are subject to work availability.

Monitoring across OECD countries

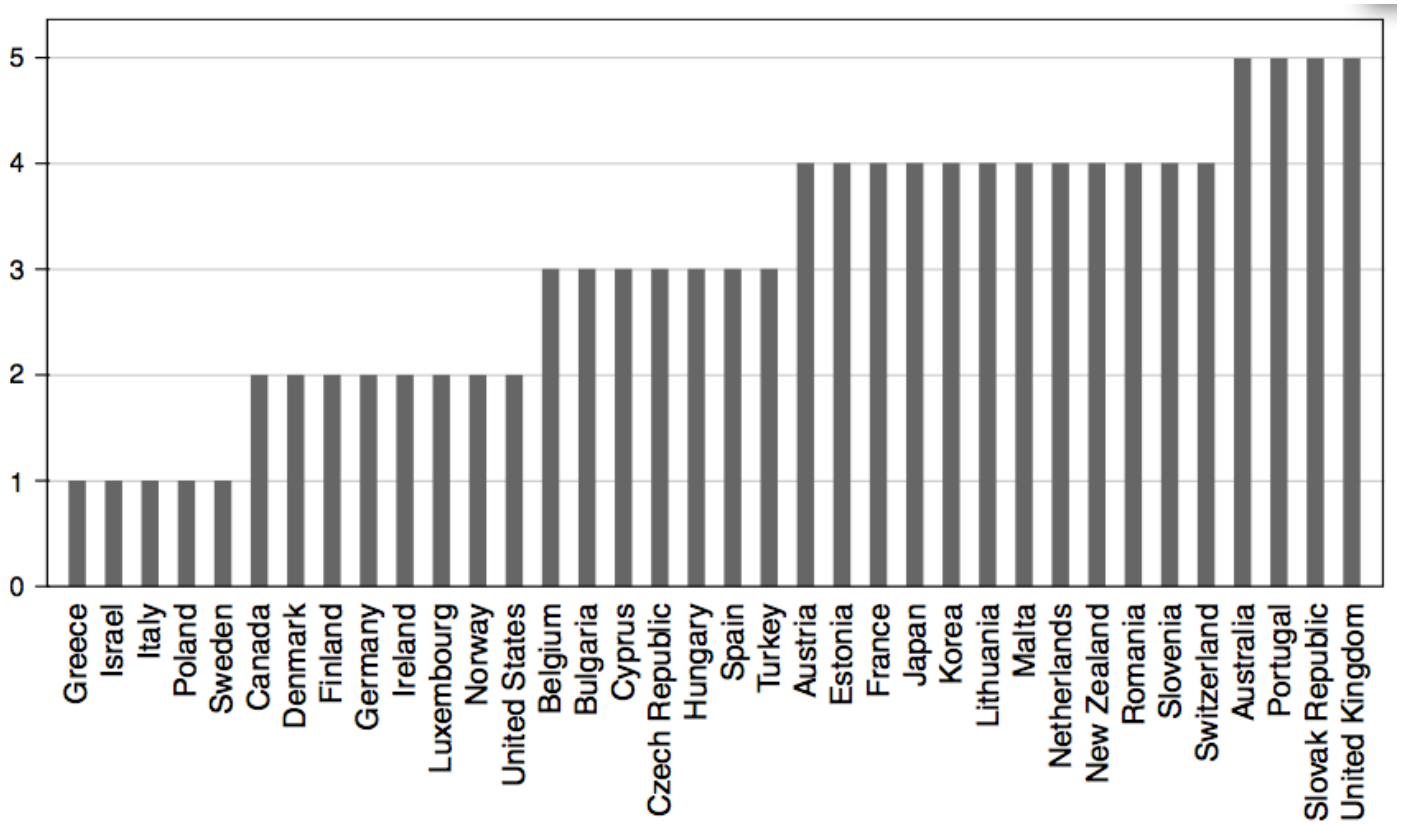


FIGURE 5.2
Strictness of job search monitoring, scored from 1 (least strict) to 5 (most strict).

Source: Venn (2012, figure 4, p. 18).

Sanctions across OECD countries

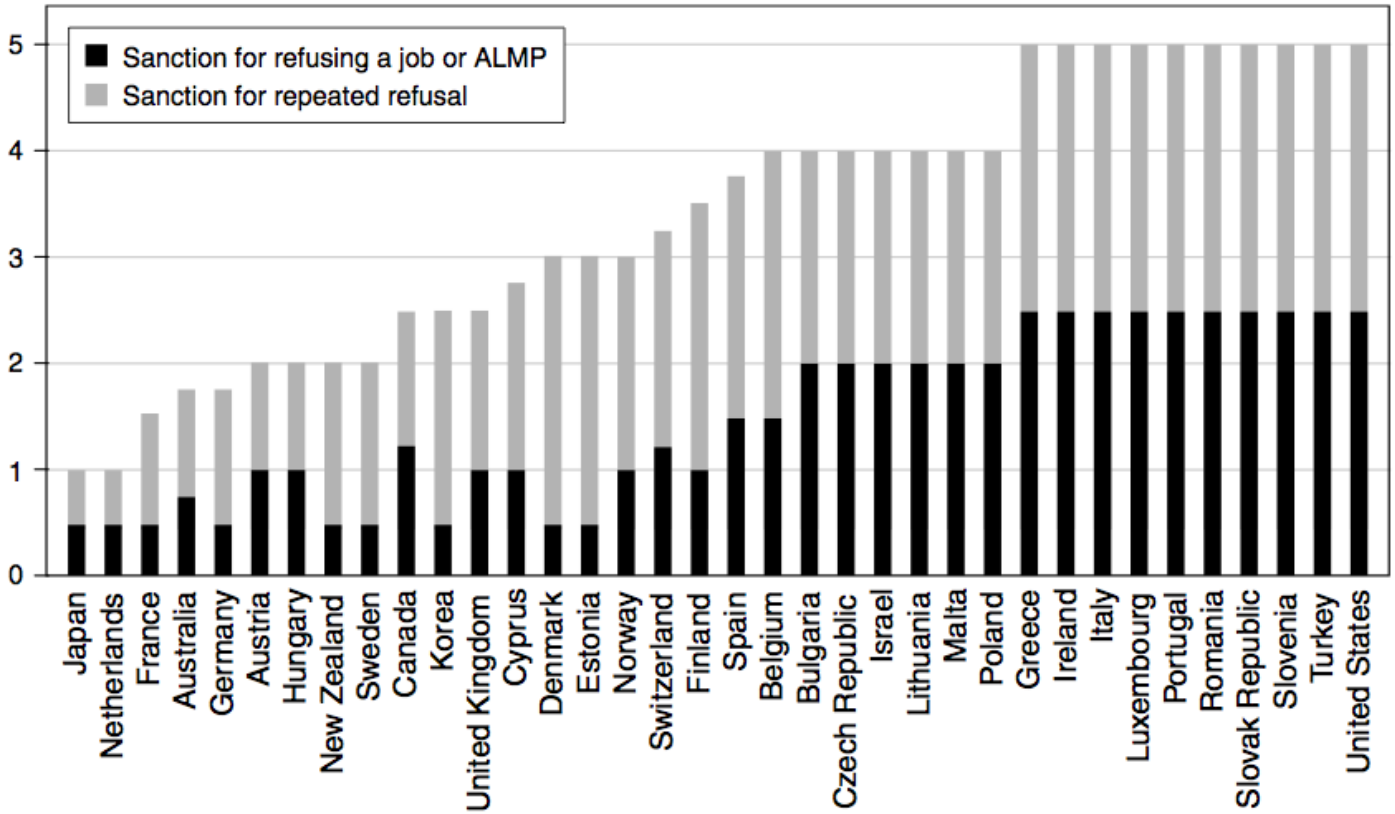


FIGURE 5.3
Strictness of sanctions, scored from 1 (least strict) to 5 (most strict).

Source: Venn (2012).

A model stripped to the bone

- The possibility of being sanctioned may be formalized with the assumption that the probability of being sanctioned decreases with search effort.
- The sanction consists of a *permanent* reduction in unemployment benefits by an amount s which occurs at rate of $\sigma(e)$ with $\sigma'(e) < 0$. [12]
- The prob. to receive an offer is $\lambda = \lambda(e) = \alpha e$.
- The search effort is :
  - e_u for a job seeker not subject to sanction
  - e_s for that of a sanctioned one
- All agents have an instantaneous death rate δ and we assume that employment is an absorbing state ($q = 0$).

12. $\sigma(e)$ is a decreasing and convex function of search effort. ▸ Appendix

- Simple **three states model**. Let us denote by :
  - N, the size of the active population (constant and exogeneous),
  - U, the number of unsanctioned job seekers,
  - S, the number of sanctioned job seekers,
  - E, the number of employees.

- The law of motion of unsanctioned and sanctioned job seekers are respectively given by :

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \underbrace{\delta N}_{\text{birth}} - \underbrace{\sigma\left(e_u\right)U}_{\text{exit to } S} - \underbrace{\alpha e_u U}_{\text{exit to } E} - \underbrace{\delta U}_{\text{death}}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \underbrace{\sigma\left(e_u\right)U}_{\text{entry into S}} - \underbrace{\alpha e_s S}_{\text{exit to } E} - \underbrace{\delta S}_{\text{death}}$$

- In a stationary environment, it implies :

$$U = \frac{\delta}{\delta + \alpha e_u + \sigma(e_u)} N$$

$$S = \frac{\sigma(e_u)}{\delta + \alpha e_s} \cdot \frac{\delta}{\delta + \alpha e_u + \sigma(e_u)} N$$

- We assume that all jobs have the same wage w and as $q = 0$, it follows that :

$$V_e = \int_0^{\infty} e^{-(\rho+\delta)t} w dt = \frac{w}{\rho + \delta}$$

- At stationary equilibrium, the expected utility of an unsanctioned unemployed person, V_u, and that of a sanctioned unemployed person, V_s verify : [13]

$$\begin{aligned}(\rho + \delta) V_u &= b - c(e_u) + \alpha e_u (V_e - V_u) + \sigma(e_u)(V_s - V_u) \\ (\rho + \delta) V_s &= b - s - c(e_s) + \alpha e_s (V_e - V_s)\end{aligned}$$

where s denotes a *permanent* reduction in unemployment benefits.

13. $c(e)$ is increasing and convex in e (*see previous section*).

- The unemployed chooses levels of search effort that maximize their respective expected utilities :

$$\max_e V_u \rightarrow e_u$$
$$\max_e V_s \rightarrow e_s$$

- The FOCs are as follows :

$$\begin{aligned} c'(e_u) &= \alpha (V_e - V_u) + \sigma'(e_u)(V_s - V_u) \\ c'(e_s) &= \alpha (V_e - V_s) \end{aligned}$$

$\Rightarrow$ marginal costs $=$ marginal benefits

- These equations allow us to study the impact of s on search efforts :

▸ Appendix

  - *Ex-post effect :* When sanction is applied, search effort increases on account of the reduction of unemployment benefit.

$$\frac{\mathrm{d}e_s}{\mathrm{d}s} > 0$$

  - *Ex-ante effect :* The threat of its imposition modifies the behavior of the unemployed who are not sanctioned.

$$\frac{\mathrm{d}e_u}{\mathrm{d}s} > 0$$

- More generally ALMPs often entail a composition of :

Ex-ante effect+*Lock-in effect*+*Ex-post effect*

- The probability of being sanctioned rises in response to reduced search effort.
- The threat of being sanctioned pushes the search effort of the unsanctioned unemployed above that of the sanctioned unemployed.
- According to Boones et al. (2009), the threat of the sanction may be more effective than the application of the sanction. [14]
- This result suggests that a well managed system of sanctions may be an effective tool for giving the unemployed an incentive to search hard while paying them a generous unemployment benefit.
- Sanction does not need to be imposed to be effective but the threat needs to be credible.

14. Boones et al. (2009), Experiments on unemployment benefit sanctions and job search behavior, *European Economic Review*. Similar results are found in e.g. Lombardi (2019).

What do we learn from empirical studies on sanctions and monitoring ?

- Lalive et al. (2005) investigates the effectiveness of unemployment benefit sanctions in reducing unemployment duration. [15]
- This paper evaluates the strength of the *ex-ante* **(threat)** effect by using variation in the actual implementation of the Swiss sanction policy across PES and the strength of the *ex-post* **(sanction)** effect.
- They find that the exit rate from unemployment :
  - increases by 25% after a warning,
  - increases by 20% after a sanction,
  - is positively correlated with the strictness of sanctions.
- Some recent contributions indicate however that these positive effects can be at the *expense of the quality of jobs*. [16]

15. Lalive, Van Ours, & Zweimuller (2005), The effect of benefit sanctions on the duration of unemployment, *Journal of the European Economic Association*.

16. See e.g. van den Berg & Vikstom (2009) or Arni et al. (2013). See also Fontaine and Malherbet (2013), chapter 5.

Non stationarity

- Until now, we assumed a **stationary environment** such that V_e and V_u were not time dependent.

- In reality, it can be the case that some of the parameters change after entry into unemployment :
  - stigma effect of duration, $\frac{\mathrm{d}\lambda}{\mathrm{d}t} < 0$.
  - declining sequences of unemployment benefits, $\frac{\mathrm{d}b}{\mathrm{d}t} < 0$.

- Following van den Berg (1990), we assume that the net instantaneous income of a job seeker diminishes over time : [17]

$$z(t) \leq z\left(t'\right) \text{ for all } t \geq t'$$

- Let $V_u(0)$ denote the discounted expected utility of a person entering unemployment where $V_u(0)$ should differ from $V_u(t)$ at later times.

17. van den Berg (1990), Nonstationarity in Job Search Theory, *The Review of Economic Studies*.

- Assume for simplicity no OTJS and $V_e(w)$ is still stationary, such that :

$$rV_e(w) = w + q\left[V_u(0) - V_e(w)\right]$$

where $V_e(w)$ is still increasing in w.

- The optimal search strategy consists of :
  - refusing all proposals lower than V_u
  - accepting all others

- The lifetime utility of the unemployed, $V_u(t)$, maximizes wrt s :

$$V_u(t) = \max_s \frac{1}{1+r\mathrm{d}t}\left(\begin{array}{c} z(t)\,\mathrm{d}t + \lambda \mathrm{d}t\left[\begin{array}{c}\int_0^s V_u(t+\mathrm{d}t)\,\mathrm{d}H(w) \\ +\int_s^\infty V_e(w)\,\mathrm{d}H(w)\end{array}\right] \\ +(1-\lambda\mathrm{d}t)\,V_u(t+\mathrm{d}t)\end{array}\right)$$

- Over a small interval of time $[t, t+dt]$, the optimal search strategy at time t consists in choosing the reservation wage s such that $Vu(t)$ is maximized.

- The optimal reservation wage, $x(t)$, is obtained by setting to zero the derivative wrt s considering $V_u(t+\mathrm{d}t)$ as given :[18]

$$V_e\left(x\left(t\right)\right) = V_u\left(t+\mathrm{d}t\right)$$

- Let $\mathrm{d}t \to 0$, then it follows :

$$V_e\left(x\left(t\right)\right) = V_u\left(t\right)$$

- As $z(t) \leq z\left(t'\right)$ for all $t \geq t'$ (i.e. $z(t)$ is decreasing) then $V_u\left(t\right) \leq V_u\left(t'\right)$. In addition as V_e is increasing in w, it follows that :

$$x(t) \leq x(t') \text{ for all } t \geq t'$$

- The reservation wages fall with the time spent searching when unemployment benefits decrease over time.

18. Put differently, the job seeker will decide on a new reservation wage at each date independently of the previous choice.

- Hence, the exit rate from unemployment $\lambda [1 - H(x(t))]$ should increase with unemployment duration.
- This is however not necessarily the case if :
  - the job arrival rate exhibits duration dependence.
  - workers are heterogeneous. [19]
- This result is confirmed by a number of studies, in particular for job seekers whose entitlement to unemployment benefits is drawing to a close.

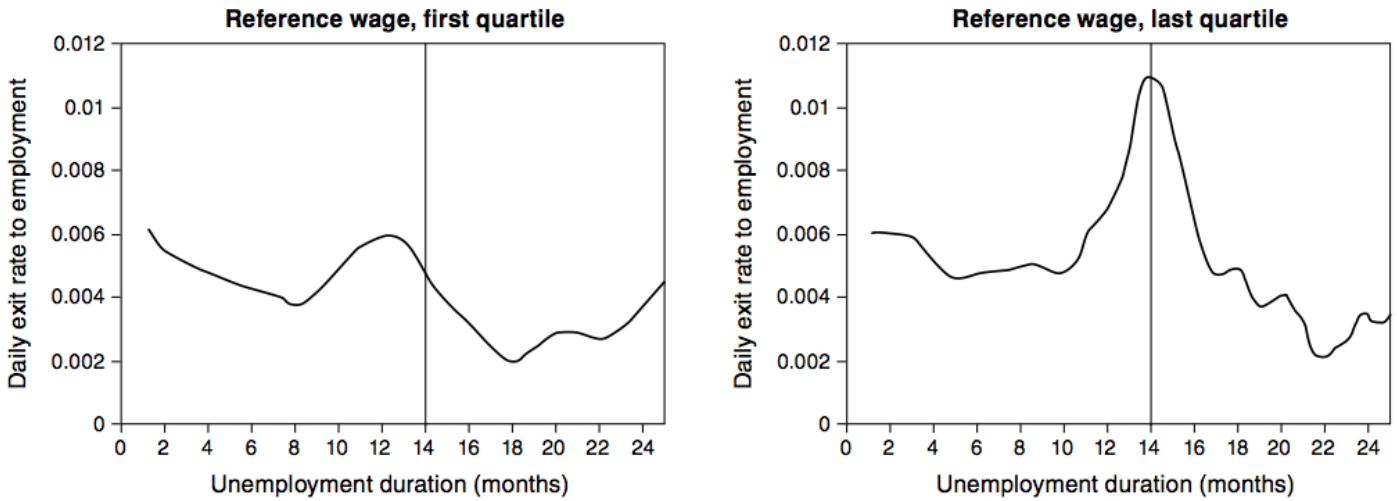


FIGURE 5.8
Exit rate from unemployment into employment and the end of entitlement to benefits. Period: 1986–1992. Population: individuals aged 25 and older. The reference wage corresponds to the average wage for the 12 months immediately preceding job loss.

Source: Dormont et al. (2001).

19. For instance, it can be the case that more skilled workers exit unemployment faster because they have a better labor market prospect.

Some empirical aspects of job search

- In this section, we consider some empirical aspects of job search models.

1. How to structurally estimate a job-search model (Flinn and Heckman, 1982)
2. How changes in the generosity of the UIS affect the duration of unemployment through the lens of job search theory (Lalive, van Ours and Zweimuller, 2006)

Flinn and Heckman (1982)

- Flinn and Heckman (1982) were the first to structurally estimate the job search model. [20]
- They consider first a simplified version of the model where employment is an absorbing state (*single unemployment spell*).
- It follows that :

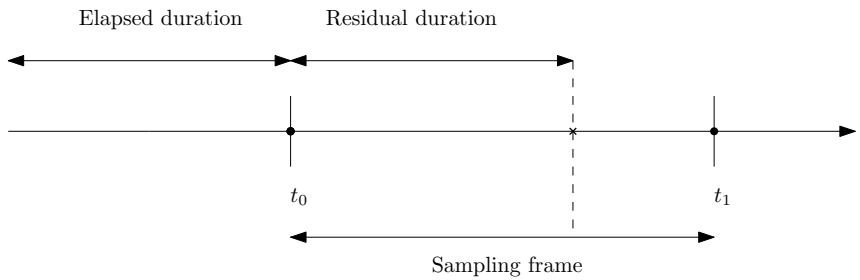
$$x = z + \frac{\lambda}{r} \int_{x}^{+\infty} (w - x) \, \mathrm{d}H(w) \tag{13}$$

- Job (wage) offers are independent realizations from the sampling distribution, $H(w)$, ($h(w)$ density of the sampling distribution).

20. Flinn and Heckman (1982), New methods for analyzing structural models of labor force dynamics, *Journal of Econometrics*.

Data

- Labor Force Surveys (LFS) usually observe individuals continuously over a given time period and have some retrospective information at the beginning of the sampling frame.
- In most countries, LFS provides information on :
  - duration of unemployment spells
  - wages
- Assume longitudinal data with N independent unemployment spells and define the sampling frame $[t_0, t_1]$ as :



- It follows that an unemployment spell is composed of : [21]
  - elapsed unemp. duration (at the beginning of the sampling frame)
  - residual unemp. duration (during the sampling frame)

21. The observed length of an unemployment spell is therefore equal to the elapsed duration plus the residual duration.

Data

- A worker may exit unemployment :
  - during the sampling frame, in this case we have a complete unemployment spell and we observe the accepted wage.
  - after the sampling frame, in this case we have an incomplete unemployment spell. The spell is then said to be right-censored.

- Let us denote by :
  - t_u the observed duration of an unemployment spell
  - $\bar{t}_u$ the length of the sampling frame

  Hence unemployment spells longer than $t_u = \bar{t}_u$ are not observed and are right-censored.

- Let d_i denote an (individual) indicator function with :

$$d_i = \begin{cases} 1 \text{ if } t_u \leq \bar{t}_u & \text{(completed spell)} \\ 0 \text{ if } t_u > \bar{t}_u & \text{(right censored spell)} \end{cases}$$

- Let us denote by $f \equiv \lambda(1 - H(x))$, the exit rate from unemployment.
- Define :

$$\begin{aligned} \text{Cumulative distribution function} &= 1 - e^{-f.t_u} \\ \text{Survival function} &= e^{-f.t_u} \\ \text{Density of the duration} &= fe^{-f.t_u} \end{aligned}$$

- Assume that wages are distributed independently of job offer arrival times.
- The density of accepted wages :

$$h(w \,|w > x) = \frac{h(w)}{1 - H(x)}$$

and the c.d.f of accepted wages :

$$H(w \,|w > x) = \frac{H(w) - H(x)}{1 - H(x)}$$

- The likelihood of an individual observation, ℓ_i, is characterized by a tuple :

$$(t_{u_i}, w_i, d_i)$$

- From the assumption that wages are distributed independently of job offer arrival times, the joint density of duration times t_u and accepted wages w satisfies :

$$\underbrace{f e^{-f \cdot t_{u_i}}}_{\text{Prob. to have an unemp. duration } t_u} \times \underbrace{\frac{h(w_i)}{1 - H(x)}}_{\text{Prob. to get a wage } w}$$

- The probability that an individual has not exited unemployment (survived) at the end of the sampling frame is :

$$\underbrace{e^{-f \cdot \bar{t}_u}}_{\text{Prob. that } t_u \text{ exceeds } \bar{t}_u \text{ (length of the sampling frame)}}$$

- The likelihood of an individual observation satisfies :

$$\ell_i\left(t_{u_i}, w_i, d_i\right) = \left[\lambda e^{-f.t_{u_i}} h\left(w_i\right)\right]^{d_i} \left[e^{-f.\bar{t}_u}\right]^{1-d_i}$$

and the log-likelihood :

$$\begin{aligned}\log\left(\ell_i\left(t_{u_i}, w_i, d_i\right)\right) &= d_i.\left[\log\left(\lambda\right) - f.t_{u_i} + \log\left(h\left(w_i\right)\right)\right] \\ &\quad + \left(1 - d_i\right).\left(-f.\bar{t}_u\right)\end{aligned}$$

- The sample log-likelihood function, $\mathcal{L}$, satisfies :

$$\begin{aligned}\mathcal{L} &= \sum_i \log\left(\ell_i\left(t_{u_i}, w_i, d_i\right)\right) \\ &= \log\left(\lambda\right)\sum_i d_i + \sum_i d_i.\log\left(h\left(w_i\right)\right) - f\sum_i t_{u_i} \qquad (14)\end{aligned}$$

with the convention that $t_{u_i} = \bar{t}_u$ for $d_i = 0$, *i.e.* when the unemployment spell is *right-censored*.

- The structural estimation of the model requires maximizing (14) subject to some restrictions and parametric assumptions.

- Distributional assumption :

$$H(w) = H(w\,|\theta)$$

where θ is a vector of parameters.

- Distribution $H(w\,|\theta)$ is recoverable.

Definition : A distribution H is recoverable from a truncated distribution with know point of truncation $(x = rV_u)$ if knowledge of $H(w\,|w > x)$ and x implies that $H(w)$ is uniquely determined. [22]

22. See Flinn and Heckman (1982) for further details.

- Sketch of the estimation procedure :
  - Set externally parameter r
  - Choose a parametric form for H, e.g. Normal, Log-Normal, ...
  - Maximize the likelihood of a sample $\{(t_{u_i}, w_i, d_i),\ i = 1...N\}$ s.t. $w_i \geq x$

- Three stages estimation procedure :
  1. x is estimated by the lower accepted wage, $\hat{x} = \min(w_i)$
  2. λ and θ are estimated by maximizing the log-likelihood $\mathcal{L}$ conditional on $x = \hat{x}$
  3. z is estimated as :

$$\widehat{z} = \widehat{x} - \frac{\widehat{\lambda}}{r} \int_{\widehat{x}}^{\overline{w}} (w - \widehat{x})\, \mathrm{d}H(w \,|\, \widehat{\theta})$$

- **Implementation :** minimize $-\mathcal{L}$
  Typically use a search algorithm (e.g. built-in `fminsearch` in Matlab).

Estimation results of a two-state model (Flinn and Heckman, 1982)

Table 1

Estimates of the two-state equilibrium search model (standard errors in parentheses).[a]

λ (rate of arrival of job offers)	rV_u (reservation wage, \$/hr)	θ (parameter of exponential match distribution)	M (mean of normal match distribution)	σ^2 (variance of normal match distribution)	σ (rate of termination of jobs)
Exponential model					
0.201	1.5	0.339	—	—	0.035
(0.008)	—	(0.038)			(0.035)
Normal model					
0.1318	1.5	—	3.325	1.709	0.035
(0.004)	—		(0.34)	(0.16)	(0.035)

The mean of the accepted wage offer distribution is $\bar{x} = 3.45$ and the variance is 1.464.

Source : Flinn and Heckman (1982), *op. cit.*, 148-150

Estimation results of a two-state model (Flinn and Heckman, 1982)

Table 2

Implications of the estimates (standard errors in parentheses).

		Exponential match	Normal match
Implied cost of search, c	$r=0.05$	1.139 (0.616)	0.327 (0.148)
	$r=0.10$	2.692 (1.86)	1.4547 (0.643)
Implied exit rate from 'unemployment', $h_u(=\lambda(1-F(rV_u))$		1.21	1.21

Source : Flinn and Heckman (1982), *op. cit.*, 148-150

Lalive, van Ours and Zweimuller (2006)

- The job search model contains a number of predictions, which can potentially help policy makers.
- A mere correlation between the generosity of unemployment insurance benefit and the duration of unemployment is not a sufficient basis for inferring a cause-and-effect sequence.
- Empirical research that merely presents correlation without any convincing strategy, *identification strategy*, must be regarded as inadequate to ground an inference of causality and should be considered simply as descriptive.

- Natural experiments take advantage of policy changes or external shocks that exert varying effects on groups of persons whose characteristics are as unvarying as possible.

- Lalive et al. (2006) identify the causal effect of benefit duration on *accepting jobs or not* using a natural experiment. [23]

- They exploit a policy change introduced in 1989 by the Austrian government, which affected various unemployed workers differently.

23. Lalive, van Ours, & Zweimuller, (2006), How changes in financial incentives affect the duration of unemployment, Review of Economic Studies.

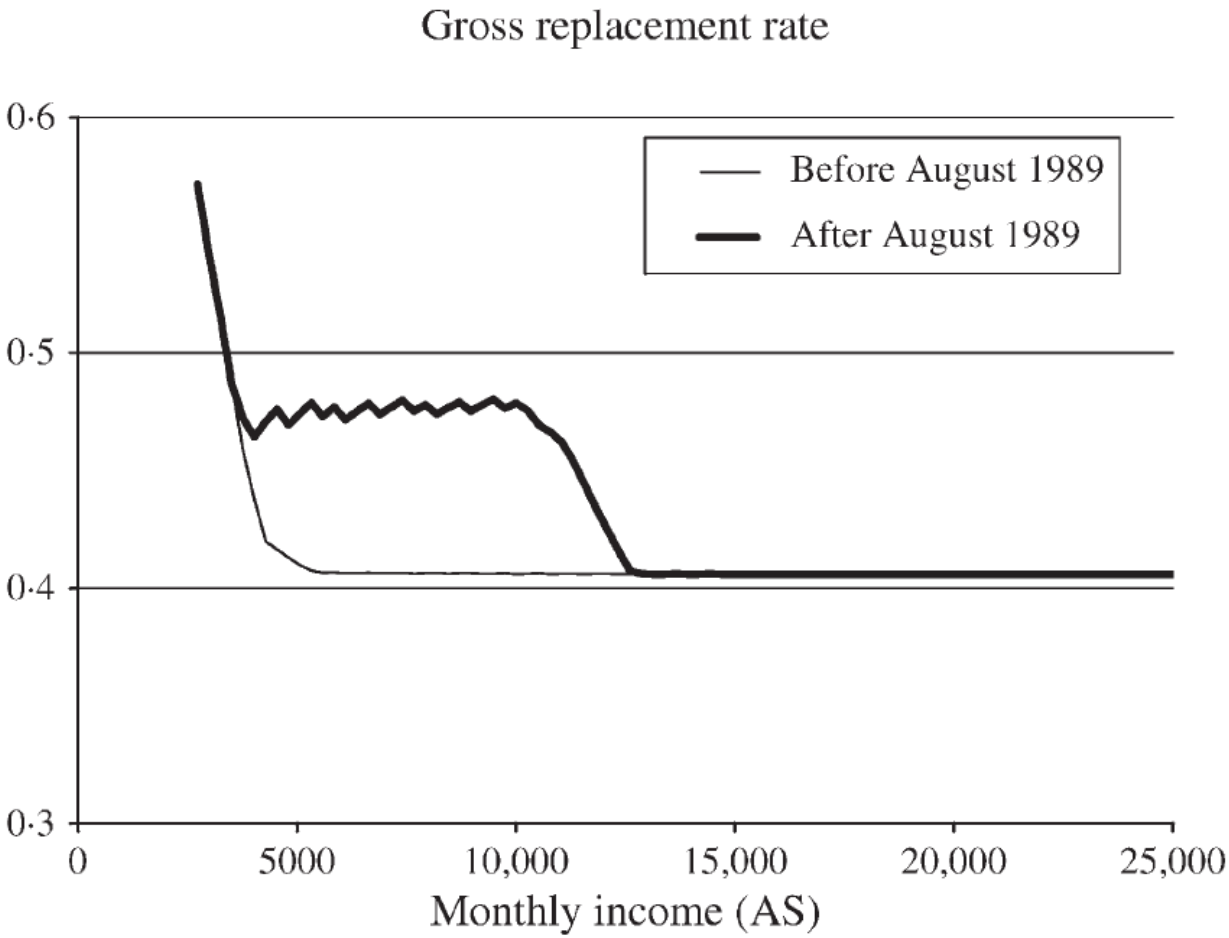
- The replacement ratio (RR) and/or the potential benefit duration (PBD) were modified depending on :
  1. age
  2. previous work experience
  3. previous gross monthly income
- In a nutshell :
  - For workers younger than 40, the PDB remained unchanged.
  - For workers older than 40 and with sufficient work experience, the PDB has increased :
    - from 30 to 39 weeks for the age group 40 to 49
    - from 30 to 52 weeks for workers aged 50 and older
  - For all workers earning below a certain threshold, the RR has increased by about 15%.

	Age			
	Younger that 40		40 and older	
	Work experience		Work experience	
Monthly income	Low	High	Low	High
$\leq 12,610$ Austrian Shillings	eRR	eRR	eRR	ePDB-RR
$> 12,610$ Austrian Shillings	Control	Control	Control	ePBD

TABLE 1 – Changes in the replacement ratio (RR) and in potential benefit duration (PBD) on 1 August 1989 in Austria)

Note : Work experience "Low" refers to less than 6 out of previous 10 years and less than 9 out of previous 15 years work experience. Work experience "High" refers to worked more than 6 out of previous 10 and worked more than 9 out of previous 15 years. ePBD : eligible for increase in potential benefit duration ; eRR : eligible for increase in replacement ratio ; ePDB-RR : eligible for increase in potential duration benefits and in replacement ratio.

Source : Lalive et al. (2006, Tab 2, p. 1018)



Source: Austrian federal laws (Bundesgessetzblätter) no. 594/1983, 364/1989.

- The policy change affects various unemployed workers *differently*.
- It is possible to distinguish four different groups :
  1. The first group experiences an increase in the replacement ratio (RR) *(treated)*.
  2. The second one experiences an extension of potential benefit duration (PDB) *(treated)*.
  3. The third group experiences both a higher replacement ratio and a longer potential benefit duration (RR-PDB) *(treated)*.
  4. The last group is not affected *(control)*.

- Let $\bar{Y}$ denote the average unemployment duration.
- The difference-in-differences (DiD) estimator : [24]

$$\tilde{\Delta}_{DD} = \left(\bar{Y}_A^T - \bar{Y}_B^T\right) - \left(\bar{Y}_A^C - \bar{Y}_B^C\right)$$

  - $\overline{Y}_B^T$ and $\overline{Y}_A^T$ are the average duration of unemployment for the treated group before and after the date of the reform.
  - $\overline{Y}_B^C$ and $\overline{Y}_A^C$ are the average duration of unemployment for the control group before and after the reform.
- Unemployment duration is defined as $t_u^{104} \equiv min(t_u, 104)$. This definition discards unemployment durations greater than 104 weeks.

24. **T** for Treated, **C** for Control, **A** for After and **B** for Before.

	Before August 1989	After August 1989	Change (after-before)	Diff-in-diff (compared to control)
ePBD group	16.25	18.67	2.42	1.13
	(0.08)	(0.09)	(0.12)	(0.18)
N	48,294	51,110		
eRR group	17.79	20.03	2.24	0.96
	(0.12)	(0.16)	(0.20)	(0.24)
N	17,160	15,310		
ePBD-RR group	19.01	23.55	4.53	3.25
	(0.17)	(0.24)	(0.20)	(0.24)
N	11,992	9,182		
Control group	15.24	16.52	1.29	
	(0.08)	(0.09)	(0.13)	
N	33,815	38,958		

TABLE 2 – Average unemployment duration in first 104 weeks (measured in weeks).

Note : Standard errors in parentheses. N : number of unemployment spells in the group. Diff-in-diff : difference-in-difference ; RR : replacement rate ; PBD : potential benefits duration ; ePBD : eligible for increase in potential benefit duration ; eRR : eligible for increase in benefit RR ; ePBD–RR : eligible for both.

Source : Lalive et al. (2006, Tab 4, p. 1020)

- Column 3 of the previous table shows that the duration of unemployment rises for all groups after August 1989, *i.e.* after the reform.

- However, column 4 reveals that the average duration of unemployment *rose more* for the groups that benefited from more generous unemployment insurance after that date.

- It is possible to go beyond average durations by estimating the impact of the reform on the rates of exit from unemployment as a function of the amount of time spent unemployed.

- Duration models (either non-parametric or parametric) are of particular interest here. [25]

- Let us denote by T a non-negative random variable representing the waiting time (survival period) until the occurrence of an event (death), say the exit rate from unemployment. [26]

- In our empirical context $F(t)$ is the prob. that the unemployment spell lasts less than t periods.

25. See e.g. Lancaster (1990) for a comprehensive presentation.
26. This random variable is characterised by its cdf, $F(t) = \Pr\{T < t\}$ and its pdf $f(t)$.

- Let us define : [27]
  - The survival function, $S(t)$:

$$S(t) = 1 - F(t)$$

  - The hazard function, $h(t)$:

$$h(t) = \frac{f(t)}{S(t)}$$

- The hazard function, $h(t)$, is the instantaneous probability of exiting from unemployment provided a worker has been unemployed for at least a period of length t.

27. See Handout #2 for details.

- The hazard function allows us to characterize the notion of "duration dependence" :
  - Positive duration dependence : the probability of getting a job increases with the amount of time t already spent unemployed
  - Negative duration dependence : the probability of getting a job diminishes with the amount of time t already passed in this state
- It should be noted that the hazard function is not necessarily monotonic : it may increase for certain values of t and diminish for others.
- The hazard function may equally be independent to the length of an unemployment spell.

- The non-parametric approach makes no hypothesis about the form of the distribution of the durations of unemployment spells and is given by the Kaplan-Meier estimator of the hazard function.
- The Kaplan-Meier estimator of the hazard function allows to visualize the rates of exit from unemployment before and after the reform.
- Presence of peaks at 30, 39 and 52 weeks suggests that the reform indeed has a causal impact on exit rates from unemployment.

Estimation results (Kaplan-Meier)

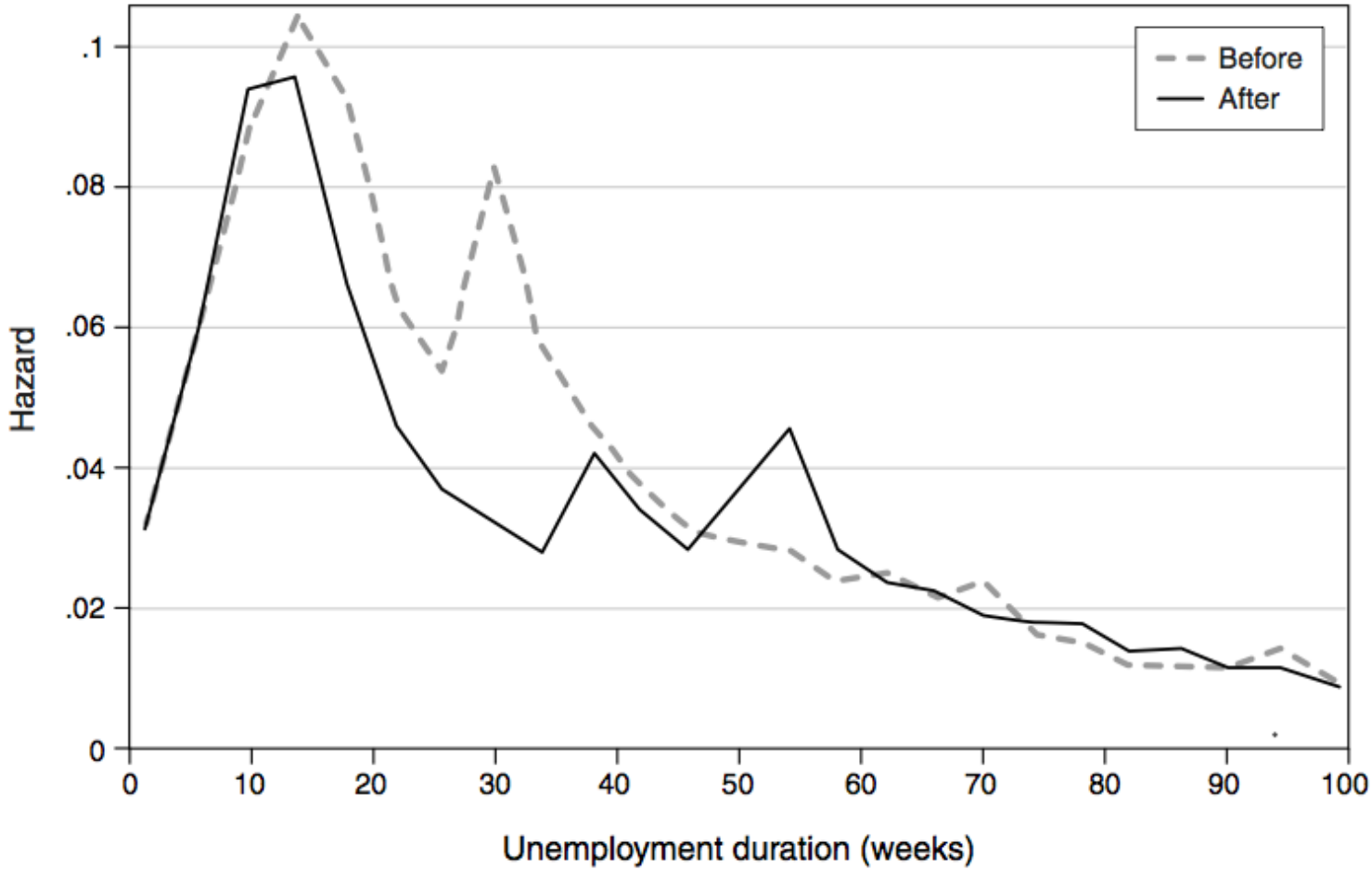


FIGURE 5.5

Kaplan-Meier hazard functions before and after the reform for the group of individuals potentially eligible to the extension of potential duration of benefits. Hazard functions are smoothed on 4-week windows.

Source: Data from Lalive et al. (2006).

- The non-parametric estimation allows us to observe the alterations in the average rates of exit from unemployment for the treated and control group.
- However, it does not allow us to estimate the impact of unemployment insurance modification conditional upon an array of explanatory variables.
- To complement the analysis, it is useful to proceed with parametric estimation methods.
- It allows to analyze the impact of changes in the UIS as a function of observable individual characteristics. [28]

28. See Lalive et al. (2006) for further details.

Estimation results (proportional hazard model)

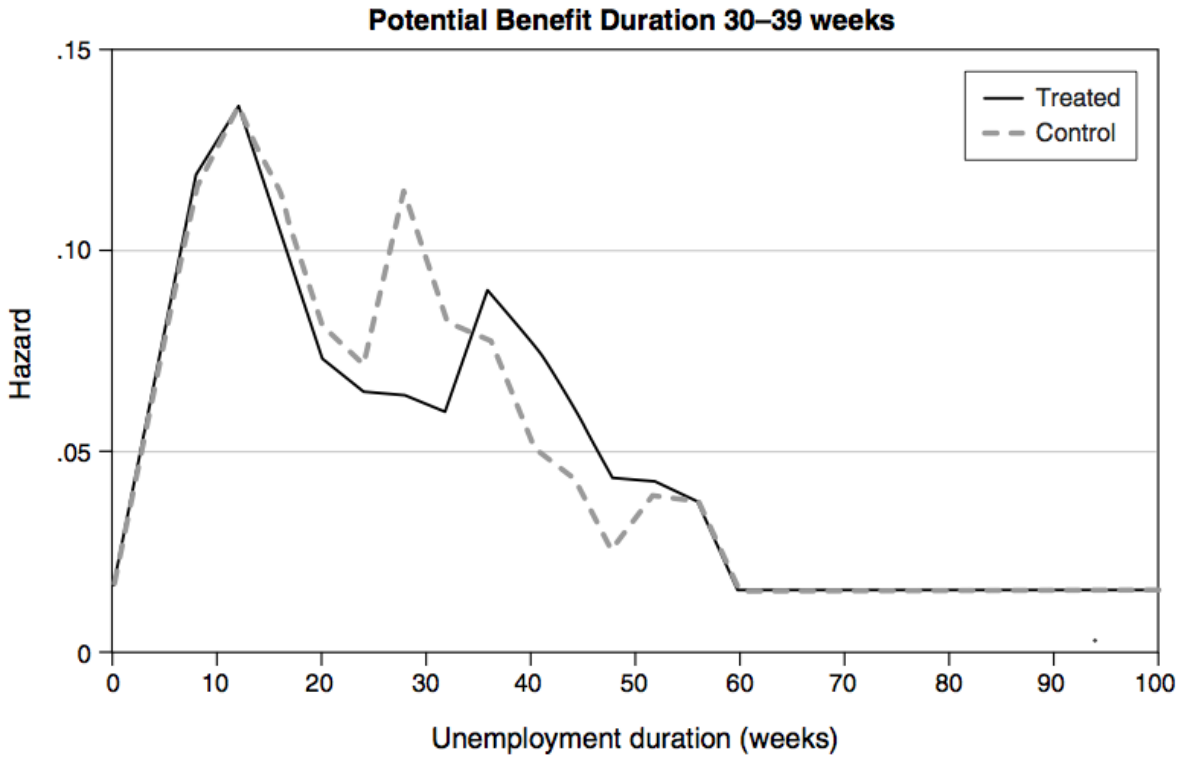


FIGURE 5.7
Estimated average treated and control hazard rates for the group who benefited from the extension of the potential benefit duration.

Source: Lalive et al. (2006, figure 5, p. 1026).

Remark : This approach allows for difference across treatments and across individuals. Here the population of individuals aged 40-49 with high work experience and high monthly income whose PDB has been increased from 30 to 39 weeks.

- Lalive et al. (2006) show that potential benefit duration and the replacement rate exert significant effects on the duration of unemployment.

- The magnitude of the effects of unemployment benefits differs for different countries and different types of policy changes : [29]
  - An increase of the replacement ratio of 1% leads to an increase in the duration of unemployment ranging between 0.4 and 1.6 of a week.
  - An increase of a week in the potential duration of benefit payments leads to an increase in the duration of unemployment ranging between 0.1 and 0.4 of a week.

- Numerous studies highlight a significant discontinuity in the exit rate from unemployment just before the exhaustion of entitlement to UB (as in Lalive et al., 2006).

29. Tatsiramos & van Ours (2014), Labor market effects of unemployment insurance, Journal of Economic Surveys.

- Does a more generous insurance system *improve* the quality of the matches ?
- **Theory :** Acemoglu and Shimer (2000) show that more generous unemployment insurance increases labor productivity by encouraging workers to seek higher productivity jobs. [30]
- **Empirics :** Tatsiramos (2009) documents a beneficial effect of unemployment insurance on employment stability. [31]
- This effect is more pronounced in countries with relatively generous benefit systems.

30. Acemoglu & Shimer (2000), Productivity gains from unemployment insurance, European Economic Review.

31. Tatsiramos (2009), Unemployment Insurance in Europe : Unemployment Duration and Subsequent Employment Stability, Journal of the European Economic Association.

Two complementary approaches : [32]

1. **Structural approach**
   - theoretical mechanisms
   - counterfactuals
   - welfare

2. **Reduced form approach**
   - fewer assumptions
   - direct evidence
   - partial equilibrium

All approaches *reduced form* and *structural* rely on theoretical assumptions. They have **complimentary roles** to play.

Recent articles in the state of the art attempt to combine the two approaches.

32. See e.g. Keane (2010), A structural perspective on the experimentalist school, Journal of Economic Perspectives for a thorough discussion.

- The job search model explains how imperfect information about job offers generate frictions on the labor market.

- The model can be extended (more or less easily) in a number of directions to make it (even) more realistic e.g. :
  - Lentz and Tranaes (2005) study how wealth affect job search.
  - Guler, Guvenen and Violante (2012) study how couple may make joint decisions about job search.
  - Schmutz and Sidibe (2019) study how spacial frictions affect job search.

- A recent strand of the literature nests the job search model in behavioral economics and is interested in a number of topics : [33]
  - What if the job seeker is impatient ?
  - What if the job seeker has wrong beliefs ?
  - What if the job seeker has reference-dependent preferences ?

- The job search model could also be readily extended to study optimal unemployment insurance (OUI).

  A good example is Shimer and Werning (2007) who extend the basic job search model to determine the optimal parameters (b, τ) of an UI system in an environment where workers are risk averse. [34]

33. See e.g. Della Vigna et al. (2017), Reference dependent job search : Evidence from Hungary, Quarterly Journal of Economics.
34. Shimer and Werning (2007), Reservation Wages and Unemployment Insurance, Quarterly Journal of Economics.

- The model is not without limits.

1. The basic job search model takes the distribution of wages as given and hence leaves the setting of wages unexplained.

   In the next lecture, we will focus on equilibrium search models that render endogeneous the wage distribution.

2. The basic job search model pays little (if any) attention to labor demand.

   In the last two lectures, we will focus on search and matching models that discipline labor demand and explain firm's behavior.

Effects of an increase of the sanctions on search efforts

- From V_s, differentiation wrt s yields (with $r = \rho + \delta$) :

$$(r + \alpha e_s) \frac{\mathrm{d}V_s}{\mathrm{d}s} = -1$$

- Making use of $\frac{\mathrm{d}V_s}{\mathrm{d}s} < 0$, the differentiation of the foc $c'(e_s) = \alpha (V_e - V_s)$ wrt s yields :

$$c''(e_s) \frac{\mathrm{d}e_s}{\mathrm{d}s} = -\alpha \frac{\mathrm{d}V_s}{\mathrm{d}s} > 0$$

- From V_u, differentiation wrt s yields :

$$(r + \alpha e_u + \sigma(e_u)) \frac{\mathrm{d}V_u}{\mathrm{d}s} = \sigma(e_u) \frac{\mathrm{d}V_s}{\mathrm{d}s} < 0$$

- Making use of $\frac{\mathrm{d}V_u}{\mathrm{d}s} < 0$, the differentiation of the foc $c'(e_u) = \alpha (V_e - V_u) + \sigma'(e_u)(V_s - V_u)$ wrt s yields :

$$\underbrace{[c''(e_u) - \sigma''(e_u)(V_s - V_u)]}_{+} \frac{\mathrm{d}e_u}{\mathrm{d}s} = \underbrace{\frac{(r + \alpha e_u)\sigma'(e_u) - \alpha\sigma(e_u)}{\sigma(e_u)}}_{-} \underbrace{\frac{\mathrm{d}V_u}{\mathrm{d}s}}_{-} > 0$$

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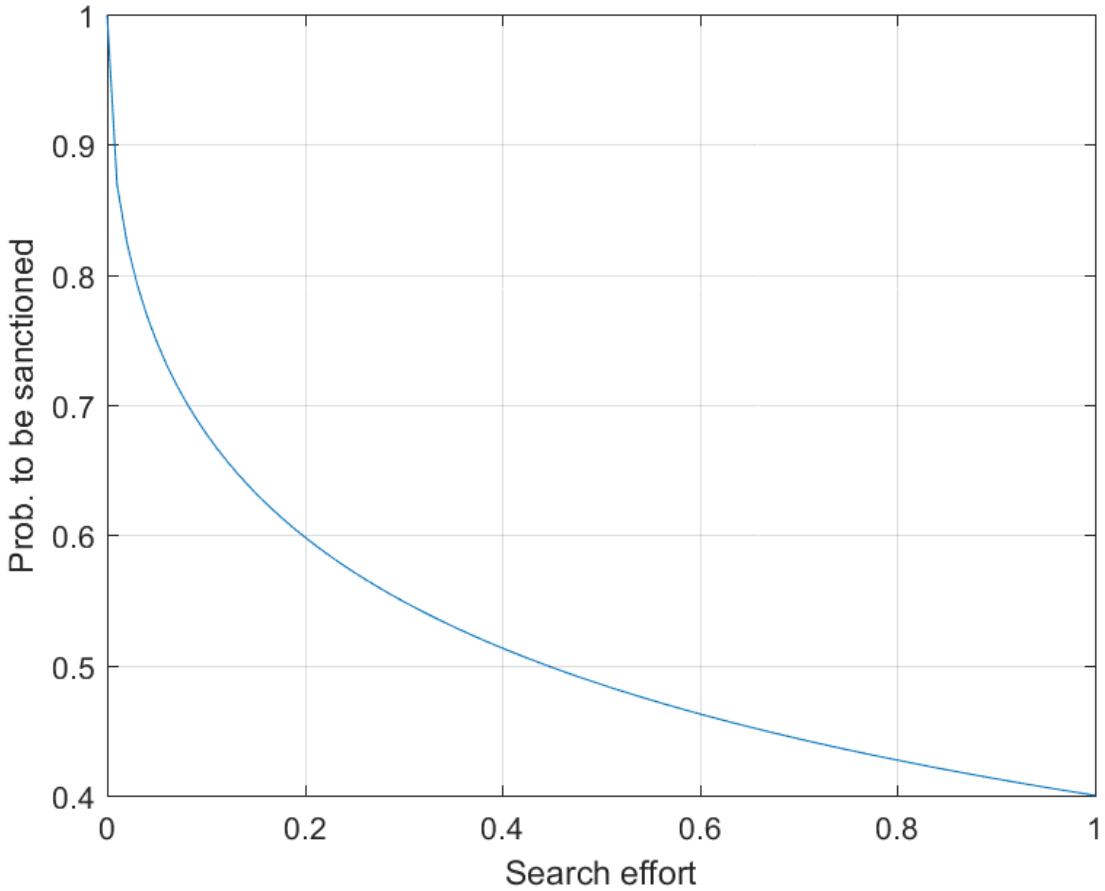
Prob. to be sanctioned $\sigma(e)$ as a function of effort e



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JTJ transitions in OECD countries (2005-2012)

Note: Labour market transitions for European countries are computed as the number of working-age individuals moving between two statuses from one year to another as a share of average employment between these two years. Job-to-Job transitions measure job changes from one job to another. Hirings from non-employment and separations to non-employment include transitions from and to both unemployment and inactivity.

Source : OECD (2019). Labour market transitions in a cross-country comparative perspective in 2019. [35]

▸ Go Back

35. Causa, Luu and Abendschein (2021), Labour market transitions across OECD countries : Stylised facts, *OECD Economics Department Working Paper 1692*.