SVAR-based supply and demand decomposition of inflation

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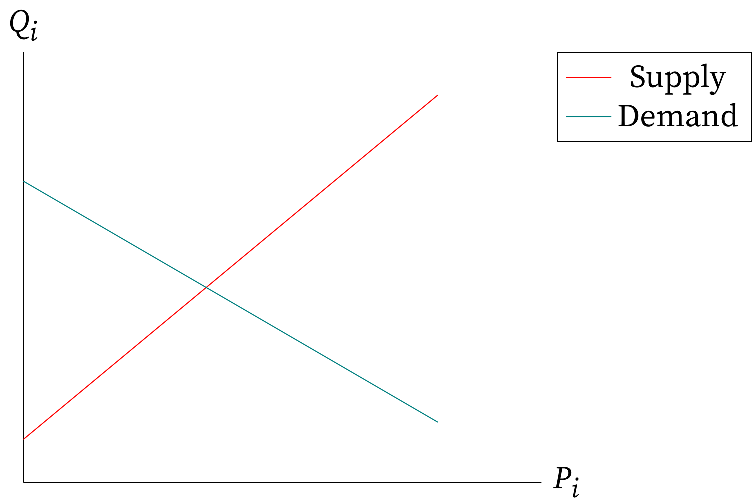
Available at https://github.com/PierreRlld/SVAR3A

1. Methodology and Data

1.1. Basis of *Shapiro (2022)*

The methodology proposed by Adam Hale Shapiro in his 2022 paper "Decomposing Supply and Demand Driven Inflation" takes its foundations in the work of Jump and Kohler (2022). The core assumption of the framework is that for each sector/section (i) of the inflation decomposition, an upward sloping in price supply curve and a downward demand curve can be assumed.



In theory, assuming $\sigma_i > 0$ and $\delta_i > 0$, we have:

$$\begin{aligned} \text{Supply :} \quad & Q_i = \sigma_i . P_i + \alpha_i \\ \text{Demand :} \quad & P_i = -\delta_i . Q_i + \beta_i \end{aligned}$$

Shocks are then defined as vertical movements of the curves :

$$\begin{aligned} \text{Supply shock : } \varepsilon_t^s = \Delta\alpha_i \ & = (Q_{i,t} - \sigma_i P_{i,t}) - (Q_{i,t-1} - \sigma_i P_{i,t-1}) \\ \rightarrow \varepsilon_t^s & = \Delta Q_{i,t} - \sigma_i \Delta P_{i,t} \\ \text{Demand shock : } \varepsilon_t^d = \Delta\beta_i \ & = (P_{i,t} + \delta_i Q_{i,t}) - (P_{i,t-1} + \delta_i Q_{i,t-1}) \\ \rightarrow \varepsilon_t^d & = \Delta P_{i,t} + \delta_i \Delta Q_{i,t} \end{aligned}$$

We can easily show that the two previous equations lead to :

$$\begin{cases} \Delta Q_{i,t} & = \frac{1}{1+\delta_i}(\varepsilon_t^s + \sigma_i.\varepsilon_t^d) \\ \Delta P_{i,t} & = \frac{1}{\sigma_i(1+\delta_i)}(\sigma_i.\varepsilon_t^d - \delta_i.\varepsilon_t^s) \end{cases}$$

As we assume $\sigma_i > 0$ and $\delta_i > 0$, we can derive expected comovements between the two variables following supply and demand shocks.

$$\begin{cases} \text{Supply shock } \Delta^+\varepsilon^s & : \Delta^+ Q \quad \& \quad \Delta^- P > \text{Negative comovements (S)} \\ \text{Demand shock } \Delta^+\varepsilon^d & : \Delta^+ Q \quad \& \quad \Delta^+ P > \text{Positive comovements (D)} \end{cases}$$

Consider the following structural VAR of (arbitrary) order p (dropping i indices). Let $z_t = \begin{bmatrix} \Delta Q_t \\ \Delta P_t \end{bmatrix}$ and the structural shocks $\varepsilon_t = \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \end{bmatrix}$:

$$A.\begin{bmatrix} \Delta Q_t \\ \Delta P_t \end{bmatrix} = \mu + \sum_{i=1}^{p} A_i.z_{t-i} + \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \end{bmatrix}$$

With ν_t the residuals of the estimated reduced-form VAR(p) we should have $\nu_t = A^{-1}.\varepsilon_t$
Let A satisfy $A \equiv \begin{pmatrix} 1 & -\alpha \\ \beta & 1 \end{pmatrix}$ with $\alpha, \beta > 0$, it follows that $A^{-1} = \frac{1}{1+\alpha\beta}\begin{pmatrix} 1 & \alpha \\ -\beta & 1 \end{pmatrix}$
Omitting the t indices we have :

$$\begin{bmatrix} \nu^s \\ \nu^d \end{bmatrix} = A^{-1}.\begin{bmatrix} \varepsilon^s \\ \varepsilon^d \end{bmatrix}$$

$$\begin{bmatrix} \nu^s \\ \nu^d \end{bmatrix} = \frac{1}{1+\alpha\beta}\begin{pmatrix} 1 & \alpha \\ -\beta & 1 \end{pmatrix}\begin{bmatrix} \varepsilon^s \\ \varepsilon^d \end{bmatrix}$$

As $\frac{1}{1+\alpha\beta} > 0$, we finally have:

$$\begin{cases} \nu^s & \propto \varepsilon^s + \alpha.\varepsilon^d \\ \nu^d & \propto -\beta.\varepsilon^s + \varepsilon^d \end{cases}$$

Which leads to:

$$\begin{cases} \varepsilon^s > 0\,,\ \varepsilon^d > 0 & \Rightarrow \nu^s > 0 \\ \varepsilon^s < 0\,,\ \varepsilon^d < 0 & \Rightarrow \nu^s < 0 \\ \varepsilon^s < 0\,,\ \varepsilon^d > 0 & \Rightarrow \nu^d > 0 \\ \varepsilon^s > 0\,,\ \varepsilon^d < 0 & \Rightarrow \nu^d < 0 \end{cases}$$

Looking at the signs of ν^s and ν^d we can then back-track and derive the signs of ε^s and ε^d. For example when $\nu^s > 0$ and $\nu^d < 0$ we have $\varepsilon^s > 0$. We derive:

$$\begin{cases} \nu^s > 0,\ \nu^d < 0 \Rightarrow \varepsilon^s > 0 & \text{+ Supply shock} \quad (1) \\ \nu^s < 0,\ \nu^d > 0 \Rightarrow \varepsilon^s < 0 & \text{- Suppply shock} \quad (2) \\ \nu^s > 0,\ \nu^d > 0 \Rightarrow \varepsilon^d > 0 & \text{+ Demand shock} \quad (3) \\ \nu^s < 0,\ \nu^d < 0 \Rightarrow \varepsilon^d < 0 & \text{- Demand shock} \quad (4) \end{cases}$$

Thus, from (1) and (2) we see that reduced-form errors of opposite sign (negative comovements of P and Q) are associated with demand shocks, which is consistent with theory (S). Similarly, from (3) and (4) reduced-form errors of the same sign (positive comovements of P and Q) are associated with demand shocks, again consistent with theory (D).

Assuming that A in the structural VAR is of the form $\begin{pmatrix} 1 & -\alpha \\ \beta & 1 \end{pmatrix}$ with $\alpha, \beta > 0$, or more generally $A = \begin{pmatrix} a_{11} > 0 & a_{12} < 0 \\ a_{21} > 0 & a_{22} > 0 \end{pmatrix}$, thus **ensures** that expected structural shocks' effects on the covariates derived from theory are consistent in the model. We can then infer aforementioned effects from the reduced-form residuals that are in practice derived from the estimation of a VAR(p) model.

1.2. Data for European countries

In the original paper from Shapiro the data used are price, quantity and expenditures from the personal consumption expenditure (PCE) data with fourth level of disaggregation of the Bureau of Economic Analysis (BEA). However it is more complicated when considering European countries. Indeed, similar quantity series are not readily available for each of the 4-digit COICOP classification component of the HICP.

To tackle this issue, we follow the indication provided by Eduardo Goncalves and Gerrit Koester in the ECB Economic Bulletin 7 of 2022. We consider the turnover series from the short-term statistics data of the ECB as proxy for demand, and once deflated as proxy for the quantity series. The main issue that remains is that the classification of sectors in the HICP (COICOP) is different from the classification of sectors used in the short-term statistics data (NACE Rev.2). We then try to match as best as possible each COICOP sector to a NACE Rev.2 counterpart. Out of the 94 4-digit COICOP sectors, we managed to create 75 pairs. On average that is around 80% of the overall HICP that has been matched. In what follows we refer for each sector the 'quantity index' as the real matched turnover series (CPI deflated).

2. Estimation

2.1. Classification from *Shapiro (2022)*

2.1.1. Baseline classification

With our price-quantity pairs matched for 75 sectors, we run the following monthly VAR for each of the sectors denoted by index (i) :

$$\begin{bmatrix} \Delta q_{i,t} \\ \Delta p_{i,t} \end{bmatrix} = \sum_{j=1}^{p} A_j \cdot \begin{bmatrix} \Delta q_{i,t-j} \\ \Delta p_{i,t-j} \end{bmatrix} + \begin{bmatrix} \nu^q_{i,t} \\ \nu^p_{i,t} \end{bmatrix}$$

Where $\Delta q_{i,t}$ and $\Delta p_{i,t}$ are the demeaned first-difference of the log-transformed quantity and price indices ($x_{i,t} = \log(X_{i,t})$). This assumes $\Delta q_{i,t}$ and $\Delta p_{i,t}$ are second-order stationary, which seems to be the case.

In our implementation of the methodology, we let the order p of the VAR be automatically selected via minimisation of a statistical criterion (AIC or BIC) for each sector instead of imposing $p = 12$ as done by Shapiro. Even though he explains the classification seems to be robust to the lag order selection we would rather let it be sector-specific than adding another constraint on the methodology.

Note that we said 75 sectors had matched proxy quantity series, but we cannot run 75 VAR. Indeed, even though we managed to match the series we still had some data missing in the series themselves. Thus, for each zone in our study (France, Germany, Spain and EU27) we flag sectors that have data problems for either the price or quantity series. A sector being flagged mainly comes from the fact that the latest data available is before 2023 (some stop in 2022 or 2020 and some even in 2015). Overall we run regressions on 52 (EU27) to 72 (France) sectors.

The estimated reduced-form residuals (ν_t^q, ν_t^p) (equivalent to (ν_t^s, ν_t^d) in the previous section) are then used to label each sector in each month using the following rule :

$$\mathbb{1}_{i,sup(+)}(t) = \begin{cases} 1 & \text{if } \nu_{i,t}^q > 0, \nu_{i,t}^p < 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{1}_{i,sup(-)}(t) = \begin{cases} 1 & \text{if } \nu_{i,t}^q < 0, \nu_{i,t}^p > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{1}_{i,dem(+)}(t) = \begin{cases} 1 & \text{if } \nu_{i,t}^q > 0, \nu_{i,t}^p > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{1}_{i,dem(-)}(t) = \begin{cases} 1 & \text{if } \nu_{i,t}^q < 0, \nu_{i,t}^p < 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \mathbb{1}_{i,sup}(t) = \mathbb{1}_{i,sup(+)}(t) + \mathbb{1}_{i,sup(-)}(t)$$

$$\Rightarrow \mathbb{1}_{i,dem}(t) = \mathbb{1}_{i,dem(+)}(t) + \mathbb{1}_{i,dem(-)}(t)$$

It follows that each month, overall month-over-month inflation $\pi_{t,t-1}$ can be devided into supply and demand driven components. As the overall CPI is a weighted average of its sectorial inflation $\pi_{i,t,t-1}$ we denote $\omega_{i,y}$ the weight of sector (i) at year y in the CPI. In the following equation y_t means the year of month t.

$$\pi_{t,t-1} = \sum_i \mathbb{1}_{i,sup}(t)\omega_{i,y_t}\pi_{i,t,t-1} + \sum_i \mathbb{1}_{i,dem}(t)\omega_{i,y_t}\pi_{i,t,t-1}$$

$$\pi_{t,t-1} = \pi_{t,t-1}^{sup} + \pi_{t,t-1}^{dem}$$

We present results for year-over-year inflation, which is simply given by:

$$\begin{cases} \pi^{sup}_{t,t-12} & = \sum_{k=0}^{11} \pi^{sup}_{t-k,t-k-1} \\ \pi^{dem}_{t,t-12} & = \sum_{k=0}^{11} \pi^{dem}_{t-k,t-k-1} \end{cases}$$

2.1.2. Robustness check

Shapiro introduced several alternative classification methods which we replicated to assess whether the methodology was robust enough. In our version, as the lag order for each VAR is automatically selected (and if we believe the selected lag is correct) there should not be specification error.

We implement the the two alternative methods that are the smoothed-errors classification (to offset possible measurement errors) and the parametric approach. In the latter, each month a sector is no longer deterministically classified as experiencing a supply or a demand shock but there rather is a probability p_t^K that it is experiencing a shock of type $K \in \{\text{supply,demand}\}$.

2.2. Classification from *Sheremirov (2022)*

The algorithm proposed by Sheremirov is as follows, for each sector i and month t :

1. Compute year-over-year inflation and (real) consumption/demand growth : $\pi_{i,t}$, $c_{i,t}$
2. Classify the sector i as experiencing a demand shock at month t if both inflation and consumption growth are over their 2001-2019 respective average $\tilde{\pi}_i$, $\tilde{c}_i$:

$$\mathbb{1}_{i,dem}(t) = \begin{cases} 1 & \text{if } (\pi_{i,t} - \tilde{\pi}_i).(c_{i,t} - \tilde{c}_i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{1}_{i,sup}(t) = 1 - \mathbb{1}_{i,dem}(t)$$

3. Classify the shock at month t as:

$$\text{Persistent demand } \mathbb{1}^{pers}_{i,dem}(t) = \begin{cases} 1 & \text{if } \sum_{k=0}^{11} \mathbb{1}_{i,dem}(t-k) \geq 11 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Persistent demand } \mathbb{1}^{pers}_{i,sup}(t) = \begin{cases} 1 & \text{if } \sum_{k=0}^{11} \mathbb{1}_{i,sup}(t-k) \geq 11 \\ 0 & \text{otherwise} \end{cases}$$

3. If neither condition is verified, month t is classified as transitory demand (if $\mathbb{1}_{i,dem}(t) = 1$) or transitory supply.

3. Results