Assignment 03

ⓘ This is a preview of the published version of the quiz

Started: Nov 12 at 4:06pm

Quiz Instructions

Fill out this assignment in Canvas. Most questions can be directly answered in Canvas. Often you have to fill out a free-form text field: make use of the HTML options and the equation editor, if you feel comfortable with it. If not, these questions can also be answered with just simple ASCII text.

Some questions require you to upload your answer as a PDF, since the solutions are too complex for a structured input in Canvas. Make sure the content of your PDF is legible.

The quiz **saves itself automatically** whenever you change any input. You can quit the browser and return later to continue filling it out. However, you do **have to submit** once you are done with everything.

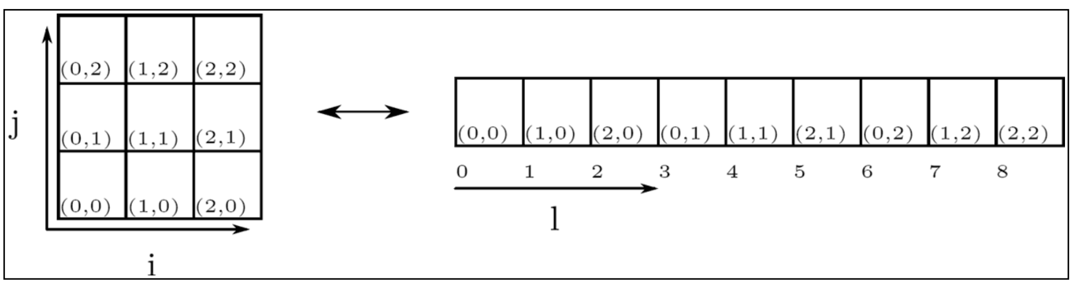
After submission, you can re-do this assignment as often as you like. However, it will not load your previous answers. The latest attempt is kept. We do not take the number of attempts into account, nor the time that you need to fill out the form.

Question 1

5 pts

Linear Index Mapping

The cells of a structured grid are usually saved linearly in memory. For example, in 2D the cells (i, j) are saved to the linear memory space at indices l according to the following scheme:



Let a structured grid in 3D consist of (n_x, n_y, n_z) grid points in each dimension.

a. Find the map $(i, j, k) \to l$ that maps 3D cell indices to linear cell indices. Assume that the first dimension (i) varies fastest in the linear memory space. *(2 points)*
b. Find the map $l \to (i, j, k)$ that maps linear cell indices to 3D cell indices. Assume again that the first dimension (i) varies fastest in the linear memory space. *(3 points)*

HTML Editor

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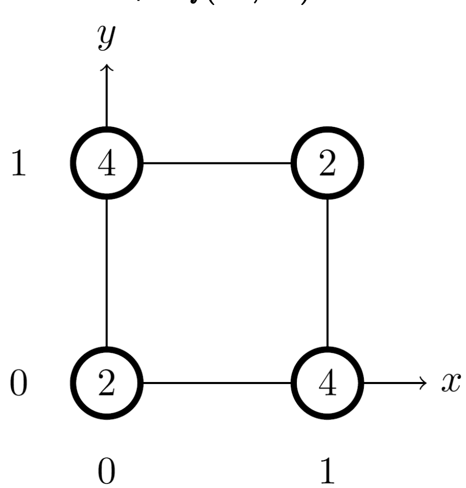
0 words

Question 2

1 pts

Bilinear Interpolation

Given is the bilinear function $f(x, y)$ with $f(0,0) = 2$, $f(1,0) = 4$, $f(0,1) = 4$, $f(1,1) = 2$ as shown in the figure below. Compute $f(0.5, 0.5)$!



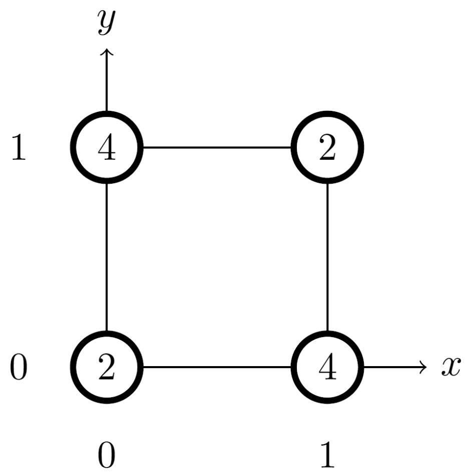
Question 3

3 pts

Bilinear Interpolation (Gradient Formula)

Given is the bilinear function $f(x, y)$ with $f(0,0) = 2$, $f(1,0) = 4$, $f(0,1) = 4$, $f(1,1) = 2$ as shown in the figure below. Determine the formula for the gradient of f.

Hint: Write down a short derivation (Härledning) and the gradient formula. You do not need to simplify the gradient formula.



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12pt Paragraph

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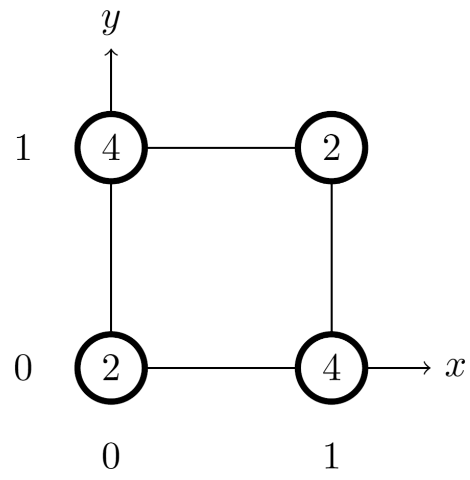
Question 4

1 pts

Bilinear Interpolation (Gradient)

Given is the bilinear function $f(x, y)$ with $f(0,0) = 2$, $f(1,0) = 4$, $f(0,1) = 4$, $f(1,1) = 2$ as shown in the figure below. Compute the gradient at $(0.5, 0.5)$!

$\nabla f(0.5, 0.5) =$



Question 5

2 pts

Interpolation I

Given are 3 points $\mathbf{p}_0 = (0,0), \mathbf{p}_1 = (1,0), \mathbf{p}_2 = (0,1)$ equipped with the scalar values $s_0 = 1, s_1 = 2, s_2 = 3$.

Determine the interpolating function $s(x,y)$ (i.e., write down the formula) for linear interpolation.

HTML Editor

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0 words

Question 6

0.5 pts

Interpolation IIa

Consider the interpolation function $s(x,y)$ from the task above *(Interpolation I)*. Compute the following value:

$s(1/2, 0) =$

Question 7

0.5 pts

Interpolation IIb

Consider the interpolation function $s(x, y)$ from the task above *(Interpolation I)*. Compute the following value:

$s(0, 1/2) =$

Question 8

0.5 pts

Interpolation IIc

Consider the interpolation function $s(x, y)$ from the task above *(Interpolation I)*. Compute the following value:

$s(1/2, 1/2) =$

Question 9

0.5 pts

Interpolation IId

Consider the interpolation function $s(x, y)$ from the task above *(Interpolation I)*. Compute the following value:

$s(2/3, 2/3) =$

Question 10

4 pts

Triangle Basis Function Gradients

Given is a 2D triangle with coefficients f_0, f_1, and f_2 at the vertices $\mathbf{x}_0$, $\mathbf{x}_1$, and $\mathbf{x}_2$ that are interpolated linearly on a triangle by

$$f(x, y) = \alpha_0(x, y)\, f_0 + \alpha_1(x, y)\, f_1 + \alpha_2(x, y)\, f_2$$

using linear barycentric coordinates α_i as basis functions.

The constant gradient of this function

$$\nabla f = \nabla\alpha_0\, f_0 + \nabla\alpha_1\, f_1 + \nabla\alpha_2\, f_2$$

is represented as the linear combination of constant basis function gradient vectors $\nabla\alpha_i \in \mathbb{R}^2$.

Determine closed form expressions of all $\nabla\alpha_i$ with respect to the vertex coordinates $\mathbf{x}_i$. Is there a geometric interpretation of these basis function gradients?

HTML Editor

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12pt Paragraph

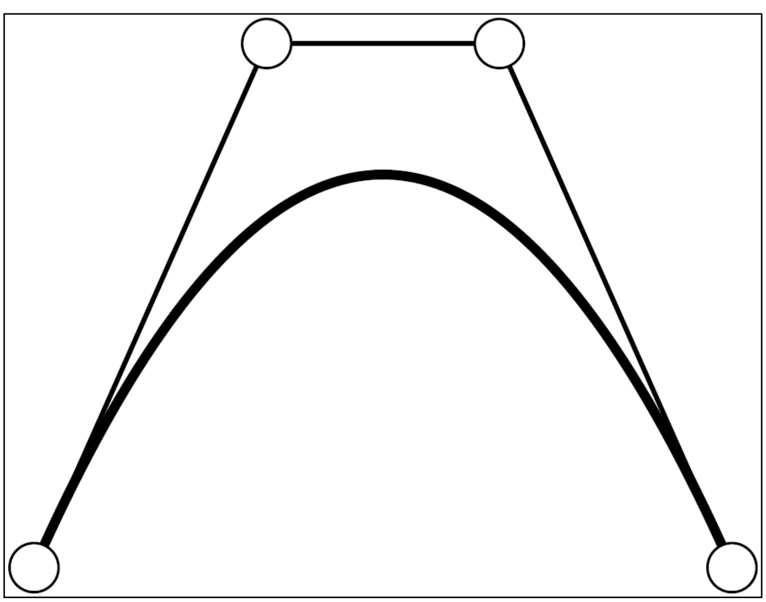
0 words

Question 11

1 pts

Bézier Curve Properties I

The following figure shows a curve and its supposed control polygon. Does the control polygon correspond to the curve? If not, check all violated properties of Bézier curves!



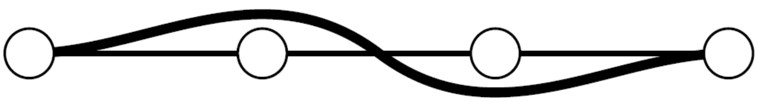
- [ ] The control polygon corresponds to the curve. No Bézier curve property is violated.
- [ ] Violates the end point interpolation property of Bézier curves.
- [ ] Violates the convex hull property of Bézier curves.
- [ ] Violates the variation diminishing property of Bézier curves.
- [ ] Violates the linear precision property of Bézier curves.
- [ ] Violates the first derivative property of Bézier curves at the end points.

Question 12

1 pts

Bézier Curve Properties II

The following figure shows a curve and its supposed control polygon. Does the control polygon correspond to the curve? If not, check all violated properties of Bézier curves!



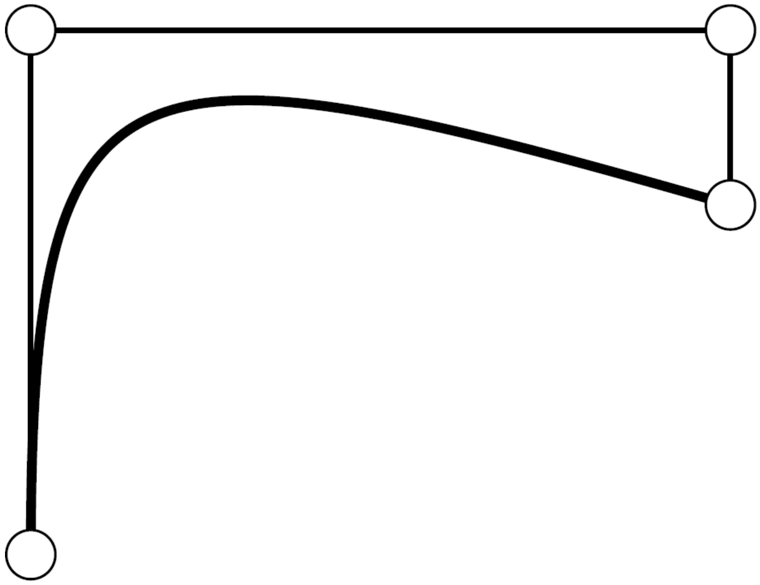
- [ ] The control polygon corresponds to the curve. No Bézier curve property is violated.
- [ ] Violates the end point interpolation property of Bézier curves.
- [ ] Violates the convex hull property of Bézier curves.
- [ ] Violates the variation diminishing property of Bézier curves.
- [ ] Violates the linear precision property of Bézier curves.
- [ ] Violates the first derivative property of Bézier curves at the end points.

Question 13

1 pts

Bézier Curve Properties III

The following figure shows a curve and its supposed control polygon. Does the control polygon correspond to the curve? If not, check all violated properties of Bézier curves!



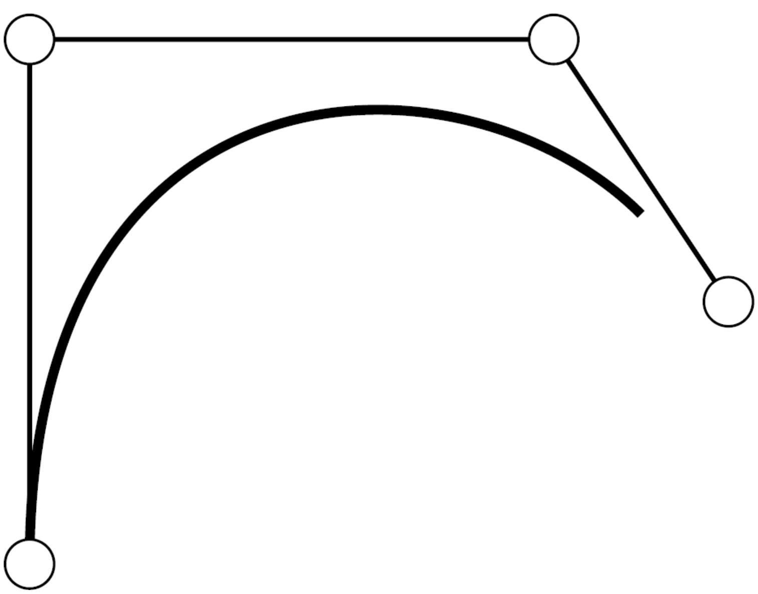
- [ ] The control polygon corresponds to the curve. No Bézier curve property is violated.
- [ ] Violates the end point interpolation property of Bézier curves.
- [ ] Violates the convex hull property of Bézier curves.
- [ ] Violates the variation diminishing property of Bézier curves.
- [ ] Violates the linear precision property of Bézier curves.
- [ ] Violates the first derivative property of Bézier curves at the end points.

Question 14

1 pts

Bézier Curve Properties IV

The following figure shows a curve and its supposed control polygon. Does the control polygon correspond to the curve? If not, check all violated properties of Bézier curves!



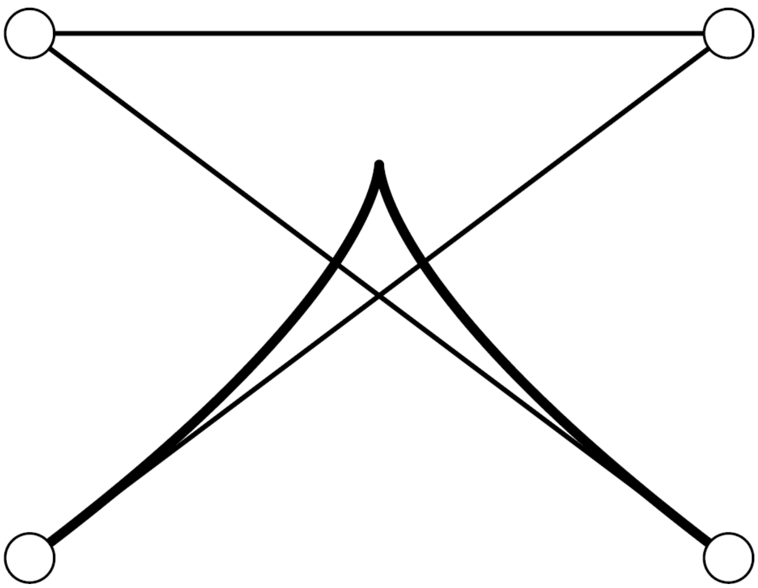
- [ ] The control polygon corresponds to the curve. No Bézier curve property is violated.
- [ ] Violates the end point interpolation property of Bézier curves.
- [ ] Violates the convex hull property of Bézier curves.
- [ ] Violates the variation diminishing property of Bézier curves.
- [ ] Violates the linear precision property of Bézier curves.
- [ ] Violates the first derivative property of Bézier curves at the end points.

Question 15

1 pts

Bézier Curve Properties V

The following figure shows a curve and its supposed control polygon. Does the control polygon correspond to the curve? If not, check all violated properties of Bézier curves!



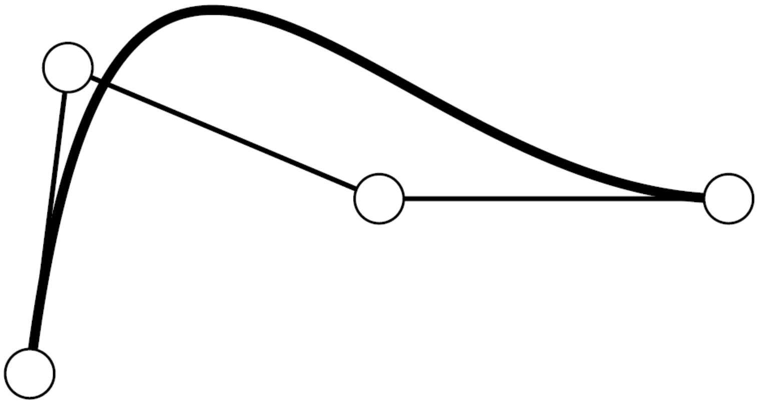
- [ ] The control polygon corresponds to the curve. No Bézier curve property is violated.
- [ ] Violates the end point interpolation property of Bézier curves.
- [ ] Violates the convex hull property of Bézier curves.
- [ ] Violates the variation diminishing property of Bézier curves.
- [ ] Violates the linear precision property of Bézier curves.
- [ ] Violates the first derivative property of Bézier curves at the end points.

Question 16

1 pts

Bézier Curve Properties VI

The following figure shows a curve and its supposed control polygon. Does the control polygon correspond to the curve? If not, check all violated properties of Bézier curves!



- [ ] The control polygon corresponds to the curve. No Bézier curve property is violated.
- [ ] Violates the end point interpolation property of Bézier curves.
- [ ] Violates the convex hull property of Bézier curves.
- [ ] Violates the variation diminishing property of Bézier curves.
- [ ] Violates the linear precision property of Bézier curves.
- [ ] Violates the first derivative property of Bézier curves at the end points.

Question 17

1 pts

De Casteljau Algorithm (Degree)

Given are the Bézier points of a Bézier curve $\mathbf{x}(t)$ with

$$\mathrm{b}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathrm{b}_1 = \begin{pmatrix} 0 \\ 12 \end{pmatrix} \quad \mathrm{b}_2 = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$$

where $\mathbf{x}(0) = \mathrm{b}_0$ and $\mathbf{x}(1) = \mathrm{b}_2$.

What is the degree of the Bézier curve?

Question 18

2 pts

De Casteljau Algorithm (1/2)

Given are the Bézier points of a Bézier curve $\mathbf{x}(t)$ with

$$\mathrm{b}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathrm{b}_1 = \begin{pmatrix} 0 \\ 12 \end{pmatrix} \quad \mathrm{b}_2 = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$$

where $\mathbf{x}(0) = \mathbf{b}_0$ and $\mathbf{x}(1) = \mathbf{b}_2$.

Use the de Casteljau algorithm to determine the value $\mathbf{x}(\frac{1}{2})$!

$\mathbf{x}(\frac{1}{2}) =$

Question 19

3 pts

De Casteljau Algorithm (3/2)

Given are the Bézier points of a Bézier curve $\mathbf{x}(t)$ with

$$\mathbf{b}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{b}_1 = \begin{pmatrix} 0 \\ 12 \end{pmatrix} \quad \mathbf{b}_2 = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$$

where $\mathbf{x}(0) = \mathbf{b}_0$ and $\mathbf{x}(1) = \mathbf{b}_2$.

Use the de Casteljau algorithm to determine the value $\mathbf{x}(\frac{3}{2})$ as well as all intermediate points!

Give proper names to the intermediate points. Compute their exact coordinates! You do not need to draw something.

HTML Editor

12pt Paragraph

0 words

Question 20

2 pts

Bézier Curve and Bézier Point Coincide

Construct a non-trivial Bézier curve $\mathbf{x}(t)$, $\mathbf{x} : [0, 1] \to \mathbb{R}^2$ of degree 4 with $\mathbf{b}_2 = \mathbf{x}(\frac{1}{2})$.

HTML Editor

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12pt Paragraph

0 words

Question 21

0 pts

Piecewise Quadratic Bézier Splines

a. Given are three points $\mathbf{a},\ \mathbf{b},\ \mathbf{c} \in \mathbb{R}^2$. Construct a closed, piecewise quadratic and $\mathbf{C}^1$-continuous Bézier spline $\mathbf{s} : [0, 3] \to \mathbb{R}^2$ with junctions at the parameter values 0, 1, and 2, such that these conditions hold: $\mathbf{s}(0) = \mathbf{a}$, $\mathbf{s}(1) = \mathbf{b}$, $\mathbf{s}(2) = \mathbf{c}$. Give a geometric description for the construction of the Bézier control points and show that there is a unique solution for this problem!
(3 Extra Points)

b. Show that, in contrast to a), for the similar problem of four points $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathbb{R}^2$, no general solution exists! Formulate the constraints for such a solution to exist!
(3 Extra Points)

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