

# 1 Center of Mass

## 1.1 Discrete

$$R_{cm} = \frac{1}{M_{total}} \sum_{n=1}^N m_n r_n \quad (1)$$

Find a point in the center of a group of points.

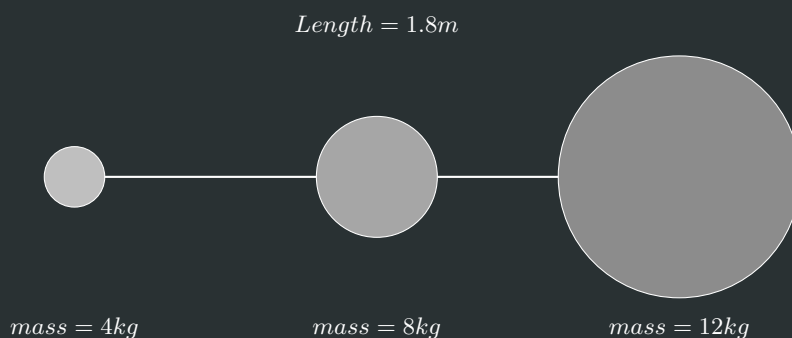
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## 1.2 Examples

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### 1.2.1 Example one

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$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{(0)(4) + (.9)(8) + (1.8)(12)}{4 + 8 + 12}$$

### 1.2.2 Example Two

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m	x	y	$v_x$	$v_y$
1	7.8	-2.8	3.2	-4.2
2	7.8	-3.7	-5.2	5.2
3	7.8	-5.7	-6.2	2.2
4	7.8	2.7	4.2	-3.2

$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}$$

### 1.3 Example Three

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$$X_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

$$X_{cm} = \frac{(4)(3) + (1)(0)}{(4+1)}$$

$$V_{LAB} = 2.4m/s$$

$$V_{CM} = -2.4m/s$$

$$V_{b_1 CM} = V_{b_1 LAB} + V_{LAB_1 CM}$$

$$3m/s - 2.4m/s$$

$$V_{b_1 cm} = .6m/s$$

$$V_{R_1 CM} = V_{R_1 LAB} + V_{LAB_1 CM}$$

$$0m/s - 2.4m/s$$

$$V_{R_1 cm} = -2.4m/s$$



## 2 Momentum

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$$\boxed{\vec{P} = m\vec{v}} \quad (2)$$

*Different version of Newtons law.*

$$\boxed{P_{total}^{\vec{}} = M_{total}V_{cm}}$$

$$P_i = P_f$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\boxed{\text{Special Case Eqs}}$$

$$v_{1f} = \frac{v_{1i}(m_1 - m_2)}{(m_1 + m_2)}$$

$$v_{2f} = \frac{v_{1i}(2m_1)}{(m_1 + m_2)}$$

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### 2.1 Elastic Collisions

- Conservation of linear Momentum
- conservation of mechanical energy
- kinetic energy of the system is conserved,
- kinetic energy of the individual bodies can change
- ex. Billiard ball collisions

### 2.2 Inelastic Collisions

- Mechanical energy not conserved
- conservation of linear Momentum
- loss of energy: sound, heat, Elastic, Etc
- bodies stick together

- paintball

In a closed system, no momentum will be lost.

- Friction is typically not considered
- typically the system will have a net force

## 2.3 Center of Mass Frame

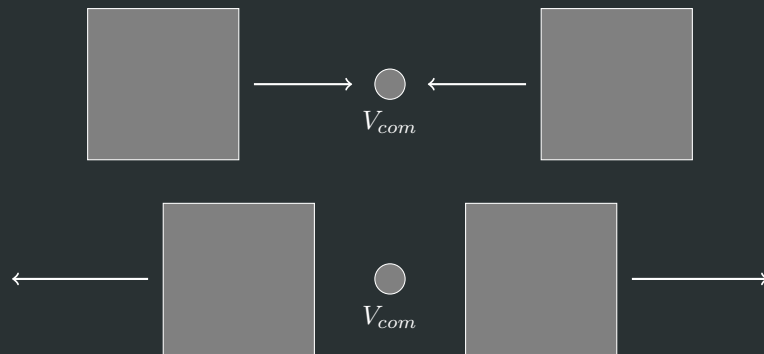
In center of mass frame, Velocity is equal to zero.

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

COM Ref frame will stay in the same spot before and after collision. Magnitude of initial  $v_1$  will be equal to  $v_{1f}$ .

$$|v_{1i}| = |v_{1f}|$$

$$|v_{2i}| = |v_{2f}|$$



Thus, you can just convert to COM ref frame and then just

## 2.4 Examples

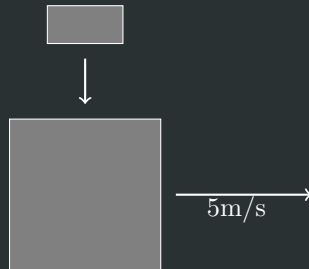
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### 2.4.1 Example 1

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A 3kg cart is rolling along when a 2kg drops on top of and sticks what is the final velocity

$$\begin{aligned}
 p_i &= m_1 v_1 + m_2 v_2 \\
 &= (3)(5) \\
 P_f &= (m_1 + m_2) V_f \\
 &\quad \rightarrow \\
 V_f &= 3m/s
 \end{aligned}$$



### 2.4.2 Example 2

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Train cars are coupled together by being bumped into each other. Supposed two loaded train cars are moving towards each other, first having a mass of  $1.5 \times 10^5 \text{ kg}$  and a velocity of  $.3m/s \hat{i}$  and the second having a mass of  $1.1 \times 10^5 \text{ kg}$  and a velocity of  $-.12m/s \hat{j}$

**Before**

$$P_i = m_1 v_1 + m_2 v_2$$

**Ater**

$$P_f = (m_1 + m_2) V_f$$

$$P_i = P_f$$

$$P_i = m_1 v_1 + m_2 v_2 = P_f = (m_1 + m_2) V_f$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

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**2.4.3**   **Example 3, Ballistic Pendulum ★**

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A projectile of mass  $m$  moving horizontally with speed  $v$  strikes a stationary mass  $M$  suspended by strings of length  $L$ . Subsequently,  $m + M$  rise to a height of  $H$   
*perfectly inelastic collision*

$$P_i = P_f$$

$$Mv_i = (m + M)v_f$$

$$V_f = \frac{mv_i}{m+M}$$

$$(m + M)gH = \frac{1}{2}(m + M)v^2$$

$$H = \frac{v^2}{2g} - \frac{\frac{(mV)^2}{(m+M)^2}}{2g} = \frac{(m^2 v^2)}{2((m+M)^2)g}$$

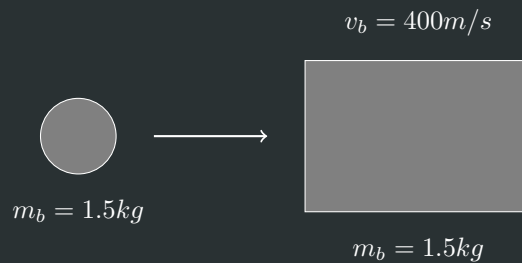


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#### 2.4.4 Example 4

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The figure below (bullet hitting block) shows a bullet of mass 200g traveling towards the east with a speed of  $400\text{m/s}$ , which strikes a block of mass 1.5kg that is intentionally at rest on a frictionless table



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#### 2.4.5 Example 5

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A glider of mass .02 kg slides on a frictionless track with initial velocity of 1.5 m/s. It hits a glider of mass .8kg moving to the left at  $v_{2i} = .2\text{m/s}$ . A spring attached to the first glider compresses and relaxes during the collision, but this is no friction (energy is conserved). What are the final velocities.

*Special Case Eqs*

$$v_{1f} = \frac{v_{1i}(m_1 - m_2)}{(m_1 + m_2)}$$

$$v_{2f} = \frac{v_{1i}(2m)}{(m_1 + m_2)}$$

$$v_{2f} = \frac{(1.5)(.4)}{(1)} = .6$$

$$v_{1f} = \frac{(1.5)(-.6)}{(1)} = -.9\text{kg}$$