1 Center of Mass

1.1 Discrete

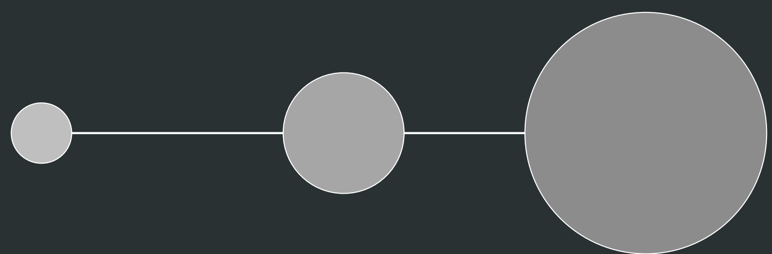
$$R_{cm} = \frac{1}{M_{total}} \sum_{n=1}^{N} m_n r_n \tag{1}$$

Find a point in the center of a group of points.

1.2 Examples

1.2.1 Example one

$Length = 1.8m$



$mass = 4kg$ $mass = 8kg$ $mass = 12kg$

$$\boxed{x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}}$$

$$\boxed{x_{cm} = \frac{(0)(4) + (.9)(8) + (1.8)(12)}{4 + 8 + 12}}$$

1.2.2 Example Two

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m	x	y	v_x	v_y
1	7.8	-2.8	3.2	-4.2
2	7.8	-3.7	-5.2	5.2
3	7.8	-5.7	-6.2	2.2
4	7.8	2.7	4.2	-3.2

$$\boxed{x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}}$$

1.3 Example Three

Square 1, 4kg $4m/2sec \rightarrow$ Square 2, 1kg

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

$$X_{cm} = \frac{(4)(3)+(1)(0)}{(4+1)}$$

$$V_{LAB} = 2.4m/s$$
$$V_{CM} = -2.4m/s$$

$$V_{b_1 CM} = V_{b_1 LAB} + V_{LAB_1 CM}$$
$$3m/s - 2.4m/s$$
$$V_{b_1} cm = .6m/s$$

$$V_{R_1 CM} = V_{R_1 LAB} + V_{LAB_1 CM}$$
$$0m/s - 2.4m/s$$
$$V_{R_1} cm = -2.4m/s$$

1.4 Continuous

$$R_{cm} = \frac{1}{M_{total}} \int \vec{r} dm$$

1.5 COM of multiple objects

2 Momentum

$$\boxed{\vec{P} = m\vec{v}} \tag{2}$$

Different version of Newtons law.

$$\boxed{\vec{P_{total}} = M_{total}V_{cm}}$$

2.1 Elastic Collisions

- Conservation of linear Momentum
- conservation of mechanical energy
- kinetic energy of the system is conserved,
- kinetic energy of the individual bodies can change
- ex. Billiard ball collisions

2.2 Inelastic Collisions

- Mechanical energy not conserved
- conservation of linear Momentum
- loss of energy: sound, heat, Elastic, Etc
- bodies stick together
- paintball

In a closed system, no momentum will be lost.

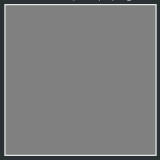
- Friction is typically not considered
- typically the system will have a net force

2.3 Examples

2.3.1 Example 1

A 3kg cart is rolling along when a at 5m per sec a 2kg drops on tp of and sticks what is the final velocity

$$p_i = m_1 v_1 + m_2 v_2$$
$$= (3)(5)$$
$$P_f = (m_1 + m_2)V_f$$
$$\rightarrow$$
$$V_f = 3m/s//[20pt]$$



2.3.2 Example 2

Train cars are coupled together by being bumped into each other. Supposed two loaded train cars are moving towards each there, first having a mass of $1.5x105kg$ and a velocity of $.3m/s\hat{i}$ and the second having a mass of $1.1x105kg$ and a velocity of $-.12m/s\hat{j}$

Before

$$P_i = m_1 v_1 + m_2 v_2$$

Bfter

$$P_f = (m_1 + m_2)V_f$$

$$P_i = P_f$$

$$P_i = m_1 v_1 + m_2 v_2 = P_f = (m_1 + m_2)V_f$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

2.3.3 Example 3, Ballistic Pendulum $\star$

A projectile of mass m moving horizontally with speed v strikes a stationary mass M suspended by strings of length L. Subsequently, $m + M$ rise to a height of H

perfectly inelastic collision

$$P_i = P_f$$

$$Mv_i = (m + M)v_f$$

$$V_f = \frac{mv_i}{m+M}$$

$$(m + M)gH = \frac{1}{2}(m + M)v^2$$

$$H = \frac{v^2}{2g} - \frac{\frac{(mV)^2}{(m+M)^2}}{2g} = \frac{(m^2v^2)}{2((m+M)^2)g}$$

2.3.4 $\boxed{\text{Example 4}}$

The figure below (bullet hitting block) shows a bullet of mass 200g traveling towards the east with a speed of $400m/s$, which stirks a block of mass 1.5kg that is intentionally at rest on a frictionless table