Calc II Notes

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Ratio Test

Suppose that $\sum_{n=1}^{\infty} a_n$ is ***any*** series,
and that

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = r$$

Then:

The series converges absolutely if $r < 1$
The series diverges if $r > 1$

Limit Comparison Test

$\lim_{a=\infty} \frac{A_n}{B_n}$, Where B_n is a series with known behavior

If $\lim_{a=\infty} \frac{A_n}{B_n} \neq 0$, then B_n and A_n must either both converge or diverge
If $\lim_{a=\infty} \frac{A_n}{B_n} = 0$ and B_n is a convergent series, then A_N must also converge
If $\lim_{a=\infty} \frac{A_n}{B_n}$ diverges and B_n is a divergent series, then A_N must diverge

Absolute Convergence

Root Test

Suppose that $\sum_{n=1}^{\infty} a_n$ is *any* series,
and that

$$\lim_{n\to\infty} (|a_n|)^{\frac{1}{n}} = L$$

Then:

The series converges absolutely if $L < 1$
The series diverges if $L > 1$

Comparison Tests

P-Series Test

$$\sum_{n=1}^{\infty} \frac{1}{x^p} \begin{cases} Converge & x \leq 1 \\ Diverge & 1 > 1 \end{cases}$$

By the integral test

Converge/Diverge

Direct Comparison Test

If B_n is a known series which converges and
$A_n \leq B_n$
then A_n must also converge

If B_n is a known series which diverges and
$A_n \geq B_n$
then A_n must also diverge

Integral Test

$\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.

and

$\sum_{n=1}^{\infty} a_n$ diverges if and only if $\int_1^{\infty} f(x)\,dx$ diverges.

Conditions for the Integral Test

I) Must be continous for all values of x
II) Must be positive
III) Must be decreasing

$$\sum_{n=1}^{\infty} a_n = \lim_{a\to\infty} \sum_{n=1}^{a} a_n$$

N^{th} test for divergnce (NTTFDO)

$\lim_{n=\infty} a_n \neq 0$
then the series must diverge

Evaluate Exactly

Telescoping Series

$$\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$$

A series where intermediate terms cancel, leaving a finite sum

- Partial Fraction Decomposition
- Find Equation for S_n
- $\lim_{n\to\infty} S_n$

Geometric Series

$$\sum_{k=0}^{\infty} (r)^k = \frac{1}{1-r},\ |r| < 1$$

or

$$\sum_{k=N}^{\infty} r^N = \frac{r^N}{1-r},\ |r| < 1$$

A series which increases by the same amount for each value of N. Ie.
$\frac{A_{n+1}}{A_n} = L$, where L is a number
Note: This does not converge for values of r greater than 1

1 Ratio Test

The Ratio Test[1] states that

Suppose that $\sum_{n=1}^{\infty} a_n$ is ***any*** series,
and that

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = r$$

Then:

The series converges absolutely if $r < 1$
The series diverges if $r > 1$

1.1 Examples

$\boxed{\textbf{ex. 1}}$

$$\boxed{\text{Simplify } \frac{(n+2)!}{(n-1)!}}$$

Ratio of factorials

$$= \frac{(n+2)!}{(n-1)!}$$

$$= \frac{(n+1)(n+1)(n)(n-1)\cdots(1)}{(n-1)(n-2)(n-3)\cdots(1)}$$

$$= \frac{(n+1)(n+1)(n)(n-1)}{(n-1)}$$

$$= (n+1)(n+1)(n)$$

[1] Very important regarding factorials

1.2 Factorials

$\boxed{\text{ex.1}}$

$$\boxed{4! = 4 \text{ x } 3! = 4\text{x}3\text{x}2! = 4\text{x}3\text{x}2\text{x}1! = 4\text{x}3\text{x}2\text{x}1\text{x}0!}$$

$$\rightarrow$$

$$\boxed{0! = 1}$$

$$\rightarrow$$

$$\boxed{4! = 24}$$

$\boxed{\textbf{ex. 2}}$

$$\boxed{\text{Simplify } \frac{(n+2)!}{(n-1)!}}$$

Ratio of factorials

$$= \frac{(n+2)!}{(n-1)!}$$

$$= \frac{(n+1)(n+1)(n)(n-1)\cdots(1)}{(n-1)(n-2)(n-3)\cdots(1)}$$

$$= \frac{(n+1)(n+1)(n)(n-1)}{(n-1)}$$

$$= (n+1)(n+1)(n)$$