1 Center of Mass

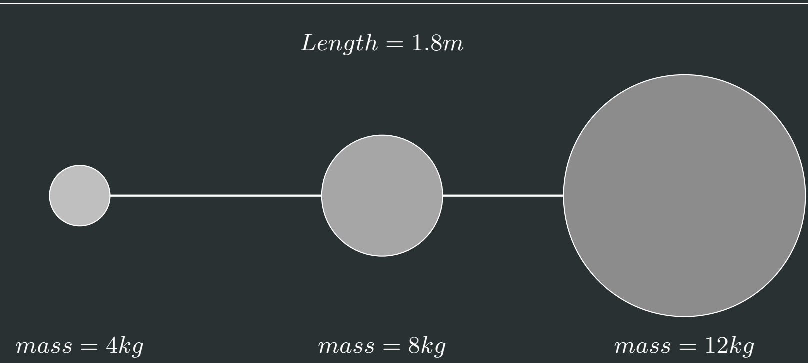
1.1 Discrete

$$R_{cm} = \frac{1}{M_{total}} \sum_{n=1}^{N} m_n r_n \tag{1}$$

Find a point in the center of a group of points.

1.2 Examples

1.2.1 Example one



$$\boxed{x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}}$$

$$\boxed{x_{cm} = \frac{(0)(4) + (.9)(8) + (1.8)(12)}{4 + 8 + 12}}$$

1.2.2 Example Two

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m	x	y	v_x	v_y
1	7.8	-2.8	3.2	-4.2
2	7.8	-3.7	-5.2	5.2
3	7.8	-5.7	-6.2	2.2
4	7.8	2.7	4.2	-3.2

$$\boxed{x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}}$$

1.3 Example Three

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

$$X_{cm} = \frac{(4)(3)+(1)(0)}{(4+1)}$$

$$V_{LAB} = 2.4m/s$$
$$V_{CM} = -2.4m/s$$

$$V_{b_1CM} = V_{b_1LAB} + V_{LAB_1CM}$$
$$3m/s - 2.4m/s$$
$$V_{b_1}cm = .6m/s$$

$$V_{R_1CM} = V_{R_1LAB} + V_{LAB_1CM}$$
$$0m/s - 2.4m/s$$
$$V_{R_1}cm = -2.4m/s$$

1.4 Continuous

$$R_{cm} = \frac{1}{M_{total}} \int \vec{r} dm$$

1.5 COM of multiple objects

2 Momentum

$$\boxed{\vec{P} = m\vec{v}} \tag{2}$$

Different version of Newtons law.

$$\boxed{\vec{P_{total}} = M_{total}V_{cm}}$$

$$P_i = P_f$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$\boxed{\text{Special Case Eqs}}$$

$$v_{1f} = \frac{v_{1f}(m_1 - m_2)}{(m_1 + m_2)}$$

$$v_{2f} = \frac{v_{1f}(2m)}{(m_1 + m_2)}$$

2.1 Elastic Collisions

- Conservation of linear Momentum
- conservation of mechanical energy
- kinetic energy of the system is conserved,
- kinetic energy of the individual bodies can change
- ex. Billiard ball collisions

2.2 Inelastic Collisions

- Mechanical energy not conserved
- conservation of linear Momentum
- loss of energy: sound, heat, Elastic, Etc
- bodies stick together

- paintball

In a closed system, no momentum will be lost.

- Friction is typically not considered
- typically the system will have a net force

2.3 Center of Mass Frame

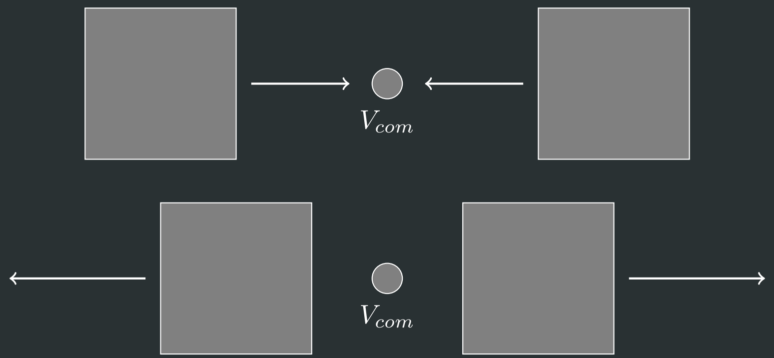
In center of mass frame, Velocity is equal to zero.

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

COM Ref frame will stay in the same spot before and after collision. Magnitude of initial v1 will be equal to v2f.

$$|v_{1_i}| = |v_{1_f}|$$

$$|v_{2_i}| = |v_{2_f}|$$



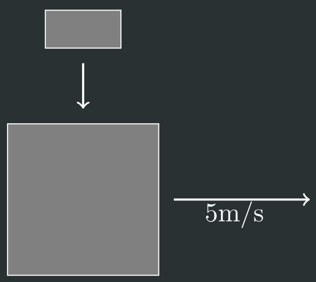
Thus, you can just convert to COM ref frame and then just

2.4 Examples

2.4.1 Example 1

A 3kg cart is rolling along when a at 5m per sec a 2kg drops on tp of and sticks what is the final velocity

$$p_i = m_1 v_1 + m_2 v_2$$
$$= (3)(5)$$
$$P_f = (m_1 + m_2) V_f$$
$$\rightarrow$$
$$V_f = 3m/s$$



2.4.2 Example 2

Train cars are coupled together by being bumped into each other. Supposed two loaded train cars are moving towards each there, first having a mass of $1.5x105kg$ and a velocity of $.3m/s\hat{i}$ and the second having a mass of $1.1x105kg$ and a velocity of $-.12m/s\hat{j}$

Before

$$P_i = m_1 v_1 + m_2 v_2$$

Ater

$$P_f = (m_1 + m_2) V_f$$

$$P_i = P_f$$

$$P_i = m_1 v_1 + m_2 v_2 = P_f = (m_1 + m_2) V_f$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

2.4.3 Example 3, Ballistic Pendulum $\star$

A projectile of mass m moving horizontally with speed v strikes a stationary mass M suspended by strings of length L. Subsequently, $m + M$ rise to a height of H

perfectly inelastic collision

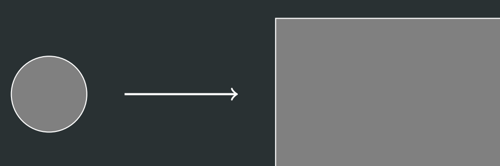
$$P_i = P_f$$

$$Mv_i = (m + M)v_f$$

$$V_f = \frac{mv_i}{m+M}$$

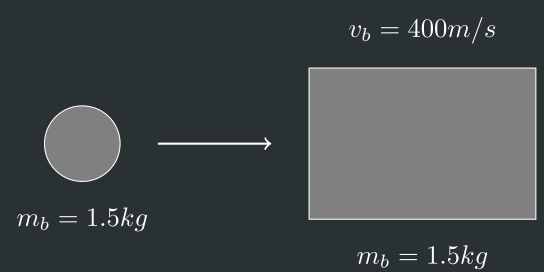
$$(m + M)gH = \frac{1}{2}(m + M)v^2$$

$$H = \frac{v^2}{2g} - \frac{\frac{(mV)^2}{(m+M)^2}}{2g} = \frac{(m^2 v^2)}{2((m+M)^2)g}$$



2.4.4 Example 4

The figure below (bullet hitting block) shows a bullet of mass 200g traveling towards the east with a speed of $400m/s$, which stirks a block of mass 1.5kg that is intentionally at rest on a frictionless table



2.4.5 Example 5

A glider of mass .02 kg slides on a frictionless track with initial velocity of 1.5 m/s. It hits a glider of mass .8kg moving to the left at v2i = .2m/s. A spring attached to the first glider compresses and relaxes during the collision, but this is no friction (energy is conserved). What are the final velocities.

Special Case Eqs

$$v_{1f} = \frac{v_{1f}(m_1 - m_2)}{(m_1 + m_2)}$$

$$v_{2f} = \frac{v_{1f}(2m)}{(m_1 + m_2)}$$

$$v_{2f} = \frac{(1.5)(.4)}{(1)} = .6$$

$$v_{1f} = \frac{(1.5)(-.6)}{(1)} = -.9kg$$

2.5 Moment of inertia

$$I = I_c + MD^2$$
$$x \to \theta$$
$$dx/dt = v \to d\theta/dt = \omega$$
$$m = I$$

2.5.1 Examples

2.5.2 $\boxed{\textbf{Example 1}}$

What is the moment of inertia of two solid spheres with radius r = 10cm and mass 10kg. Attached to a rod of mass 2kg and length of d = 25cm

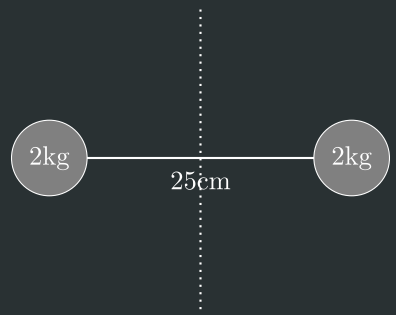
$$I_{system} = I_{sphere} + I_{sphere_2} + I_{rod} \text{ v}$$

$$= 2(i_{sphere_c m} + MD^2) + I_{rod_{cm}}$$

$$= 2(\tfrac{2}{5}MR^2 + M(R + \tfrac{d}{2})^2 + \tfrac{1}{12}md^2)$$

$$= 2(\tfrac{2}{5})(10)(.1)^{(}.1) + (10)(.225)^2 + \tfrac{1}{12}(2)(.25)^2$$

$$I_{system} = 1.103kgm^2$$



Axis of rotation

2.5.3 Torque

$$|\tau| = |r||F|\sin\theta$$

$$\vec{\tau} = \vec{r} X \vec{F}$$
$$W = \vec{\tau} \Delta \vec{\theta}$$
$$KE_{total} = KE_{tras} + KE_{rot}$$
$$= 1/2 m v_{com}^2 + 1/2 I_{com} W^2$$

Kinematic equations

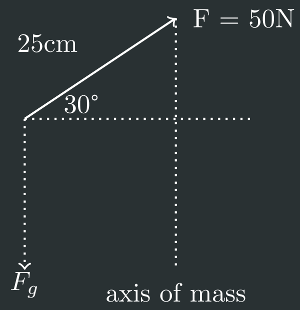
$$\omega = \omega_0 + \alpha t \tag{3}$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \tag{4}$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta \tag{5}$$
$$\theta = \frac{1}{2}(\omega + \omega_0)t \tag{6}$$
$$\theta = \omega t - \frac{1}{2}\alpha t^2 \tag{7}$$

2.6 Examples

2.6.1 Example 1

A 50n force is placed on one end of a 25cm long wrench as shown. what is the torque applied by this force if it rotates about the other end?

$$\tau = (.25m)(50n)(\sin 120)$$



2.6.2 Example 2

A long length of string is wrapped around 5kg drum with a radius of 30cm. The drum is free to spin around a frictionless axle. The other end of the string is attached to a 10kg mass. If the mass is allowed to drop, what is the acceleration?

$$a_t = R\alpha \rightarrow \alpha = \frac{a}{r}$$

$$I_{soliddisk} = \frac{1}{2}M_1R^2$$

$$\sum F = M_2a$$

$$M_2g - T = M_2a$$

$$T = m_2g = m_2a$$

$$\sum \tau = I\alpha$$

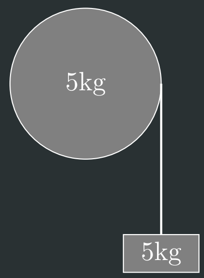
$$\cancel{\tau_{fg}} + \cancel{\tau_{Fs}} + \tau_T = \frac{1}{2}M_1R^2\frac{a}{R}$$

$$T\cancel{R} = \frac{1}{2}M_1\cancel{R^2}\frac{a}{\cancel{R}}$$

$$T = \frac{1}{2}m_1a$$

$$\frac{1}{2}m_1a = m_2g = m_2a$$

$$a = \frac{M_2g}{\frac{1}{2}M_1+M_2}$$



2.6.3 Example 3

A long length of string is wrapped around a 5kg drum with a radius of 30cm. The drum is free to spin around a frictionless axle. After the string has been pulled with a 10n force for a distance of 5m. What is the angular velocity of the drum.

$$W_{net} = \Delta\ KE$$

$$F \times d = RKEf - \cancel{Rkei}$$

$$F \times d = \frac{1}{2} I W_f^2$$

$$I = \frac{1}{2} M R^2 = \frac{1}{2}(5kg)(.3m)^2$$

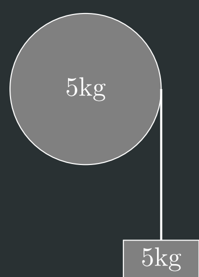
$$I = .225 Kgm^2$$

$$(10N)(5m) = \frac{1}{2}(.225kgm^2) W_f^2$$

$$W_f = 21.1 \frac{Rad}{S}$$

2.6.4 $\boxed{\textbf{Example 4}}$

A length of string is wrapped around a 5kg drum with a radius of 30cm. The drum is free to spin around a frictionless axle. The other end of the string is attached to a 10kg mass. What is the angular velocity of the pully after the mass has dropped 2m?



$$W_f^2 = \cancel{W_0^2} + 2\alpha \; \Delta \; \theta$$

$$W_f = \sqrt{2(26.1 \frac{rad}{s^2}(6.66 rad))}$$

$$W_f = 18.6 \frac{rad}{s}$$
$$rev = 2\pi \; R$$

$$= 2\pi \; (.3)$$

$$= .6\pi\omega$$

$$= 2.0\omega$$