

1 Center of Mass

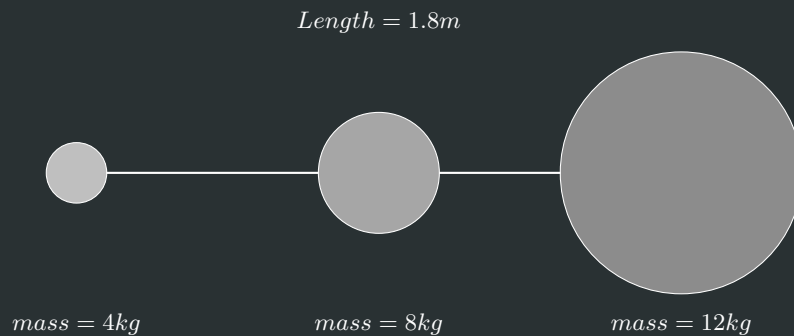
1.1 Discrete

$$R_{cm} = \frac{1}{M_{total}} \sum_{n=1}^N m_n r_n \quad (1)$$

Find a point in the center of a group of points.

1.2 Examples

1.2.1 Example one



$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{(0)(4) + (.9)(8) + (1.8)(12)}{4 + 8 + 12}$$

1.2.2 Example Two

m	x	y	v_x	v_y
1	7.8	-2.8	3.2	-4.2
2	7.8	-3.7	-5.2	5.2
3	7.8	-5.7	-6.2	2.2
4	7.8	2.7	4.2	-3.2

$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}$$

1.3 Example Three



$$X_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

$$X_{cm} = \frac{(4)(3) + (1)(0)}{(4+1)}$$

$$V_{LAB} = 2.4m/s$$

$$V_{CM} = -2.4m/s$$

$$V_{b_1 CM} = V_{b_1 LAB} + V_{LAB_1 CM}$$

$$3m/s - 2.4m/s$$

$$V_{b_1 cm} = .6m/s$$

$$V_{R_1 CM} = V_{R_1 LAB} + V_{LAB_1 CM}$$

$$0m/s - 2.4m/s$$

$$V_{R_1 cm} = -2.4m/s$$

2 Momentum

$$\boxed{\vec{P} = m\vec{v}} \quad (2)$$

Different version of Newton's law.

$$\boxed{\vec{P}_{total} = M_{total}V_{cm}}$$

2.1 Elastic Collisions

- Conservation of linear Momentum
- conservation of mechanical energy
- kinetic energy of the system is conserved,
- kinetic energy of the individual bodies can change
- ex. Billiard ball collisions

2.2 Inelastic Collisions

- Mechanical energy not conserved
- conservation of linear Momentum
- loss of energy: sound, heat, Elastic, Etc
- bodies stick together
- paintball

In a closed system, no momentum will be lost.

- Friction is typically not considered
- typically the system will have a net force

2.3 Examples

2.3.1 Example 1

A 3kg cart is rolling along when a 2kg drops on top of and sticks
what is the final velocity

$$p_i = m_1 v_1 + m_2 v_2$$

$$= (3)(5)$$

$$P_f = (m_1 + m_2) V_f$$

$$V_f = 3m/s / [20pt]$$



2.3.2 Example 2

Train cars are coupled together by being bumped into each other. Supposed
two loaded train cars are moving towards each other, first having a mass of
 $1.5 \times 10^5 kg$ and a velocity of $.3m/s \hat{i}$ and the second having a mass of $1.1 \times 10^5 kg$
and a velocity of $-.12m/s \hat{j}$

Before

$$P_i = m_1 v_1 + m_2 v_2$$

After

$$P_f = (m_1 + m_2) V_f$$

$$P_i = P_f$$

$$P_i = m_1 v_1 + m_2 v_2 = P_f = (m_1 + m_2) V_f$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

2.3.3 Example 3, Ballistic Pendulum ★

A projectile of mass m moving horizontally with speed v strikes a stationary mass M suspended by strings of length L . Subsequently, $m + M$ rise to a height of H
perfectly inelastic collision

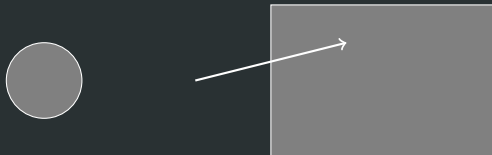
$$P_i = P_f$$

$$Mv_i = (m + M)v_f$$

$$V_f = \frac{mv_i}{m+M}$$

$$(m + M)gH = \frac{1}{2}(m + M)v^2$$

$$H = \frac{v^2}{2g} - \frac{\frac{(mV)^2}{(m+M)^2}}{2g} = \frac{(m^2 v^2)}{2((m+M)^2)g}$$



2.3.4 Example 4

The figure below (bullet hitting block) shows a bullet of mass 200g traveling towards the east with a speed of 400m/s , which strikes a block of mass 1.5kg that is intentionally at rest on a frictionless table

