1 Center of Mass

1.1 Discrete

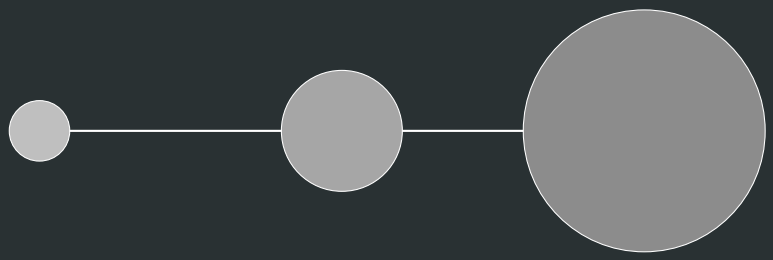
$$R_{cm} = \frac{1}{M_{total}} \sum_{n=1}^{N} m_n r_n \tag{1}$$

Find a point in the center of a group of points.

1.2 Examples

1.2.1 Example one

$Length = 1.8m$



$mass = 4kg$ $mass = 8kg$ $mass = 12kg$

$$\boxed{x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}}$$

$$\boxed{x_{cm} = \frac{(0)(4) + (.9)(8) + (1.8)(12)}{4 + 8 + 12}}$$

1.2.2 Example Two

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m	x	y	v_x	v_y
1	7.8	-2.8	3.2	-4.2
2	7.8	-3.7	-5.2	5.2
3	7.8	-5.7	-6.2	2.2
4	7.8	2.7	4.2	-3.2

$$\boxed{x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + x_4 m_4}{m_1 + m_2 + m_3 + m_4}}$$

1.3 Example Three

Square 1, 4kg $4m/2sec \rightarrow$ Square 2, 1kg

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$V_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$$

$$X_{cm} = \frac{(4)(3)+(1)(0)}{(4+1)}$$

$$V_{LAB} = 2.4m/s$$
$$V_{CM} = -2.4m/s$$

$$V_{b_1 CM} = V_{b_1 LAB} + V_{LAB_1 CM}$$
$$3m/s - 2.4m/s$$
$$V_{b_1} cm = .6m/s$$

$$V_{R_1 CM} = V_{R_1 LAB} + V_{LAB_1 CM}$$
$$0m/s - 2.4m/s$$
$$V_{R_1} cm = -2.4m/s$$

1.4 Continuous

$$R_{cm} = \frac{1}{M_{total}} \int \vec{r} dm$$

1.5 COM of multiple objects

2 Momentum

$$\boxed{\vec{P} = m\vec{v}} \tag{2}$$

Different version of Newtons law.

$$\boxed{\vec{P_{total}} = M_{total}V_{cm}}$$

2.1 Elastic Collisions

- Conservation of linear Momentum
- conservation of mechanical energy
- kinetic energy of the system is conserved,
- kinetic energy of the individual bodies can change
- ex. Billiard ball collisions

2.2 Inelastic Collisions

- Mechanical energy not conserved
- conservation of linear Momentum
- loss of energy: sound, heat, Elastic, Etc
- bodies stick together
- paintball

In a closed system, no momentum will be lost.

- Friction is typically not considered
- typically the system will have a net force