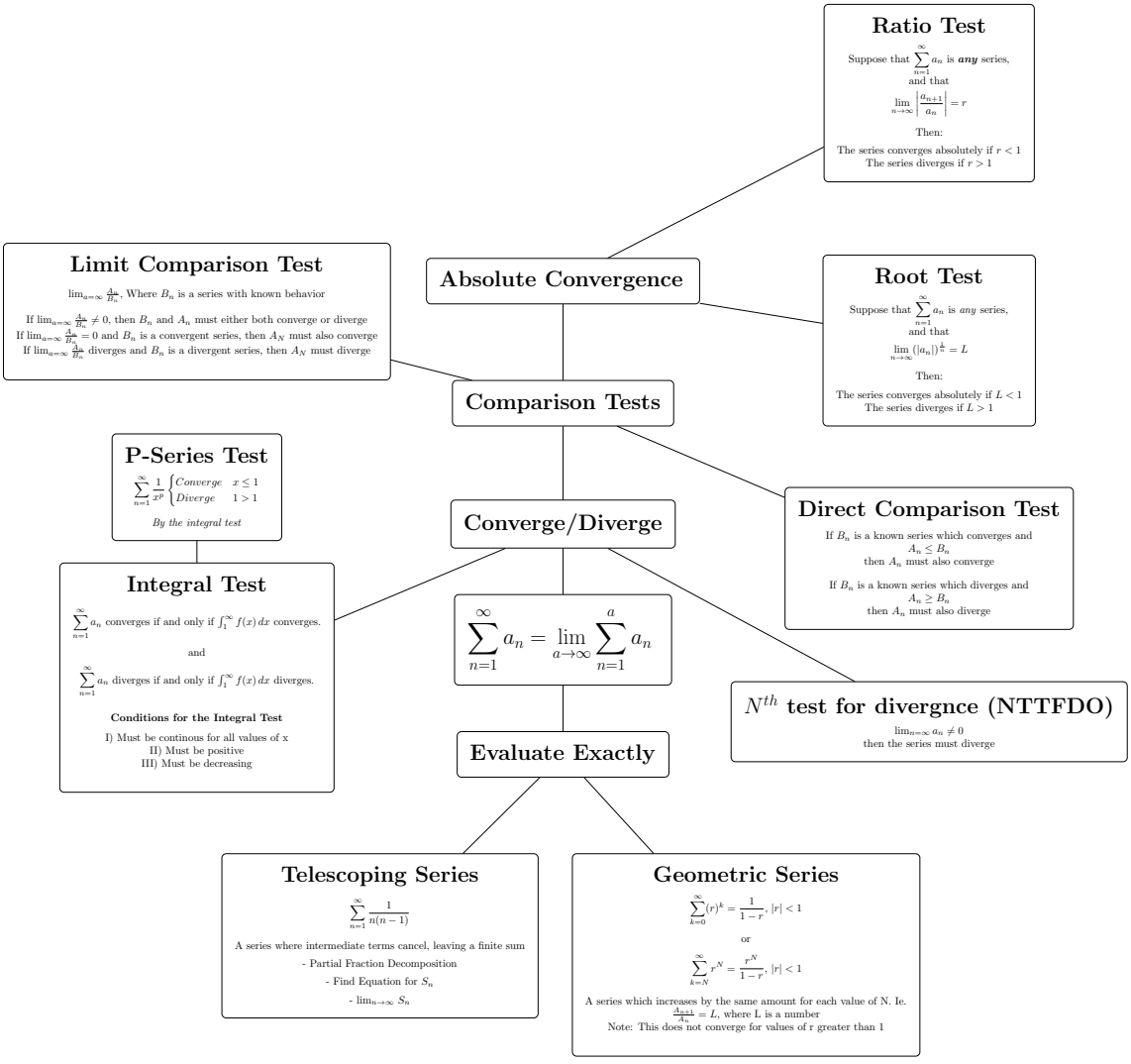
Calc II Notes
Pierson L


Ratio Test
Suppose that $\sum_{n=1}^{\infty} a_n$ is ***any*** series,
and that
$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = r$
Then:
The series converges absolutely if $r < 1$
The series diverges if $r > 1$
Limit Comparison Test
$\lim_{a=\infty} \frac{A_n}{B_n}$, Where B_n is a series with known behavior
If $\lim_{a=\infty} \frac{A_n}{B_n} \neq 0$, then B_n and A_n must either both converge or diverge
If $\lim_{a=\infty} \frac{A_n}{B_n} = 0$ and B_n is a convergent series, then A_N must also converge
If $\lim_{a=\infty} \frac{A_n}{B_n}$ diverges and B_n is a divergent series, then A_N must diverge
Absolute Convergence
Root Test
Suppose that $\sum_{n=1}^{\infty} a_n$ is *any* series,
and that
$\lim_{n\to\infty} (|a_n|)^{\frac{1}{n}} = L$
Then:
The series converges absolutely if $L < 1$
The series diverges if $L > 1$
Comparison Tests
P-Series Test
$\sum_{n=1}^{\infty} \frac{1}{x^p} \begin{cases} Converge & x \leq 1 \\ Diverge & 1 > 1 \end{cases}$
By the integral test
Converge/Diverge
Direct Comparison Test
If B_n is a known series which converges and
$A_n \leq B_n$
then A_n must also converge
If B_n is a known series which diverges and
$A_n \geq B_n$
then A_n must also diverge
Integral Test
$\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.
and
$\sum_{n=1}^{\infty} a_n$ diverges if and only if $\int_1^{\infty} f(x)\,dx$ diverges.
Conditions for the Integral Test
I) Must be continous for all values of x
II) Must be positive
III) Must be decreasing
$\sum_{n=1}^{\infty} a_n = \lim_{a\to\infty} \sum_{n=1}^{a} a_n$
N^{th} test for divergnce (NTTFDO)
$\lim_{n=\infty} a_n \neq 0$
then the series must diverge
Evaluate Exactly
Telescoping Series
$\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$
A series where intermediate terms cancel, leaving a finite sum
- Partial Fraction Decomposition
- Find Equation for S_n
- $\lim_{n\to\infty} S_n$
Geometric Series
$\sum_{k=0}^{\infty} (r)^k = \frac{1}{1-r}, |r| < 1$
or
$\sum_{k=N}^{\infty} r^N = \frac{r^N}{1-r}, |r| < 1$
A series which increases by the same amount for each value of N. Ie.
$\frac{A_{n+1}}{A_n} = L$, where L is a number
Note: This does not converge for values of r greater than 1

1 Absolute Convergence

10.5

1.1 Good Practice Problems

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1.2 Ratio Test

The Ratio Test[1] states that

$$\text{Suppose that } \sum_{n=1}^{\infty} a_n \text{ is } \textbf{\textit{any}} \text{ series,}$$
$$\text{and that}$$
$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = r$$

Then:

The series converges absolutely if $r < 1$
The series diverges if $r > 1$

1.2.1 Examples

ex. 1

Simplify $\frac{(n+2)!}{(n-1)!}$

Ratio of factorials

$$= \frac{(n+2)!}{(n-1)!}$$

$$= \frac{(n+1)(n+1)(n)(n-1)\cdots(1)}{(n-1)(n-2)(n-3)\cdots(1)}$$

$$= \frac{(n+1)(n+1)(n)(n-1)}{(n-1)}$$

$$= (n+1)(n+1)(n)$$

[1] Very important regarding factorials

1.3 Misc

1.3.1 Factorials

ex.1

$4! = 4 \text{ x } 3! = 4\text{x}3\text{x}2! = 4\text{x}3\text{x}2\text{x}1! = 4\text{x}3\text{x}2\text{x}1\text{x}0!$

$\rightarrow$

$0! = 1$

$\rightarrow$

$4! = 24$

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ex. 2

Simplify $\frac{(n+2)!}{(n-1)!}$

Ratio of factorials

$$= \frac{(n+2)!}{(n-1)!}$$

$$= \frac{(n+1)(n+1)(n)(n-1)\cdots(1)}{(n-1)(n-2)(n-3)\cdots(1)}$$

$$= \frac{(n+1)(n+1)(n)(n-1)}{(n-1)}$$

$$= (n+1)(n+1)(n)$$