P-Series Test

$\sum_{n=1}^{\infty} \frac{1}{x^p} \begin{cases} Diverge & x \leq 1 \\ Converge & 1 > 1 \end{cases}$

By the integral test

Integral Test

$\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.

and

$\sum_{n=1}^{\infty} a_n$ diverges if and only if $\int_1^{\infty} f(x)\,dx$ diverges.

Conditions for the Integral Test

I) Must be continous for all values of x
II) Must be positive
III) Must be decreasing

Direct Comparison Test

If B_n is a known series which converges and
$A_n \leq B_n$
then A_n must also converge

If B_n is a known series which diverges and
$A_n \geq B_n$
then A_n must also diverge

Limit Comparison Test

$\lim_{a=\infty} \frac{A_n}{B_n}$, Where B_n is a series with known behavior

If $\lim_{a=\infty} \frac{A_n}{B_n} \neq 0$, then B_n and A_n must either both converge or diverge
If $\lim_{a=\infty} \frac{A_n}{B_n} = 0$ and B_n is a convergent series, then A_N must also converge
If $\lim_{a=\infty} \frac{A_n}{B_n}$ diverges and B_n is a divergent series, then A_N must diverge

N^{th} test for divergnce (NTTFDO)

$\lim_{n=\infty} a_n \neq 0$
then the series must diverge

Comparison Tests

Converge/Diverge

$$\sum_{n=1}^{\infty} a_n = \lim_{a} \sum_{n=1}^{a}$$

Evaluate Exactly

Telescoping Series

$$\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$$

A series where intermediate terms cancel, leaving a finite sum
- Partial Fraction Decomposition
- Find Equation for S_n
- $\lim_{n\to\infty} S_n$

Geometric Series

$$\sum_{k=0}^{\infty} (r)^k = \frac{1}{1-r}, \ |r| < 1$$

or

$$\sum_{k=N}^{\infty} r^N = \frac{r^N}{1-r}, \ |r| < 1$$

A series which increases by the same amount for each value of N. Ie.
$\frac{A_{n+1}}{A_n} = L$, where L is a number
Note: This does not converge for values of r greater than 1