







# Notes

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## 0 Class overview

Office hours tues and thurs 11-12 astro 237

ta office hours

wed 3:30:530 astro 2367

hw due wed

hw posted week before

- solar system
- stellar evo
- compact objects
- galaxy quasi darkmatter
- cosmic web
- big bang

course goals

- apply pys to universe
- understand foundations of modern astro, astrophys, and cosmology
- conceptual understanding of the uni based on physical principles

## 1 Early Astronomy

### 1.0.1 Greek

- Aristotle
  - earth is spherical
  - partial lunar eclipses
  - some stars visible from southern locations but not northern and vice versa
  - had ideas regarding perfect geo influenced by Pythagoras and Plato
- Aristarchus (310-230 BC):

- unpreceded heliocentric framework
- trig distances earth-moon-sun system
- angular diameters  $\theta_{sun} \approx \theta_{moon} \therefore \frac{A}{C} = \frac{D_{moon}}{D_{moon}}$
- diameters from lunar eclipses  $D_{moon} < D_{earth}$
- Eratosthenes (176-195 BC):
  - Determined radius of spherical earth  $R_E$
  - Sun at zenith at noon on summer solstice at Aswan
  - But further north in Alexandria, Egypt, the sun is south of the zenith by angle  $\alpha$
- Hipparchus (190-120 BC):
  - Discover precession of the equinoxes from examination of star catalogs over centuries
  - established the magnitude system
- Copernicus (1473-1543):
  - heliocentric
  - earth rotates
  - still assumed uniform circular celestial motion
  - inferior planets: orbit smaller than earths
  - superior planets: orbits larger than earths

## 1.1 Emergence of modern Astro

### Inferior planets

- $B/C = \sin \theta_E$
- $B = C \sin \theta_E$
- C is AU
- Early astronomers didnt know C, so they could only infer ratios of B/C. Ie. Orbital radii measured in AU

### Superior Planets

- Measure time between opposition and eastern quadrature
- want angle  $\theta$  between opp and east quad
- $\theta = (\omega_E - \omega_p)$  and  $C/B = \cos \theta$
- measure  $\tau$  and synodic period, calculate sidereal period and  $\omega_p$ ; know  $\omega_E$  and infer  $C/B$

### Galilean Revolution

- Galileo Galilei (1564 -1642)
- improved and used a basic refracting telescoping

- def publication of early results 1610 "*starry messenger*"
- - Moon is cratered; not a perfect Sphere
  - milkyway is made out of stars
  - Jupiter has moons (or as he thought, stars)
  - measured phases of Venus

## Phases of Venus

- direct confrontation with Ptolemaic geocentric models
- in Ptolemaic models you only see crescent phases

## Tycho Brahe (1546-1601)

- Denmark, later Prague
- Given island by king Fredrick (and staff)
- made a accurate and vast database of celestial motion
- had a lead nose?
- Threw giant ragers
- supernova named after him

## Johannes Kepler (1571–1630, Prague)

- 'Inherited' (maybe stole) Brahe's data
- also has a SN
- Kepler fit a new empirical model of heliocentric orbits, abandoning perfect circles
  - "*It was as if I awoke from sleep and saw a new light*" (Kepler, New astronomy)

## Kepler's Laws

### First law

- The planets travel on elliptical orbits with the sun at one focus
- Semimajor axis, half the major axis
- eccentricity: how elliptical (stretched) an orbit is - distance between foci divided by major axis.

### second law

- A line drawn from the sun to a planet sweeps out equal areas in equal time intervals'
- perihelion: orbital point closet to the sun
- aphelion: furthest orbital point from the sun

### third law

Def: *The square of the sidereal orbital periods of the planets are prop to the cubes of the Semimajor axis of their orbits*

$$p^2 = Ka^3$$

P = planets sidereal period  
a = length of semimajor axis  
K = constant

## Consequences of heliocentric model

- retrograde motion of outer planets
- positions of outer and inner planets wrt sun
- annual parallax
- aberration of starlight
- Coriolis effect

## Parallax

- annual parallax: change in the apparent position when seen from two diff locations due to earth revolving around the sun. First measured by Bessel in 1838

## Aberration of starlight

- deflection of apparent stellar positions in the direction of the observers motion
- analog: running throw rain and getting wet in the front and not in the back
- detected (Picard, 1680); explained (Bradley, 1729)
- telescope is moving along orbital vector around the sun; translation along orbit cannot exceed transit time of light through telescope

## Coriolis effect: evidence of earth rotation

- Coriolis acceleration is perp to the direction of motion

-

$$a_{cor}^{\vec{}} = s\vec{v} \times \vec{\omega}$$

- can be deduced from a pendulum
- and in hurricanes!

## 2 Orbital Mechanics I

### 2.1 Newtonian mechanics

Parametric vectors

Displacement  $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

distance:  $r(t) = |\vec{r}(t)| = \sqrt{\vec{r} \cdot \vec{r}}$

## 2.2 Newtons laws

First law

- Isaac newton(1642-1727)
- an objects' velocity remains constant unless a net outside force acts upon it
- $\vec{v}(t) = \vec{v}_0 = \text{const}$

second law

- $\vec{F} = m\vec{a}(t)$
- $\vec{F} = \frac{d\vec{p}(t)}{dt}$
- $d\vec{v}/dt = \vec{f}/m$
- force changes velocity
- used a lot in computational math

third law

- forces come in pairs, equal in magnitude, and opposite in direction

Newtonian gravity

- a force, grav, exists between any two objects having mass m and M, prop to the product of their masses mM and inversely proportional to the square of the separation distance r of their centers
- for coordinates centered on M:
- $\vec{F} = -G \frac{Mm}{|\vec{r}|^2} \hat{r}$

## 2.3 Displacement vector and polar coordinates

- cartesian coordinates are often written as (x,y,z) in a coordinate system centered on mass M
- Axis orientations are chosen so that the planet orbits in the x-y plane
- Displacement  $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

velocity vector and polar coordinates

- unit vectors in polar coordinates vary with  $\theta(t)$

-

$$\frac{d\hat{r}(t)}{dt} = \frac{d\hat{r}(t)}{d\theta} \frac{d\theta(t)}{dt} = \frac{d\theta(t)}{dt} \hat{\theta}(t)$$

- .

- .

- .

-

$$\vec{v}(t) = v_r \hat{r} + v_t \hat{\theta}$$

- two velocity components in polar coords

## 2.4 Kepler laws: angular momentum

-

## 2.5 keplers 2nd law = consv, angular momentum

-

$$d\vec{L}/dt = 0$$

-

$$\vec{L} = \vec{R} \times \vec{p} = \vec{r} \times m\vec{v} = \text{const}$$

-

$$\Rightarrow |\vec{v}| = L = mrv_1$$

## 2.6 Keplers Laws

### 2.6.1 Keplers First Law

-

$$\frac{d\vec{v}}{dt} = -\frac{GM}{r^2} \hat{r}$$

-

$$\frac{L}{GMm} \frac{d\vec{v}}{dt} = \frac{d\hat{\theta}}{dt}$$

-

$$\frac{L}{GMm} \vec{v} = \hat{\theta} + e\hat{j}$$

- take dot product of both sides with unit vector  $\hat{\theta}$ , using

$$\hat{j} \cdot \hat{\theta} = \cos \theta$$

-

$$\vec{v} \cdot \hat{\theta} = v_t = \frac{L}{mr}$$

## 2.7 Kepler III

- we know that  $\frac{dA}{dt} = \frac{l}{2m} = \text{const}$

- area of a ellipse  $A = \pi ab$  of orb period  $P$ .

-

$$\therefore \frac{A}{P} = \frac{\pi ab}{P} = \frac{L}{2m}$$

- eclipse geo :  $b^2 = a^2(1 - e^2)$

- also,  $\frac{L^2}{m^2} GMa(1 - e^2)$

-

$$P^2 = \frac{4\pi^2}{GM} a^3$$

### 3 Orbital energetics

- total energy  $E$  is conserved

- sum of  $K$  and  $U$

-

$$E = K + U$$

-

$$= \frac{1}{2}mv^2 - \frac{GMm}{r}$$

- total  $E$  is conserved

-

$$E = \left(\frac{GMm}{L}\right)^2 \frac{m}{2}(e^2 - 1)$$

- Hyperbolic orbit:  $e > 1, E > 0, K > |U|$ 
  - open orbit, unbound; single perihelion passage at  $\theta = 0$
- Parabolic orbit:  $e = 1, E = 0, K = |U|$ 
  - marginally unbound; velocity approach zero at infinite time
- elliptical orbit:  $e < 1, E < 0, K = |U|$
- objects originating outside our solar system are easily identified by their total energy
  - measure total energy (how far away it is, how fast is it moving)

#### 3.1 Checking energy in circular orbits

- governing equation for circular orbits in scalar form

-

$$f = ma$$

-

$$\frac{GM}{r^2} = \frac{v^2}{r} = \omega^2 r$$

-

$$v = \sqrt{\frac{GM}{r}}$$

#### 3.2 Negative total energy orbits

- bound orbits have  $E < 0$
- must add energy to break “unbind” the orbits

### 3.3 Parabolic orbits: escape speeds

- Escape speed is the speed that will bring your total pot energy to 0
- velocity becomes zero at infinite distance

-

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

-

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

### 3.4 Hohmann transfer orbit

- Elliptical transfer orbit from earth to superior planet
  - earth's orbit becomes the transfer orbit's perihelion passage
  - inserted into superior planet orbit at aphelion. This constrains launch windows
  - theoretically requires only two burns: at launch and aphelion insertion point
- semimajor axis of transfer orbit

-

$$a_{to} = \frac{a + a_{sup}}{2}; Earth$$

## 4 Earth-Moon System

### 4.1 Motion of the moon

- 27.3 sidereal orbit
- 29.5 synodic orbit
- rises in east and sets in west diurnally, but moves eastwards by about 12 deg per day rel to stars
- rises hour later per night

### 4.2 Precession

- earth is an oblate spheroid with equatorial bulge of .3% cause by separation
- sun, moon, and planets exert a torque  $\tau$  on earth

-

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- results in precession of spin axis of earth around ecliptic pole
- NCP moves. Polaris will not always be at NCP
- moves through stars with  $P \approx 28500yr$

- opening angle

-

$$47^\circ (= 2 \times 23.5^\circ)$$

### 4.3 Tidal Forces

- Moon exerts diff tidal forces on matter on earth
- esp noticeable on earths ocean surface as tides
- when sun and moon align (sun-earth-moon at  $0^\circ$  and  $180^\circ$ ) high-amp tides result, called spring tides
- when sun and moon are at  $90^\circ$  they sum destructively, producing neap tides

#### 4.3.1 Diff gravitational tidal forces

- arise from the  $r^{-2}$  dependence of grav force
- Taylor expansion about center of earth  $r_0$
- 

$$\delta F = \frac{2GM_{moon}m}{r^3}(r - r_0)$$

- Sun exerts about half as strong as moon tidal forces

#### 4.3.2 Rotation of tides

- tidal bulges produced on earths by the moon rotate at the same angular rate as the moons orbit around earth
- but the earth is rotating faster at once per sidereal day by 4 minutes. Drags the tides forward from where they would otherwise be by about  $10^\circ$  by friction
  - therefore high tides occur shortly after upper transit of moon
  - the misalignment drives angular momentum transfer between earth and Moon
    - moon pulls strongly on nearer tidal bulge than farther tidal bulge
    - net torque to slow earth rotation
    - but conversely the tidal bugle pulls more strongly on the moon, pulling it forward, increasing its angular momentum

### 4.4 Earth Shape

- moon stretches earth in a prolate deformation
- spin of the earth causes an oblate deformation
- oblate is much greater the prolate def

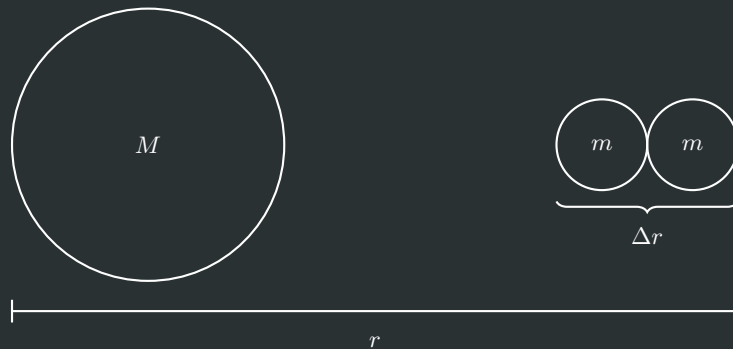
## 4.5 Roche Limit

- object get too close, forces on one side much greater then other, rip object apart
- approx a planet as two spheres 2m

$$\Delta F = \frac{dF}{dr} \Delta r = \frac{2GMm}{r^3} \Delta r$$

- Is there a force holding 2m together? yes, self grav

$$F = -\frac{Gmm}{(\Delta r)^2}$$



## 4.6 Hill radius

- Tidal forces of sun on earth-moon systems means that there is a maximum orbital distance for the moon, if it is to remain bound to the earth

## 4.7 Plane of lunar orbit

- Inclined by  $5.1^\circ$
- the moon is near the cele equator so the moon is above the horizon about 50% of the time for most observers on earth
- moves north and south in the sky in addition to its motion around the earth. greatest dec is  $23.5 + 5.1 = 28.6$  and min is  $-23.5 - 5.1 = -28.6$
- causes eclipses to be retrograde

## 4.8 Tidal forces: earth vs moon

- earth exerts greater tidal forces on moon than the moon does on the earth.

$$\Delta g_{moon \rightarrow earth} = \frac{\Delta F}{m} \propto \frac{M_{Moon} R_{Earth}}{r^3}$$

-

$$\Delta g_{\text{earth} \rightarrow \text{moon}} \propto \frac{M_{\text{Earth}} R_{\text{Moon}}}{r^3}$$

-

$$\frac{\Delta g_{\text{moon} \rightarrow \text{earth}}}{\Delta g_{\text{earth} \rightarrow \text{moon}}} \frac{M_{\text{moon}} R_{\text{earth}}}{M_{\text{earth}} r_{\text{moon}}} \approx \frac{1}{20}$$

## 4.9 lunar librations

- E-w and n-s nodding motions of the moon seen from earth, caused by parallax
- tidal locking is not perfect, so the libration happens in longitude
- because the rotation axis is inclined there is libration in lat

# 5 Waves

## 5.1 Spectra (How do we know what the universe is made out of?)

### Multi-messenger astronomy

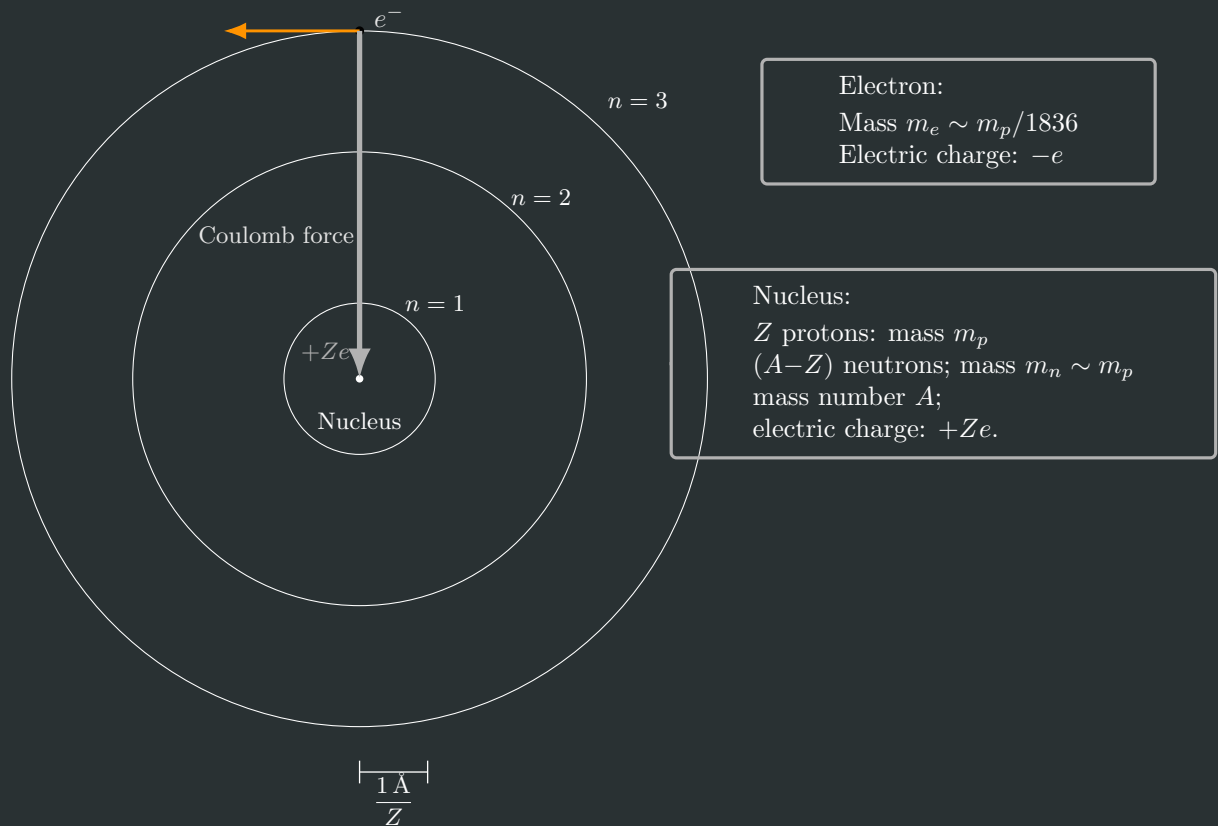
- Electromagnetic radiation
- cosmic rays
- meteorites
- neutrinos
- gravitational waves

## 5.2 Atoms and spectra

Hydrogen gas exhibits emission lines at discrete visible wavelengths, fit by empirical relation by Balmer in 1885

$$\frac{1}{\lambda} = R(1/4 - 1/n^2)$$
$$n = \text{integer} > 2$$

### 5.3 Bohrs model



Because orbital angular momentum is quantized, so is  $r_n$  and  $E_n \rightarrow$  discrete orbital levels

### 5.4 Atomic transition processes

- Transitions to free unbounded states behave similarly, however, they have diff names
- ionization and recombination
- photoionization (electron is knocked free). photon knocks electron free
- collisional ionization (electron becomes free). any other particle knocks electron free
- a positively charged ion may combine with a free electron, and atom emits radiation (photons) as the electrons drop to lower levels. called **recombination**

### 5.5 Kirchoff's Laws

- blackbody
- emission lines
- absorption lines

## 5.6 Temperature affects internal states of atoms and molecules

- temp of gas determines the kinetic energies of the colliding particles
- and the incoming photons
- we observe outgoing photons

## 5.7 Temperature vs velocity

- Thermal motions: emitted or absorbed photons inherit their energy from the Doppler velocities of the thermal motions of particles/atoms
- equilibrium distribution of particle speeds in an ideal gas is given by the Maxwell-Boltzmann distribution

## 5.8 mean free path and optical depth

- mean free path  $x_m$ . Distance which intensity decreases by a factor of  $1/e$
- optical depth  $\tau = x/x_m$ . Thickness of slab in units of mean free path  $x_m$
- column density,  $N(x)$ : total number of absorbing particles in a column with cross-section area  $1 \text{ m}^2$  and length  $x$

# 6 Telescopes

## 6.1 photoelectric effect

- Photoemission emission of an electron from a material in response to an incident photon
  - photoemissive material (underlying material)
  - work function (min energy required to produce light)
  - photoelectric effect (photoemission from atoms in certain materials)
  - photoelectron (released electron)
- particle energy of EM radiation

## 6.2 Sun

### 6.2.1 Chromosphere

- Very sparse layer of gas above the photosphere
- very hot gas, emission spectra by Kirchhoff laws
- easily seen during total eclipse or with an H $\alpha$  filter

### 6.2.2 Corona

- Low density out layer of suns atmosphere. Most easily visible during total eclipse
- $T = 2 \times 10^6$
- Emission lines from highly ionized atoms
- x-ray emission from thermal Bremsstrahlung (not black body)
- Optical continuum is originally from photosphere
- scattered by free electrons in coronal plasma
- coronal streamers show how plasma follows magnetic field lines

### 6.2.3 Solar Wind

- Bunch of protons and other various particles ejected from the sun
- Speed of protons in the corona

$$V_{rms} = \frac{3kT^{1/2}}{m_p} \approx 160km/s$$

- Escape speed as function of distance

$$\frac{GM_{\odot}}{r}^{1/2} \approx 620km/s$$

- Sun produces a solar wind with  $v = 400km/s$ , density  $10^{-21}kgm^{-3}$  earth

$$\Delta M = (4\pi r^2 \Delta r) \rho$$

- therefore mass flux through shell

$$\frac{dM}{dt} = 4\pi r^2 \frac{dr}{dt} \rho$$

$$\dot{M} = 4\pi r^2 v \rho$$

$$\dot{M} \sim 10^8 kg s^{-1}; t_m \sim 10^{14}$$

- Maybe try this myself with various sizes of stars?? Seems easy to verify large stars ejecting large amounts of wind

#### 6.2.4 Magnetic Fields

- Lorentz force  $\vec{F} = q\vec{v} \times \vec{B}$
- Charged particles follow curved helical paths
- Magnetic field energy

-

$$P_B = \varepsilon_B = \frac{B^2}{2\mu_0}$$

#### 6.2.5 Sunspots

- Cooler than surroundings because the magnetic field is enhanced in the spot
- Pressure due to a magnetic field

-

$$P_B = 4 \times 10^5 \text{ Nm}^{-2} \left(\frac{B}{1T}\right)^2$$

- pressure due to ideal gas

-

$$P_{gas} = nkT$$

#### Pressure balance in sunspots

- gas and magnetic pressure inside sunspot must equal surrounding gas pressure

-

$$\frac{\rho k T_s}{m_p} + \frac{B^2}{2\mu_0} = \frac{\rho k T_P}{m_p}$$

- $B \approx .1T$

#### Sunspot Cycle

- Star near 30° N/S, migrate towards solar equator
- more numerous every 11 years

## 10 The planets

### 10.1 Mercury

- always 30° from the sun
- strong tidal forces, permanent prolate tidal bulge
- sidereal rotation  $P_{rot} = 58.65d$
- $P_{orb} = 87.97$
- Orbit is tidally locked at perihelion

## 10.2 Venus

- Retrograde Motion
- atmosphere
  - Clouds are sulfuric acid
  - $\sim 96.5\%$   $\text{CO}_2$  and  $\sim 3.5\%$   $\text{N}_2$
  - Very strong greenhouses
- Earth's liquid water ocean dissolves  $\text{CO}_2$
- runaway greenhouse

## 10.3 Earth

- Temperature and pressure on earth allow significant quantities of gas, liquid, and solid water.  
Not true for Venus or Mars

## 10.4 Mars

- $a = 1.52\text{AU}$
- $P_{\text{sidereal}} = 1.88$
- $24\text{h}40\text{m}$
- Atmosphere
  - Pressure  $\sim .006\text{atm}$  (earth)
  - $95\%$   $\text{CO}_2$
  - UV photodissociates  $\text{H}_2\text{O}$ , the oxygen oxidizes iron in soil
- Seasons caused by obliquity
- polar caps of  $\text{CO}_2$  form and melt in winter and summer

## 10.5 Jupiter and Saturn

- they are in hydrostatic equilibrium
- $\frac{dP}{dr} = -\frac{GM_r\phi}{r^2}$
- $M_r$  is the mass within radius  $r$  assuming constant density  $\phi$
- $\int_{P_c}^0 dP = \frac{4\pi}{3}\phi^2 G \int_0^R$
- solving shows that there must be very high pressures in centers of Jovian planets
- Metallic hydrogen is a conductor
- convection + rotation in a conducting fluid produce magnetic dynamos and magnetic fields

### 10.5.1 energy deficit for jupiter

- Jupiter's luminosity is twice its rate of solar irradiation from the sun
- Consider gravitational potential energy of a shell of thickness
- extra energy comes from the radius shrinking

### 10.6 Fast rotation

- Polar diameter of Jupiter is 6.5% shorter than equatorial diameter
- Saturn is 10% shorter than equatorial diameter

### 10.7 rings

- all Jovian planets have rings; Saturn are the most prominent
- tidally disrupted satellite inside Roche limit
- internal structures caused by orbital resonances

## 12 Solar system in perspective

### 12.1 Solar system config

- Planetary orbits are coplanar
- Sun's equator close to orbit plane
- nearly circular orbits
- planets all orbit in same direction
- most planets rotate in the same direction as their orbital motion

### 12.2 Protostellar Neb

- Cloud of gas compressing and spinning

### 12.3 Comparative planetology

- Massive colder planets retain atmospheres
- planet masses and compositions were driven by the temperature gradient in the protoplanetary disk and its relation to the condensation temperature profile

### 12.4 Origin of the solar system

- Giant Jovian satellites are a mini version of the whole solar system
- rings are temporary and provide evidence for a dynamic, evolving planetary system
  - all Jovian planets have rings
  - produced by bodies that failed to form or were disrupted due to tidal forces
  - Triton is scheduled to explode

## 12.5 Dynamic solar system

- our moon, large satellite of a small planet
- triton captured by Neptune
- retrograde rotation of Venus
- large impact craters

## 12.6 Detecting exoplanets

### 12.6.1 Transit

- can theoretically resolve some based on angular resolution, but very hard to separate from the star itself
- $L_{jup} = (\frac{L_{\odot}}{r\pi a^2})(\pi R_{jup}^2 A)$
- if you look at infrared radiation it is “easier” to resolve to planet

### 12.6.2 Indirect

- Wobble due to the planet pulling on the star
- They orbit their COM

### 12.6.3 Doppler effect

- Measure redshift and blue shift on the star

## 12.7 Unknown orbital inclination angle

- We do not know the inclination of the plane of the system relative to our line of sight
- Doppler can only measure velocities projected onto the line of sight
- if  $i = 90^\circ$ , the plane is viewed edge on, and we measure true velocity. Otherwise, we measure a smaller velocity, giving a lower limit of planet masses.

## 12.8 Radial Velocity method

- we need the complete more accurate version of Kepler III

-

$$p^2 = \frac{4\pi^2(a_A + a_B)^3}{G(M_A + M_B)}$$

- can find  $p$  by looking at when the velocity pattern repeats
- we want  $a_b$  and  $M_B$ . get  $a_B$  from

-

$$a_b^3 \approx \frac{GM_A P^2}{4\pi^2}$$
$$M_A \gg M_B$$

-

$$\frac{a_A}{a_B} = \frac{M_B}{M_A}$$

-

$$M_B = \left( \frac{4\pi^2 M_A^2 a_A^3}{GP^2} \right)^{1/3}$$

## 12.9 Transit method

- Detected by planet passing in front of the star, causing a dip in the brightness
- find mind transit flux reduction
- find planet velocity (same method at radial vel)
- time before  $t_1$  and  $t_2$  yields planet radius or measuring the fraction of the starlight blocked (if  $r_s$  is known)

### 12.9.1 Chance of detection

- Range of inclinations for which transit is observed

-

$$\cos i \leq \frac{R_A + R_B}{a}$$

## 12.10 Surprising exoplanet discoveries

- **Hot Jupiter's** - exoplanets with Jupiter like masses which are very close to their host stars
- very eccentric and/or inclined planet orbits
- current perspective biased by observational methods

## 14 Stellar Atmospheres

## 23 Galaxies and Super clusters

### Stellar streams

- Galaxies ( $r \sim 10$  kpc)
- Groups ( $r \sim 1$ Mpc)
- Supercluster ( $r \sim 100$ Mpc)

### Coma Cluster

- rich cluster dominated by elliptical Galaxies

### Cluster ISM

- Bright x-ray emitting gas

- low density, but very large quality
- Total mass of ISM is around double that of the galaxies
- Motion through the ISM causes ram pressure stripping

### 23.1 Dark matter in galaxy clusters

- Use viral theorem...
- 

$$M \approx 7.5 \frac{\sigma^2 r_h}{G}$$

- Where  $\sigma$  is a velocity distribution, and  $r_h$  is radius of the cluster
- If we look at the mass to light ratio of the Coma Cluster, which is 250 (in solar units), we find a very different distribution as compared to our sun ( $= 1$ ), and the Milky Way ( $\sim 10$ )

### 23.2 Large Scale Structure

- Cosmic web structure
- Sloan great wall
- Few galaxies measured far away, simply because they're hard to measure

### 23.3 Galaxy collisions

- Can estimate velocity in cluster from velocity dispersion ( $\sigma$ ) of galaxy motions
  -
- $$v \sim \sqrt{3}\sigma$$
- Can then compute collision rates
  - In rich clusters about a 50% chance

#### 23.3.1 Rates of collisions

Consider a galaxy of radii  $R$  moving at velocity  $v$ , in an elapsed time  $t$  it will collide with any other galaxy within a cylindrical volume  $V(t)$  of radius  $R^2$  and length  $vt$

$$V(t) = (4\pi R^2)(vt)$$

If galaxy density is  $n$  then the total collisions is  $N_*$  and the average time to collision is  $t_*$

$$N_* = nV(t) = v(4\pi R^2)(vt)$$

$$N_* = 1 \rightarrow t_* = \frac{1}{nv(4\pi R^2)}$$

#### 23.3.2 Galaxies collide, but stars to not

If you repeat the previous equation but for stars, you find a 1 in  $10^8$  chance of stellar collision, in the age of the entire universe

### 23.3.3 Physics of collisions

- Raise tidal bulges and resulting tidal tails
- They are inelastic, the kinetic energy is converted into funny motions of the stars themselves

## 24 Cosmology

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### 24.1 CMB

- $ct \Rightarrow$  theoretical size of the universe
- Does not account for expansion of the uni while the light is travelling
- **Surface of last scattering**, point where the universe becomes opaque, also the CMB

#### 24.1.1 olbers paradox

- We do not see a galaxy everywhere we look
- this is because the universe is finite, not infinitely large

#### 24.1.2 Equivalence principle

##### Gravitational Mass

$$F_{grav} = \frac{-GM_g m_g}{r^2} \quad (1)$$

##### Interial mass

$$a = \frac{F}{m_i} \quad (2)$$

##### Equivalence Principle

$$m_g = m_i \quad (3)$$

#### 24.1.3 Einstein v Newton

- In a small volume of space the downward pull of gravity cannot be distinguished from an upward acceleration of the observer

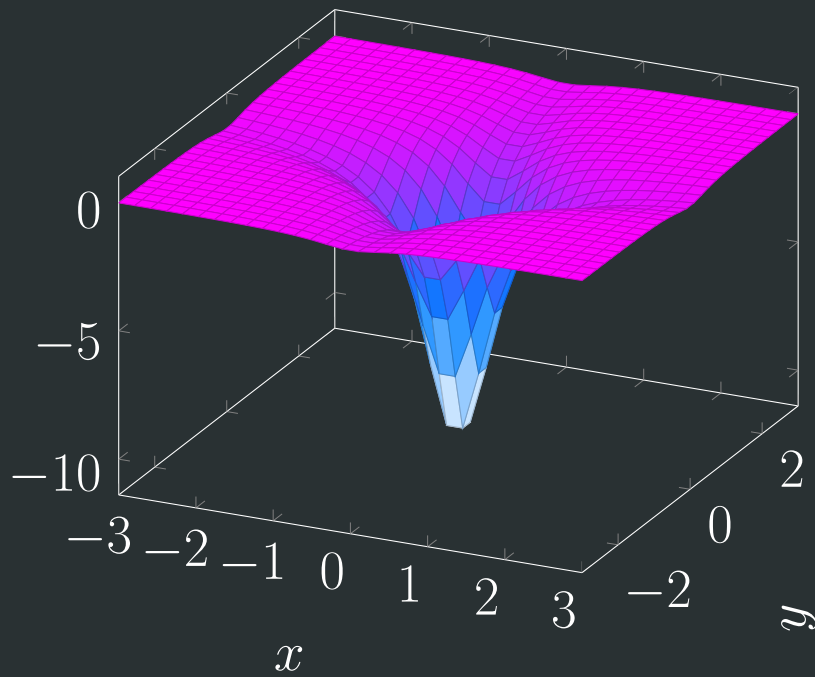
### 24.2 General relativity

- Space and time bind together to form curved spacetime
- ‘free-fall’ is motion in a straight line in 4d spacetime (this is grav acceleration)
- Matter (Mass-energy) tells spacetime how to curve
- Curved spacetime tells matter how to move

### 24.2.1 Predictions of GR

- Gravitational field deflects light (grav lensing)
- more accurate orbit for Mercury
- grav fields slow clocks
- moving objects will produce gravitational waves

## Gravity in spacetime



### 24.3 Surface of spacetime

- How do we know the curvature and angles?
- If we draw a triangle what do the angles add up to?

#### Curvature and Angles of a triangle in 4d space

$$\alpha + \beta + \gamma = \pi \frac{\kappa A}{r_{c,0}^2} \quad (4)$$

#### Newtonian Friedmann Equation

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho(t) + 2 \frac{k}{r^2} \frac{1}{a^2(t)} \quad (5)$$

Newtonian version of Friedman equation

#### Relativistic Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} - \frac{\kappa c^2}{r_{c,0}^2} \frac{1}{a^2(t)} + \frac{\alpha}{3} \quad (6)$$

## 25 Equations

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$$F_{grav} = \frac{-GM_g m_g}{r^2} \quad (1) \text{ (Gravitational Mass)}$$

$$a = \frac{F}{m_i} \quad (2) \text{ (Inertial mass)}$$

$$m_g = m_i \quad (3) \text{ (Equivalence Principle)}$$

$$\alpha + \beta + \gamma = \pi \frac{\kappa A}{r_{c,0}^2} \quad (4) \text{ (Curvature and Angles of a triangle in)}$$

$$\frac{\dot{a}^2}{a} = \frac{8\pi G}{3} \rho(t) + 2 \frac{k}{r^2} \frac{1}{a^2(t)} \quad (5) \text{ (Newtonian Friedmann Equation)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} - \frac{\kappa c^2}{r_{c,0}^2} \frac{1}{a^2(t)} + \frac{\alpha}{3} \quad (6) \text{ (Relativistic Friedmann equation)}$$

## Glossary

**Stellar streams** Stars following a path away from a galaxy [22](#)

**Surface of last scattering** Point looking deep into the universe where the universe becomes opaque. Also where the CMB is [25](#)