

Cram

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1 Polar

$$x \rightarrow r \cos(\theta)$$

$$y \rightarrow r \sin(\theta)$$

$$da \rightarrow rdrd\theta$$

$$x^2 + y^2 = r^2$$

Converting Cords

$$x = \rho \sin \varphi \cos \theta$$

$$x = \rho \sin \varphi \sin \theta$$

$$z = \rho \cos \varphi$$

$$\rho^2 = x^2 + y^2 + z^2$$

To cylindrical . . .

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$z = z$$

To spherical . . .

$$\rho^2 = x^2 + y^2 + z^2$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$\varphi = \arccos\left(\frac{z}{\rho}\right)$$

2 Vector fields and such

A curve C given by... $r = \langle x(t), y(t) \rangle$, $a \leq t \leq b$

$$\begin{aligned} \int_C \sqrt{1 + 9xy} ds, \quad C = r(t) = \langle t, t^3 \rangle, \quad 0 \leq t \leq 1 \\ \int_0^1 \sqrt{1 + 9t^4} \sqrt{1 + 9t^4} dr, \quad r(t)' = \langle 1, 3t^2 \rangle, \quad r(t)' = \sqrt{1 + 9t^4} \\ \int_1^4 1 + 9t^4 \\ t + \frac{9t^5}{5} \Big|_0^1 \\ 1 + \frac{9}{5} \end{aligned}$$

2.1 Greens theorem

$$\int_c P dx + Q dy = \int \int_D \frac{\partial Q}{\partial x} \frac{\partial P}{\partial y} dxdy$$

Example ...

$$\begin{aligned} \int_0^1 xy + y^2 dxdy &\rightarrow \int_0^1 \int_0^1 \frac{\partial}{\partial x} x^2 - \frac{\partial}{\partial y} xy dxdy \\ &\int_0^1 \int_0^1 2x - x dxdy \\ &\int_0^1 \int_0^1 x dxdy \\ &\int_0^1 1/2 dy \\ &= 1/2 \end{aligned}$$