

Calc III Notes

Pierson

August 27, 2025

1 Multivariable calc goals

- vectors and geometry of \mathbb{R}^n
- functions of several variables diff and int
- higher dimensional versions of FTC
- find them for line integrals
- greens them
- stokes them
- divergence them

12 Chapter 12: vectors and geo of space

12.1

-

$$\mathbb{R} = \text{line}$$

$$\text{dist}(x,y) = |y - x|$$

-

$$\mathbb{R}^2 = \text{Plane}$$

Ordered pairs of (x,y) of real numbers

-

$$\mathbb{R}^3 = \text{space}$$

ordered triples of (x,y,z) of real numbers

right-hand rule: point fingers toward pos x-axis and curl towards pos y-axis, thumb should point to positive z

R^n = n-tuples (x_1, x_2, \dots, x_n) n-dim space

Distance D from (0,0) to (x,y)

$$D^2 = x^2 + y^2$$

Distance D from (0,0,0) to (x,y,z)

$$D = \sqrt{x^2 + y^2 + z^2}$$

Sphere with center X_0, Y_0, Z_0 and radius R has equation:

$$(X - X_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

Find equation of the set of points equidistant from the points a1,4,2, and b 3,3,3

$$AX = \sqrt{(x-1)^2 + (y-4)^2 + (z-2)^2}$$

$$BX = \sqrt{(x-3)^2 + (y-3)^2 + (z-3)^2}$$

set $AX = BX$

Result is a **plane**

12.2 vectors and geo of space

scaler = magnitude ≤ 0

vector = magnitude + direction

$$\vec{u} = \langle 5, 3 \rangle, \vec{u} = 5\hat{i} + 3\hat{j}$$



Scaler multiple

Changes the length of the vectors, ie, stretches them in a direction.

Length of $k\vec{v} = |K| * |V|$

direction of $k\vec{V} = \text{same as } \vec{V} \text{ if } k > 0$

real numbers work like scaling factors

- 13 Chapter 13: vector functions
- 14 Chapter 14: partial derivate
- 15 Chapter 15: multiple integrals
- 16 Chapter 16: vector calculus