

Notes

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0 Class overview

Office hours tues and thurs 11-12 astro 237

ta office hours

wed 3:30:530 astro 2367

hw due wed

hw posted week before

- solar system
- stellar evo
- compact objects
- galaxy quasi darkmatter
- cosmic web
- big bang

course goals

- apply pys to universe
- understand foundations of modern astro, astrophys, and cosmology
- conceptual understanding of the uni based on physical principles

1 Early Astronomy

1.0.1 Greek

- Aristotle
 - earth is spherical
 - partial lunar eclipses
 - some stars visible from southern locations but not northern and vice versa
 - had ideas regarding perfect geo influenced by Pythagoras and Plato
- Aristarchus (310-230 BC):

- unpreceded heliocentric framework
- trig distances earth-moon-sun system
- angular diameters $\theta_{sun} \approx \theta_{moon} \therefore \frac{A}{C} = \frac{D_{moon}}{D_{earth}}$
- diameters from lunar eclipses $D_{moon} < D_{earth}$
- Eratosthenes (176-195 BC):
 - Determined radius of spherical earth R_E
 - Sun at zenith at noon on summer solstice at Aswan
 - But further north in Alexandria, Egypt, the sun is south of the zenith by angle α
- Hipparchus (190-120 BC):
 - Discover precession of the equinoxes from examination of star catalogs over centuries
 - established the magnitude system
- Copernicus (1473-1543):
 - heliocentric
 - earth rotates
 - still assumed uniform circular celestial motion
 - inferior planets: orbit smaller than earths
 - superior planets: orbits larger than earths

1.1 Emergence of modern Astro Inferior planets

- $B/C = \sin \theta_E$
- $B=C \sin \theta_E$
- C is AU
- Early astronomers didn't know C, so they could only infer ratios of B/C. I.e. Orbital radii measured in AU

Superior Planets

- Measure time between opposition and eastern quadrature
- want angle θ between opp and east quad
- $\theta = (\omega_E - \omega_p)$ and $C/B = \cos\theta$
- measure τ and synodic period, calculate sidereal period and ω_p ; know ω_E and infer C/B

Galilean Revolution

- Galileo Galilei (1564 -1642)
 - improved and used a basic refracting telescope

- def publication of early results 1610 "starry messenger"
- - Moon is cratered; not a perfect Sphere
- milkyway is made out of stars
- Jupiter has moons (or as he thought, stars)
- measured phases of Venus

Phases of Venus

- direct confrontation with Ptolemaic geocentric models
- in Ptolemaic models you only see crescent phases

Tycho Brahe (1546-1601)

- Denmark, later Prague
- Given island by king Fredrick (and staff)
- made a accurate and vast database of celestial motion
- had a lead nose?
- Threw giant ragers
- supernova named after him

Johannes Kepler (1571–1630, Prague)

- 'Inherited' (maybe stole) Brahe's data
- also has a SN
- Kepler fit a new empirical model of heliocentric orbits, abandoning perfect circles
 - "*It was as if I awoke from sleep and saw a new light*" (Kepler, New astronomy)

Kepler's Laws

First law

- The planets travel on elliptical orbits with the sun at one focus
- Semimajor axis, half the major axis
- eccentricity: how elliptical (stretched) an orbit is - distance between foci divided by major axis.

second law

- A line drawn from the sun to a planet sweeps out equal areas in equal time intervals'
- perihelion: orbital point closest to the sun
- aphelion: furthest orbital point from the sun

third law

Def: *The square of the sidereal orbital periods of the planets are prop to the cubes of the Semimajor axis of their orbits*

$$p^2 = Ka^3$$

P = planets sidereal period

a= length of semimajor axis

K = constant

Consequences of heliocentric model

- retrograde motion of outer planets
- positions of outer and inner planets wrt sun
- annual parallax
- aberration of starlight
- Coriolis effect

Parallax

- annual parallax: change in the apparent position when seen from two diff locations due to earth revolving around the sun. First measured by Bessel in 1838

Aberration of starlight

- deflection of apparent stellar positions in the direction of the observers motion
- analog: running throw rain and getting wet in the front and not in the back
- detected (Picard, 1680); explained (Bradley, 1729)
- telescope is moving along orbital vector around the sun; translation along orbit cannot exceed transit time of light through telescope

Coriolis effect: evidence of earth rotation

- Coriolis acceleration is perp to the direction of motion
- $$\vec{a}_{cor} = s\vec{v} \times \vec{\omega}$$
- can be deduced from a pendulum
- and in hurricanes!

2 Orbital Mechanics I

2.1 Newtonian mechanics

Parametric vectors

Displacement $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$
distance: $r(t) = |\vec{r}(t)| = \sqrt{\vec{r} \cdot \vec{r}}$

2.2 Newtons laws

First law

- Isaac newton(1642-1727)
 - an objects' velocity remains constant unless a net outside force acts upon it
 - $\vec{v}(t) = \vec{v}_0 = \text{const}$

second law

- $\vec{F} = m\vec{a}(t)$
 - $\vec{F} = \frac{d\vec{p}(t)}{dt}$
 - $d\vec{v}/dt = \vec{f}/m$
 - force changes velocity
 - used a lot in computational math

third law

- forces come in pairs, equal in magnitude, and opposite in direction

Newtonian gravity

- a force, grav, exists between any two objects having mass m and M , prop to the product of their masses MM and inversely proportional to the square of the separation distance r of their centers
 - for coordinates centered on M :
 - $\vec{F} = -G \frac{Mm}{|\vec{r}|^2} \hat{r}$

2.3 Displacement vector and polar coordinates

2.4 Kepler laws: angular momentum

-

2.5 keplers 2nd law = consv, angular momentum

-

$$d\vec{L}/dt = 0$$

-

$$\vec{L} = \vec{R} \times \vec{p} = \vec{r} \times m\vec{v} = const$$

-

$$\Rightarrow |\vec{v}| = L = mr v_1$$

2.6 Keplers Laws

2.6.1 Keplers First Law

- $\frac{d\vec{v}}{dt} = -\frac{GM}{r^2} \hat{r}$

-

$$\frac{L}{GMm} \frac{d\vec{v}}{dt} = \frac{d\hat{\theta}}{dt}$$

-

$$\frac{L}{GMm} \vec{v} = \hat{\theta} + e\hat{j}$$

- take dot product of both sides with unit vector $\hat{\theta}$, *using*

- $\hat{j} \cdot \hat{\theta} = \cos \theta$

-

$$\vec{v} \cdot \hat{\theta} = v_t = \frac{L}{mr}$$

2.7 Kepler III

- we know that $\frac{dA}{dt} = \frac{l}{2m} = const$

- area of a ellipse $a = \pi ab$ of orb period p.

-

$$\therefore \frac{A}{P} = \frac{\pi ab}{P} = \frac{L}{2m}$$

- eclipse geo : $b^2 = a^2(1 - e^2)$

- also, $\frac{L^2}{m^2} GM a (1 - e^2)$

-

$$P^2 = \frac{4\pi^2}{GM} a^3$$

3 Orbital energetics

- total energy e is conserved
 - sum of K and U
 - $$E = K + U$$
 - $$= \frac{1}{2}mv^2 - \frac{GMm}{r}$$
- total E is conserved
 - $$E = (\frac{GMm}{L})^2 \frac{m}{2} (e^2 - 1)$$
- Hyperbolic orbit: $e > 1, E > 0, K > |U|$
 - open orbit, unbound;, single perihelion passage at $\theta = 0$
- Parabolic orbit: $e = 1, E = 0, K = |U|$
 - marginally unbound; velocity approach zero at infinite time
- elliptical orbit: $e < 1, E < 0, K = |U|$
 - objects originating outside our solar system are easily identified by their total energy
 - measure total energy (how far away it is, how fast is it moving)

3.1 Checking energy in circular orbits

- governing equation for circular orbits in scalar form
 - $$f = ma$$
 - $$\frac{GM}{r^2} = \frac{v^2}{r} = \omega^2 r$$
 - $$v = \sqrt{\frac{GM}{r}}$$

3.2 Negative total energy orbits

- bound orbits have $E \downarrow 0$
- must add energy to break “unbind” the orbits

3.3 Parabolic orbits: escape speeds

- Escape speed is the speed that will bring your total pot energy to 0
- velocity becomes zero at infinite distance
- $$\frac{1}{2}mv^2 = \frac{GMm}{r}$$
- $$v_{esc} = \sqrt{\frac{2GM}{r}}$$

3.4 Hohmann transfer orbit

- Elliptical transfer orbit from earth to superior planet
 - earth's orbit becomes the transfer orbits perihelion passage
 - inserted into superior planet orbit at aphelion. This constrains launch windows
 - theoretically requires only two burns: at launch and aphelion insertion point
- semimajor axis os transfer orbit
- $$a_{to} = \frac{a + a_{sup}}{2}; Earth$$

4 Earth-Moon System

4.1 Motion of the moon

- 27.3 sidereal orbit
- 29.5 synodic orbit
- rises in east and sets in west diurnally, but moves eastwards by about 12 deg per day rel to stars
- rises hour later per night

4.2 Precession

- earth is an oblate spheroid with equatorial bulge of .3% cause by separation
- sun, moon, and planets exert a torque τ on earth
- $$\vec{\tau} = \vec{r} \times \vec{F}$$
- results in precession of spin axis of earth around ecliptic pole
- NCP moves. Polaris will not always be at NCP
- moves through stars with $P \approx 28500\text{yr}$

- opening angle
- $47^\circ (= 2 \times 23.5^\circ)$

4.3 Tidal Forces

- Moon exerts diff tidal forces on matter on earth
- esp noticeable on earths ocean surface as tides
- when sun and moon align (sun-earth-moon at 0° and 180°) high-amp tides result, called spring tides
- when sun and moon are at 90° they sum destructively, producing neap tides

4.3.1 Diff gravitational tidal forces

- arise from the r^{-2} dependence of grav force
- Taylor expansion about center of earth r_0
-
$$\delta F = \frac{2GM_{moon}m}{r^3}(r - r_0)$$
- Sun exerts about half as strong as moon tidal forces

4.3.2 Rotation of tides

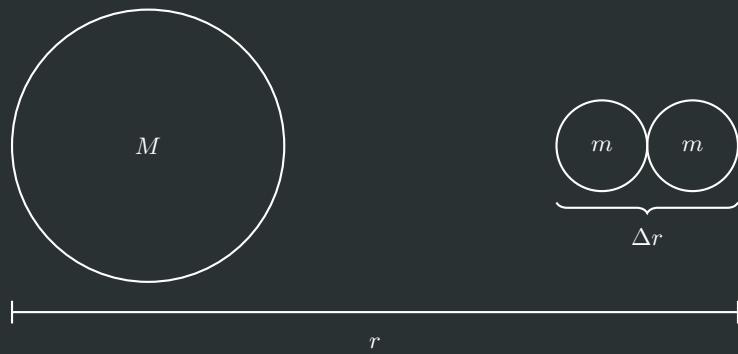
- tidal bulges produced on earths by the moon rotate at the same angular rate as the moons orbit around earth
- but the earth is rotating faster at once per sidereal day by 4 minutes. Drags the tides forward from where they would otherwise be by about 10° by friction
 - therefore high tides occur shortly after upper transit of moon
 - the misalignment drives angular momentum transfer between earth and Moon
 - moon pulls strongly on nearer tidal bulge than farther tidal bulge
 - net torque to slow earth rotation
 - but conversely the tidal bulge pulls more strongly on the moon, pulling it forward, increasing its angular momentum

4.4 Earth Shape

- moon stretches earth in a prolate deformation
- spin of the earth causes an oblate deformation
- oblate is much greater the prolate def

4.5 Roche Limit

- object get too close, forces on one side much greater than other, rip object apart
- approx a planet as two spheres $2m$
-
- $$\Delta F = \frac{dF}{dr} \Delta r = \frac{2GMm}{r^3} \Delta r$$
- Is there a force holding $2m$ together? yes, self grav
-
- $$F = -\frac{Gmm}{(\Delta r)^2}$$



4.6 Hill radius

- Tidal forces of sun on earth-moon systems means that there is a maximum orbital distance for the moon, if it is to remain bound to the earth

4.7 Plane of lunar orbit

- Inclined by 5.1°
- the moon is near the celestial equator so the moon is above the horizon about 50% of the time for most observers on earth
- moves north and south in the sky in addition to its motion around the earth. greatest dec is $23.5 + 5.1 = 28.6$ and min is $-23.5 - 5.1 = -28.6$
- causes eclipses to be retrograde

4.8 Tidal forces: earth vs moon

- earth exerts greater tidal forces on moon than the moon does on the earth.
-

$$\Delta g_{moon \rightarrow earth} = \frac{\Delta F}{m} \propto \frac{M_{Moon} R_{Earth}}{r^3}$$

$$\Delta g_{\text{earth} \rightarrow \text{moon}} = \propto \frac{M_{\text{Earth}} R_{\text{Moon}}}{r^3}$$

$$\frac{\Delta g_{\text{moon} \rightarrow \text{earth}}}{\Delta g_{\text{earth} \rightarrow \text{moon}}} \frac{M_{\text{Moon}} R_{\text{Earth}}}{M_{\text{Earth}} r_{\text{Moon}}} \approx \frac{1}{20}$$

4.9 lunar librations

- E-w and n-s nodding motions of the moon seen from earth, caused by parallax
- tidal locking is not perfect, so the libration happens in longitude
- because the rotation axis is inclined there is libration in lat

5 Waves

5.1 Spectra (How do we know what the universe is made out of?)

Multi-messenger astronomy

- Electromagnetic radiation
- cosmic rays
- meteorites
- neutrinos
- gravitational waves

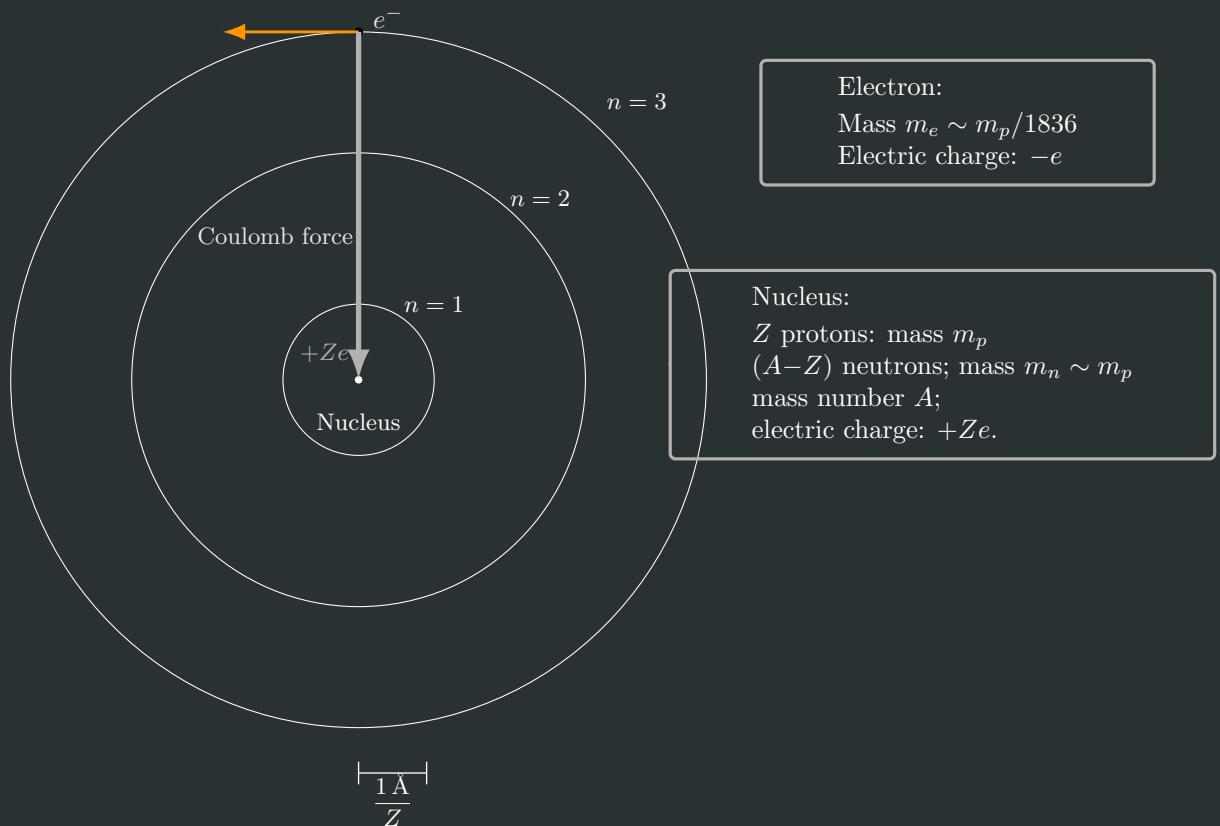
5.2 Atoms and spectra

Hydrogen gas exhibits emission lines at discrete visible wavelengths, fit by empirical relation by Balmer in 1885

$$\frac{1}{\lambda} = R(1/4 - 1/n^2)$$

$n = \text{integer} > 2$

5.3 Bohrs model



Because orbital angular momentum is quantized, so is r_n and $E_n \rightarrow$ discrete orbital levels

5.4 Atomic transition processes

- Transitions to free unbounded states behave similarly, however, they have diff names
- ionization and recombination
- photoionization (electron is knocked free). photon knocks electron free
- collisional ionization (electron becomes free). any other particle knocks electron free
- a positively charged ion may combine with a free leectron, and atom emits radiation (photons) as the electrons drop to lower levels. called **recombination**

5.5 Kirchoff's Laws

- blackbody
- emission lines
- absorption lines

5.6 Temperature affects internal states of atoms and molecules

- temp of gas determines the kinetic energies of the colliding particles
- and the incoming photons
- we observe outgoing photons

5.7 Temperature vs velocity

- Thermal motions: emitted or absorbed photons inherit their energy from the Doppler velocities of the thermal motions of particles/atoms
- equilibrium distribution of particle speeds in an ideal gas is given by the Maxwell-Boltzmann distribution

5.8 mean free path and opt depth

- mean free path x_m . Distance which intensity decreases by a factor of $1/e$
- optical depth $\tau = x/x_m$. Thickness of slab in units of mean free path x_m
- column density, $N(x)$: total number of absorbing particles in a column with cross-section area 1 m^2 and length x

6 Telescopes

6.1 photoelectric effect

- Photoemission emission of an electron from a material in response to an incident photon
 - photoemissive material (underlying material)
 - work function (min energy required to produce light)
 - photoelectric effect (photoemission from atoms in certain materials)
 - photoelectron (released electron)
- particle energy of EM radiation

6.2 Sun

6.2.1 Chromosphere

- Very sparse layer of gas above the photosphere
- very hot gas, emission spectra by Kirchhoff laws
- easily seen during total eclipse or with an H α filter

6.2.2 Corona

- Low density outer layer of sun's atmosphere. Most easily visible during total eclipse
- $T = 2 \times 10^6$
- Emission lines from highly ionized atoms
- x-ray emission from thermal Bremsstrahlung (not black body)
- Optical continuum is originally from photosphere
- scattered by free electrons in coronal plasma
- coronal streamers show how plasma follows magnetic field lines

6.2.3 Solar Wind

- Bunch of protons and other various particles ejected from the sun
- Speed of protons in the corona
- $$V_{rms} = \frac{3kT^{1/2}}{m_p} \approx 160 \text{ km/s}$$
- Escape speed as function of distance
- $$\frac{GM_{\odot}^{1/2}}{r} \approx 620 \text{ km/s}$$
- Sun produces a solar wind with $v = 400 \text{ km/s}$, density $\rho = 10^{-21} \text{ kg m}^{-3}$ earth
- $$\Delta M = (4\pi r^2 \Delta r) \rho$$
- therefore mass flux through shell
- $$\frac{dM}{dt} = 4\pi r^2 \frac{dr}{dt} \rho$$
- $$\dot{M} = 4\pi r^2 v \rho$$
- $$\dot{M} \sim 10^8 \text{ kg s}^{-1}; t_m \sim 10^{14}$$
- Maybe try this myself with various sizes of stars?? Seems easy to verify large stars ejecting large amounts of wind

6.2.4 Magnetic Fields

- Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$
- Charged particles follow curved helical paths
- Magnetic field energy
- $$P_B = \epsilon_B = \frac{B^2}{2\mu_0}$$

6.2.5 Sunspots

- Cooler than surroundings because the magnetic field is enhanced in the spot
- Pressure due to a magnetic field
- $$P_B = 4 \times 10^5 Nm^{-2} \left(\frac{B}{1T}\right)^2$$
- pressure due to ideal gas
- $$P_{gas} = nkT$$

Pressure balance in sunspots

- gas and magnetic pressure inside sunspot must equal surrounding gas pressure
- $$\frac{\rho k T_s}{m_p} + \frac{B^2}{2\mu_0} = \frac{\rho k T_P}{m_p}$$
- $B \approx .1T$

Sunspot Cycle

- Star near 30° N/S, migrate towards solar equator
- more numerous every 11 years

10 The planets

10.1 Mercury

- always 30° from the sun
- strong tidal forces, permanent prolate tidal bulge
- sidereal rotation $P_{rot} = 58.65d$
- $P_{orb} = 87.97$
- Orbit is tidally locked at perihelion

10.2 Venus

- Retrograde Motion
- atmosphere
 - Clouds are sulfuric acid
 - $\sim 96.5\text{ co2\%}$ and $\sim 3.5\text{ N2\%}$
 - Very strong greenhouses
- Earths liquid water ocean dissolves co2
- runaway greenhouse

10.3 Earth

- Temperature and pressure on earth allow significant qualities of gas, liquid, and solid water.
Not true for Venus or Mars

10.4 Mars

- $\alpha = 1.52\text{AU}$
- $P_{sidereal} = 1.88$
- $24h40m$
- Atmosphere
 - Pressure $\sim .006\text{atm}$ (earth)
 - 95% CO_2
 - UV photodissociates H_2O , the oxygen oxidizes iron in soil
- Seasons caused by obliquity
- polar caps of CO_2 form and melt in winter and summer

10.5 Jupiter and Saturn

- they are in hydrostatic equilibrium
- $\frac{dP}{dr} = -\frac{GM_r\phi}{r^2}$
- M_r is the mass within radius r assuming constant density ϕ
- $\int_{P_c}^0 dP = \frac{4\pi}{3}\phi^2 G \int_0^R$
- solving shows that there must be very high pressures in centers of Jovian planets
- Metallic hydrogen is a conductor
- convection + rotation in a conducting fluid produce magnetic dynamos and magnetic fields

10.5.1 energy deficit for jupiter

- Jupiter's luminosity is twice its rate of solar irradiation from the sun
- Consider gravitational potential energy of a shell of thickness
- extra energy comes from the radius shrinking

10.6 Fast rotation

- Polar diameter of jupiter is 6.5 shorter than equatorial bulge
- saturn is 10% shorter than equatorial diam

10.7 rings

- all Jovian planets have rings; Saturn are the most prominent
- tidally disrupted satellite inside Roche limit
- internal structures caused by orbital resonances

12 Solar system in perspective

12.1 Solar system config

- Planetary orbits are coplanar
- suns equator close to orbit Plane
- nearly circular orbits
- planets all orbit in same direction
- most planets rotate in the same direction as their orbital motion

12.2 Protostellar Neb

- Cloud of gas compressing and spinning

12.3 Comparative planetology

- Massive colder planets retain atmospheres
- planet masses and compositions were driven by the temperature gradient in the protoplanetary disk and its relation to the condensation Temperature profile

12.4 Origin of the solar system

- Giant jovian satellites are a mini version of the whole solar system
- rings are temporary and provide evidence for a dynamic, evolving planetary system
 - all Jovian planets have rings
 - produced by bodies that failed to form or were disrupted due to tidal forces
 - triton is scheduled to explode

12.5 Dynamic solar system

- our moon, large satellite of a small planet
- triton captured by Neptune
- retrograde rotation of Venus
- large impact craters

12.6 Detecting exoplanets

12.6.1 Transit

- can theoretically resolve some based on angular resolution, but very hard to separate from the star itself
- $L_{jup} = \left(\frac{L_\odot}{r\pi a^2} (\pi R_{jup}^2 A) \right)$
- if you look at infrared radiation it is “easier” to resolve to planet

12.6.2 Indirect

- Wobble due to the planet pulling on the star
- They orbit their COM

12.6.3 Doppler effect

- Measure redshift and blue shift on the star

12.7 Unknown orbital inclination angle

- We do not know the inclination of the plane of the system relative to our line of sight
- Doppler can only measure velocities projected onto the line of sight
- if $i = 90^\circ$, the plane is viewed edge on, and we measure true velocity. Otherwise, we measure a smaller velocity, giving a lower limit of planet masses.

12.8 Radial Velocity method

- we need the complete more accurate version of Kepler III
- $$p^2 = \frac{4\pi^2(a_A + a_B)^3}{G(M_A + M_B)}$$
- can find p by looking at when the velocity pattern repeats
- we want a_b and M_B . get a_B from
- $$a_b^3 \approx \frac{GM_AP^2}{4\pi^2}$$
$$M_A \gg M_B$$

- $\frac{a_A}{a_B} = \frac{M_B}{M_A}$
- $M_B = \left(\frac{4\pi^2 M_A^2}{GP^2} a_A^3\right)^{1/3}$

12.9 Transit method

- Detected by planet passing in front of the star, causing a dip in the brightness
- find mind transit flux reduction
- find planet velocity (same method at radial vel)
- time before t_1 and t_2 yields planet radius or measuring the fraction of the starlight blocked (if r_s is known)

12.9.1 Chance of detection

- Range of inclinations for which transit is observed
- $\cos i \leq \frac{R_A + R_B}{a}$

12.10 Surprising exoplanet discoveries

- **Hot Jupiter's** - exoplanets with Jupiter like masses which are very close to their host stars
- very eccentric and/or inclined planet orbits
- current perspective biased by observational methods

14 Stellar Atmospheres

23 Galaxies and Super clusters

Stellar streams

- Galaxies ($r \sim 10$ kpc)
- Groups ($r \sim 1$ Mpc)
- Supercluster ($r \sim 100$ Mpc)

Coma Cluster

- rich cluster dominated by elliptical Galaxies

Cluster ISM

- Bright x-ray emitting gas

- low density, but very large quality
- Total mass of ISM is around double that of the galaxies
- Motion through the ISM causes ram pressure stripping

23.1 Dark matter in galaxy clusters

- Use viral theorem...
-
- $$M \approx 7.5 \frac{\sigma^2 r_h}{G}$$
- Where σ is a velocity distribution, and r_h is radius of the cluster
- If we look at the mass to light ratio of the Coma Cluster, which is 250 (in solar units), we find a very different distribution as compared to our sun (= 1), and the Milky Way (~ 10)

23.2 Large Scale Structure

- Cosmic web structure
- Sloan great wall
- Few galaxies measured far away, simply because they're hard to measure

23.3 Galaxy collisions

- Can estimate velocity in cluster from velocity dispersion (σ) of galaxy motions
-
- $$v \sim \sqrt{3}\sigma$$
- Can then compute collision rates
- In rich clusters about a 50% chance

23.3.1 Rates of collisions

Consider a galaxy of radii R moving at velocity v , in an elapsed time t it will collide with any other galaxy within a cylindrical volume $V(t)$ of radius R^2 and length vt

$$V(t)(4\pi R^2)(vt)$$

If galaxy density is n then the total collisions is N_\star and the average time to collision is t_\star

$$N_\star = nV(t) = v(4\pi R^2)(vt)$$

$$N_\star = 1 \rightarrow t_\star = \frac{1}{nv(4\pi R)}$$

23.3.2 Galaxies collide, but stars to not

If you repeat the previous equation but for stars, you find a 1 in 10^8 chance of stellar collision, in the age of the entire universe

23.3.3 Physics of collisions

- Raise tidal bulges and resulting tidal tails
- They are inelastic, the kinetic energy is converted into funny motions of the stars themselves

24 Cosmology

24.1 CMB

- $ct \Rightarrow$ theoretical size of the universe
- Does not account for expansion of the universe while the light is travelling
- **Surface of last scattering**, point where the universe becomes opaque, also the CMB

24.1.1 olbers paradox

- We do not see a galaxy everywhere we look
- this is because the universe is finite, not infinitely large

24.1.2 Equivalence principle

Gravitational Mass

$$F_{grav} = \frac{-GM_g m_g}{r^2} \quad (1)$$

Inertial mass

$$a = \frac{F}{m_i} \quad (2)$$

Equivalence Principle

$$m_g = m_i \quad (3)$$

24.1.3 Einstein v Newton

- In a small volume of space the downward pull of gravity cannot be distinguished from an upward acceleration of the observer

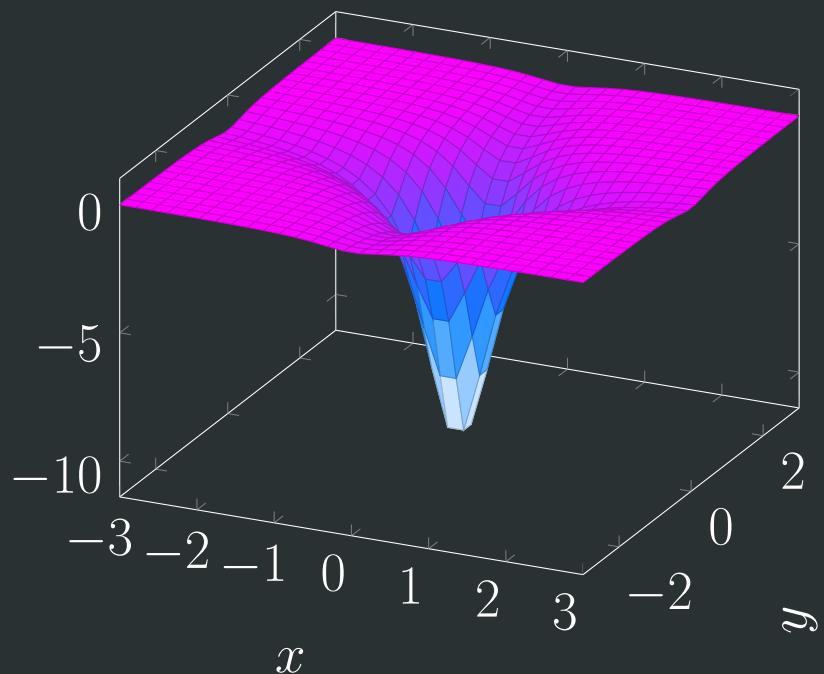
24.2 General relativity

- Space and time bind together to form curved spacetime
- ‘free-fall’ is motion in a straight line in 4d spacetime (this is grav acceleration)
- Matter (Mass-energy) tells spacetime how to curve
- Curved spacetime tells matter how to move

24.2.1 Predictions of GR

- Gravtional field sdeflect light (grav lensing)
- more accurate orbit for Mercury
- grav fields slow clocks
- moving objects will produce gravitational waves

Gravity in spacetime



24.3 Surface of spacetime

- How do we know the curvature and angles?
- If we draw a triangle what do the angles add up to?

Curvature and Angles of a triangle in 4d space

$$\alpha + \beta + \gamma = \pi + \frac{\kappa A}{r_{c,0}^2} \quad (4)$$

Where $\alpha + \beta + \gamma$ are angles of an equilateral triangle.
Flat is $\kappa = 0$, spherical is $\kappa = 1$, Hyperbolic is $\kappa = -1$

Newtonian Friedmann Equation

$$\frac{\dot{a}^2}{a} = \frac{8\pi G}{3}\rho(t) + 2\frac{k}{r^2} \frac{1}{a^2(t)} \quad (5)$$

Newtonian version of Friedman equation

Relativistic Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho(t) - \frac{\kappa c^2}{r_{c,0}^2} \frac{1}{a^2(t)} + \frac{\alpha}{3} \quad (6)$$

where κ is curvature parameter

Currently, $\dot{a} > 0$, thus the universe is expanding

24.3.1 Measuring curvature

- Look at angular diam vs distance
- We look at something which we know the size of, a ‘standard ruler’
- if flat... $\Rightarrow \alpha = \frac{D}{d}$
- Can’t use galaxies, non standardized size
- Use sound waves before recombination
 - Use sound waves! $t < 380000\text{yr}$
 - Sound waves in early fluid like plasma in universe
 - wave equation for sound waves relates wavelength to the properties of the gas
- Apply the sound wave logic to the CMB
 - the reason that the CMB is not homogeneous is because of the sound waves traveling through the plasma in early universe
 - If universe was curved, the light traveling from the CMB would take a different path, resulting in a different angular resolution

24.3.2 Predicting angular scale distribution in CMB from sound waves

- large fluctuations are caused by patches of varying temperature
- small fluctuations are from
- Look at how CMB is ‘distorted’ from the sound waves
- Then infer wavelength of soundwaves
- Then apply previous logic for measuring angular res and comparing it to known wavelength

24.4 Density of the universe

- $\Omega_m = \frac{M/V}{\rho_c}$
- Matter ...
- $\Omega_m \sim .3$
- Radiation ...
- $\Omega_{rad} \sim .00007$
- Something else ...
- $\Omega_{rest} \sim .7$
- Because the universe is flat we know that
- $\Omega_m + \Omega_{rad} + \Omega_{res} \sim 1$

24.4.1 Energy density components

- Looking for a solution for $a(t)$
- Wavelength of a photon scales with the size of the universe
- Lambda cosmological constant
- $\Omega_\Lambda \sim .7$
- Non-Relativistic particles:
- $\Omega_{m,0} \sim .3$
- $\Omega_{(bary,0)} \sim .04$

- Relativistic particles
-
- $\Omega_{r,0} = \Omega_{cmb,0} + \Omega_{v,0} \sim 8.4 \times 10^{-5}$

Behavior of nonrelativistic particles

$$n(t) \propto V^{-1}(t) \propto a^{-3}(t) \quad (7)$$

If matter is conserved

24.4.2 Cosmological redshift

Redshift Z

$$Z = \frac{\lambda_0 - \lambda_e}{\lambda_e} \quad (8)$$

If we use 8 combined with expansion of the universe ...

$$1 + Z = \frac{1}{a(t)}$$

24.4.3 Universe at different stages

- Solve 6 for different Ω s
- We find that all are decelerating, except for Ω_Λ (Dark Energy)

Friedmann Consensus Model

$$\dot{a} = H_0 \left[\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\lambda,0} a^2 \right]^{1/2} \quad (9)$$

unitless acceleration of the universe

24.4.4 Standard candles for expanding universe

Flux

$$L = \frac{L}{4\pi r^2} \quad (10)$$

Photon Energy

$$\epsilon = \frac{hc}{\lambda_e}; \quad \epsilon_0 = \frac{hc}{\lambda_0} \quad (11)$$

- One can combine 8 and 11

25 Equations

$$F_{grav} = \frac{-GM_g m_g}{r^2} \quad (1) \text{ (Gravitational Mass)}$$

$$a = \frac{F}{m_i} \quad (2) \text{ (Inertial mass)}$$

$$m_g = m_i \quad (3) \text{ (Equivalence Principle)}$$

$$\alpha + \beta + \gamma = \pi + \frac{\kappa A}{r_{c,0}^2} \quad (4) \text{ (Curvature and Angles of a triangle in}$$

$$\frac{\dot{a}^2}{a} = \frac{8\pi G}{3}\rho(t + 2\frac{k}{r^2}\frac{1}{a^2(t)}) \quad (5) \text{ (Newtonian Friedmann Equation)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} - \frac{\kappa c^2}{r_{c,0}^2}\frac{1}{a^2(t)} + \frac{\alpha}{3} \quad (6) \text{ (Relativistic Friedmann equation)}$$

$$n(t) \propto V^{-1}(t) \propto a^{-3}(t) \quad (7) \text{ (Behavior of nonrelativistic particles)}$$

$$Z = \frac{\lambda_0 - \lambda_e}{\lambda_e} \quad (8) \text{ (Redshift Z)}$$

$$\dot{a} = H_0 \left[\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\lambda,0} a^2 \right]^{1/2} \quad (9) \text{ (Friedmann Consensus Model)}$$

$$L = \frac{L}{4\pi r^2} \quad (10) \text{ (Flux)}$$

$$\epsilon = \frac{hc}{\lambda_e}; \quad \epsilon_0 = \frac{hc}{\lambda_0} \quad (11) \text{ (Photon Energy)}$$

Glossary

Stellar streams Stars following a path away from a galaxy [22](#)

Surface of last scattering Point looking deep into the universe where the universe becomes opaque. Also where the CMB is [25](#)