Contents

Astro Notes

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1 Class overview

Office hours tues and thurs 11-12 astro 237
ta office hours
wed 3:30:530 astro 2367
hw due wed
hw posted week before

- solar system
- steller evo
- compact objects
- galaxy quasi darkmatter
- cosmic web
- big bang

course goals

- apply pys to universe
- understand foundations of modern astro, astrophys, and cosmology
- conceptual understanding of the uni based on physical principles

2 Early Astronomy

2.0.1 Greek

- Aristotle
  - earth is spherical
  - partial lunar eclipses
  - some stars visible from southern locations but not northern and vice versa

- had ideas regarding perfect geo influenced by Pythagoras and Plato

- Aristarchus (310-230 BC):
  - unpreceded heliocentric framework
  - trig distances earth-moon-sun system
  - angular diameters $\theta_{sun} \approx \theta_{moon} \therefore \frac{A}{C} = \frac{D_{moon}}{D_{moon}}$
  - diameters from lunar eclipses $D_{moon} < D_{earth}$

- Eratoshs (176-195 BC):
  - Determined radius of spherical earth R_E
  - Sun at zenith at noon on summer solstice at Aswan
  - But further north in Alexandria, Egypt, the sun is south of the zenith by angle α

- Hipparchus (190-120 BC):
  - Discover precession of the equinoxes from examination of star catalogs over centuries
  - established the magnitude system

- Copernicus (1473-1543):
  - heliocentric
  - earth rotates
  - still assumed uniform circular celestial motion
  - inferior planets: orbit smaller than earths
  - superior planets: orbits larger than earths

2.1 Emergence of modern Astro

Inferior planets

- B/C = sin θ_E
- B=C Sin θ_E
- C is AU
- Early astronomers didnt know C, so they could only infer rations of B/C. Ie. Orbital radii measured in AU

Superior Planets

- Measure time between opposition and eastern quadrature
- want angle θ between opp and east quad

- $\theta = (\omega_E - \omega_p)$ and $C/B = cos\theta$
- measure τ and synodic period, calculate sidereal period and ω_p; know ω_E and infer C/B

Galilean Revolution

- Galileo Galilei (1564 -1642)
- - improved and used a basic refracting telescoping
- def publication of early results 1610 *"starry messenger"*
- - Moon is cratered; not a perfect Sphere
  - milkyway is made out of stars
  - Jupiter has moons (or as he thought, stars)
  - measured phases of Venus

Phases of Venus

- direct confrontation with Ptolemaic geocentric models
- in Ptolemaic models you only see crescent phases

Tycho Brahe (1546-1601)

- Denmark, later Prague
- Given island by king Fredrick (and staff)
- made a accurate and vast database of celestial motion
- had a lead nose?
- Threw giant ragers
- supernova named after him

Johannes Kepler (1571–1630, Prague)

- 'Inherited' (maybe stole) Brahe's data
- also has a SN
- Kepler fit a new empirical model of heliocentric orbits, abandoning perfect circles
  - "*It was as if I awoke from sleep and saw a new light*" (Kepler, New astronomy)

Kepler's Laws

First law

- The planets travel on elliptical orbits with the sun at one focus
- Semimajor axis, half the major axis
- eccentricity: how elliptical (stretched) an orbit is - distance between foci divided by major axis.

second law

- A line drawn from the sun to a planet sweeps out equal areas in equal time intervals'
- perihelion: orbital point closet to the sun
- aphelion: furthest orbital point from the sun

third law

Def: *The square of the sidereal orbital periods of the planets are prop to the cubes of the Semimajor axis of their orbits*

$$p^2 = Ka^3$$

P = planets sidereal period
a= length of semimajor axis
K = constant

Consequences of heliocentric model

- retrograde motion of outer planets
- positions of outer and inner planets wrt sun
- annual parallax
- aberration of starlight
- Coriolis effect

Parallax

- annual parallax: change in the apparent position when seen from two diff locations due to earth revolving around the sun. First measured by Bessel in 1838

Aberration of starlight

- deflection of apparent stellar positions in the direction of the observers motion
- analog: running throw rain and getting wet in the front and not in the back
- detected (Picard, 1680); explained (Bradley, 1729)

- telescope is moving along orbital vector around the sun; translation along orbit cannot exceed transit time of light through telescope

Coriolis effect: evidence of earth rotation

- coriolis acceleration is perp to the direction of motion
- $$a_{\vec{cor}} = s\vec{v} \times \vec{\omega}$$
- can be deduced from a pendulum
- and in hurricanes!

3 Orbital Mechanics I

3.1 Newtonian mechanics

Parametric vectors
Displacement $\vec{r}(t) = x(t)\hat{i}y(t)\hat{j} + z(t)\hat{k}$
distance: $r(t) = |\vec{r}(t)| = \sqrt{\vec{r} \cdot \vec{r}}$

3.2 Newtons laws

First law

- Isaac newton(1642-1727)
- an objects' velocity remains constant unless a net outside force acts upon it
- $\vec{v}(t) = \vec{v_0} = const$

second law

- $\vec{F} = m\vec{a}(t)$
- $\vec{F} = \frac{d\vec{p}(t)}{dt}$
- $d\vec{v}/dt = \vec{f}/m$
- force changes velocity
- used a lot in computational math

third law

- forces come in pairs, equal in magnitude, and opposite in direction

Newtonian gravity

- a force, grav, exsits between any two objects having mass m and M, prop to the product of their masses mM and inversely proportinal tothe square of the separtation distance r of their centers

- for coordinates centered on M:

- $\vec{F} = -G\frac{Mm}{|\vec{r}|^2}\hat{r}$

3.3 Displacement vector and polar coordinates

- cartesian coordinates are often written a (x,y,z) in a coordinate system centered on mass M

- Axis orientations are chosen so that the planet orbits in the x-y plane

- Displacement $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

velocity vector and polar coordinates

- unit vectors in polar coordinates vary with $\theta(t)$

- $$\frac{d\hat{r}(t)}{dt} = \frac{d\hat{r}(t)}{d\theta}\frac{d\theta(t)}{dt} = \frac{d\theta(t)}{dt}\hat{\theta}(t)$$

- .

- .

- .

- $$\vec{v}(t) = v_r\hat{r} + v_t\hat{\theta}$$

- two velocity components in polar coords

3.4 Kepler laws: angular momentum

- .

3.5 keplers 2nd law = consv, angular momentum

- $$d\vec{L}/dt = 0$$

- $$\vec{L} = \vec{R} \times \vec{p} = \vec{r} \times m\vec{v} = const$$

- $$\rightrightarrows |\vec{v}| = L = mrv_1$$

3.6 Keplers Laws

3.6.1 Keplers First Law

- $\frac{d\vec{v}}{dt} = -\frac{GM}{r_2}\hat{r}$
-
$$\frac{L}{GMm}\frac{d\vec{v}}{dt} = \frac{d\hat{\theta}}{dt}$$
-
$$\frac{L}{GMm}\vec{v} = \hat{\theta} + e\hat{j}$$
- take dot product of both sides with unit vector $\hat{\theta}, using$
- $\hat{j} \cdot \hat{\theta} = \cos\theta$
-
$$\vec{v} \cdot \hat{\theta} = v_t = \frac{L}{mr}$$

3.7 Kepler III

- we know that $\frac{dA}{dt} = \frac{l}{2m} = const$
- area of a ellipse $a = \pi ab$ of orb period p.
-
$$\therefore \frac{A}{P} = \frac{\pi ab}{P} = \frac{L}{2m}$$
- eclipse geo : $b^2 = a^2(1 - e^2)$
- also, $\frac{L^2}{m^2}GMa(1 - e^2)$
-
$$P^2 = \frac{4\pi^2}{GM}a^3$$

4 Orbital energetics

- total energy e is conserved
  - sum of K and U
  -
$$E = K + U$$
  -
$$= \frac{1}{2}mv^2 - \frac{GMm}{r}$$
- total E is conserved

-
$$E = (\frac{GMm}{L})^2 \frac{m}{2}(e^2 - 1)$$

- Hyperbolic orbit: $e > 1, E > 0, K > |U|$
  - open orbit, unbound;, single perihelion passage at $\theta = 0$
- Parabolic orbit: $e = 1, E = 0, K = |U|$
  - marginally unbound; velocity approach zero at infinite time
- elliptical orbit: $e < 1, E < 0, K = |U|$
- objects originating outside our solar system are easily identified by their total energy
  - measure total energy (how far away it is, how fast is it moving)

4.1 Checking energy in circular orbits

- governing equation for circular orbits in scalar from

-
$$f = ma$$

-
$$\frac{GM}{r^2} = \frac{v^2}{r} = \omega^2 r$$

-
$$v = \sqrt{\frac{GM}{r}}$$

4.2 Negative total energy orbits

- bound orbits have E ¡ 0
- must add energy to break "unbind" the orbits

4.3 Parabolic orbits: escape speeds

- Escape speed is the speed that will bring your total pot energy to 0
- velocity becomes zero at infinite distance

-
$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

-
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

4.4 Hohmann transfer orbit

- Elliptical transfer orbit from earth to superior planet
  - earths orbit becomes the transfer orbits perihelion passage
  - inserted into superior planet orbit at aphelion. This constrains launch windows
  - theoretically requires only two burns: at launch and aphelion insertion point
- semimajor axis os transfer orbit
- $$a_{to} = \frac{a + a_{sup}}{2}; Earth$$

5 Earth-Moon Systen

5.1 Motion of the moon

- 27.3 sidereal orbit
- 29.5 synodic orbit
- rises in east and sets in west diurnally, but moves eastwards by about 12 deg per day rel to stars
- rises hour later per night

5.2 Precession

- earth is an oblate spheroid with equatorial bulge of .3% cause by separation
- sun, moon, and planets exert a torque τ on earth
- $$\vec{\tau} = \vec{r} \times \vec{F}$$
- results in precession of spin axis of earth around ecliptic pole
- NCP moves. Polaris will not always be at NCP
- moves through stars with $P \approx 28500yr$
- opening angle
- $$47^\circ (= 2 \times 23.5^\circ)$$

5.3 Tidal Forces

- Moon exerts diff tidal forces on matter on earth
- esp noticeable on earths ocean surface as tides
- when sun and moon align (sun-earth-moon at $0° and 180°$) high-amp tides result, called spring tides
- when sun and moon are at 90° they sum destructively, producing neap tides

5.3.1 Diff gravitational tidal forces

- arise from the r^{-2} dependence of grav force
- Taylor expansion about center of earth r_0
-
$$\delta F = \frac{2GM_{moon}m}{r^3}(r - r_0)$$
- Sun exerts about half as strong as moon tidal forces

5.3.2 Rotation of tides

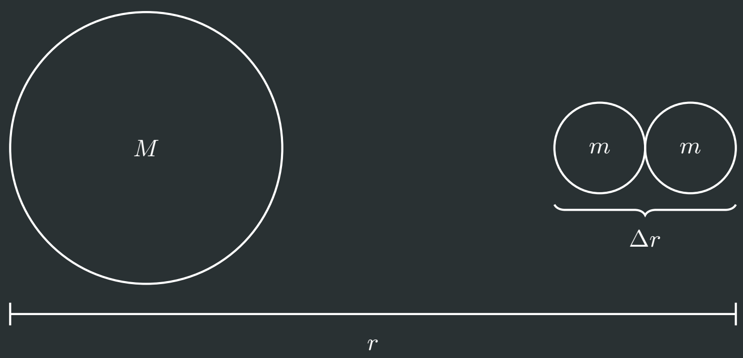
- tidal bulges produced on earths by the moon rotate at the same angular rate as the moons orbit around earth
- but the earth is rotating faster at once per sidereal day by 4 minutes. Drags the tides forward from where they would otherwise be by about 10° by friction
  - therefore high tides occur shortly after upper transit of moon
  - the misalignment drives angular momentum transfer between earth and Moon
    - moon pulls strongly on nearer tidal bulge than farther tidal bulge
    - net torque to slow earth rotation
    - but conversely the tidal bugle pulls more strongly on the moon, pulling it forward, increasing its angular momentum

5.4 Earth Shape

- moon stretches earth in a prolate deformation
- spin of the earth causes an oblate deformation
- oblate is much greater the prolate def

5.5 Roche Limit

- object get too close, forces on one side much greater then other, rip object apart
- approx a planet as two spheres 2m
- $$\Delta F = \frac{dF}{dr}\Delta r = \frac{2GMm}{r^3}\Delta r$$
- Is there a force holding 2m together? yes, self grav
- $$F = -\frac{Gmm}{(\Delta r)^2}$$



5.6 Hill radius

- tidal forces of sun on earth-moon systems means that there is a maximum orbital distance for the moon, if it is to remain bound to the earth

6 Glossary

Synodic period

- time elapsed between success conjunctions or oppositions
- this is the period we observe from earth, which is moving

Sidereal Period

- elapsed time of full orbit relative to the fixed stars (inertial ref frame)
- This is the one we will want to put in keplers laws