Contents

Notes

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0 Class overview

Office hours tues and thurs 11-12 astro 237
ta office hours
wed 3:30:530 astro 2367
hw due wed
hw posted week before

- solar system
- steller evo
- compact objects
- galaxy quasi darkmatter
- cosmic web
- big bang

course goals

- apply pys to universe
- understand foundations of modern astro, astrophys, and cosmology
- conceptual understanding of the uni based on physical principles

1 Early Astronomy

1.0.1 Greek

- Aristotle
  - earth is spherical
  - partial lunar eclipses
  - some stars visible from southern locations but not northern and vice versa
  - had ideas regarding perfect geo influenced by Pythagoras and Plato
- Aristarchus (310-230 BC):

- unpreceded heliocentric framework
- trig distances earth-moon-sun system
- angular diameters $\theta_{sun} \approx \theta_{moon} \therefore \frac{A}{C} = \frac{D_{moon}}{D_{moon}}$
- diameters from lunar eclipses $D_{moon} < D_{earth}$

- Eratoshs (176-195 BC):
  - Determined radius of spherical earth R_E
  - Sun at zenith at noon on summer solstice at Aswan
  - But further north in Alexandria, Egypt, the sun is south of the zenith by angle α

- Hipparchus (190-120 BC):
  - Discover precession of the equinoxes from examination of star catalogs over centuries
  - established the magnitude system

- Copernicus (1473-1543):
  - heliocentric
  - earth rotates
  - still assumed uniform circular celestial motion
  - inferior planets: orbit smaller than earths
  - superior planets: orbits larger than earths

1.1 Emergence of modern Astro

Inferior planets

- B/C = sin θ_E
- B=C Sin θ_E
- C is AU
- Early astronomers didnt know C, so they could only infer rations of B/C. Ie. Orbital radii measured in AU

Superior Planets

- Measure time between opposition and eastern quadrature
- want angle θ between opp and east quad
- $\theta = (\omega_E - \omega_p)$ and $C/B = cos\theta$
- measure τ and synodic period, calculate sidereal period and ω_p; know ω_E and infer C/B

Galilean Revolution

- Galileo Galilei (1564 -1642)
- - improved and used a basic refracting telescoping

- def publication of early results 1610 *"starry messenger"*
-
  - Moon is cratered; not a perfect Sphere
  - milkyway is made out of stars
  - Jupiter has moons (or as he thought, stars)
  - measured phases of Venus

Phases of Venus

- direct confrontation with Ptolemaic geocentric models
- in Ptolemaic models you only see crescent phases

Tycho Brahe (1546-1601)

- Denmark, later Prague
- Given island by king Fredrick (and staff)
- made a accurate and vast database of celestial motion
- had a lead nose?
- Threw giant ragers
- supernova named after him

Johannes Kepler (1571–1630, Prague)

- 'Inherited' (maybe stole) Brahe's data
- also has a SN
- Kepler fit a new empirical model of heliocentric orbits, abandoning perfect circles
  - "*It was as if I awoke from sleep and saw a new light*" (Kepler, New astronomy)

Kepler's Laws

First law

- The planets travel on elliptical orbits with the sun at one focus
- Semimajor axis, half the major axis
- eccentricity: how elliptical (stretched) an orbit is - distance between foci divided by major axis.

second law

- A line drawn from the sun to a planet sweeps out equal areas in equal time intervals'
- perihelion: orbital point closet to the sun
- aphelion: furthest orbital point from the sun

third law

Def: *The square of the sidereal orbital periods of the planets are prop to the cubes of the Semimajor axis of their orbits*

$$p^2 = Ka^3$$

P = planets sidereal period
a= length of semimajor axis
K = constant

Consequences of heliocentric model

- retrograde motion of outer planets
- positions of outer and inner planets wrt sun
- annual parallax
- aberration of starlight
- Coriolis effect

Parallax

- annual parallax: change in the apparent position when seen from two diff locations due to earth revolving around the sun. First measured by Bessel in 1838

Aberration of starlight

- deflection of apparent stellar positions in the direction of the observers motion
- analog: running throw rain and getting wet in the front and not in the back
- detected (Picard, 1680); explained (Bradley, 1729)
- telescope is moving along orbital vector around the sun; translation along orbit cannot exceed transit time of light through telescope

Coriolis effect: evidence of earth rotation

- Coriolis acceleration is perp to the direction of motion
- $$\vec{a_{cor}} = s\vec{v} \times \vec{\omega}$$
- can be deduced from a pendulum
- and in hurricanes!

2 Orbital Mechanics I

2.1 Newtonian mechanics

Parametric vectors
Displacement $\vec{r}(t) = x(t)\hat{i}y(t)\hat{j} + z(t)\hat{k}$
distance: $r(t) = |\vec{r}(t)| = \sqrt{\vec{r} \cdot \vec{r}}$

2.2 Newtons laws

First law

- Isaac newton(1642-1727)
- an objects' velocity remains constant unless a net outside force acts upon it
- $\vec{v}(t) = \vec{v_0} = const$

second law

- $\vec{F} = m\vec{a}(t)$
- $\vec{F} = \frac{d\vec{p}(t)}{dt}$
- $d\vec{v}/dt = \vec{f}/m$
- force changes velocity
- used a lot in computational math

third law

- forces come in pairs, equal in magnitude, and opposite in direction

Newtonian gravity

- a force, grav, exsits between any two objects having mass m and M, prop to the product of their masses mM and inversely proportinal tothe square of the separtation distance r of their centers
- for coordinates centered on M:
- $\vec{F} = -G\frac{Mm}{|\vec{r}|^2}\hat{r}$

2.3 Displacement vector and polar coordinates

- cartesian coordinates are often written a (x,y,z) in a coordinate system centered on mass M
- Axis orientations are chosen so that the planet orbits in the x-y plane
- Displacement $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

velocity vector and polar coordinates

- unit vectors in polar coordinates vary with $\theta(t)$
- $$\frac{d\hat{r}(t)}{dt} = \frac{d\hat{r}(t)}{d\theta}\frac{d\theta(t)}{dt} = \frac{d\theta(t)}{dt}\hat{\theta}(t)$$
- .
- .
- .
- $$\vec{v}(t) = v_r\hat{r} + v_t\hat{\theta}$$
- two velocity components in polar coords

2.4 Kepler laws: angular momentum

- .

2.5 keplers 2nd law = consv, angular momentum

-
$$d\vec{L}/dt = 0$$
-
$$\vec{L} = \vec{R} \times \vec{p} = \vec{r} \times m\vec{v} = const$$
-
$$\rightrightarrows |\vec{v}| = L = mrv_1$$

2.6 Keplers Laws

2.6.1 Keplers First Law

- $\frac{d\vec{v}}{dt} = -\frac{GM}{r_2}\hat{r}$
-
$$\frac{L}{GMm}\frac{d\vec{v}}{dt} = \frac{d\hat{\theta}}{dt}$$
-
$$\frac{L}{GMm}\vec{v} = \hat{\theta} + e\hat{j}$$
- take dot product of both sides with unit vector $\hat{\theta}, using$
- $\hat{j} \cdot \hat{\theta} = \cos\theta$
-
$$\vec{v} \cdot \hat{\theta} = v_t = \frac{L}{mr}$$

2.7 Kepler III

- we know that $\frac{dA}{dt} = \frac{l}{2m} = const$
- area of a ellipse $a = \pi ab$ of orb period p.
-
$$\therefore \frac{A}{P} = \frac{\pi ab}{P} = \frac{L}{2m}$$
- eclipse geo : $b^2 = a^2(1 - e^2)$
- also, $\frac{L^2}{m^2}GMa(1 - e^2)$
-
$$P^2 = \frac{4\pi^2}{GM}a^3$$

3 Orbital energetics

- total energy e is conserved
  - sum of K and U
  - $$E = K + U$$
  - $$= \frac{1}{2}mv^2 - \frac{GMm}{r}$$
- total E is conserved
- $$E = (\frac{GMm}{L})^2 \frac{m}{2}(e^2 - 1)$$
- Hyperbolic orbit: $e > 1, E > 0, K > |U|$
  - open orbit, unbound;, single perihelion passage at $\theta = 0$
- Parabolic orbit: $e = 1, E = 0, K = |U|$
  - marginally unbound; velocity approach zero at infinite time
- elliptical orbit: $e < 1, E < 0, K = |U|$
- objects originating outside our solar system are easily identified by their total energy
  - measure total energy (how far away it is, how fast is it moving)

3.1 Checking energy in circular orbits

- governing equation for circular orbits in scalar from
- $$f = ma$$
- $$\frac{GM}{r^2} = \frac{v^2}{r} = \omega^2 r$$
- $$v = \sqrt{\frac{GM}{r}}$$

3.2 Negative total energy orbits

- bound orbits have E ¡ 0
- must add energy to break “unbind” the orbits

3.3 Parabolic orbits: escape speeds

- Escape speed is the speed that will bring your total pot energy to 0
- velocity becomes zero at infinite distance
- $$\frac{1}{2}mv^2 = \frac{GMm}{r}$$
- $$v_{esc} = \sqrt{\frac{2GM}{r}}$$

3.4 Hohmann transfer orbit

- Elliptical transfer orbit from earth to superior planet
  - earths orbit becomes the transfer orbits perihelion passage
  - inserted into superior planet orbit at aphelion. This constrains launch windows
  - theoretically requires only two burns: at launch and aphelion insertion point
- semimajor axis os transfer orbit
- $$a_{to} = \frac{a + a_{sup}}{2}; Earth$$

4 Earth-Moon Systen

4.1 Motion of the moon

- 27.3 sidereal orbit
- 29.5 synodic orbit
- rises in east and sets in west diurnally, but moves eastwards by about 12 deg per day rel to stars
- rises hour later per night

4.2 Precession

- earth is an oblate spheroid with equatorial bulge of .3% cause by separation
- sun, moon, and planets exert a torque τ on earth
- $$\vec{\tau} = \vec{r} \times \vec{F}$$
- results in precession of spin axis of earth around ecliptic pole
- NCP moves. Polaris will not always be at NCP
- moves through stars with $P \approx 28500yr$

- opening angle
- $$47°(= 2 \times 23.5°)$$

4.3 Tidal Forces

- Moon exerts diff tidal forces on matter on earth
- esp noticeable on earths ocean surface as tides
- when sun and moon align (sun-earth-moon at $0° and 180°$) high-amp tides result, called spring tides
- when sun and moon are at 90° they sum destructively, producing neap tides

4.3.1 Diff gravitational tidal forces

- arise from the r^{-2} dependence of grav force
- Taylor expansion about center of earth r_0
- $$\delta F = \frac{2GM_{moon}m}{r^3}(r - r_0)$$
- Sun exerts about half as strong as moon tidal forces

4.3.2 Rotation of tides

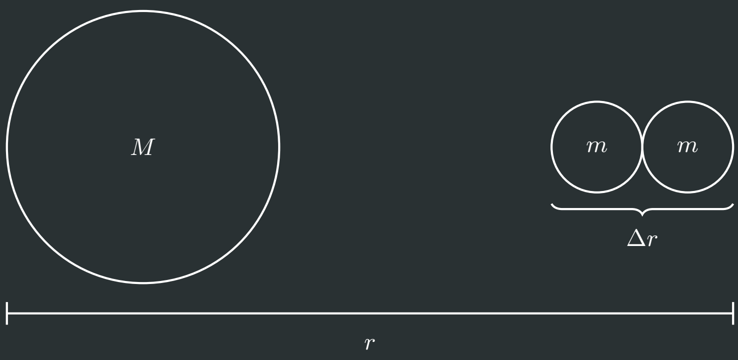
- tidal bulges produced on earths by the moon rotate at the same angular rate as the moons orbit around earth
- but the earth is rotating faster at once per sidereal day by 4 minutes. Drags the tides forward from where they would otherwise be by about 10° by friction
    - therefore high tides occur shortly after upper transit of moon
    - the misalignment drives angular momentum transfer between earth and Moon
        - moon pulls strongly on nearer tidal bulge than farther tidal bulge
        - net torque to slow earth rotation
        - but conversely the tidal bugle pulls more strongly on the moon, pulling it forward, increasing its angular momentum

4.4 Earth Shape

- moon stretches earth in a prolate deformation
- spin of the earth causes an oblate deformation
- oblate is much greater the prolate def

4.5 Roche Limit

- object get too close, forces on one side much greater then other, rip object apart
- approx a planet as two spheres 2m
- $$\Delta F = \frac{dF}{dr}\Delta r = \frac{2GMm}{r^3}\Delta r$$
- Is there a force holding 2m together? yes, self grav
- $$F = -\frac{Gmm}{(\Delta r)^2}$$



4.6 Hill radius

- Tidal forces of sun on earth-moon systems means that there is a maximum orbital distance for the moon, if it is to remain bound to the earth

4.7 Plane of lunar orbit

- Inclined by 5.1°
- the moon is near the cele equator so the moon is above the horizon about 50% of the time for most observers on earth
- moves north and south in the sky in addition to its motion around the earth. greatest dec is $23.5 + 5.1 = 28.6$ and min is $-23.5 - 5.1 = -28.6$
- causes eclipses to be retrograde

4.8 Tidal forces: earth vs moon

- earth exerts greater tidal forces on moon than the moon does on the earth.
- $$\Delta g_{moon \to earth} = \frac{\Delta F}{m} \propto \frac{M_{Moon} R_{Earth}}{r^3}$$

-
$$\Delta g_{earth\to moon} = \propto \frac{M_{Earth}R_{Moon}}{r^3}$$

-
$$\frac{\Delta g_{moon\to earth}}{\Delta g_{earth\to moon}}\frac{M_{moon}R_{earth}}{M_{earth}r_{moon}} \approx \frac{1}{20}$$

4.9 lunar librations

- E-w and n-s nodding motions of the moon seen from earth, caused by parallax
- tidal locking is not perfect, so the libration happens in longitude
- because the rotation axis is inclined there is libration in lat

5 Waves

5.1 Spectra (How do we know what the universe if made out of?)

Multi-messenger astronomy

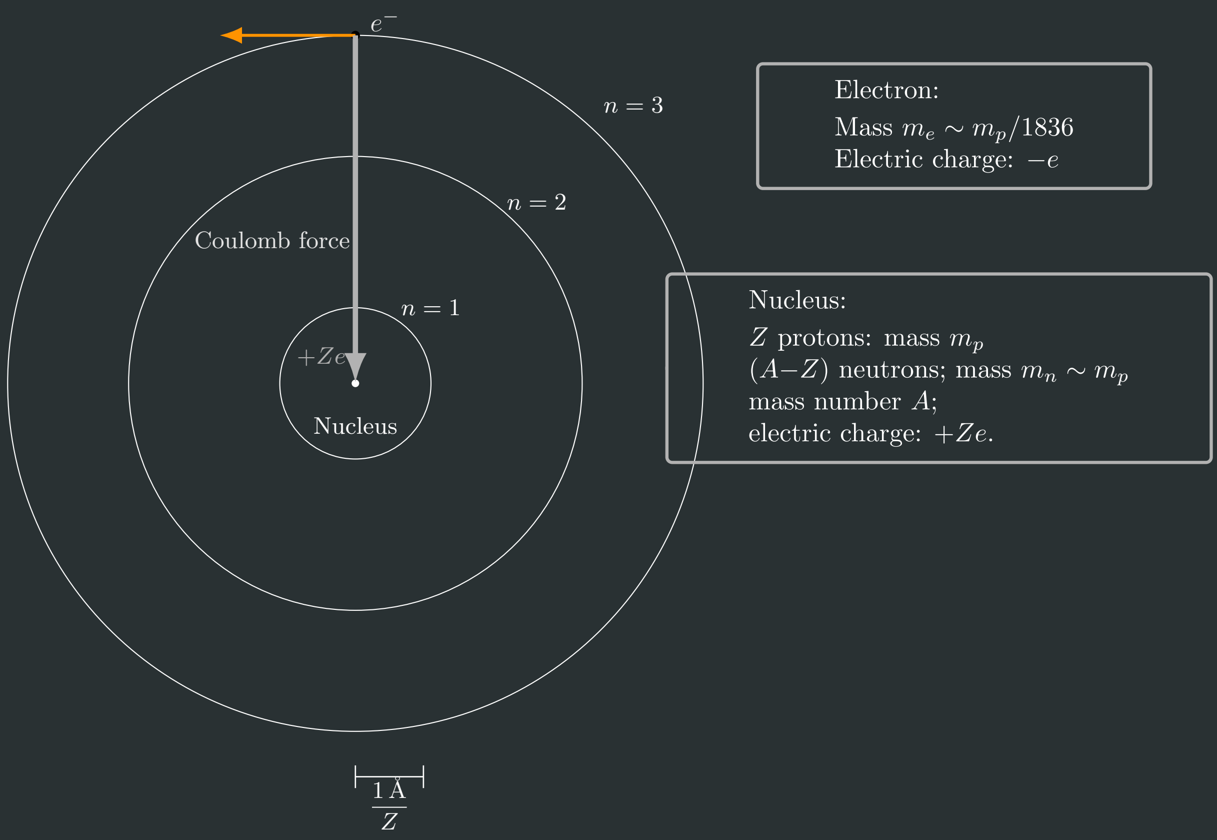
- Electromagnetic radiation
- cosmic rays
- meteorites
- neutrinos
- gravitational waves

5.2 Atoms and spectra

Hydrogen gas exhibits emission lines at discrete visible wavelengths, fit by empirical relation by Balmer in 1885

$$\frac{1}{\lambda} = R(1/4 - 1/n^2)$$
$$n = integer > 2$$

5.3 Bohrs model



Because orbital angular momentum is quantized, so is r_n and $E_n \rightarrow$ discrete orbtial levels

5.4 Atomic transition processes

- Transitions to free unbounded states behave similarly, however, they have diff names
- ionization and recombination
- photoionization (electron is knocked free). photon knocks electron free
- collisional ionization (electron becomes free). any other particle knocks electron free
- a positively charged ion may combine with a free leectron, and atom emits radiation (photons) as the electrons drop to lower levels. called **recombination**

5.5 Kirchoff's Laws

- blackbody
- emission lines
- absorption lines

5.6 Tempature affects internal states of atoms and molecules

- temp of gas dermines the kinetic energies of the colliding particles
- and the incoming photons
- we observe outgoing photons

5.7 Temperature vs velocity

- Thermal motions: emitted or absorbed photos inherent their energy from the doppler velocites of the thermal motions of particles/atoms
- equilibrium distribution of particles speeds in an ideal gas is given by the Maxwell-Boltzmann distribution

5.8 mean free path and opt depth

- mean free path x_m. Distance which intensity decrease by a factor of 1/e
- optical depth $\tau = x/x_m$. Thickness of slab in units of mean free path x_m
- column density, $N(x)$: total number of absorbing particle sin a column with cross-section area 1 m^2 and length x

6 Telescopes

6.1 photoeletric effect

- Photoemission emisison of a electron from a material in response to a incident photon
  - photoemissive material (underlying material)
  - work function (min energy requried to produce light)
  - photoeletric effect (photoemission from atoms in certain materials)
  - photoelectron (release electron)
- particle energy of EM radiation

6.2 Sun

6.2.1 Chromosphere

- Very sparse layer of gas above the photosphere
- very hot gas, emission spectra by Kirchhoff laws
- easily seen during total eclipse or with an Hα filter

6.2.2 Corona

- Low density out layer of suns atmosphere. Most easily visible during total eclipse
- T = 2×10^6
- Emission lines from highly ionized atoms
- x-ray emission from thermal Bremsstrahlung (not black body)
- Optical continuum is originally from photosphere
- scattered by free electrons in coronal plasma
- coronal streamers show how plasma follows magnetic field lines

6.2.3 Solar Wind

- Bunch of protons and other various particles ejected from the sun
- Speed of protons in the corona
- $$V_{rms} = \frac{3kT}{m_p}^{1/2} \approx 160km/s$$
- Escape speed as function of distance
- $$\frac{GM_\odot}{r}^{1/2} \approx 620km/s$$
- Sun produces a solar wind with $v = 400km/s$, density$p$ $10^{-21}kgm^{-3}$ earth
- $$\Delta M = (4\pi r^2 \Delta r)\rho$$
- therefore mass flux through shell
- $$\frac{dM}{dt} = 4\pi r^2 \frac{dr}{dt}\rho$$
- $$\dot{M} = 4\pi r^2 v\rho$$
- $$\dot{M} \sim 10^8 kgs^{-1}; t_m \sim 10^{14}$$
- Maybe try this myself with various sizes of stars?? Seems easy to verify large stars ejecting large amounts of wind

6.2.4 Magnetic Fields

- Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$
- Charged particles follow curved helical paths
- Magnetic field energy
- $$P_B = \varepsilon_B = \frac{B^2}{2\mu_0}$$

6.2.5 Sunspots

- Cooler than surroundings because the magnetic field is enhanced in the spot
- Pressure due to a magnetic field
- $$P_B = 4 \times 10^5 Nm^{-2} (\frac{B}{1T})^2$$
- pressure due to ideal gas
- $$P_{gas} = nkT$$

Pressure balance in sunspots

- gas and magnetic pressure inside sunspot must equal surrounding gas pressure
- $$\frac{\rho k T_s}{m_p} + \frac{B^2}{2\mu_0} = \frac{\rho k T_P}{m_p}$$
- $B \approx .1T$

Sunspot Cycle

- Star near 30° N/S, migrate towards solar equator
- more numerous every 11 years

10 The planets

10.1 Mecury

- always 30° from the sun
- strong tidal forces, permanent prolate tidal bulge
- sidereal rotation $P_{rot} = 58.65d$
- $P_{orb} = 87.97$
- Orbit is tidally locked at perihelion

10.2 Venus

- Retrograde Motion
- atmosphere
  - Clouds are sulfuric acid
  - $\sim$ 96.5 co2% and $\sim$ 3.5% N2
  - Very strong greenhouses
- Earths liquid water ocean dissolves co2
- runaway greenhouse

10.3 Earth

- Temperature and pressure on earth allow significant qualities of gas, liquid, and solid water. Not true for Venus or Mars

10.4 Mars

- $\alpha = 1.52$AU
- $P_{sidereal} = 1.88$
- $24h40m$
- Atmosphere
  - Pressure $\sim$ .006atm (earth)
  - 95% CO_2
  - UV photodissociastes H_2O, the oxygen oxidizes iron in soil
- Seasons caused by obliquity
- polar caps of CO_2 form and melt in winter and summer

10.5 Jupiter and Saturn

- they are in hydrostatic equilibrium
- $\frac{dP}{dr} = -\frac{GM_r\phi}{r^2}$
- M_r is the mass within radius r assuming constant density ϕ
- $\int_{P_c}^{0} dP = \frac{4\pi}{3}\phi^2 G \int_0^R$
- solving shows that there must be very high pressures in centers of Jovian planets
- Metallic hydrogen is a conductor
- convection + rotation in a conducting fluid produce magnetic dynamos and magnetic fields

10.5.1 energy deficit for jupiter

- Jupiter's luminosity is twice its rate of solar irradiation from the sun
- Consider gravitational potential energy of a shell of thickness
- extra energy comes from the radius shrinking

10.6 Fast rotation

- Polar diamter of jupiter is 6.5 shorter thne equatorial bulge
- saturn is 10% shorter than equatorial diam

10.7 rings

- all Jovian planets have rings; Saturn are the most prominent
- tidally disrupted satellite inside Roche limit
- internal structures caused by orbital resonances

12 Solar system in perspective

12.1 Solar system config

- Planetary orbits are coplanar
- suns equator close to orbit Plane
- nearly circular orbits
- planets all orbit in same direction
- most planets rotate in the same direction as their orbital motion

12.2 Porotstellar Neb

- Cloud of gas compressing and spinning

12.3 Compartive planetology

- Massive colder planets retain atmospheres
- planet masses and compositions were driven by the temperature gradient in the protoplanetary disk and its relation to the condensation Temperature profile

12.4 Origin of the solar system

- Giant jovian satellites are a mini version of the whole solar system
- rings are temporary and provide evidence for a dynamic, evolving planetary system
  - all Jovan planets have rings
  - produced by bodies that failed to form or were disrupted due to tidal forces
  - triton is scheduled to explode

12.5 Dynamic solar system

- our moon, large satellite of a small planet
- triton captured by Neptune
- retrograde rotation of Venus
- large impact craters

12.6 Detecting exoplanets

12.6.1 Transit

- can theoretically resolve some based on angular resolution, but very hard to separate from the star itself
- $L_{jup} = (\frac{L_\odot}{r\pi a^2}(\pi R_{jup}^2 A))$
- if you look at infrared radiation it is "easier" to resolve to planet

12.6.2 Indirect

- Wobble due to the planet pulling on the star
- They orbit their COM

12.6.3 Doppler effect

- Measure redshift and blue shift on the star

12.7 Unknown orbital inclination angle

- We do not know the inclination of the plane of the system relative to our line of sight
- Doppler can only measure velocities projected onto the line of sight
- if $i = 90°$, the plane is viewed edge on, and we measure true veloicty. Otherwise, we measure a smaller velocity, giving a lower limit of planet masses.

12.8 Radial Velocity method

- we need the complete more accurate version of Kepler III
- $$p^2 = \frac{4\pi^2(a_A + a_B)^3}{G(M_A + M_B)}$$
- can find p by looking at when the velocity pattern repeats
- we want a_b and M_B. get a_B from
- $$a_b^3 \approx \frac{GM_A P^2}{4\pi^2}$$ $$M_A \gg M_B$$

-
$$\frac{a_A}{a_B} = \frac{M_B}{M_A}$$

-
$$M_B = (\frac{4\pi^2 M_A^2}{GP^2} a_A^3)^{1/3}$$

12.9 Transit method

- Detected by planet passing in front of the star, causing a dip in the brightness
- find mind transit flux reduction
- find planet velocity (same method at radial vel)
- time before t_1 and t_2 yields planet radius or measuring the fraction of the starlight blocked (if r_s is known)

12.9.1 Chance of detection

- Range of inclinations for which transit is observed
-
$$\cos i \leq \frac{R_A + R_B}{a}$$

12.10 Surprising exoplanet discoveries

- **Hot Jupiter's** - exoplanets with Jupiter like masses which are very close to their host stars
- very eccentric and/or inclined planet orbits
- current perspective biased by observational methods

14 Stellar Atmospheres

23 Galaxies and Super clusters

Stellar streams

- Galaxies (r $\sim$ 10 kpc)
- Groups (r $\sim$ 1Mpc)
- Supercluster (r $\sim$ 100Mpc)

Coma Cluster

- rich cluster dominated by elliptical Galaxies

Cluster ISM

- Bright x-ray emitting gas

- low density, but very large quality
- Total mass of ISM is around double that of the galaxies
- Motion through the ISM causes ram pressure stripping

23.1 Dark matter in galaxy clusters

- Use viral theorem…
- $$M \approx 7.5\frac{\sigma^2 r_h}{G}$$
- Where σ is a velocity distribution, and r_h is radius of the cluster
- If we look at the mass to light radio of the Coma Cluster, which is 250 (in solar units), we find a very different distribution as compared to our sun (= 1), and the Milky Way (~ 10)

23.2 Large Scale Structure

- Cosmic web structure
- Sloan great wall
- Few galaxies measured far away, simply because they're hard to measure

23.3 Galaxy collisions

- Can estimate velocity is cluster from velocity dispersion (σ) of galaxy motions
- $$v \sim \sqrt{3}\sigma$$
- Can then compute collision rates
- In rich clusters about a 50% chance

23.3.1 Rates of collisions

Consider a galaxy of radii R moving at velocity in, in an elapsed time t it will collide with any other galaxy within a cylindrical volume $V(t)$ of radius R^2 and length vt

$$V(t)(4\pi^2)(vt)$$

If galaxy density is n then the total collisions is $N_\star$ and the average time to collision is $t_\star$

$$N_\star = nV(t) = v(4\pi R^2)(vt)$$

$$N_\star = 1 \to t_\star = \frac{1}{nv(4\pi R)}$$

23.3.2 Galaxies collide, but stars to not

If you repeat the previous equation but for stars, you find a 1 in 10^8 chance of stellar collision, in the age of the entire universe

23.3.3 Physics of collisions

- Raise tidal bulges and resulting tidal tails
- They are inelastic, the kinetic energy is converted into funny motions of the stars themselves

24 Cosmology

24.1 CMB

- $ct \rightrightarrows$ theoretical size of the universe
- Does not account for expansion of the uni while the light is travelling
- Surface of last scattering, point where the universe becomes opaque, also the CMB

24.1.1 olbers paradox

- We do not see a galaxy everywhere we look
- this is because the universe is finite, not infinitly large

24.1.2 Equivalence principle

Gravitational Mass

$$F_{grav} = \frac{-GM_g m_g}{r^2} \tag{1}$$

Interial mass

$$a = \frac{F}{m_i} \tag{2}$$

Equivalence Principle

$$m_g = m_i \tag{3}$$

24.1.3 Einstein v Newton

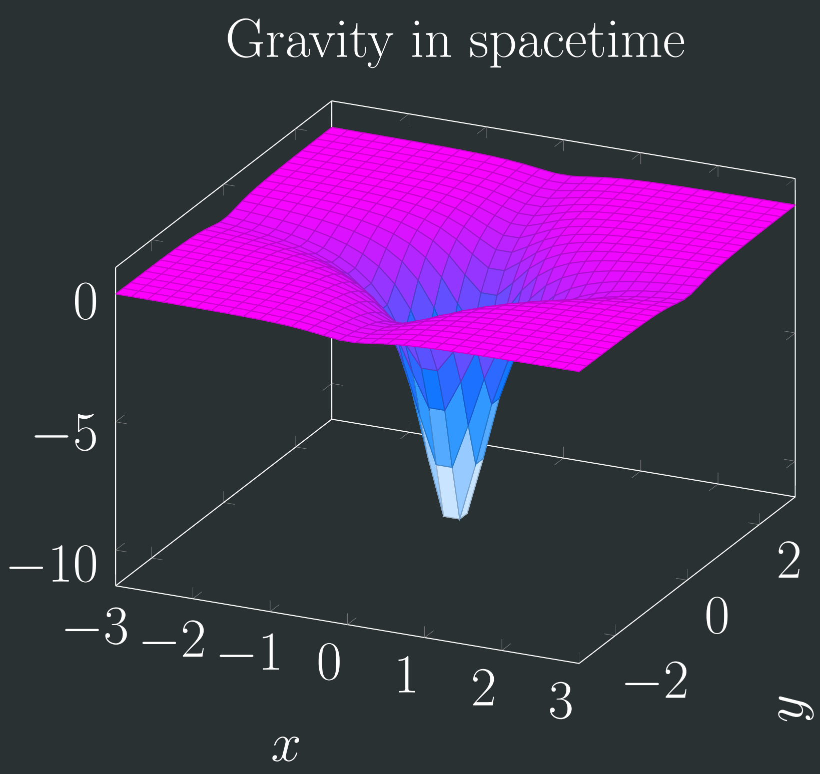
- In a small volume of space the downward pull of gravity cannot be distinguished from an upward acceleration of the observer

24.2 General relativity

- Space and time bind together to form curved spacetime
- 'free-fall' is motion in a straight line in 4d spacetime (this is grav acceleration)
- Matter (Mass-energy) tells spacetime how to curve
- Curved spacetime tells matter how to move

24.2.1 Predictions of GR

- Gravtional field sdeflect light (grav lensing)
- more accurate orbit for Mercury
- grav fields slow clocks
- moving objects will produce gravitational waves



24.3 Surface of spacetime

- How do we know the curvature and angles?
- If we draw a triangle what do the angles add up to?

Curvature and Angles of a triangle in 4d space

$$\alpha + \beta + \gamma = \pi \frac{\kappa A}{r_{c,0}^2} \tag{4}$$

Newtonian Friedmann Equation

$$\frac{\dot{a}}{a}^2 = \frac{8\pi G}{3} \rho(t + 2\frac{k}{r^2}\frac{1}{a^2(t)}) \tag{5}$$

Newtonian version of Friedman equation

Relativistic Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} - \frac{\kappa c^2}{r_{c,0}^2}\frac{1}{a^2(t)} + \frac{\alpha}{3} \tag{6}$$

where κ is curvature parameter

Currently, $\dot{a} > 0$, thus the universe is expanding

24.3.1 Measuring curvature

- Look at angular diam vs distance
- We look at something which we know the size of, a 'standard ruler'
- if flat... $\Rightarrow \alpha = \frac{D}{d}$
- Can't use galaxies, non standardized size
- Use sound saves before recombination
  - Use sound waves! $t < 380000yr$
  - Sound waves in early fluid like plasma in universe
  - wave equation for sound waves relates wavelength to the properties of the gas
- Apply the sound wave logic to the CMB
- the reason that the CMB is not homogeneous is because of the sound waves traveling through the plasma in early universe
- If universe was curved, the light traveling from the CMB would take a different path, resulting in a different angular resolution

24.3.2 Predicting angular scale distribution in CMB from sound waves

- large fluctuations are caused by patches of varying temperature
- small fluctuations are from
- Look at how CMB is 'distorted' from the sound waves
- Then infer wavelength of soundwaves
- Then apply previous logic for measuring angular res and comparing it to known wavelength

24.4 Density of the universe

-
$$\Omega_m = \frac{M/V}{\rho_c}$$
- Matter ...
-
$$\Omega_m \sim .3$$
- Radiation ...
-
$$\Omega_{rad} \sim .00007$$
- Something else ...
-
$$\Omega_{rest} \sim .7$$
- Because the universe is flat we know that
-
$$\Omega_m + \Omega_{rad} + \Omega_{res} \sim 1$$

24.4.1 Energy density components

- Looking for a solution for $a(t)$
- Wavelength of a photon scales with the size of the universe
- Lambda cosmological constant
-
$$\Omega_\Lambda \sim .7$$
- Non-Relativistic particles:
-
$$\Omega_{m,0} \sim .3$$
-
$$\Omega_(bary, 0) \sim .04$$

- Relativistic particles
- $$\Omega_{r,0} = \Omega_{cmb,0} + \Omega_{v,0} \sim 8.4 \times 10^{-5}$$

Behavior of nonrelatvistic partciles

$$n(t) \propto V^{-1}(t) \propto a^{-3}(t) \tag{7}$$

If matter is conserved

24.4.2 Cosmological redshift

Redshift Z

$$Z = \frac{\lambda_0 - \lambda_e}{\lambda_e} \tag{8}$$

If we use 8 combined with expansion of the universe ...

$$1 + Z = \frac{1}{a(t)}$$

24.4.3 Universe at different stages

- Solve 6 for different Ωs
- We find that all are decelerating, except for Ω_Λ (Dark Energy)

24.4.4 Standard candles for expanding universe

Flux

$$L = \frac{L}{4\pi r^2} \tag{9}$$

Photon Energy

$$\epsilon = \frac{hc}{\lambda_e}; \quad \epsilon_0 = \frac{hc}{\lambda_0} \tag{10}$$

- One can combine 8 and 10
- We can use type 1a SN as a standard candle
- Plot the redshift and distance, see what expected path it lies on (based on DE DM content)

25 Equations

$$F_{grav} = \frac{-GM_g m_g}{r^2} \qquad (1)\ (Gravitational\ Mass)$$

$$a = \frac{F}{m_i} \qquad (2)\ (Interial\ mass)$$

$$m_g = m_i \qquad (3)\ (Equivalence\ Principle)$$

$$\alpha + \beta + \gamma = \pi \frac{\kappa A}{r_{c,0}^2} \qquad (4)\ (Curvature\ and\ Angles\ of\ a\ triangle\ in$$

$$\frac{\dot{a}}{a}^2 = \frac{8\pi G}{3}\rho(t + 2\frac{k}{r^2}\frac{1}{a^2(t)}) \qquad (5)\ (Newtonian\ Friedmann\ Equation)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} - \frac{\kappa c^2}{r_{c,0}^2}\frac{1}{a^2(t)} + \frac{\alpha}{3} \qquad (6)\ (Relativistic\ Friedmann\ equation)$$

$$n(t) \propto V^{-1}(t) \propto a^{-3}(t) \qquad (7)\ (Behavior\ of\ nonrelatvistic\ partciles)$$

$$Z = \frac{\lambda_0 - \lambda_e}{\lambda_e} \qquad (8)\ (Redshift\ Z)$$

$$L = \frac{L}{4\pi r^2} \qquad (9)\ (Flux)$$

$$\epsilon = \frac{hc}{\lambda_e}; \quad \epsilon_0 = \frac{hc}{\lambda_0} \qquad (10)\ (Photon\ Energy)$$

Glossary

Stellar streams Stars follwing a path away from a galaxy 22

Surface of last scattering Point looking deep into the universe where the universe becomes opaque. Also where the CMB is 25