Astro HW 8

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8.1 Stellar Properties

Given two stars A and B with properties ...

Name	**Spectral type**	m_B (mag)	m_V (mag)	**Diameter** ($\times 10^{-3}$")	**Parallax** (")	**Mass** ($M_\odot$)
Star A	M2 Ia	2.27	0.42	43.43	0.00451	19.6
Star B	M2 V	8.96	7.52	1.43	0.393	0.46

(a) Distances

$$d(\text{pc}) = \frac{1}{\theta_{\text{parallax}}('')}$$

$$d_A = \frac{1}{0.00451} \boxed{\approx 221.72 \text{ pc}}, \qquad d_B = \frac{1}{0.393} \boxed{\approx 2.544 \text{ pc}}$$

(b) Radii

Given the small-angle relation ...

$$\theta_{\text{diam}} = \frac{2R}{d} \qquad \Rightarrow \qquad R = \frac{\theta_{\text{diam}}}{2} d$$

We find ...

$$R_A = \frac{43.43 \times 10^{-3}}{2 \times 206265} (221.72)(3.086 \times 10^{16}) \boxed{\approx 7.20 \times 10^{11} \text{ m} \approx 1.035 \times 10^3 R_\odot}$$

$$R_B = \frac{1.43 \times 10^{-3}}{2 \times 206265} (2.544)(3.086 \times 10^{16}) \boxed{\approx 2.72 \times 10^8 \text{ m} \approx 0.391 R_\odot}$$

*If there's anything I can do to make grading easier please let me know :)

(c) Surface Gravity

$$g = \frac{GM}{R^2}$$

$$M_A = 19.6\,M_\odot = 3.89 \times 10^{31}\,\mathrm{kg}, \qquad M_B = 0.46\,M_\odot = 9.15 \times 10^{29}\,\mathrm{kg}$$

$$g_A = \frac{6.67 \times 10^{-11}(3.89 \times 10^{31})}{(7.20 \times 10^{11})^2} \boxed{\approx 5.0 \times 10^{-3}\ \mathrm{m/s^2}}$$

$$g_B = \frac{6.67 \times 10^{-11}(9.15 \times 10^{29})}{(2.72 \times 10^{8})^2} \boxed{\approx 8.2 \times 10^{2}\ \mathrm{m/s^2}}$$

(d) Luminosity Class Comparison

We expect the main-sequence dwarf (V) to have much higher surface gravity than the supergiant (Ia). Indeed, $g_B \gg g_A$, consistent with their classifications.

(e) Temperatures

Using

$$B - V = m_B - m_V \tag{13.35}$$

and

$$T \approx \frac{9000\ \mathrm{K}}{(B - V) + 0.93} \tag{13.36}$$

$$T_A = \frac{9000}{(2.27 - 0.42) + 0.93} \boxed{\approx 3.24 \times 10^3\ \mathrm{K}}, \qquad T_B = \frac{9000}{(8.96 - 7.52) + 0.93} \boxed{\approx 3.80 \times 10^3\ \mathrm{K}}$$

(f) Luminosities

From the Stefan-Boltzmann law:

$$L = 4\pi R^2 \sigma_{\mathrm{SB}} T^4$$

$$L_A = 4\pi(7.20 \times 10^{11})^2(5.67 \times 10^{-8})(3.24 \times 10^3)^4 \boxed{\approx 4.06 \times 10^{31}\ \mathrm{W}}$$

$$L_B = 4\pi(2.72 \times 10^{8})^2(5.67 \times 10^{-8})(3.80 \times 10^3)^4 \boxed{\approx 1.10 \times 10^{25}\ \mathrm{W}}$$

(g) H-R Diagram Location [1]

Star A: very cool, extremely luminous → upper right of the H–R diagram (red supergiant). Star B: cooler main-sequence dwarf → lower right region (red dwarf). Both positions are consistent with their spectral classifications.

Name	Distance (pc)	Radius (m)	Radius ($R_\odot$)	g (m/s^2)	Temp (K)	$L_{\mathbf{tot}}$ (W)
Star A	221.72	7.20×10^{11}	1.04×10^3	5.0×10^{-3}	3.24×10^3	4.06×10^{31}
Star B	2.544	2.72×10^8	3.91×10^{-1}	8.2×10^2	3.80×10^3	1.10×10^{25}

8.2 Stellar main sequence lifetimes

Given ...

$$N_H = \frac{M}{m_p} \tag{15.52}$$

$$E_{fus} = \frac{N_h}{4} \Delta E \tag{15.53}$$

$$t_{fus} = \frac{E_{fus}}{L} \tag{15.54}$$

We can derive ...

$$t_{fus} = \frac{1}{L}\frac{1}{4}\frac{\Delta E}{1}\frac{M}{m_p}$$

$$t_{fus} = \frac{\Delta E \times M}{4L \times m_p}$$

Where ...

$$\Delta E = 4.1 \times 10^{-12}\text{J}, \qquad m_p = 1.67 \times 10^{-27}\text{kg}$$

Additionally, we are given ...

$$\tau = 10\text{Gyr}\left(\frac{M}{M_\odot}\right)^{-3} \tag{15.55}$$

a) High-Mass

Given ...

$$M = 100 M_\odot \approx 1.989 \times 10^{32}\text{kg}, \qquad L = 10^6 L\odot \approx 3.83 \times 10^{32}\text{W}$$

We find that ...

$$t_{fus} = \frac{4.1 \times 10^{-12}\text{J} \times 1.989 \times 10^{32}\text{kg}}{4(3.83 \times 10^{32}\text{W}) \times 1.67 \times 10^{-27}\text{kg}} \boxed{\approx 3.187 \times 10^{14}\text{s} \approx 1 \times 10^7\text{yr}}$$

[1] If whoever is grading this is horribly bored, and has nothing better to do, check out the script I wrote to streamline H-R diagram graphing of POSYDON Data using matplotlib and bokeh :) `https://github.com/PiersonLip/Posydon_HR_Graphing_Script`

Using equation 15.55 ...

$$\tau = 10\text{Gyr}\left(\frac{100\text{M}_\odot}{\text{M}_\odot}\right)^{-3} = \frac{1}{100000} \times 10\text{Gyr} = 1 \times 10^4\text{yr}$$

Thus, we find that...

$$\frac{t_{fus}}{\tau} = 10^3$$

This means our approximation is off by 3 orders of magnitude

b) Low-mass

Given ...

$$M = .5M_\odot \approx 9.945 \times 10^{29}\text{kg}, \qquad L = .1L\odot \approx 3.83 \times 10^{25}\text{W}$$

We find that ...

$$t_{fus} = \frac{4.1 \times 10^{-12}\text{J} \times 9.945 \times 10^{29}\text{kg}}{4(3.83 \times 10^{25}\text{W}) \times 1.67 \times 10^{-27}\text{kg}} \approx \boxed{1.58 \times 10^{19}\text{s} \approx 5.006 \times 10^{11}\text{yr}}$$

Using equation 15.55 we find ...

$$\tau = 10\text{Gyr}\left(\frac{.5\text{M}_\odot}{\text{M}_\odot}\right)^{-3} = 8 \times 10\text{Gyr} = \boxed{8 \times 10^{10}\text{yr}}$$

Thus, we find that ...

$$\frac{t_{fus}}{\tau} = \frac{5.06 \times 10^{11}}{8 \times 10^{10}} \approx 6.25$$

Hence, our approximation is only off by a factor of ≈ 6.25

8.3 Stellar structure from opacity

Given ...

$$\kappa \propto \rho T^{-3.5}$$

$$\frac{dT}{dr} \approx \frac{T}{R}$$

$$\rho = M/(\frac{4}{3}\pi R^2)$$

$$L = 4\pi R^2 \sigma_{sb} T^4 \rightarrow T = \left(\frac{L}{4\pi R^2 \sigma_{sb}}\right)^{1/4}$$

a) Deriving $L \propto M^{5.5}R^{-.5}$

$$\kappa = \frac{3M}{4\pi R^2}\left(\frac{L}{4\pi R^2 \sigma_{sb}}\right)^{1/4} R$$

8.4 Extra Credit

a)

The parallax error would affect its location on the y-axis. This is because the parallax would affect the distance measured, which in turn effect the absolute luminosity, hence shifting the location on the y-axis.[2]

b)

Given ...

$$d = \frac{1}{\theta_{par}} \qquad F = \frac{L_{abs}}{4\pi d^2}$$

We can find the difference as ...

$$\frac{L_{\text{meas}}}{L_{\text{true}}} = \left(\frac{d_{\text{meas}}}{d_{\text{true}}}\right)^2 = \left(\frac{1}{0.9}\right)^2 = 1.234\ldots$$

c)

$$\approx 23\%_{error}$$

This means that the L value on the y-axis of the HR diagram should be 23% brighter, meaning the star should be further **up** on the HR diagram.

[2]It could also affect the x-axis through redshift + Hubble const, but if a parallax error is that big you might have bigger problems.