ASTR 210 Fall 2025 — Homework set 3
50 points plus optional extra credit

Solutions need to show all important intermediate calculations, diagrams, and explanations in addition to the final answer to receive full credit. Please check the legibility of homework solutions uploaded to the class website as we cannot grade or give credit for illegible or unreadable work. Units do not need to be specified if an expression or equation is requested; for numerical calculations please use the specified units for the answer, or if not specified, SI or cgs units. Please use Table A.1 for the value of physical constants and Table A.2 for the value of astronomical constants, unless otherwise specified in the problem. Please use the correct number of significant figures.

3.1 Ellipse Geometry

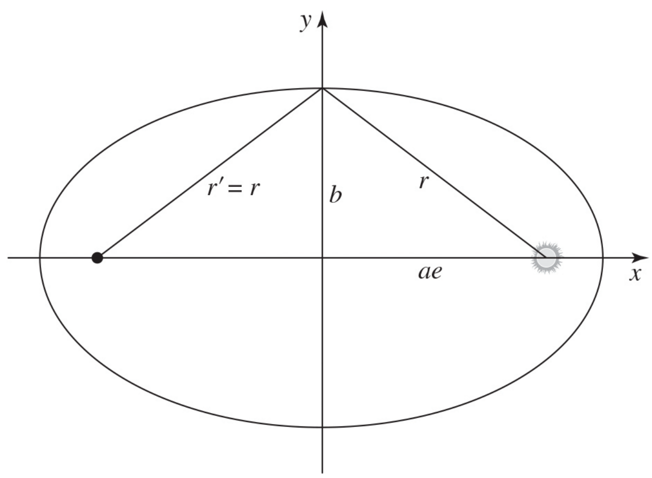

Figure 1: Figure 3.7 from Ryden & Peterson (2010).

Figure 1 reproduces Figure 3.7 from the text. The semimajor axis is a, the semiminor axis is b, and the eccentricity is e. The points on the ellipse satisfy $r + r' = 2a$, which is the "string" definition of an ellipse.

(a) Using the information provided, show that the perihelion distance is $r_{pe} = a(1 - e)$ and the aphelion distance is $r = a(1 + e)$. Also show that the eccentricity obeys the relation $e = \sqrt{1 - b^2/a^2}$. [4 pts]

(b) Zoozve is an asteroid that starred in a charming RadioLab episode here. It is also a quasi-moon of Venus because it never gets very far away from Venus, so it moves through Venus's sky like a moon would. Its orbital semimajor axis is 0.7237 AU and its eccentricity is 0.4103. Show that Zoozve is an Earth-crossing asteroid (its orbit crosses Earth's). Aside: If Earth is elsewhere at the time these orbit crossings do not cause trouble, at least in the short term. In the long term such orbit-crossing asteroids tend to have highly unstable orbits and Zoozve could ultimately end up almost anywhere in the solar system. [3 pts]

(c) Recently there have been some controversial suggestions that our solar system may host another planet, the hypothetical Planet 9.[1] Planet 9 might have a mass of $4.4 \pm 1.1\ M_\oplus$, an

[1] If you like you can read all about it here though that paper is as technical as papers ever get.

orbital semimajor axis of 290 ± 30 AU, and an orbital eccentricity of 0.29 ± 0.13. If those orbital parameters are correct, what would be its orbital period? [2 pts]

3.2 Energy and elliptical orbits

I've been encouraging you to do the missing algebra steps in the text, as an excellent study strategy. Here's an example of the kind of thing I mean.

(a) Show how to combine equations 3.58 and 3.59 in your text to produce equation 3.60. [4 pts]

(b) Combine equations 3.60 and 3.43 to produce a simple relation that gives the total energy E of an elliptical orbit in terms of M, m, and a. [3 pts]

3.3 Vis Viva Equation

The figure below shows a Keplerian elliptical orbit with polar coordinates (r, θ) for an object of mass m in orbit around the Sun (mass M).

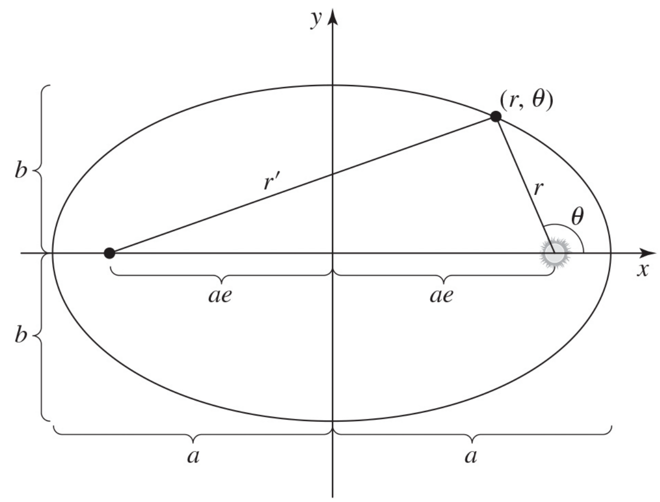

Figure 2: Figure 3.6 from Ryden & Peterson (2010).

(a) The vis viva equation yields the orbital speed v of the planet:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$

For an elliptical Keplerian orbit show that

$$v(\theta) = \sqrt{\frac{GM}{a(1-e^2)}(1 + 2e\cos\theta + e^2)}.$$

You may use the polar form of an ellipse, as given in the text, without derivation. That derivation is on the extra credit. [4 pts]

(b) Consider a well-behaved function $f(x)$, with first derivative df/dx and second derivative d^2f/dx^2. Recall that a local minimum in $f(x)$ is defined by $df/dx = 0$ and $d^2f/dx^2 > 0$, and the local maximum has a similar definition with the second derivative < 0. Using these definitions, show that the planet's orbital speed is a maximum at perihelion and a minimum at aphelion. [4 pts]

(c) Using the result from (a), derive expressions for the orbital speed at perihelion and at aphelion. Show that these yield equations (3.46) and (3.47) in the text. [4 pts]

(d) Compute Zoozve's orbital speed at aphelion, when it is close to Earth's orbit. Compare that value to Earth's orbital speed. For simplicity, you can pretend that Earth's orbit is circular. If Zoozve ever collides with Earth, will it be because Earth is overtaking Zoozve or because Zoozve is overtaking Earth? [5 pts]

3.4 Roche limit for Jupiter's moons.

(a) Use the mass and radius of Jupiter (Table A.3) to calculate its mean density (kg m^{-3}). [4 pts]

(b) Considering a moon made of ice, estimate the Roche limit for the moons of Jupiter. Assume that ice has a density of about 1000 kg m^{-3}, and quote the Roche limit radius in km. [4 pts]

(c) The innermost moons of Jupiter have orbital semimajor axes around 128,000 km. How do those compare to the Roche limit you just found? Do you expect these moons to be stable against tidal disruption? [4 pts]

(d) From the semimajor axis value quoted in part c, estimate the orbital period of these innermost moons of Jupiter. [5 pts]

[OPTIONAL EXTRA CREDIT PROBLEMS]

3.5 Elliptical orbits [5 pts]

An ellipse in Cartesian coordinates takes the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Derive the equation for an ellipse in polar coordinates as given in equation (3.42) in the text:

$$r = \frac{a(1-e^2)}{1+e\cos\theta}.$$

Hint: The polar coordinates used to derive equation (3.42) have the origin at one focus of the ellipse, but x and y have their origins at the center of the ellipse. The first thing you'll need to do is to write the relationships between x, y, r, and θ. See Figure 3.6 in your text.

3.6 Tidal Forces [5 pts]

Figure 3 replicates Figure 4.3 from the text showing the Earth (at left) and the Moon (at right) in one-dimensional projection at a central separation r_0. The Earth's radius is $R_\oplus$. Both the Earth and the Moon are considered perfectly spherical bodies in this problem; the mass of the Moon is M_M and the mass of the Earth is $M_\oplus$.

(a) Consider an infinitesimal test mass m located at the center of the Earth. Write down an expression for the magnitude of the Newtonian gravitation force F_C exerted by the Moon on this test mass.

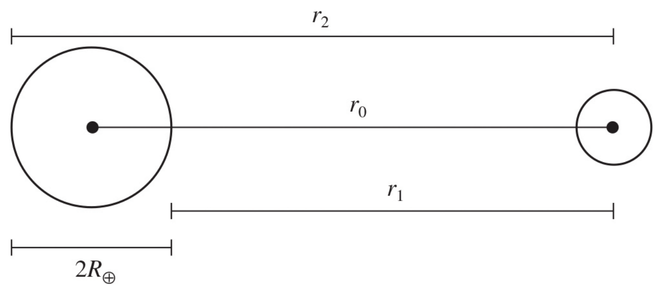

Figure 3: Figure 4.3 from Ryden & Peterson (2010).

(b) Now imagine m is located on the near side of the Earth on the line connecting the center of the Moon and the center of the Earth. Write down an expression for the magnitude of the Newtonian gravitation force F_R exerted by the Moon on this test mass.

(c) Using the results from (a) and (b), derive an expression for the differential (tidal) force on the test mass m. Show that it simplifies to $\Delta F \approx \frac{2GM_M m R_\oplus}{r_0^3}$. Hint: Use a first-order Taylor series expansion to approximate $\left(1 - \frac{R_\oplus}{r_0}\right)^{-2}$.