ASTR 210 Fall 2025 — Homework set 1
50 points plus optional extra credit

Solutions need to show all important intermediate calculations, diagrams, and explanations in addition to the final answer to receive full credit. Please check the legibility of homework solutions uploaded to the class website as we cannot grade or give credit for illegible or unreadable work. Units do not need to be specified if an expression or equation is requested; for numerical calculations please use the specified units for the answer, or if not specified, SI or cgs units. Please use Table A.1 for the value of physical constants and Table A.2 for the value of astronomical constants, unless otherwise specified in the problem. Please use the correct number of significant figures.

1.1 Ethics: collaboration and academic integrity.

Please read the course syllabus statements on collaboration and academic integrity and follow the link provided to the associated section of the UIUC Student Code. These are common-sense policies but it is always good to recall our ethical responsibilities. Answer the following questions in your own words. I recommend not having the syllabus in view as you write.

(a) Summarize in 1-2 sentences how much collaboration is and is not allowed. [3 pts]

(b) Summarize in 1-2 sentences your responsibilities regarding academic integrity. [3 pts]

(c) Explain in 1-2 sentences why these policies and the UIUC Student Code forbid copying problem solutions found online. [3 pts]

1.2 You are standing approximately 384 meters away from some trees that are 61 meters tall. (The ground is relatively flat.) What is the elevation of an astronomical source that you will just barely be able to observe as it skims the top of the trees? Quote your answer in both radians and degrees. [5 pts]

1.3 Show that Newton's gravitational constant G has a value of approximately 39.5 $\mathrm{AU}^3\ \mathrm{M}_\odot{}^{-1}\ \mathrm{yr}^{-2}$, which means that $G/4\pi^2 \approx 1.00\ \mathrm{AU}^3\ \mathrm{M}_\odot{}^{-1}\ \mathrm{yr}^{-2}$. Those funky units will be useful when we work with stellar and planetary orbits. AU is an astronomical unit and $\mathrm{M}_\odot$ is the mass of the Sun; you can find values for those constants in the back of your book. [6 pts]

1.4 The Celestial Sphere and Local Horizon Coordinates

Here we are working on the mathematical relationships between celestial and horizon coordinates. Yes, there's a lot of verbiage, but most of it just defines the coordinate systems.

Figure 1 shows the Earth in 2D projection, viewed in the plane of the equator. The (x, z) axes have their origin at the Earth's center (G) and define coordinates on the celestial sphere. The z-axis lies along the polar axis of rotation of the Earth and intersects the South and North Celestial Poles (SCP & NCP); this axis has unit vector $\hat{z}$. In this 2D projection, the x-axis lies in the celestial equator and is chosen to pass through the point on the celestial sphere with right ascension $\alpha = 0$ (the vernal equinox). This axis has unit vector $\hat{x}$. The observer's zenith meridian is in the plane of the page. A unit vector $\hat{s}$ is shown, pointing toward an astronomical source on the celestial sphere with coordinates right ascension $\alpha = 0$ and declination δ (at zero hour angle in this configuration).

An observer O at latitude ℓ on the surface of the Earth is shown. The local observer horizon coordinate system is defined by axes $\hat{x}'$ and $\hat{z}'$. Unit vector $\hat{z}'$ points in the direction of the observer zenith. Unit vector $\hat{x}'$ lies in the observer horizon plane. The tangent horizon plane is parallel-transported to the horizon circle through the coordinate system origin (G).

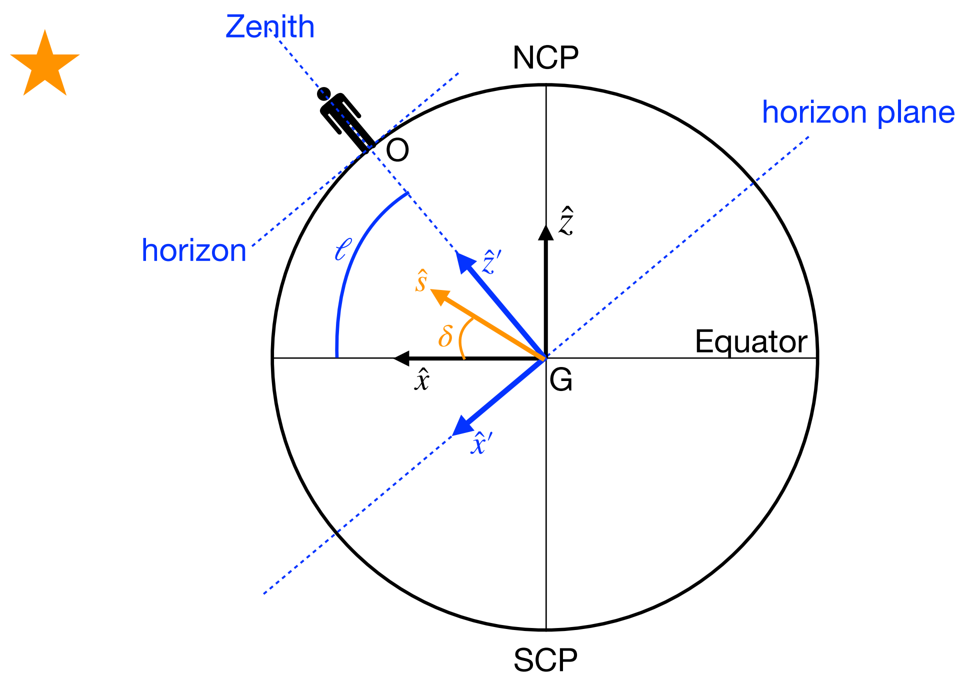

Figure 1: The Earth viewed in 2D projection along the East-West axis.

(a) Express the source unit vector $\hat{s}$ as components along the unit vectors $(\hat{x}, \hat{z})$. I find it helpful to draw a little figure that's just a subset of Figure 1 with only $\hat{s}$, $\hat{x}$ and $\hat{z}$. [3 points]

(b) From examination of Figure 1 and basic trigonometry, derive an expression for the equatorial unit vector $\hat{x}$ in terms of the horizon coordinate axis unit vectors $(\hat{x}', \hat{z}')$. [3 points]

(c) Analogously to (b), derive an expression for the polar unit vector $\hat{z}$ in terms of $(\hat{x}', \hat{z}')$. [3 points]

(d) Using the above results, derive an expression for the source vector $\hat{s}$ in terms of the horizon coordinate unit vectors $(\hat{x}', \hat{z}')$. Hint: The trigonometric angle addition identities are helpful. [6 points]

(e) In the horizon system the meridional altitude (the maximum altitude of a source in its diurnal motion) is the angle θ in the range $[0, \pi/2]$ between the source vector $\hat{s}$ and the horizon plane, as the source transits the zenith meridian. Consider the two cases (i) $\delta > \ell$ and (ii) $\delta < \ell$, and derive two separate expressions for altitude θ in these two cases. [6 points]

(f) Use the result from (e) to derive an expression for the altitude θ_{NCP} of the NCP above the observer's horizon plane. You should obtain the result stated on page 6 of the text. [4 points]

(g) Assume our location to be Champaign-Urbana, with latitude $\ell = 40°07'$. Compute the southernmost declination δ_{lim} visible from Champaign-Urbana. Explain briefly (1-2 sentences) whether we can or cannot observe the galaxy NGC 1316 (Dec $= -37°12.5'$) from Champaign-Urbana. [3 points]

(h) By inspection of Figure 1, what is the declination of the zenith vector (here unit vector $\hat{z}'$) for an observer at latitude ℓ? [2 pts]

Optional extra credit problem

1.5 Solid angle on the sphere

The differential element of solid angle on the sphere in spherical polar coordinates is given by $d\Omega = \sin\theta \, d\theta \, d\phi$, where θ is the colatitude (polar angle) and ϕ is the azimuthal angle. Derive an expression for the solid angle of one polar cap on the celestial sphere, of angular radius θ_r. Show that in the small-angle limit, this approximates to $\Omega \simeq \pi\theta_r^2$. Hint: A full sphere has a solid angle of 4π sr. You can check that you have the correct answer by verifying that you obtain $\Omega(\theta_r = \pi/2) = 2\pi$. In the small-angle limit, use a second-order Taylor series for the cosine. [5 pts]

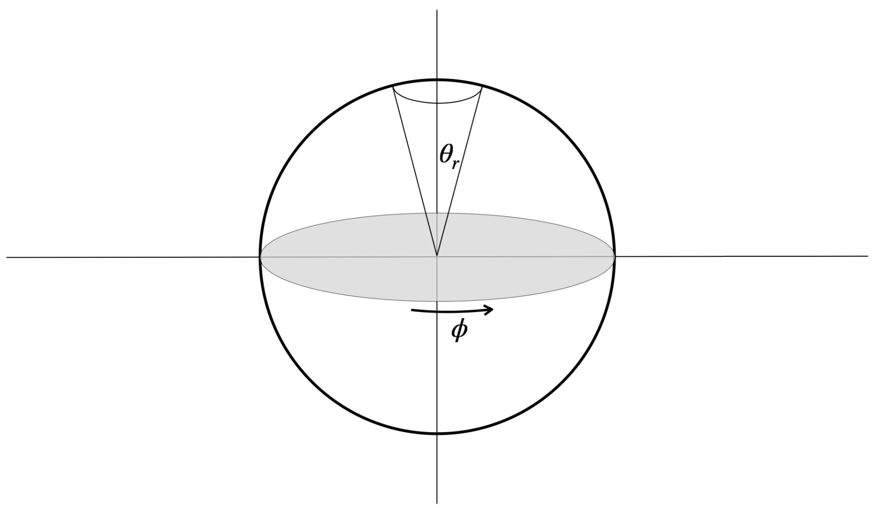

Figure 2: For the extra credit solid angle.