ASTR 210 Fall 2025 — Homework set 2
50 points plus optional extra credit

Solutions need to show all important intermediate calculations, diagrams, and explanations in addition to the final answer to receive full credit. Please check the legibility of homework solutions uploaded to the class website as we cannot grade or give credit for illegible or unreadable work. Units do not need to be specified if an expression or equation is requested; for numerical calculations please use the specified units for the answer, or if not specified, SI or cgs units. Please use Table A.1 for the value of physical constants and Table A.2 for the value of astronomical constants, unless otherwise specified in the problem. Please use the correct number of significant figures.

2.1 Stellar parallax

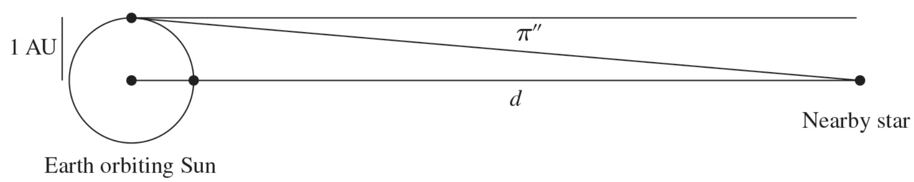

Figure 1: Figure 2.23 from Ryden & Peterson (2010).

(a) The best naked-eye (non-telescopic) observations had an accuracy of approximately 1 arcmin. Generously assuming that an annual parallax of 3 arcmin could therefore marginally be detected, what maximum stellar distance d (in AU) could be detected using naked-eye parallax observations? Find the closest star to Earth in Table A.7 of the text. Express the distance limit d as a fraction of the distance to the nearest star. Based on this fraction, was there any chance that naked-eye parallax measurement could succeed? [4 points]

(b) Current radio telescopes can measure annual parallaxes as small as about 0.1 milliarcsec (0.1×10^{-3} arcsec).[1] How far away is an object with that parallax, and how does it compare to the distance from Earth to the center of the Galaxy (about 8300 pc)? [3 points]

2.2 Sidereal and synodic orbital periods

In Figure 2 we consider two planets, A and B, orbiting the Sun in circular orbits at constant angular velocity. They have sidereal orbital periods P_A and P_B respectively. We describe their positions with the angles θ_A and θ_B.

(a) Assume that at reference time t_0 planet A is at angle $\theta_A(t_0) = \beta_A$. Similarly for planet B and β_B. Write down the general expressions for $\theta_A(t)$ and $\theta_B(t)$ at some future time t. (Don't reset those angles to zero when the planets complete an orbit; let them continue to increase past 2π.) [3 points]

(b) The synodic period is the time interval P_{syn} between consecutive appearances of the same planetary configuration (e.g. successive conjunction for a superior planet or successive inferior conjunctions for an inferior planet; see p.41 of text). The synodic period can be defined

[1] If you want to dig into the nitty-gritty, it is here. https://iopscience.iop.org/article/10.3847/1538-4357/ab4a11/pdf

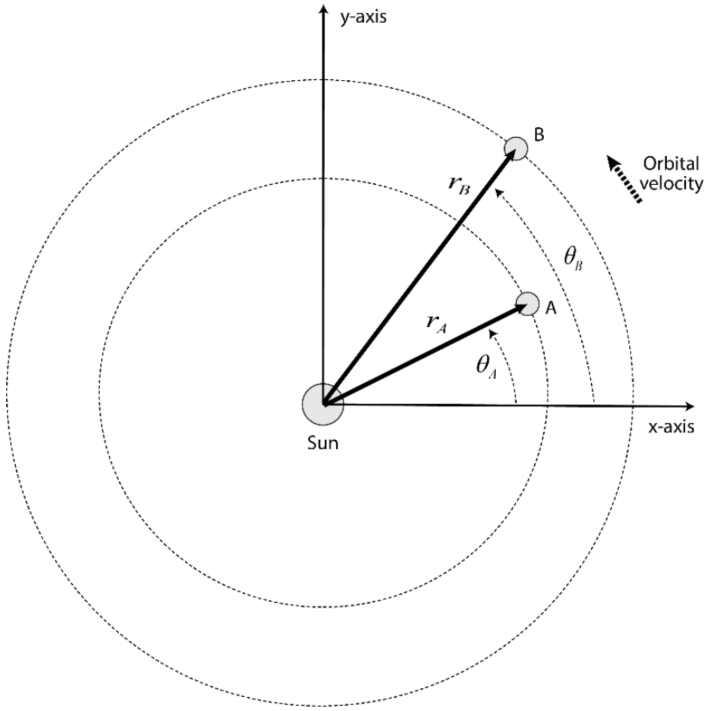

Figure 2: Planets A and B orbiting in the ecliptic plane.

therefore as $P_{syn} = t_2 - t_1$, where t_1 is defined by the condition $\theta_A(t_1) - \theta_B(t_1) = 0$ and t_2 is defined by $\theta_A(t_2) - \theta_B(t_2) = 2\pi$.

For Keplerian orbits, planet A orbits at a higher angular velocity than planet B. Thus, after one synodic period, planet A has "lapped" planet B like runners can "lap" other runners on a racetrack. Using the expressions from (a), solve these equations to show: $1/P_{syn} = 1/P_A - 1/P_B$. [6 points]

(c) Assume the Earth has a sidereal orbital period P_E and that an inferior planet has a sidereal orbital period P_P. Use the relation obtained in (b) to derive equation (2.8) in the text for inferior planets: $1/P_P = 1/P_E + 1/P_{syn}$. [3 points]

(d) Now take P_P to refer to a superior planet. Use the relation obtained in (b) to derive equation (2.10) in the text for superior planets: $1/P_P = 1/P_E - 1/P_{syn}$. [3 points]

(e) Re-arrange equation (2.10) derived in (d) to yield the synodic period of the superior planet as a function of P_E and P_P. Treating the Earth's sidereal orbital period as fixed, what will be the limiting synodic period P_{syn} for very distant planets, as $P_P \to \infty$? Explain briefly why this makes sense intuitively. [5 points]

(f) For an inferior planet as in (c), hold P_E fixed as before, and determine the limiting synodic period P_{syn} as $P_P \to P_E$. Explain briefly why this solution makes sense intuitively. [3 points]

(g) The sidereal orbital period of Mars is $1.881 P_E$ and that of Neptune is $164.79 P_E$, where P_E = 365.256 days. Calculate the synodic periods of Mars and Neptune (in days). Are these results consistent with the predictions of (e) and (f)? [5 points]

2.3 Estimating the radius of Mars's orbit

Consider a setup similar to Figure 2. Planet A will be the Earth and planet B will be Mars. On a particular day (call it day 0) Mars is at opposition, as seen from Earth. Mars's sidereal period is 687 Earth days; thus, 687 Earth days later, Mars has completed a full orbit with respect to the fixed stars and Earth has completed something less than two full orbits. It happens that on (Earth) day 687, Mars is at western quadrature as seen from Earth. Use these facts to estimate the distance of Mars from the Sun, in units of AU. Specifically:

(a) Draw two sketches indicating the configurations of Earth and Mars on day 0 and on day 687. The statement that day 687 is Mars's western quadrature means that you can draw a right triangle between Earth, Mars, and the Sun. Be sure to label which of the angles is the right angle. [3 pts]

(b) What is the total angle through which Earth has orbited in those 687 days? Use that information to find the values of the other two angles in the right triangle of interest. Be sure to clearly label them so that we can follow which one is which. [5 pts]

(c) Use your result from the previous part to find Mars's distance from the Sun in units of AU. You should find a value a bit less than 1.5 AU, which is the value quoted in Table A.3, and the reason is that Mars's orbit is not circular. [4 pts]

(d) Mars's radius is also given in Table A.3. When Mars is at opposition, what is its angular diameter in arcseconds? (Show the calculation, don't just look it up.) [3 pts]

[OPTIONAL EXTRA CREDIT PROBLEM]

2.4 *Uncertainties in early Greek astronomy.*

Aristarchus derived an expression [Eq. 2.1 in text] for the distance C to the Sun in terms of the distance A to the Moon based on their measured angular separation at first-quarter lunar phase: $C = A/\cos(\theta)$. We know today that the true value of θ is 89.853° [p.31 of text]. Treating A as an unknown constant and assuming the unsigned error in the measured value of θ is ϵ_θ, an approximate expression for the resulting unsigned error in C is

$$\epsilon_C = \left|\frac{dC}{d\theta}\right|\epsilon_\theta,$$

where ϵ_θ is in radians. Compute the expression for ϵ_C and show, by sampling at θ=[87°, 88°, 89°] that this error rises sharply as $\theta \to \pi/2$. Thus, the final estimate for C is very sensitive to errors in the measurement of θ near 90°. [5 points]