ASTR 210 Fall 2025 — Homework set 8
50 points plus optional extra credit

Solutions need to show all important intermediate calculations, diagrams, and explanations in addition to the final answer to receive full credit. Please check the legibility of homework solutions uploaded to the class website as we cannot grade or give credit for illegible or unreadable work. Units do not need to be specified if an expression or equation is requested; for numerical calculations please use the specified units for the answer, or if not specified, SI or cgs units. Please use Table A.1 for the value of physical constants and Table A.2 for the value of astronomical constants, unless otherwise specified in the problem. Please use the correct number of significant figures.

8.1 Stellar properties

Consider two stars A and B with parameters as tabulated below. For each star, this table contains the spectral type, apparent magnitude in the V-band, apparent magnitude in the B-band, angular diameter (from interferometry), annual stellar parallax, and mass M.

Identifier	Spectral type	m_B	m_V	Diameter	Parallax	Mass
		mag	mag	$\times 10^{-3}$ ″	″	$M_\odot$
Star A	M2 Ia	2.27	0.42	43.43	0.00451	19.6
Star B	M2 V	8.96	7.52	1.43	0.393	0.46

(a) Calculate the distances of the stars. [2 points, i.e. 1 pt each]

(b) Calculate the radii of the stars; quote both in m and in units of $R_\odot$. [4 pts, i.e. 2 pts each]

(c) Calculate the surface gravities of the stars (the gravitational acceleration g at their surfaces). Quote those values in m s^{-2}. [5 pts]

(d) As the text notes (see material around equation 14.26), the surface gravity of a star is related to its luminosity class. Which of the two stars should we expect to have the larger g value, and is that consistent with your calculations? [4 pts]

(e) Using the definition in equation 13.35 and the empirical relation in equation 13.36, compute approximate temperatures for the stars. (Why would you do this instead of using a proper temperature measurement from the spectrum? You may not have good quality spectra.) [5 pts]

(f) Using the prior results, and assuming the stars have blackbody spectra, calculate the total luminosities of the stars. [5 pts]

(g) Based on the luminosities and temperatures, in what region of the HR diagram do stars A and star B live? Are their luminosities and temperatures consistent with their spectral classifications? [5 pts]

To make this easy for the people reading your work, you might like to create your own small table with all of your computed quantities for the two stars. There's an H-R diagram for reference at the end of the assignment.

8.2 Stellar main sequence lifetimes

Estimate the main sequence (hydrogen-burning) lifetimes of two different stars, assuming they start as pure hydrogen and they are able to convert all of that hydrogen into helium.

Also assume that their luminosities are constant throughout their main sequence lifetimes. None of those assumptions are perfect, of course, but they'll work for order-of-magnitude estimates. Give the lifetimes in years. DO NOT simply use equation 15.55; make your own explicit calculation from the fuel reserves and the rate of fuel expenditure, and then compare your calculations to the results of equation 15.55.

(a) A high-mass star, with mass $M = 100\ \mathrm{M}_\odot$ and luminosity $L = 10^6\ \mathrm{L}_\odot$. [5 pts]

(b) A low-mass star, with mass $M = 0.5\ \mathrm{M}_\odot$ and luminosity $L = 0.1\ \mathrm{L}_\odot$. [5 pts]

8.3 Stellar structure from opacity

An assumption about the opacity κ can be used to give us constraints on stellar properties. For example, when the opacity is caused by photoionization of "heavy elements" (astronomers often put elements like carbon, oxygen, and iron in that category), the opacity can be written in terms of density and temperature as $\kappa \propto \rho T^{-3.5}$.

Use that information with the equations of stellar structure to show that in this case a star's luminosity will be related to its mass and radius as

$$L \propto M^{5.5} R^{-0.5}.$$

For this problem, you don't have to solve any differential equations. You should approximate, for example, by writing $dT/dr \approx T/R$. Why is this approximation even legal, or under what circumstances is it justified? Well, if the central temperature is T and the surface temperature is basically 0 (compared to the central temperature), then $\Delta T/\Delta R$ evaluated from the center to the outside edge is T/R. In that sense, T/R is an estimate of the average gradient. Use approximations at that level to arrive at the result quoted above. Just be careful that you probably shouldn't make this kind of an approximation in a math class. [10 pts]

[OPTIONAL EXTRA CREDIT PROBLEM]

8.4 Distance errors and the H-R diagram

When we make a Hertzsprung-Russell diagram from real data, the stars that are plotted have uncertainties in their distance estimates and those errors propagate into uncertainties in their positions on the HR diagram. Thus, part of the width of the main sequence is caused by uncertain distances. Suppose a star has an uncertainty of 10% in its parallax measurement – for specificity, you can assume that the measured parallax is 10% lower than the true value.

(a) Will the parallax error primarily affect the star's horizontal coordinate in the HR diagram, or its vertical coordinate, or both? Explain. [3 pts]

(b) Estimate the star's displacement (distance and direction) from its true position, both as a percentage of the accurate value and in magnitudes. [4 pts]

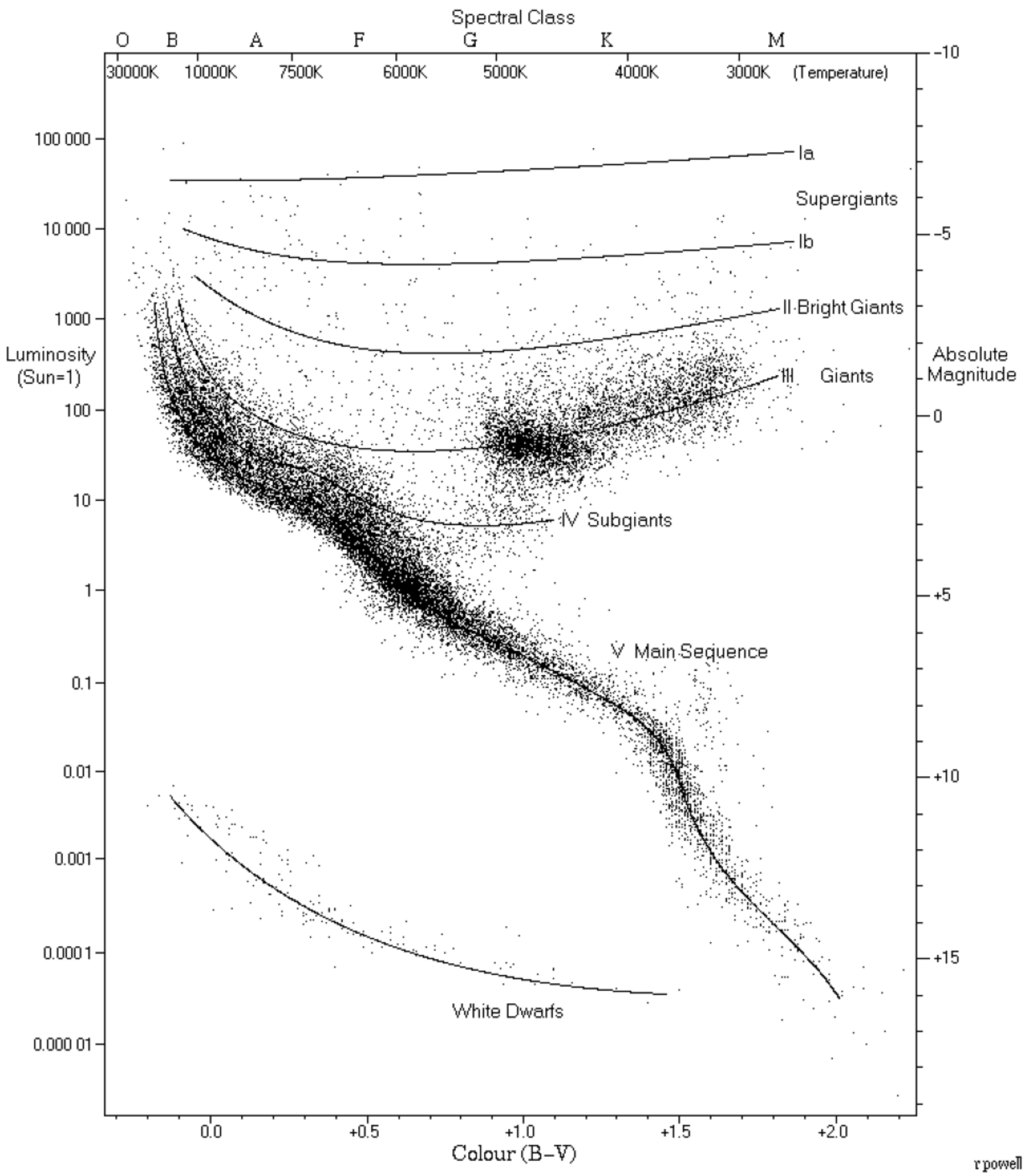

Figure 1: H-R diagram. (Credit: Creative Commons).