

Astro HW 1

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September 5, 2025

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1.1

- A) Collaboration and working with fellow students is encouraged, however, work still must be that of the individual, and copying answers is not allowed.
- B) Copying from others, cheating, enabling others to cheat, all are violations of academic honesty.
- C) Because they are the words/work of someone other than myself, meaning that I am not truly learning the material.

1.2

$$\tan^{-1}\left(\frac{61}{384}\right) \approx 9.0 \text{ deg}$$

$$9.0 * \frac{\pi}{180} \approx .16 \text{ rad}$$

1.3

$$G = 6.67430 * 10^{-11} m^3 kg^{-1} s^{-2} = 39.5 AU^3 M_{\odot}^{-1} yr^{-2}$$

$$6.67430 * 10^{-11} m^3 kg^{-1} s^{-2} * 1.989 * 10^{30} kg M_{\odot}$$

$$6.67430 * 10^{-11} m^3 \cancel{kg} s^{-2} * 1.989 * 10^{30} \cancel{kg} M_{\odot}$$

$$6.67430 * 10^{-11} m^3 s^{-2} * 1.989 * 10^{30} M_{\odot} * (1.496 * 10^{11})^3 AU^3 m^{-3}$$

$$\begin{aligned}
& 6.67430 * 10^{-11} \cancel{m^3} s^{-2} * 1.989 * 10^{30} M_{\odot} * (1.496 * 10^{11})^3 AU^3 \cancel{m^3} \\
& 6.67430 * 10^{-11} s^{-2} * 1.989 * 10^{30} M_{\odot} * (1.496 * 10^{11})^3 AU^3 * (3.154 * 10^7)^2 s^2 y^{-2} \\
& 6.67430 * 10^{-11} \cancel{s^2} * 1.989 * 10^{30} M_{\odot} * (1.496 * 10^{11})^3 AU^3 * (3.154 * 10^7)^2 \cancel{s^2} y^{-2} \\
& 6.67430 * 10^{-11} * 1.989 * 10^{30} M_{\odot} * (1.496 * 10^{11})^3 AU^3 * (3.154 * 10^7)^2 y^{-2} \\
& = 39.5 AU^3 M_{\odot}^{-1} yr^{-2}
\end{aligned}$$

1.4

1.4.1 a

$$\begin{aligned}
\frac{x}{\hat{s}} &= \cos(\delta) \\
x &= \cos(\delta) \\
x &= \hat{x} \cos(\delta) \\
\frac{z}{\hat{s}} &= \sin(\delta) \\
z &= \sin(\delta) \\
z &= \hat{z} \sin(\delta) \\
\hat{s} &= \hat{x} \cos(\delta) + \hat{z} \sin(\delta)
\end{aligned}$$

1.4.2 b

$$\begin{aligned}
\frac{z'}{\hat{x}} &= \cos(\ell) \\
z' &= \cos(\ell) \\
z' &= \hat{z}' \cos(\ell) \\
\frac{x}{\hat{x}} &= \sin(\ell) \\
x' &= \sin(\ell) \\
x' &= \hat{x}' \sin(\ell) \\
\hat{x} &= \hat{z}' \cos(\ell) + \hat{x}' \sin(\ell)
\end{aligned}$$

1.4.3 c

$$\frac{z'}{\hat{z}} = \cos(90 - \ell)$$

$$z' = \cos(90 - \ell)$$

$$z' = \hat{z}' \cos(90 - \ell)$$

$$\frac{-x'}{\hat{z}} = \sin(90 - \ell)$$

$$-x' = \sin(90 - \ell)$$

$$-x' = \hat{x}' \sin(90 - \ell)$$

$$\hat{z} = \hat{z}' \cos(90 - \ell) - \hat{x}' \sin(90 - \ell)$$

$$\hat{z} = \hat{z}' \cos(90 - \ell) - \hat{x}' \sin(90 - \ell)$$

$$\hat{z} = \hat{z}' \sin(\ell) - \hat{x}' \cos(\ell)$$

1.4.4 d

$$\hat{s} = \hat{x} \cos(\delta) + \hat{z} \sin(\delta)$$

$$\hat{s} = \cos(\delta)(\hat{z}' \cos(\ell) + \hat{x}' \sin(\ell)) + \sin(\delta)(\hat{z}' \sin(\ell) - \hat{x}' \cos(\ell))$$

$$\hat{s} = \hat{z}' \cos(\ell) \cos(\delta) + \hat{x}' \sin(\ell) \cos(\delta) + \hat{z}' \sin(\ell) \sin(\delta) - \hat{x}' \cos(\ell) \sin(\delta)$$

$$\hat{s} = \hat{z}'(\cos(\ell) \cos(\delta) + \sin(\ell) \sin(\delta)) + \hat{x}'(\sin(\ell) \cos(\delta) - \cos(\ell) \sin(\delta))$$

$$\hat{s} = \hat{z}' \cos(\ell + \delta) + \hat{x}' \sin(\ell - \delta)$$

1.4.5 e

$$\theta = \frac{\pi}{2} - (\ell + \delta), \ell < \delta$$

$$\theta = \frac{\pi}{2} - (\ell - \delta), \ell > \delta$$

1.4.6 f

$$\theta_{NCP} = \ell$$

1.4.7 g

$$\delta_{lim} = 40.116^{\circ} - 90^{\circ}$$

$$\delta_{lim} \approx -49.884^{\circ}$$

$$\delta_{lim} \approx -49^{\circ}53'$$

Yes, you can see NGC 1316 from Champaign-Urbana. This is because $\delta_{NGC1316} > \delta_{lim}$, or $37^{\circ}12.5' > 49^{\circ}53'$, meaning that it is above the horizon at some point.

1.4.8 h

$$\delta = \ell$$

1.5

$$d\Omega = \sin \theta d\theta d\phi$$

$$\int d\Omega = \int_0^{2\pi} \int_0^{\theta_r} \sin \theta d\theta d\phi$$

$$\Omega = \int_0^{2\pi} (-\cos \theta_r + 1) d\phi$$

$$\Omega = (-\cos \theta_r + 1)$$

$$\approx 2\pi(-1 - \frac{\theta_r^2}{2} + 1)$$

$$\Omega = -\pi\theta_r^2$$