In[1]:=
```
Clear[g, V]
```

In[2]:=
```
g_xy[x_, y_] := (y - x) / L
g_yx[x_, y_] := 1 + (y - x) / L
```

In[4]:=
```
V[x_] := Piecewise[{{ΔV x / (a L), x < a L}, {ΔV (L - x) / ((1 - a) L), x ≥ a L}}]
```

In[5]:=
```
Z[β_] := L / (β ΔV) ( 1 - Exp[-β ΔV ] )
```

In[6]:=
```
inner[f_] := Integrate[g_yx[x, y] f[y], {y, 0, x}, Assumptions → x > 0] +
  Integrate[g_xy[x, y] f[y], {y, x, L}, Assumptions → x < L]
```

In[7]:=
```
full[f_, g_] := Integrate[f[x] inner[g], {x, 0, L}, Assumptions → L > 0]
```

In[8]:=
```
fVV = Simplify[full[Exp[β V[#]] &, Exp[-β V[#]] &],
  Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0]
```

Out[8]=

$$\frac{1}{\beta^3\,\Delta V^3}\, e^{-\frac{a\,\beta\,\Delta V}{-1+a}}\, L^2 \Big(-2\, e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V} + 5\, a\, e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V} - 3\, a^2\, e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V} - 2\, e^{\frac{\beta\,\Delta V}{-1+a}} + 5\, a\, e^{\frac{\beta\,\Delta V}{-1+a}} - 3\, a^2\, e^{\frac{\beta\,\Delta V}{-1+a}} + 4\, e^{\frac{a\,\beta\,\Delta V}{-1+a}} - 8\, a\, e^{\frac{a\,\beta\,\Delta V}{-1+a}} + e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V}\,\beta\,\Delta V - 2\, a\, e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V}\,\beta\,\Delta V + a^2\, e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V}\,\beta\,\Delta V + a\, e^{\frac{\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V - a^2\, e^{\frac{\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V - e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V + e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2 - 2\, a\, e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2 + a\, e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,(-2+6\,a+\beta\,\Delta V)\,\mathrm{Cosh}[\beta\,\Delta V] - a\,(-1+2\,a)\, e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]\Big)$$

In[9]:=
```
fVc = Simplify[full[Exp[β V[#]] &, Exp[-β c V[#]] &],
  Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]
```

$$-\frac{1}{(-1+c)\,c^2\,\beta^3\,\Delta V^3}\, e^{-\frac{(1-2\,c+a\,(-1+3\,c))\,\beta\,\Delta V}{-1+a}}\, L^2 \Big((-1+c)\, e^{\frac{(-2+3\,a)\,c\,\beta\,\Delta V}{-1+a}}\,(1+c-c\,\beta\,\Delta V+a\,(-2+c\,(-2+\beta\,\Delta V))) - e^{\frac{(-1+2\,a)\,c\,\beta\,\Delta V}{-1+a}}\left(-1+c^2\,(1-\beta\,\Delta V)+a\,(1+c)\,(2+c\,(-2+\beta\,\Delta V))\right) + e^{\frac{(1-2\,c+a\,(-1+3\,c))\,\beta\,\Delta V}{-1+a}}\left(1-c^2-c\,\beta\,\Delta V+a\,(1+c)\,(-2+c\,(2+\beta\,\Delta V))\right) - (-1+c)\, e^{\left(-1+\left(2+\frac{1}{-1+a}\right)c\right)\beta\,\Delta V}\,(-1-c+a\,(2+c\,(2+\beta\,\Delta V)))\Big)$$

```
j1 = -Simplify[(fVc / (Z[β c] Z[-β]) - fVV / (Z[β] Z[-β])),
   Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]
```

$$\frac{1}{\left(-1+e^{\beta\Delta V}\right)\beta\Delta V}\left(\frac{1}{-1+e^{\beta\Delta V}}\left(-2+e^{2\beta\Delta V}(-2+\beta\Delta V)+e^{\beta\Delta V}\left(4-\beta\Delta V+\beta^2\Delta V^2\right)+a\left(4+\beta\Delta V+e^{2\beta\Delta V}(4-\beta\Delta V)-2e^{\beta\Delta V}\left(4+\beta^2\Delta V^2\right)\right)\right)+\frac{1}{(-1+c)\,c\left(1-e^{-c\beta\Delta V}\right)}e^{-\frac{(1-2c+a(-1+3c))\beta\Delta V}{-1+a}}\left((-1+c)\,e^{\frac{(-2+3a)c\beta\Delta V}{-1+a}}(1+c-c\beta\Delta V+a(-2+c(-2+\beta\Delta V)))-e^{\frac{(-1+2a)c\beta\Delta V}{-1+a}}\left(-1+c^2(1-\beta\Delta V)+a(1+c)(2+c(-2+\beta\Delta V))\right)+e^{\frac{(1-2c+a(-1+3c))\beta\Delta V}{-1+a}}\left(1-c^2-c\beta\Delta V+a(1+c)(-2+c(2+\beta\Delta V))\right)-(-1+c)\,e^{\left(-1+\left(2+\frac{1}{-1+a}\right)c\right)\beta\Delta V}(-1-c+a(2+c(2+\beta\Delta V)))\right)\right)$$

```
fcV = Simplify[full[Exp[ β c V[#]] &, Exp[-β V[#]] &],
  Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]
```

$$-\frac{1}{(-1+c)\,c^2\beta^3\Delta V^3}e^{-\frac{(-2+2a+c)\beta\Delta V}{-1+a}}L^2\left((-1+c)\,e^{\frac{(-2+a(2+c))\beta\Delta V}{-1+a}}(1+c-c\beta\Delta V+a(-2+c(-2+\beta\Delta V)))-e^{\frac{(-2+2a+c)\beta\Delta V}{-1+a}}\left(-1+c^2(1-\beta\Delta V)+a(1+c)(2+c(-2+\beta\Delta V))\right)+e^{\frac{(-1+a+ac)\beta\Delta V}{-1+a}}\left(1-c^2-c\beta\Delta V+a(1+c)(-2+c(2+\beta\Delta V))\right)-(-1+c)\,e^{\left(1+\frac{c}{-1+a}\right)\beta\Delta V}(-1-c+a(2+c(2+\beta\Delta V)))\right)$$

```
fcc = Simplify[full[Exp[ β c V[#]] &, Exp[-β c V[#]] &],
  Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]
```

$$\frac{1}{c^3\beta^3\Delta V^3}e^{-\frac{(-2+3a)c\beta\Delta V}{-1+a}}L^2\left(e^{\frac{(-3+4a)c\beta\Delta V}{-1+a}}(-2+c\beta\Delta V+a(4-c\beta\Delta V))+e^{\left(1+\frac{a}{-1+a}\right)c\beta\Delta V}(-2+a(4+c\beta\Delta V))+e^{\frac{(-2+3a)c\beta\Delta V}{-1+a}}\left(4-c\beta\Delta V+c^2\beta^2\Delta V^2-2a\left(4+c^2\beta^2\Delta V^2\right)\right)\right)$$

```
jc = -FullSimplify[(fcV / (Z[-β c] Z[β]) - fcc / (Z[c β] Z[-c β])),
   Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]
```

$$-\left(\left(2(-1+2a)\,e^{\frac{1}{2}(1+2c)\beta\Delta V}\left(-c^2\beta\Delta V\,\mathrm{Cosh}\left[\frac{\beta\Delta V}{2}\right](-1+\mathrm{Cosh}[c\beta\Delta V])+\mathrm{Sinh}\left[\frac{\beta\Delta V}{2}\right]\left((-1+c)\left(2+c\left(-2+c\beta^2\Delta V^2\right)\right)+2(-1+c)^2\mathrm{Cosh}[c\beta\Delta V]+c^2\beta\Delta V\,\mathrm{Sinh}[c\beta\Delta V]\right)\right)\right)\Big/\left((-1+c)\,c\left(-1+e^{\beta\Delta V}\right)\left(-1+e^{c\beta\Delta V}\right)^2\beta\Delta V\right)\right)$$

jtot = FullSimplify[j1 + jc, Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]

$$\frac{1}{8\,(-1+c)}(-1+2\,a)\,\mathrm{Csch}\left[\frac{c\,\beta\,\Delta V}{2}\right]^2\left(\mathrm{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\,(-(-1+c)\,\beta\,\Delta V\,(-1-c+c\,\mathrm{Cosh}[\beta\,\Delta V]+\mathrm{Cosh}[c\,\beta\,\Delta V])+(1+c)\,(-1+\mathrm{Cosh}[c\,\beta\,\Delta V])\,\mathrm{Sinh}[\beta\,\Delta V])-2\,(1+c)\,\mathrm{Sinh}[c\,\beta\,\Delta V]\right)$$

Limit[j1, c → 0]

$$-\frac{(-1+2\,a)\left(-4-\beta\,\Delta V+e^{2\,\beta\,\Delta V}\,(-4+\beta\,\Delta V)+2\,e^{\beta\,\Delta V}\,(4+\beta^2\,\Delta V^2)\right)}{2\left(-1+e^{\beta\,\Delta V}\right)^2\beta\,\Delta V}$$

Limit[jc, c → 0]

0

Series[j1, {c, 1, 2}]

$$\frac{1}{2\left(-1+e^{\beta\,\Delta V}\right)^3\beta\,\Delta V}e^{\beta\,\Delta V}\Big(-4+8\,a+4\,e^{\beta\,\Delta V}-8\,a\,e^{\beta\,\Delta V}+2\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}-4\,a\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}-2\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}+$$
$$4\,a\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}+2\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}-4\,a\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}-$$
$$2\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}+4\,a\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}+4\,\beta\,\Delta V-8\,a\,\beta\,\Delta V+$$
$$4\,e^{\beta\,\Delta V}\,\beta\,\Delta V-8\,a\,e^{\beta\,\Delta V}\,\beta\,\Delta V-4\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V+8\,a\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V-$$
$$4\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V+8\,a\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V-2\,a\,\beta^2\,\Delta V^2-2\,e^{\beta\,\Delta V}\,\beta^2\,\Delta V^2+$$
$$2\,a\,e^{\beta\,\Delta V}\,\beta^2\,\Delta V^2+2\,a\,e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2+2\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2-$$
$$2\,a\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}+\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2+\beta^3\,\Delta V^3-2\,a\,\beta^3\,\Delta V^3+e^{\beta\,\Delta V}\,\beta^3\,\Delta V^3-2\,a\,e^{\beta\,\Delta V}\,\beta^3\,\Delta V^3\Big)\,(c-1)+$$

$$\frac{1}{\left(-1+e^{\beta\,\Delta V}\right)\beta\,\Delta V}\Bigg(\Bigg(-e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,(-1+2\,a+a\,\beta\,\Delta V)+e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,(-1+2\,a+a\,\beta\,\Delta V)-$$
$$\frac{(-1+2\,a)\,e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(-2+4\,a+a\,\beta\,\Delta V)}{-1+a}+$$
$$\frac{(-2+3\,a)\,e^{\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(2-4\,a-\beta\,\Delta V+a\,\beta\,\Delta V)}{-1+a}-e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,(1-2\,a-\beta\,\Delta V+a\,\beta\,\Delta V)+$$
$$e^{\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,(1-2\,a-\beta\,\Delta V+a\,\beta\,\Delta V)-\frac{(-1+2\,a)^2\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2\,(-\beta\,\Delta V+2\,a\,\beta\,\Delta V)}{2\,(-1+a)^2}+$$
$$\frac{(-2+3\,a)^2\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^2\,\Delta V^2\,(-\beta\,\Delta V+2\,a\,\beta\,\Delta V)}{2\,(-1+a)^2}-\frac{1}{-1+a}(-1+2\,a)\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V$$

$$(2-4\,a-2\,\beta\,\Delta V+3\,a\,\beta\,\Delta V)+\frac{1}{-1+a}(-2+3\,a)\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(-2+4\,a-\beta\,\Delta V+3\,a\,\beta\,\Delta V)\Bigg)$$

$$\left(-\frac{e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V}{\left(-1+e^{\beta\,\Delta V}\right)^{2}}+\frac{-e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}+\frac{(2-3\,a)\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V}{-1+a}}{1-e^{-\beta\,\Delta V}}\right)+\frac{1}{1-e^{-\beta\,\Delta V}}\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}$$

$$\Bigg(-\frac{(-1+2\,a)\,e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(-1+2\,a+a\,\beta\,\Delta V)}{-1+a}+\frac{(-2+3\,a)\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(-1+2\,a+a\,\beta\,\Delta V)}{-1+a}-$$

$$\frac{1}{2}\,e^{\frac{a\,\beta\,\Delta V}{-1+a}}\left(2\,\beta\,\Delta V+\frac{\beta\,\Delta V}{-1+a}\right)^{2}(-2+4\,a+a\,\beta\,\Delta V)-\frac{1}{-1+a}(-1+2\,a)\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta$$

$$\Delta V\,(1-2\,a-\beta\,\Delta V+a\,\beta\,\Delta V)-\frac{(-1+2\,a)^{3}\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^{3}\,\Delta V^{3}\,(-\beta\,\Delta V+2\,a\,\beta\,\Delta V)}{6\,(-1+a)^{3}}+$$

$$\frac{(-2+3\,a)^{3}\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^{3}\,\Delta V^{3}\,(-\beta\,\Delta V+2\,a\,\beta\,\Delta V)}{6\,(-1+a)^{3}}-\frac{1}{2\,(-1+a)^{2}}$$

$$(-1+2\,a)^{2}\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^{2}\,\Delta V^{2}\,(2-4\,a-2\,\beta\,\Delta V+3\,a\,\beta\,\Delta V)+\frac{1}{2\,(-1+a)^{2}}(-2+3\,a)^{2}$$

$$e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta^{2}\,\Delta V^{2}\,(-2+4\,a-\beta\,\Delta V+3\,a\,\beta\,\Delta V)+\frac{1}{2\,(-1+a)^{2}}\,(-2+3\,a)\,e^{\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V$$

$$\left(-2+6\,a-4\,a^{2}-2\,\beta\,\Delta V+10\,a\,\beta\,\Delta V-10\,a^{2}\,\beta\,\Delta V+2\,\beta^{2}\,\Delta V^{2}-5\,a\,\beta^{2}\,\Delta V^{2}+3\,a^{2}\,\beta^{2}\,\Delta V^{2}\right)\Bigg)+$$

$$\Bigg(-e^{\frac{a\,\beta\,\Delta V}{-1+a}}\,(-2+4\,a+a\,\beta\,\Delta V)+e^{\frac{(-2+3\,a)\,\beta\,\Delta V}{-1+a}}\,(2-4\,a-\beta\,\Delta V+a\,\beta\,\Delta V)-$$

$$\frac{(-1+2\,a)\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(-\beta\,\Delta V+2\,a\,\beta\,\Delta V)}{-1+a}+$$

$$\frac{(-2+3\,a)\,e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V\,(-\beta\,\Delta V+2\,a\,\beta\,\Delta V)}{-1+a}-$$

$$e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,(2-4\,a-2\,\beta\,\Delta V+3\,a\,\beta\,\Delta V)+e^{\frac{(-1+2\,a)\,\beta\,\Delta V}{-1+a}}\,(-2+4\,a-\beta\,\Delta V+3\,a\,\beta\,\Delta V)\Bigg)$$

$$\Bigg(\frac{e^{\beta\,\Delta V+\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}\left(1+e^{\beta\,\Delta V}\right)\beta^{2}\,\Delta V^{2}}{2\left(-1+e^{\beta\,\Delta V}\right)^{3}}-\frac{e^{\beta\,\Delta V}\,\beta\,\Delta V\left(-e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}+\frac{(2-3\,a)\,e^{\frac{(1-2\,a)\,\beta\,\Delta V}{-1+a}}\,\beta\,\Delta V}{-1+a}\right)}{\left(-1+e^{\beta\,\Delta V}\right)^{2}}+\frac{1}{1-e^{-\beta\,\Delta V}}$$

$$\left.\left.\left(e^{\frac{(1-2a)\beta\Delta V}{-1+a}} - \frac{(2-3a)\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}}\,\beta\,\Delta V}{-1+a} + \frac{(2-3a)^2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}}\,\beta^2\,\Delta V^2}{2\,(-1+a)^2}\right)\right)\right)(c-1)^2 + O[c-1]^3$$

```
Series[jc, {c, 1, 2}]
```

$$-\frac{1}{\left(-1+e^{\beta\Delta V}\right)^3\beta\,\Delta V}2\left((-1+2a)\,e^{\frac{3\beta\Delta V}{2}}\left(\beta\,\Delta V\,\mathrm{Cosh}\left[\frac{\beta\,\Delta V}{2}\right]\left(1-\mathrm{Cosh}[\beta\,\Delta V]-\frac{1}{2}\beta^2\,\Delta V^2\,\mathrm{Cosh}[\beta\,\Delta V]-2\,\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]\right)+\mathrm{Sinh}\left[\frac{\beta\,\Delta V}{2}\right]\left(-2+2\,\beta^2\,\Delta V^2+2\,\mathrm{Cosh}[\beta\,\Delta V]+2\,\beta^2\,\Delta V^2\,\mathrm{Cosh}[\beta\,\Delta V]+\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]+\frac{1}{2}\beta^3\,\Delta V^3\,\mathrm{Sinh}[\beta\,\Delta V]\right)\right)\right)(c-1)-$$

$$\frac{1}{\left(-1+e^{\beta\Delta V}\right)\beta\,\Delta V}2\left((-1+2a)\left(\left(-\frac{2\,e^{\frac{5\beta\Delta V}{2}}\,\beta\,\Delta V}{\left(-1+e^{\beta\Delta V}\right)^3}+\frac{-e^{\frac{3\beta\Delta V}{2}}+e^{\frac{3\beta\Delta V}{2}}\,\beta\,\Delta V}{\left(-1+e^{\beta\Delta V}\right)^2}\right)\left(\beta\,\Delta V\,\mathrm{Cosh}\left[\frac{\beta\,\Delta V}{2}\right]\left(1-\mathrm{Cosh}[\beta\,\Delta V]-\frac{1}{2}\beta^2\,\Delta V^2\,\mathrm{Cosh}[\beta\,\Delta V]-2\,\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]\right)+\mathrm{Sinh}\left[\frac{\beta\,\Delta V}{2}\right]\left(-2+2\,\beta^2\,\Delta V^2+2\,\mathrm{Cosh}[\beta\,\Delta V]+2\,\beta^2\,\Delta V^2\,\mathrm{Cosh}[\beta\,\Delta V]+\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]+\frac{1}{2}\beta^3\,\Delta V^3\,\mathrm{Sinh}[\beta\,\Delta V]\right)\right)+\frac{1}{\left(-1+e^{\beta\Delta V}\right)^2}e^{\frac{3\beta\Delta V}{2}}\left(\beta\,\Delta V\,\mathrm{Cosh}\left[\frac{\beta\,\Delta V}{2}\right]\left(-\beta^2\,\Delta V^2\,\mathrm{Cosh}[\beta\,\Delta V]-\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]-\frac{1}{6}\beta^3\,\Delta V^3\,\mathrm{Sinh}[\beta\,\Delta V]\right)+\mathrm{Sinh}\left[\frac{\beta\,\Delta V}{2}\right]\left(\beta^2\,\Delta V^2+\beta^2\,\Delta V^2\,\mathrm{Cosh}[\beta\,\Delta V]+\frac{1}{6}\beta^4\,\Delta V^4\,\mathrm{Cosh}[\beta\,\Delta V]+2\,\beta\,\Delta V\,\mathrm{Sinh}[\beta\,\Delta V]+\beta^3\,\Delta V^3\,\mathrm{Sinh}[\beta\,\Delta V]\right)\right)\right)\right)(c-1)^2+O[c-1]^3$$

$$\mathbf{j11} = \frac{1}{2\left(-1+e^{\beta\,\Delta V}\right)^{3}\beta\,\Delta V}\,e^{\beta\,\Delta V}$$

$$\Bigl(-4+8a+4e^{\beta\Delta V}-8a\,e^{\beta\Delta V}+2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}-4a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}-2\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}+$$

$$4a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}+2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}-4a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}-2\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}+$$

$$4a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}+4\beta\Delta V-8a\beta\Delta V+4e^{\beta\Delta V}\beta\Delta V-8a\,e^{\beta\Delta V}\beta\Delta V-$$

$$4\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}\beta\Delta V+8a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}\beta\Delta V-4\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta\Delta V+$$

$$8a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta\Delta V-2a\beta^2\Delta V^2-2e^{\beta\Delta V}\beta^2\Delta V^2+2a\,e^{\beta\Delta V}\beta^2\Delta V^2+$$

$$2a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}\beta^2\Delta V^2+2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta^2\Delta V^2-2a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta^2\Delta V^2+$$

$$\beta^3\Delta V^3-2a\beta^3\Delta V^3+e^{\beta\Delta V}\beta^3\Delta V^3-2a\,e^{\beta\Delta V}\beta^3\Delta V^3\Bigr)\,(c-1)$$

$$\frac{1}{2\left(-1+e^{\beta\Delta V}\right)^{3}\beta\Delta V}$$

$$(-1+c)\,e^{\beta\Delta V}\Bigl(-4+8a+4e^{\beta\Delta V}-8a\,e^{\beta\Delta V}+2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}-4a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}-2\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}+$$

$$4a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}+2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}-4a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}-$$

$$2\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}+4a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}+4\beta\Delta V-8a\beta\Delta V+$$

$$4e^{\beta\Delta V}\beta\Delta V-8a\,e^{\beta\Delta V}\beta\Delta V-4\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}\beta\Delta V+8a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}\beta\Delta V-$$

$$4\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta\Delta V+8a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta\Delta V-2a\beta^2\Delta V^2-2e^{\beta\Delta V}\beta^2\Delta V^2+$$

$$2a\,e^{\beta\Delta V}\beta^2\Delta V^2+2a\,e^{\beta\Delta V+\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{a\beta\Delta V}{-1+a}}\beta^2\Delta V^2+2\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta^2\Delta V^2-$$

$$2a\,e^{\frac{(1-2a)\beta\Delta V}{-1+a}+\frac{(-2+3a)\beta\Delta V}{-1+a}}\beta^2\Delta V^2+\beta^3\Delta V^3-2a\beta^3\Delta V^3+e^{\beta\Delta V}\beta^3\Delta V^3-2a\,e^{\beta\Delta V}\beta^3\Delta V^3\Bigr)$$

$$\mathbf{jc1} = -\frac{1}{\left(-1+e^{\beta\Delta V}\right)^{3}\beta\Delta V}\,2$$

$$\Biggl((-1+2a)\,e^{\frac{3\beta\Delta V}{2}}\Bigl(\beta\Delta V\,\mathrm{Cosh}\Bigl[\frac{\beta\Delta V}{2}\Bigr]\Bigl(1-\mathrm{Cosh}[\beta\Delta V]-\frac{1}{2}\beta^2\Delta V^2\,\mathrm{Cosh}[\beta\Delta V]-2\beta\Delta V\,\mathrm{Sinh}[\beta\Delta V]\Bigr)+$$

$$\mathrm{Sinh}\Bigl[\frac{\beta\Delta V}{2}\Bigr]\Bigl(-2+2\beta^2\Delta V^2+2\,\mathrm{Cosh}[\beta\Delta V]+2\beta^2\Delta V^2\,\mathrm{Cosh}[\beta\Delta V]+$$

$$\beta\Delta V\,\mathrm{Sinh}[\beta\Delta V]+\frac{1}{2}\beta^3\Delta V^3\,\mathrm{Sinh}[\beta\Delta V]\Bigr)\Bigr)\Biggr)\,(c-1)$$

$$-\frac{1}{\left(-1+e^{\beta\Delta V}\right)^{3}\beta\Delta V}\,2\,(-1+2a)\,(-1+c)\,e^{\frac{3\beta\Delta V}{2}}$$

$$\Bigl(\beta\Delta V\,\mathrm{Cosh}\Bigl[\frac{\beta\Delta V}{2}\Bigr]\Bigl(1-\mathrm{Cosh}[\beta\Delta V]-\frac{1}{2}\beta^2\Delta V^2\,\mathrm{Cosh}[\beta\Delta V]-2\beta\Delta V\,\mathrm{Sinh}[\beta\Delta V]\Bigr)+$$

$$\mathrm{Sinh}\Bigl[\frac{\beta\Delta V}{2}\Bigr]\Bigl(-2+2\beta^2\Delta V^2+2\,\mathrm{Cosh}[\beta\Delta V]+$$

$$2\beta^2\Delta V^2\,\mathrm{Cosh}[\beta\Delta V]+\beta\Delta V\,\mathrm{Sinh}[\beta\Delta V]+\frac{1}{2}\beta^3\Delta V^3\,\mathrm{Sinh}[\beta\Delta V]\Bigr)\Bigr)$$

```
Simplify[j11 + jc1, Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0 && 0 < c < 1]
```

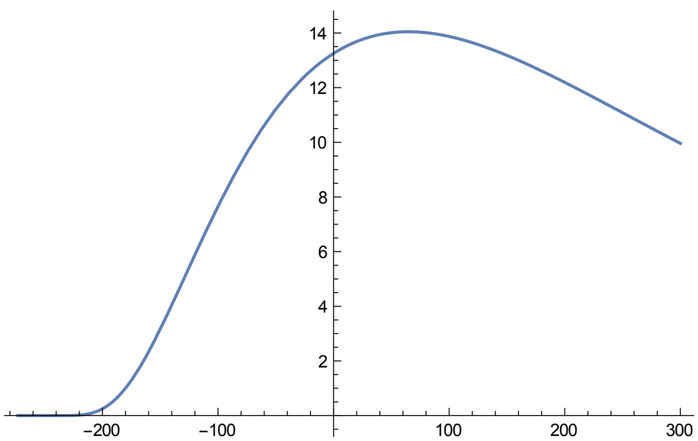
0

```
Series[Log[jtot], {c, 0, 3}]
```

$$
\begin{aligned}
&\text{Log}\Big[-\frac{1}{4\,\beta\,\Delta V}\,(-1+2\,a)\,\Big(-4+2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\\
&\qquad 2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big]-\\
&\Big(2\,\Big(6+\beta^2\,\Delta V^2-3\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+3\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-3\,\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\\
&\quad c\Big)\Big/\Big(3\,\Big(-4+2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\\
&\qquad 2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big)+\\
&\frac{1}{2}\,\Big(-\Big(\Big(4\,\Big(6+\beta^2\,\Delta V^2-3\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+3\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-3\,\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\\
&\qquad\quad \text{Sinh}[\beta\,\Delta V]\Big)^2\Big)\Big/\Big(9\,\Big(-4+2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\\
&\qquad\quad 2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)^2\Big)\Big)+\\
&\quad\Big(-24-6\,\beta^2\,\Delta V^2+12\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-12\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\\
&\qquad \beta^2\,\Delta V^2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+12\,\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big/\\
&\quad\Big(3\,\Big(-4+2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\\
&\qquad \beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big)\Big)\,c^2+\\
&\frac{1}{3}\,\Big(\Big(2\,\Big(6+\beta^2\,\Delta V^2-3\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+3\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-3\,\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\\
&\qquad\Big(-24-6\,\beta^2\,\Delta V^2+12\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-12\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\\
&\qquad \beta^2\,\Delta V^2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+12\,\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big)\Big/\\
&\quad\Big(9\,\Big(-4+2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\\
&\qquad \beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)^2\Big)+\\
&\quad\Big(-360-90\,\beta^2\,\Delta V^2+2\,\beta^4\,\Delta V^4+180\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-15\,\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\\
&\qquad 180\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+15\,\beta^2\,\Delta V^2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\\
&\qquad 180\,\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big/\Big(30\,\Big(-4+2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta^2\,\Delta V^2\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2-\\
&\qquad 2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2+\beta\,\Delta V\,\text{Csch}\big[\tfrac{\beta\,\Delta V}{2}\big]^2\,\text{Sinh}[\beta\,\Delta V]\Big)\Big)-
\end{aligned}
$$

$$
\begin{aligned}
&\Bigg(2\,\Bigg(6+\beta^2\,\Delta V^2-3\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+3\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-\\
&\quad 3\,\beta\,\Delta V\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\,\text{Sinh}[\beta\,\Delta V]\Bigg)\\
&\quad\Bigg(\Bigg(4\,\Bigg(6+\beta^2\,\Delta V^2-3\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+3\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-3\,\beta\,\Delta V\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\\
&\qquad\text{Sinh}[\beta\,\Delta V]\Bigg)^2\Bigg)\Big/\Bigg(9\,\Bigg(-4+2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\beta^2\,\Delta V^2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-\\
&\qquad 2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\beta\,\Delta V\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\,\text{Sinh}[\beta\,\Delta V]\Bigg)^2\Bigg)-\\
&\quad\Bigg(-24-6\,\beta^2\,\Delta V^2+12\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-\beta^2\,\Delta V^2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-12\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\\
&\qquad\beta^2\,\Delta V^2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+12\,\beta\,\Delta V\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\,\text{Sinh}[\beta\,\Delta V]\Bigg)\Big/\\
&\quad\Bigg(6\,\Bigg(-4+2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\beta^2\,\Delta V^2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\\
&\qquad\beta\,\Delta V\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\,\text{Sinh}[\beta\,\Delta V]\Bigg)\Bigg)\Bigg)\Bigg)\Big/\\
&\Bigg(3\,\Bigg(-4+2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\beta^2\,\Delta V^2\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2-2\,\text{Cosh}[\beta\,\Delta V]\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2+\\
&\quad\beta\,\Delta V\,\text{Csch}\left[\frac{\beta\,\Delta V}{2}\right]^2\,\text{Sinh}[\beta\,\Delta V]\Bigg)\Bigg)\,c^3+\text{O}[c]^4
\end{aligned}
$$

```
Plot[lu lb / (lu + lb) L jtot //.
  {a → 1 / 4, ΔV → 40, β → 310.15 / (4.28 (T + 273.15)), L → 8, lu → 40, lb → 80, c → 0.2},
 {T, -273.15, 300}, PlotRange → All]
```




```
Limit[lu lb / (lu + lb) L jtot //.
  {a → 1 / 4, ΔV → 40, β → 1 / 4.28, L → 8, lu → 40, lb → 80}, c → 0.3]
```

```
7.85977
```

```
Series[j1, {β, 0, 3}]
```

$$\frac{1}{720}\left(2\,\Delta V^3-4\,a\,\Delta V^3-c\,\Delta V^3+2\,a\,c\,\Delta V^3-c^2\,\Delta V^3+2\,a\,c^2\,\Delta V^3\right)\beta^3+O[\beta]^4$$

```
Series[jc, {β, 0, 3}]
```

$$-\frac{1}{720}\left((-1+c)\left(-c\,\Delta V^3+2\,a\,c\,\Delta V^3-2\,c^2\,\Delta V^3+4\,a\,c^2\,\Delta V^3\right)\right)\beta^3+O[\beta]^4$$

```
fVVV = Simplify[full[Exp[β V[#]] &, V[#] Exp[-β V[#]] &],
  Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0]
```

$$-\frac{1}{2\,\beta^4\,\Delta V^3}\,e^{-\beta\,\Delta V}\,L^2\left(6+4\,\beta\,\Delta V+e^{2\,\beta\,\Delta V}(6-2\,\beta\,\Delta V)-e^{\beta\,\Delta V}\left(12+2\,\beta\,\Delta V+\beta^3\,\Delta V^3\right)+2\,a\left(-6-5\,\beta\,\Delta V-\beta^2\,\Delta V^2+e^{2\,\beta\,\Delta V}(-6+\beta\,\Delta V)+e^{\beta\,\Delta V}\left(12+4\,\beta\,\Delta V+\beta^2\,\Delta V^2+\beta^3\,\Delta V^3\right)\right)\right)$$

```
FullSimplify[β (fVVV + Simplify[D[Log[Z[β]], β]] fVV) / (Z[β] Z[-β]),
 Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0]
```

$$-\frac{1}{2\left(-1+e^{\beta\,\Delta V}\right)^3\beta\,\Delta V}(-1+2\,a)\,e^{-\frac{a\,\beta\,\Delta V}{-1+a}}\left(2\,e^{\frac{a\,\beta\,\Delta V}{-1+a}}-2\,e^{\frac{(-3+4\,a)\,\beta\,\Delta V}{-1+a}}+e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V}\left(-6+\beta^3\,\Delta V^3\right)+e^{\left(3+\frac{1}{-1+a}\right)\beta\,\Delta V}\left(6+\beta^3\,\Delta V^3\right)\right)$$

```
FullSimplify[% / (a - 1 / 2) (1 - Exp[-β ΔV])^3,
 Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0]
```

$$\frac{1}{\beta\,\Delta V}e^{-\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V}\left(2\,e^{\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V}-2\,e^{\frac{a\,\beta\,\Delta V}{-1+a}}+e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V}\left(6-\beta^3\,\Delta V^3\right)-e^{\left(3+\frac{1}{-1+a}\right)\beta\,\Delta V}\left(6+\beta^3\,\Delta V^3\right)\right)$$

```
Expand[%]
```

$$\frac{2}{\beta\,\Delta V}+\frac{6\,e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V-\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V}}{\beta\,\Delta V}-\frac{6\,e^{\left(3+\frac{1}{-1+a}\right)\beta\,\Delta V-\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V}}{\beta\,\Delta V}-\frac{2\,e^{-\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V+\frac{a\,\beta\,\Delta V}{-1+a}}}{\beta\,\Delta V}-e^{\left(2+\frac{1}{-1+a}\right)\beta\,\Delta V-\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V}\beta^2\,\Delta V^2-e^{\left(3+\frac{1}{-1+a}\right)\beta\,\Delta V-\left(4+\frac{1}{-1+a}\right)\beta\,\Delta V}\beta^2\,\Delta V^2$$

```
Map[Simplify[#] &, %]
```

$$\frac{2}{\beta\,\Delta V}-\frac{2\,e^{-3\,\beta\,\Delta V}}{\beta\,\Delta V}+\frac{6\,e^{-2\,\beta\,\Delta V}}{\beta\,\Delta V}-\frac{6\,e^{-\beta\,\Delta V}}{\beta\,\Delta V}-e^{-2\,\beta\,\Delta V}\beta^2\,\Delta V^2-e^{-\beta\,\Delta V}\beta^2\,\Delta V^2$$

```
FullSimplify[β (fVVV + Simplify[D[Log[Z[β]], β]] fVV) / (Z[β] Z[-β]) -
  (a - 1 / 2) (2 / (β ΔV) - (β ΔV)^2 Exp[-β ΔV] (1 + Exp[-β ΔV]) (1 - Exp[-β ΔV])^-3),
 Assumptions → L > 0 && 0 < a < 1 && ΔV > 0 && β > 0]
```

0

```
Series[(a - 1 / 2) (2 / (β ΔV) - (β ΔV)^2 / 4 Cosh[β ΔV / 2] (Sinh[β ΔV / 2])^-3), {β, 0, 4}]
```

$$\left(-\frac{\Delta V^3}{240}+\frac{a\,\Delta V^3}{120}\right)\beta^3+O[\beta]^5$$

```
NSolve[(D[jtot, β] == 0) /. {ΔV → 40, a → 1 / 4, c → 3 / 10}, β]
```

ReplaceAll::reps : $\left\{\left\{\Delta V\to 40, a\to \frac{1}{4}, c\to \frac{3}{10}\right\} == 0\right\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. ≫

ReplaceAll::reps : $\left\{(\Delta V\to 40) == 0 \;\&\&\; \left(a\to \frac{1}{4}\right) == 0 \;\&\&\; \left(c\to \frac{3}{10}\right) == 0\right\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. ≫

NSolve::nsmet : This system cannot be solved with the methods available to NSolve. ≫

$$\text{NSolve}\Big[-\frac{1}{8\,(-1+c)}\,(-1+2\,a)\,c\,\Delta V\,\text{Coth}\Big[\frac{c\,\beta\,\Delta V}{2}\Big]\,\text{Csch}\Big[\frac{c\,\beta\,\Delta V}{2}\Big]^2 \Big(\text{Csch}\Big[\frac{\beta\,\Delta V}{2}\Big]^2\,(-(-1+c)\,\beta\,\Delta V\,(-1-c+c\,\text{Cosh}[\beta\,\Delta V]+\text{Cosh}[c\,\beta\,\Delta V])+(1+c)\,(-1+\text{Cosh}[c\,\beta\,\Delta V])\,\text{Sinh}[\beta\,\Delta V])-2\,(1+c)\,\text{Sinh}[c\,\beta\,\Delta V]\Big)+\frac{1}{8\,(-1+c)}\,(-1+2\,a)\,\text{Csch}\Big[\frac{c\,\beta\,\Delta V}{2}\Big]^2\,\Big(-2\,c\,(1+c)\,\Delta V\,\text{Cosh}[c\,\beta\,\Delta V]-\Delta V\,\text{Coth}\Big[\frac{\beta\,\Delta V}{2}\Big]\,\text{Csch}\Big[\frac{\beta\,\Delta V}{2}\Big]^2\,(-(-1+c)\,\beta\,\Delta V\,(-1-c+c\,\text{Cosh}[\beta\,\Delta V]+\text{Cosh}[c\,\beta\,\Delta V])+(1+c)\,(-1+\text{Cosh}[c\,\beta\,\Delta V])\,\text{Sinh}[\beta\,\Delta V])+\text{Csch}\Big[\frac{\beta\,\Delta V}{2}\Big]^2\,((1+c)\,\Delta V\,\text{Cosh}[\beta\,\Delta V]\,(-1+\text{Cosh}[c\,\beta\,\Delta V])-(-1+c)\,\Delta V\,(-1-c+c\,\text{Cosh}[\beta\,\Delta V]+\text{Cosh}[c\,\beta\,\Delta V])+c\,(1+c)\,\Delta V\,\text{Sinh}[\beta\,\Delta V]\,\text{Sinh}[c\,\beta\,\Delta V]-(-1+c)\,\beta\,\Delta V\,(c\,\Delta V\,\text{Sinh}[\beta\,\Delta V]+c\,\Delta V\,\text{Sinh}[c\,\beta\,\Delta V]))\Big)\;/.\;\Big\{\Delta V\to 40,\,a\to\frac{1}{4},\,c\to\frac{3}{10}\Big\}==0,\,\beta\Big]$$