Final Assignment: Optimizing the Energy Transition

Engineering Optimization – Concepts and Applications

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by

Pieter Bijl - 4492102
&
Toon Stolk - 4488458

Table 1: Statement of Contribution

Topic	Involvement Toon Stolk	Involvement Pieter Bijl
Introduction	(together)	(together)
Problem Formulation	Market Function and Data gathering	Market Function and Scenario
Initial Problem Investigation	Investigation	Checked results
Simplified Optimization	Analysis and SQP	2D version of problem and genetic algorithm
Complete Optimization	Checked Results	Created Genetic Algorithm
Conclusion	(together)	(together)

TU Delft
Delft University of Technology

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1 Introduction

In recent years it has become more and more evident that the energy needs of humanity will need to be provided by more sustainable resources. While many agree that this should happen, few can agree on the way that we should be heading to meet to our sustainable goal and what this exact goal should entail. In this report it is proposed that these goals should be quantified in terms of cost for the government and the public cost, caused by CO2. According to these cost functions, what is the most optimal trajectory that a country can take to meet its energy requirements? The goal of this project is to assess this by using the policy of the government on energy production on a macroscopic level as an input for a model and to try and optimize this input to make the energy transition run as smoothly as possible and to create a framework which can be expanded upon in the future.

The energy market of a hypothetical country has been modelled by estimating the ideal state according to given costs and subsidies/excises for each power plant. This market behavior is used to create a scenario in which the energy state of a hypothetical country will move from its initial state in 2020 to its final state in 2050[1]. This scenario can be influenced by the government by creating subsidies and excises for 11 energy sources. In order to assess how well a scenario performs, given the influence of the government, both the cumulative cost for the government as well as the social cost that comes with the cumulative emission of CO2 is combined to give a fitness evaluation. These models are given and explained in chapter 2. This is followed by an investigation into the boundedness of the different models in chapter 3. This has given insight into what algorithms can be used for optimization and which cannot. This investigation has been done for the complete scenario model as well as the market model that is within the scenario model. In chapter 4, a simplified optimization of a 2D case is presented, where the information obtained from analyzing the problem is used to give a simplified solution, based on various methods, such as sequential linear programming, sequential quadratic programming and a genetic algorithm, `ga` of the Global Optimization Toolbox from matlab. The complete optimization is then fully explored for the 66D case and the results are analyzed and presented along with a sensitivity and robustness analysis in chapter 5. The conclusion of the findings in this report and some recommendations with respect to the methods and findings are presented in chapter 6.

[1] In line with the Paris agreement: `https://ec.europa.eu/clima/policies/strategies/2050_en`, accessed at 05-07-2020.

2
Problem Formulation

As suggested in the introduction, the entire model consists of two parts. One part represents the energy market, the energy market strives to reach the optimum state in terms of costs. The energy market determines the optimum using the Matlab function `fmincon`, which is a gradient-based algorithm that works for problems where both the objective function and the constraint function are continuous and have continuous derivatives. The second part represents the government, which strives to make a compromise between costs and CO2 emission. The government can influence the market by providing subsidies or excise on either the costs of power or the building of power plants.

2.1. Assumptions

In order to make a mathematical model of the energy market and the influence of the government, some assumptions had to be made. These assumptions are listed in below.

1. The energy production of the country is sufficiently large that the discretization of power plants has no effect. Such that the energy production of each energy source can be modelled by a continuous function.
2. The energy production remains constant over time. this includes daily, seasonal, annual fluctuations and yearly increase in power production.
3. It is assumed that no energy is imported or exported, such that the system can be analysed as a closed system.
4. Energy sources can be constructed and dismantled instantaneously and construction and dismantling has equal cost.
5. It is assumed that the behaviour of the energy market can be modelled with the use of a second order optimization function.
6. It is assumed that the market has perfect knowledge about the market, with regard to prices and the optimal state and will always move towards its perceived optimal state.
7. The government can only influence the energy market by subsidizing or levying excise on certain energy sources.
8. Efficiency of the power grid is assumed to be 100%.
9. Energy prices for consumers remain constant over time and are independent for all powerplants.
10. The government is assumed to be independent in its decision making and has complete autonomy over all subsidies and excises.

2.2. Market Model

The complete model tries to find the optimal stimulation that the market needs in order to minimize the cost function, which consists out of the cost in CO2 and the money that the government needs to spend. The market itself is modelled in a different way, it merely tries to make a profit and while many companies claim to be 'sustainable' most of them have shareholders that demand a solid return on investment. That is why it has been chosen to model the market only using costs in dollars that are not related to CO2. The market wants to move towards the state were the operating costs are lowest and the profits are thus highest. The market model determines from the given costs per kW and kWh the most optimal solution for a given subsidy of the government. The cost function is defined below:

$$f = \sum_{i=1}^{n} \Delta t \cdot x_i \cdot \left(M_i \cdot \left(\frac{b_i}{t_l} + e_i \right) + s_i \right) \tag{2.1}$$

Where f is the cost in USD, δt is the the timestep equal to 1 hour, x_i is the state which is a 11-by-1 vector containing the fractions of all power plants. M_i is the multiplication factor vector which is introduced for certain energy sources to introduce costs that would occur if the fraction of certain energy source is relatively small or large. For instance, if a large fraction of the power would be supplied by wind energy, a more extensive power grid would be necessary to connect all the different turbines, which brings extra (hidden) costs with it. The multiplication factor aims to estimate these extra cost based on the usage of each type of powerplant. Additionally, the multiplication factor makes the cost function per power source a non-linear function, which might imply that a non-boundary-optimum is found.

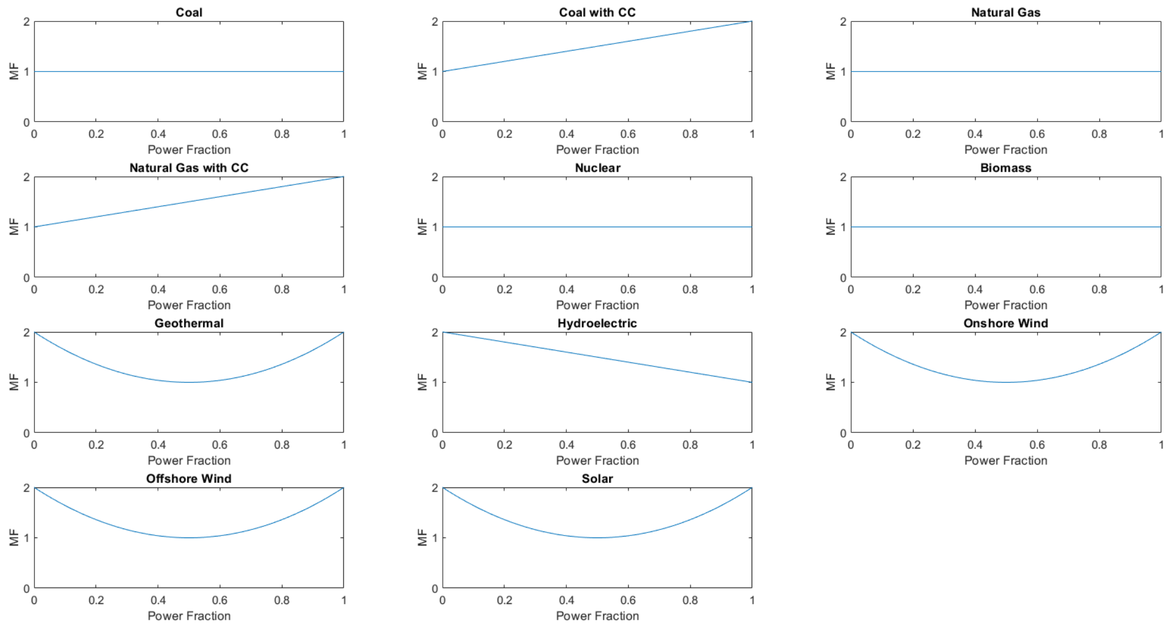


Figure 2.1: Overview of al Multiplication Factors (MF) for all powerplant types.

b_i stands for the building cost vector in USD per kWh, which together with the capital loan duration t_l, which is in hours signify the costs that powerplants have for construction and the repayment of loans and investments. e_c is the energy cost vector in USD per kWh for each of the powerplants and s_i is the subsidy/excise vector for each of the powerplants in USD per kWh. This last term is where the influence of the government comes in and can directly influence what the market sees as ideal.

This function acts as the 'invisible hand' of the market that tries to push it towards this optimum. The MATLAB function that represents the cost function of the market model is `objfun.m`

2.3. Scenario Model

The next step is to model a scenario. A scenario runs from 2020 to 2050 and starts with a state $\bar{x}_0$ that is given an input through the subsidies/excises of the government. These subsidies have an effect on what the 'market' thinks is the ideal state as described in the section about the market model. The state of the energy market is allowed to vary each timestep, which is set to 600 hours. Every timestep the market tries to move towards its optimum, i.e. the state where it has the lowest costs for a given amount of energy production so that it can make a maximum amount of profit.

The subsidies/excises that the government levies should not be constant over time as the energy market changes. That is why the subsidies are allowed to change every 5 years, to allow more flexible strategies by the government. This increases the number of variable inputs for the scenario model to 66.

To model the effect of emissions, a cost needs to be attached to the amount of emitted CO2. The costs for this per kg was found to be 0.05$ [1]. The marginal costs however would increase over time, if it is assumed that damage to the environment and public scales linearly with the amount of emitted CO2. This is modelled using an exponential function that increases the weight of CO2 with increasing emissions. All these factors are combined in one function that returns a cost for a given scenario: `scenario.m`. The steps that scenario takes are given below:

1. Calculate the ideal 'market' state given the build costs, the energy costs and the subsidies/excises. Do this every time the input of the government changes (once every 5 years.)
2. Calculate the difference between the current state and the ideal state:

$$\Delta\bar{x} = \bar{x}_{ideal} - \bar{x} \tag{2.2}$$

3. Create a unit vector of $\Delta\bar{x}$ and calculate what the cost is of moving in the direction towards $\bar{x}_{ideal}$.
4. Calculate the real change in x given a budget:

$$\Delta\bar{x}_{real} = \frac{b_t}{\hat{c}}\Delta\hat{\bar{x}} \tag{2.3}$$

   Where b_t is the budget for the given timestep, $\hat{c}$ is the unit cost vector and $\Delta\hat{\bar{x}}$ is the unit vector of the change in x.
5. Calculate the government subsidies for the given timestep:

$$c_t = -P\Delta t \sum_{i=1}^{n} \bar{x}_{i,t} \cdot \bar{s_i} \tag{2.4}$$

   Where P is the amount of power that needs to be generated, Δt is the timestep in hours, and s_i is the subsidy/excise for each powerplant in USD/kWh. The minus sign is added because a positive value for s_i is excise and generates negative cost for the government and vice versa for subsidies.
6. Calculate the emitted CO2 in kg for the timestep:

$$CO2_t = P\Delta \sum_{i=1}^{n} \bar{x}_i \cdot CO2_i \tag{2.5}$$

   Where $CO2_i$ is the emitted CO2 in kg per kWh for each power plant.
7. Add the government cost and CO2 cost to total government cost and CO2 cost.
8. Repeat steps 1 to 7 until 30 years have passed.
9. Calculate the cost in USD for the emitted CO2. For this a weight is necessary, which becomes exponentially greater as more CO2 is emitted (the marginal costs become higher). the weight of CO2 is calculated by using:

$$w_{CO2} = e^{\frac{ln(8) \cdot W_{CO2}}{10^{12}}} \tag{2.6}$$

   Where W_{CO2} is the total CO2 in kg that is emitted for a scenario, this is multiplied by the natural logarithm of 8 and divided by a Gigaton in order to achieve the earlier mentioned multiplication of 8 for every gigaton emitted.
10. Combine the government cost and the cost in CO2 to get the fitness function for a scenario:

$$f = c_{gov} + c_{CO2} w_{CO2} W_{CO2} \tag{2.7}$$

   Where c_{gov} is the total cost for the government in USD and c_{CO2} is the social cost of CO2 per kg.

The last step of the scenario is the function that forms the mathematical problem statement: "To minimize the costs for the energy transition, for both the government and the public for a given subsidy/excise."

2.4. Reference Data

In order to perform this optimization, accurate data of several powerplants is needed. This data should include the capital costs of each powerplant per unit of power generated, fixed costs for energy production and variable costs for energy production. In order to accurately asses the effect on the environment and climate change data about CO2 emissions should be known as well. Reference data for the lower bounds, upper bounds and the initial state x_0 are taken from a reference country, which in this case is based on the Netherlands[1]. This data was obtained and is shown in Table 2.1 [2]:

Table 2.1: Energy Sources Specifications

Power Source	Capital Costs ($/kW)	Fixed Costs ($/kWy)	Variable Costs ($/kWh)	CO2 Emmission (kg/kWh)	L.B.[1]	U.B.[1]	x_0[1]
Coal	3676	40.58	0.0045	0.31883061	0	1	0.5
Coal with 90% C.C.[1]	5876	59.54	0.01098	0.031883061	0	1	0
Natural Gas	1084	14.1	0.00255	0.181083405	0	1	0.27
Natural Gas with 90% C.C.	2481	27.6	0.00584	0.01810834	0	1	0
Nuclear	6041	121.64	0.00237	0	0	1	0.03
Biomass	4097	125.72	0.00483	0.31883061	0	1	0.04
Geothermal	2521	128.544	0.00116	0	0	0.5	0.02
Hydroelectric	5316	29.86	0	0	0	0.1	0.01
Onshore Wind	1677	35.14	0	0	0	0.1	0.05
Offshore Wind	4375	110	0	0	0	0.3	0.05
Solar	1313	15.25	0	0	0	0.3	0.03

The lower bounds and upper bounds represent to what extent the hypothetical country is able to use each type of powerplant. These bounds can vary between countries and for the presented case they have been estimated and should not be taken too literally. In the most right column of the table the initial state x_0 can be seen, where each entry represents the fraction of the total power needs of the hypothetical country that is supplied by the power plant specified in the first column. Not only data about the power plants is necessary, but also the hypothetical country that is going through the energy transition and the timespan. The size of the energy market is estimated to be around 30GW of power. The timespan for the energy transition is 30 years and is from 2020 to 2050. The budget that is available each year for the energy market is equal to 8 billion USD.

[1]Based on information available at: `http://eduweb.eeni.tbm.tudelft.nl/TB141E/?elektriciteit-conversie`, accessed at 01-07-2020.

[1]L.B.: Lower Bound; U.B. Upper Bound; x_0: Initial state; C.C.: Carbon capture.

3
Initial Problem Investigation

Before attempting a numerical optimization procedure, the model boundedness should be assessed to verify that the model is well posed. Additionally, this investigation gives insight into the choice of optimization algorithm.

3.1. Problem Investigation of the Market Model

First, the boundedness is assessed of the market model. This is done by analyzing the model and particularly the cost function as presented in Section 2.2. As discussed previously, the multiplication factor is the cause of nonlinearity in the cost function. With respect to this multiplication factor, there are four separate cases and these will be assessed separately. The first case is when the multiplication factor is constant ($M_f = 1$), the second when the multiplication factor is linearly increasing ($M_f = x+1$), the third is when the multiplication factor is linearly decreasing ($M_f = 2 - x$) and the fourth is when the multiplication factor is parabolic ($M_f = 4 \cdot x^2 - 4 \cdot x + 2$).

To analyze the boundedness of the four cases, equation 2.1 is expanded for a single power plant, indicated with x_i. To aid in readability, two constants are introduced, $c_1 = \frac{b}{t_l} + e$, which is strictly positive. $c_2 = s$, which is equal to the subsidies/excise, which can be positive or negative. It is apparent that each part of the sum in equation 2.1 consists of a polynomial of an order that is dependent on the multiplication factor presented earlier. Since all parts of the cost function are described by continuous polynomials, the boundedness of each case can be evaluated by differentiation, all analyses are based on an interval from 0 to 1. As a power plant is only producing electricity and no energy is exported (sum of $x_i = 1$ for $x_i \geq 0$).

Constant Multiplication Factor For the case with a constant multiplication factor, the cost function, first order derivative and second order derivative are given by:

$$f = x_i \cdot (c_1 + c_2) \tag{3.1}$$

$$\frac{\partial f}{\partial x_i} = c_1 + c_2 \tag{3.2}$$

$$\frac{\partial^2 f}{\partial x_i^2} = 0 \tag{3.3}$$

As can be seen, the derivative is given by a constant and the second order derivative is equal to 0. This makes the function monotonically increasing for $c_2 > -c_1$ and monotonically decreasing for $c_2 < -c_1$ and convex (not strictly), which is beneficial for optimization algorithms.

Linearly Increasing Multiplication Factor For the case with a linearly increasing multiplication factor, the cost function, first order derivative and second order derivative are given by:

$$\begin{aligned} f &= x_i \cdot ((1 + x_i) \cdot c_1 + c_2) \\ &= x_i^2 \cdot c_1 + (c_1 + c_2) \cdot x_i \end{aligned} \tag{3.4}$$

$$\frac{\partial f}{\partial x_i} = 2 \cdot c_1 \cdot x_i + c_1 + c_2 \tag{3.5}$$

$$\frac{\partial^2 f}{\partial x_i^2} = 2 \cdot c_1 \tag{3.6}$$

The derivative is described by a linearly increasing function and the second derivative is equal to a positive constant. This makes the function monotonically increasing and convex for $c_2 > -c_1$ and non-monotonic and not convex for $c_2 > -c_1$. The latter, non-monotonic and non convex is not beneficial for optimization.

Linearly Decreasing Multiplication Factor For the case with a linearly decreasing multiplication factor, the cost function, first order derivative and second order derivative are given by:

$$\begin{aligned} f &= x_i \cdot ((2 - x_i) \cdot c_1 + c_2) \\ &= -c1 \cdot x_i^2 + (2 \cdot c_1 + c_2) \cdot x_i \end{aligned} \tag{3.7}$$

$$\frac{\partial f}{\partial x_i} = -2 \cdot c1 \cdot x_i + 2 \cdot c_1 + c_2 \tag{3.8}$$

$$\frac{\partial^2 f}{\partial x_i^2} = -2 \cdot c_1 \tag{3.9}$$

The derivative is given by a linearly decreasing function and the second order derivative is equal to a negative constant. The function is non-monotonic and not convex, however it can be monotonic and convex locally for certain combinations of c_1 and c_2.

Parabolic Multiplication Factor For the case with a parabolic multiplication factor, the cost function, first order derivative and second order derivative are given by:

$$\begin{aligned} f &= x_i \cdot \left((4 \cdot x_i^2 - 4 \cdot x_i + 2) \cdot c_1 + c_2\right) \\ &= 4 \cdot c_1 \cdot x_i^3 - 4 \cdot c_2 \cdot x_i^2 + (2 \cdot c_1 + c_2) \cdot x_i \end{aligned} \tag{3.10}$$

$$\frac{\partial f}{\partial x_i} = 12 \cdot c_1 \cdot x_i^2 - 8 \cdot c_1 \cdot x_i + (2 \cdot c_1 + c_2) \tag{3.11}$$

$$\frac{\partial^2 f}{\partial x_i^2} = 24 \cdot c_1 \cdot x_i - 8 \cdot c_1 \tag{3.12}$$

For the case with the parabolic multiplication factor, the first order derivative is described by a second order polynomial, which makes the function non-monotonic. Since the first order derivative is also non-monotonic, the function is non-convex. Although, with certain combinations the function can be monotonic and convex locally.

The four different cases can be combined to make the entire cost function of the cost function. It is known that a sum of positive power functions, explained extensively in geometric programming problems [3], is a convex function. However, it was determined that not all parts are convex and therefore general convexity of the cost function can not be assumed. This also holds for monotonicity, a sum of both monotonic increasing and decreasing function is not necessarily monotonic itself.

This analysis has shown that all functions are differentiable and thus can be an input for the optimization process. However, due to computational reasons explained later in the report, the second order derivative, in form of the Hessian matrix, will not be included and will thus be approximated numerically. If a derivative is to be approximated numerically, it is important to determine the stable regions of finite difference steps. To determine the second derivative, a forward finite-difference approximation is used as given in the equation below:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f'}{\partial x} = \frac{f'(x+h) - f'(x)}{h} \tag{3.13}$$

The forward finite-difference approximation is based on a first order Taylor expansion. The second order derivative is plotted for the four different cases in the same graph, the second order derivative is approximated around $x_i = 0.5$ with the same coefficients c_1 (based on the costs of coal) and $c_2 = 0$ for all four cases. As the true second order derivative is known, these are plotted as well as a reference.

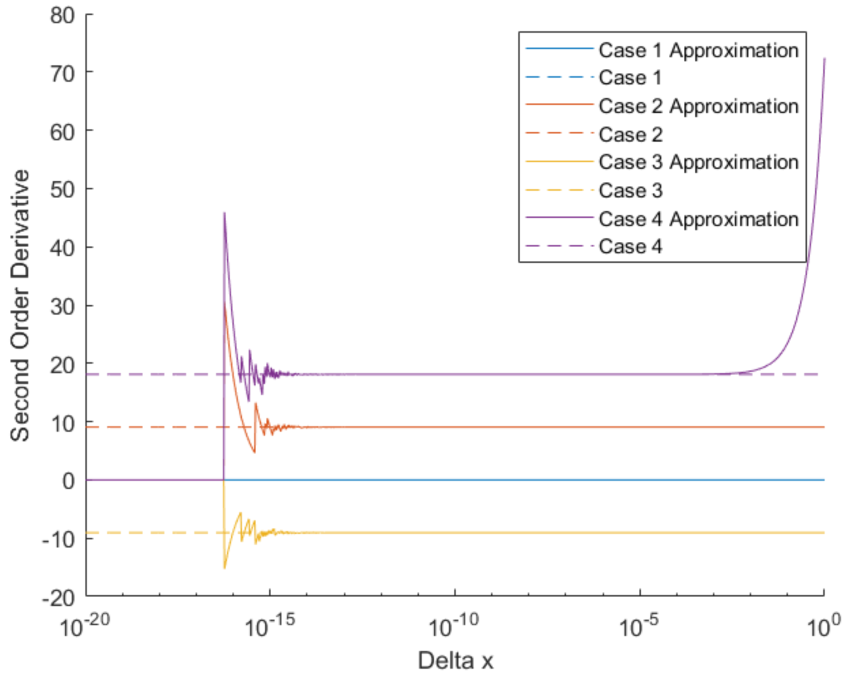


Figure 3.1: Second order derivative approximation using a forward finite-difference scheme. Approximated around $x_i = 0$ with the constant c_1 equal that of the coal power plant and c_2 equal to 0.

As is apparent from the figure, the forward finite-difference scheme provides a good approximations of the second order derivatives for finite difference steps of order of 10^{-14} to 10^{-3}. For case 4 a truncation error can be identified for larger discretization step sizes. For cases 1, 2 and 3 however, the derivatives are either constant or linear, thus a truncation error is not evident from the graph. For cases 2, 3 and 4, the condition error can be observed, caused by numerical noise or in some cases ill-conditioned matrices. For the smallest step sizes, all derivatives are approximated to be equal to 0, as the original value and the discretized value are represented by the same number in the program.

Naturally, the results will be different around other values of x_i and for other coefficients. However, it is expected that the derivatives will be stable to the same order of magnitude, due to the fact that the inputs are similar.

With respect to the constraints, the analysis is relatively straightforward, as the constraints are only constants. For instance the constraints that the sum of the fractions of energy sources is equal to 1 and that an energy source can only provide a fraction of 0 to 1 of the total demand. These constraints can differ per energy source based on the possibilities of the country, these are presented in Table 2.1 for the Netherlands.

With regard to sensitivity, the data and the multiplication factor directly influence the cost function. However, due to the nature of the problem and the nested optimization loop, the sensitivities of this data is not examined further, as this can differ in a case by case analysis. Furthermore, it is expected that the data is in of the same order of magnitude and will therefore not affect the previously mentioned numerical stability significantly.

3.2. Problem Investigation of the Scenario Model

As presented in the formulation of the model, the cost function of the model can not be defined by a single or a set of formulas. Rather, the cost function is described by a complete model in which another optimization is nested. This nested optimization and the fact that the subsidies can change every 5 years makes the cost function discontinuous. A discontinuous function is not necessarily unoptimizable, however it makes the success of first and second order methods highly unlikely. It should be noted that a discontinuous function may still be monotonic or convex locally[1], making other methods more applicable, however this can not be guaranteed in the domain of this application.

Even though it is expected that first and second order methods are not applicable, for the simplified scenario model in Section 4 the first order derivative is used, the stability analysis of a the derivative of the scenario model is to be analysed. Similar to the second order derivative of the market model, a forward finite-difference approximation is applied.

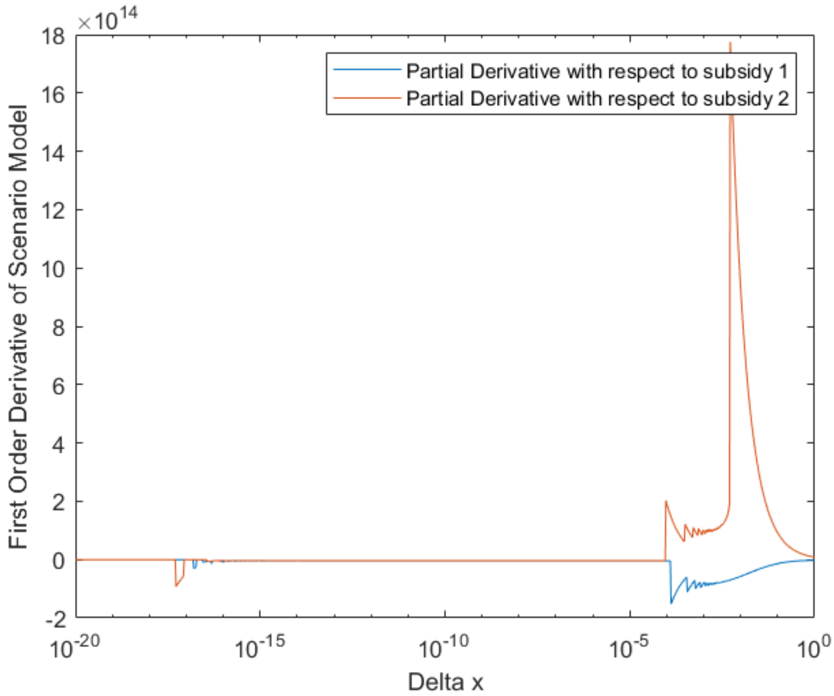


Figure 3.2: First order derivative approximation of the scenario model using a forward finite-difference scheme. Approximated around $x = [0, 0]$.

The figure shows that for finite difference steps of order of 10^{-13} to 10^{-5}, the derivative is stable.

[1]The functions may be discretized into separate domains to optimize the function in that way. In this case this is not applicable due to the fact that a optimal first period (of 5 years) is not necessarily optimal for the entire time span.

4
Simplified Optimization

In this chapter, a simplified version of the algorithm is presented. This makes the problem easier to understand and serves as a verification step. The two cost functions that have been used are presented as 2D cases and give insight how the algorithm works for more complex cases. The data of the 2 separate energy sources used for this simplified analysis is that of coal and offshore wind energy, one difference being with the upper bound for the hypothetical country is that wind energy is now allowed to produce 100% of the energy supply.

As previously stated, the optimization consists of two separate optimizations, one based on the market model and one based on the scenario model. The optimization of the scenario model is primarily the one of interest, since the optimization of the market model is relatively straightforward. To provide insight in to the entire optimization process, the market model will be discussed briefly, followed by three separate approaches to the scenario model.

4.1. Market Function

As was shown in the investigation of the problem, the cost function of the market is differentiable twice, this allows a second order algorithm to be used. Even though, both the first and second order derivatives are defined, the second order derivative will be computed numerically. This is due to the reason that the optional algorithms that were available for `fmincon` that allowed for information about the Hessian, `interior-point` and `trust-region-reflective`, resulted in ill-conditionedes matrices, possibly due to the small domain we are interested in. Therefore, it was decided to turn to other optimization algorithms that did not allow for second order derivative information to be included, such as `sqp`. For that reason, the numerical stability of the second order derivative has been investigated in Section 3.1.

As pointed out, the optimization of the market is not the main interest, therefore only the 2D scope of the cost function is plotted for the considered energy sources, coal and offshore wind, without any subsidies. Imposed on the graph is the equality constraint that the sum of the fractions is equal to one.

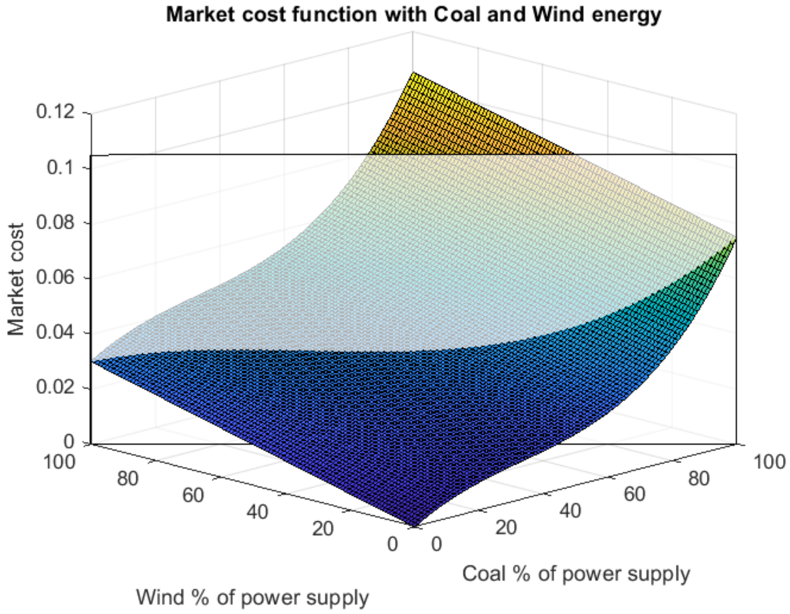


Figure 4.1: Overview of the market function for coal and offshore wind energy, without government subsidies, and the equality constraint in transparent white.

4.2. Scenario Model

The market function is but one part in the complete optimization function and is used in the `scenario.m` function to represent the behavior of the market. This is the actual function that needs to be optimized by changing the subsidies/excises that the government can levy. Even though that the scenario cost function can not be described by a single function, three separate methods are applied to show the distinction. For all approaches an initial point of [0,0] is used, which can be interpreted as no subsidies for both coal and offshore wind energy. The lower bound and upper bound are equal to -100% and 100% subsidy respectively. Additionally, for this simplified model, the subsidy is constant over the entire duration (30 years). Similar to the market model, the 2D scope is plotted to give insight in the to-be optimized function. In this simplified case the subsidies remain constant over time, so that the input is 2D.

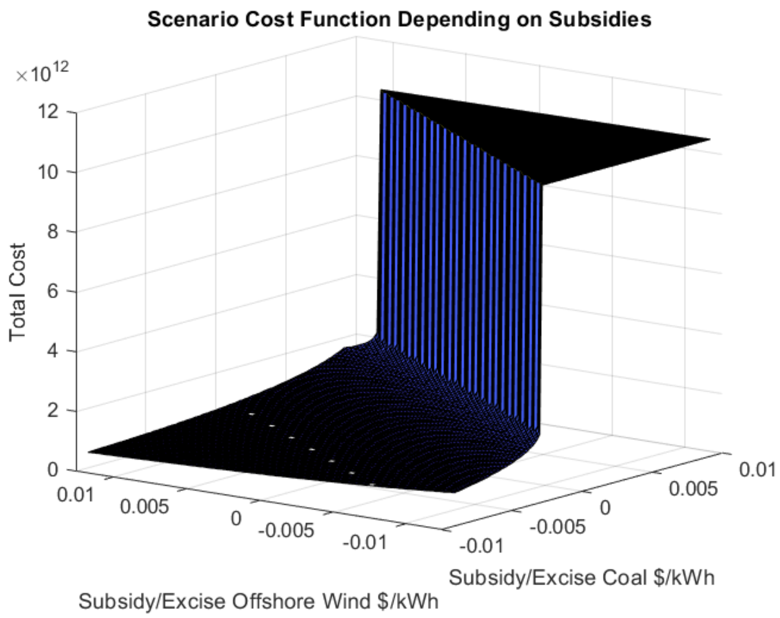


Figure 4.2: Overview of the scenario function for coal and offshore wind energy as a function of government subsidies.

It can be seen that this function is already more complicated than the market function and that it has discontinuities.

4.2.1. Sequential Linear Programming

First, sequential linear programming is used to determine the optimum. As suggested in Section 3.2, the derivatives are computed numerically with a forward finite-difference scheme, with a finite-difference step size of 10^{-8}. This results in the following optimum [0.0091, 0.0126], with a total cost of 1.6501 Trillion USD and resulted in the following market model.

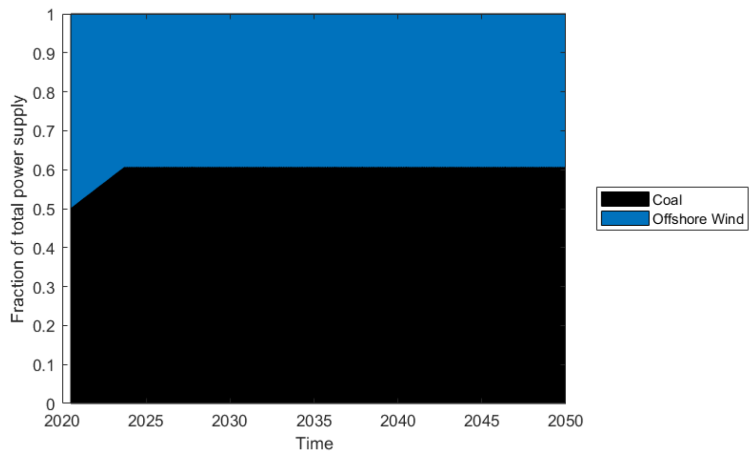


Figure 4.3: Market model as a result of optimizing the scenario model with the use of sequential linear programming.

As can be seen, the government only levies excise, which is a way to reduce (government) cost, but it does not do much to prevent the emissions of CO2.

4.2.2. Sequential Quadratic Programming

The sequential linear programming algorithm is expanded to a sequential quadratic programming algorithm using the function `fmincon`, the scenario model is optimized using sequential linear programming. This results in the optimum [0.0002425, 0.0003334] which is roughly equal to no subsidies at all. The total cost is approximated to be 1.2959 Trillion USD, which is better compared to the result of the sequential linear programming.

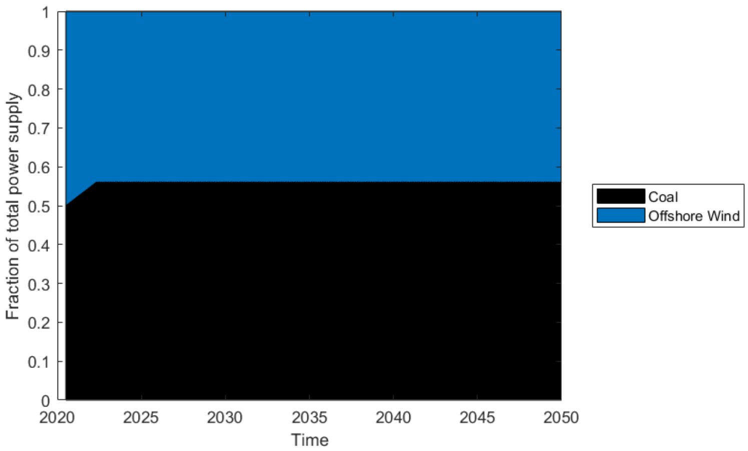


Figure 4.4: Market model as a result of optimizing the scenario model with the use of sequential quadratic programming.

It can be seen that a little less coal is used, but that the market still wants to move more towards fossil fuels.

4.2.3. Genetic Algorithm

Finally, a genetic algorithm is applied to find the optimum. The genetic algorithm is expected to work significantly better due to it being a zeroth order algorithm, able to handle discontinuous functions. For the simplified case, the algorithm is allowed a 100 generations. The algorithm resulted in the optimum [0.0091, -0.125] with a total cost of 518.44 billion USD, which is significantly better than the first two algorithms. This is due to the fact that this results in a market that relies more on renewable energy, making the costs induced by CO2 emmission lower. It is worth noting that the genetic algorithm is based on stochastic methods, and will therefore not produce the same result, unless a RNG seed is specified.

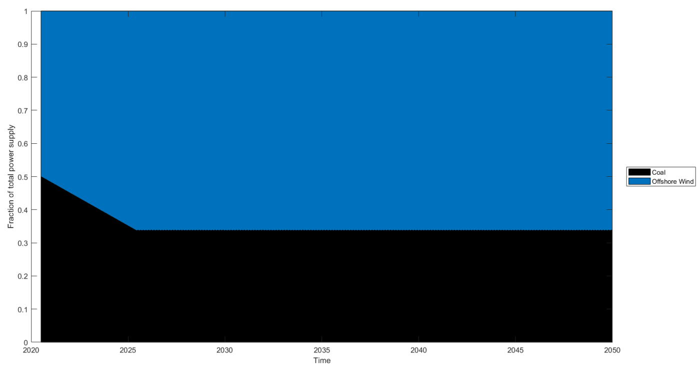


Figure 4.5: Market model as a result of optimizing the scenario model with the use of a genetic algorithm.

It can be concluded that the genetic algorithm is a more appropriate choice of algorithm for the optimization of the scenario model. Although, this was expected due to the nature of the scenario model being discontinuous. That is why the genetic algorithm is being chosen as the algorithm for the complete optimization.

5
Complete Optimization

It was chosen in chapter 4 to use a genetic algorithm for the complete optimization. In this chapter this optimization is fully explored along with an analysis into its results, its sensitivity and robustness. This is concluded by a reflection on the framework that has been created for this report and its usability for real life applications.

5.1. Problem Set Up

As discussed in previous chapters, the goal of this optimization is for the government to find the ideal subsidies/excise that will result in the lowest cost for the government in terms of CO2 and overall cost for the government. In order to allow the government to change its approach over time it has been decided to allow a change in subsidies every five years. This allows for example for heavy excise to be levied on on some resources, which can be rescinded at later time. This changes the input from 11D tot 66D as there are six times during the simulation that the input can be changed. In order to compare the result of the optimization to the current status of the system, a baseline model was created where all subsidies were set to 0, this gave the following result:

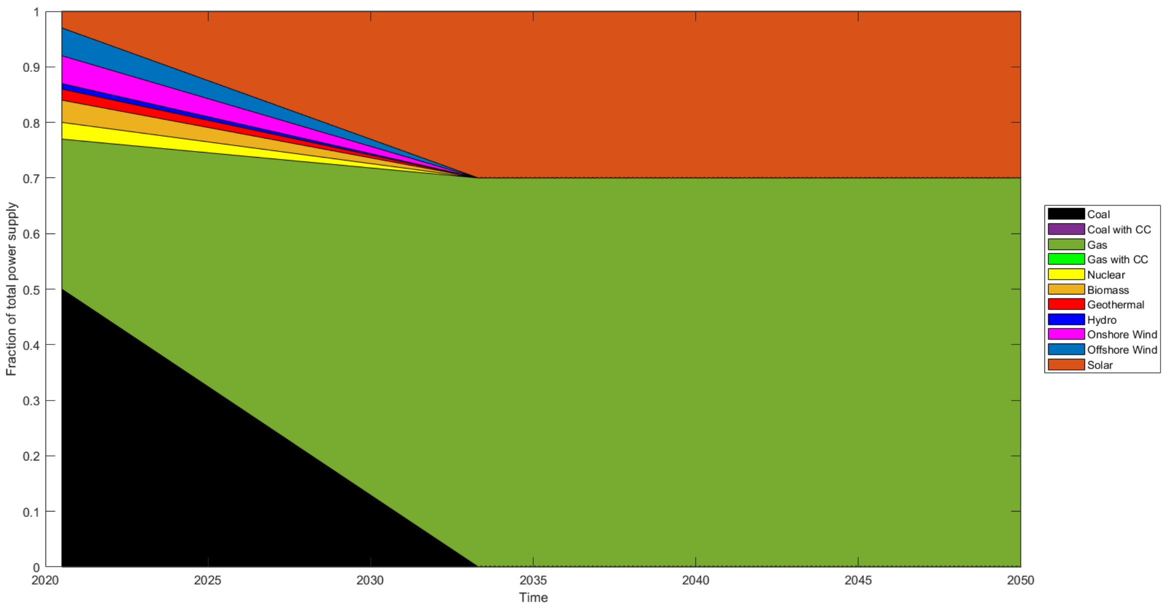


Figure 5.1: Baseline energy transition.

The total costs for the government in this scenario was 0 USD as there are no subsidies, but the damage to the environment in terms of social cost was equal to 645.5 billion USD. It can be seen in the figure that the market is not in equilibrium at the beginning and that it moves away from coal. The rest of this chapter is devoted to finding a better solution to this energy transition.

5.2. Genetic Algorithm

Because of the nature of this problem and its high dimensionality and discontinuity it is hard to use the more 'classical' first-order and second-order approaches with regard to optimization, which is why the genetic algorithm will be used. A Genetic algorithm creates a 'population' of solutions and evaluates the fitness of each member of the population according to its genes, which are the input values for the fitness function. After a generation is created a certain 'Elite' part of the population, the one with the best genes that lead to the lowest cost values of the fitness function are taken and are allowed to 'reproduce' and pass on their genes. Of the remaining part of the population a 'CrossOverFraction' is taken, which also pass on their genes. These two parts of the population form the children of the next generation and the process starts anew.

In order to select that the best settings a few simulations were run that were limited in the amount of generations (100) in order to select the parameters that would lead to the fastest reduction of the fitness function. The used parameters are:

Table 5.1: Overview of the settings of the genetic algorithm

CrossoverFraction	PopulationSize	Selection Function
0.8	200	Stochastic uniform

It was also decided not to use mutation in this genetic algorithm, because of problems with boundedness.

5.3. Results

The genetic algorithm was run for 1000 generations with the initial state vector as described in chapter 2. The constraints that were put on the inputs were that the subsidies/excises could not exceed 100% of the energy cost of each power source. This was to ensure that the government could not profit too much from levying excises nor that certain energy sources could be subsidized to an unrealistic amount. Multiple runs were done with the same input settings to verify that an optimum was reached, these can be seen below in Table 5.2:

Table 5.2: Overview of the runs

Run	Total Cost (Billion USD)
1	118.4
2	120.7
3	105.3
4	105.3
5	105.3

The first two runs did not reach the absolute optimum, but the last three runs seemed to have reached an optimum, although it cannot be verified with absolute certainty that this is the global optimum. More generations could be used, but for the goal of this optimization problem this was deemed not necessary as the runs already took 3 hours to complete. Below the progression of the genetic algorithm for the last run is shown:

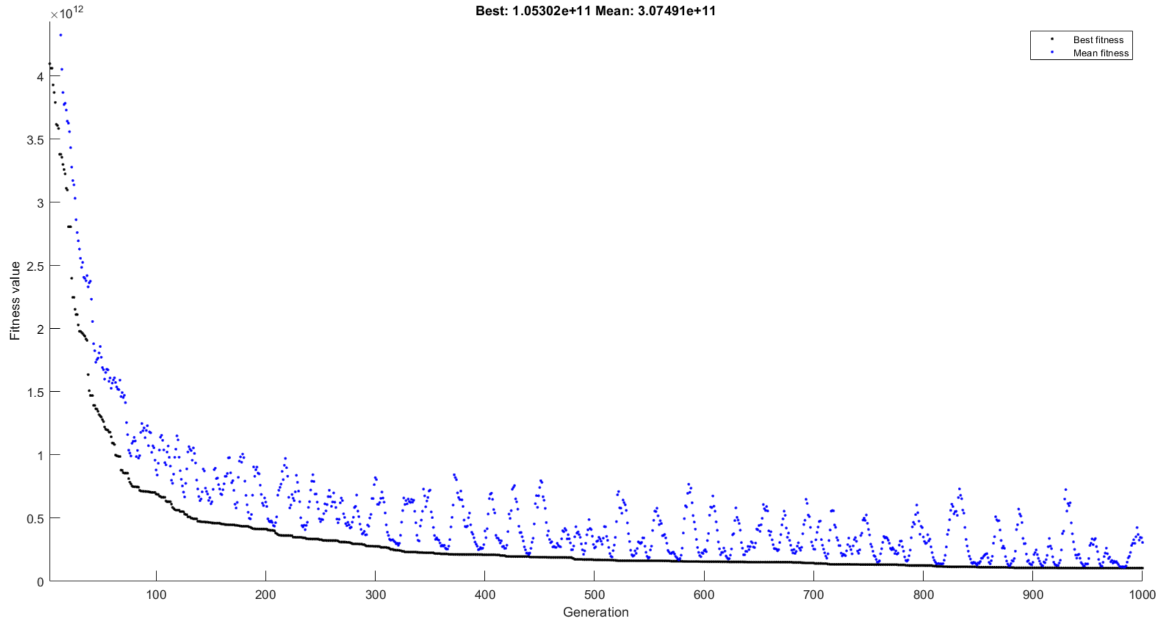


Figure 5.2: Caption

It can be seen that initially the cost function is steadily going down before flattening off, which is to be expected. The black dots represent the best members of each generation and the blue dots the mean. The output of the genetic algorithm is a row vector with the 66 values for subsidies/excises, which can be used in `scenario.m` to show how the energy transition has taken place for the hypothetical country:

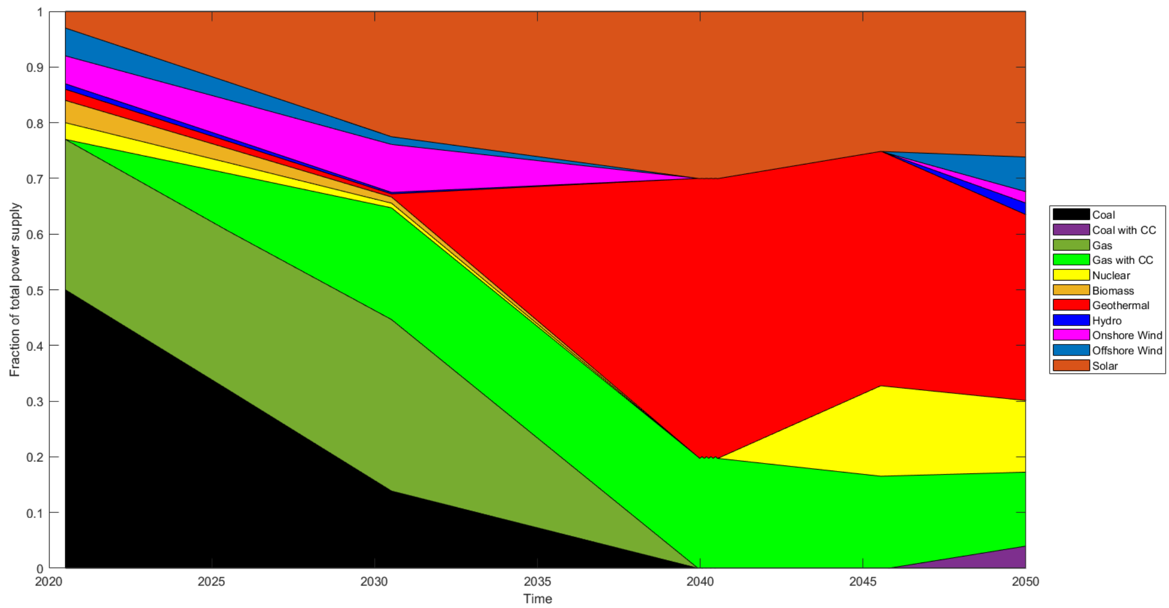


Figure 5.3: Graphical result of the energy transition with the solution provided by the genetic algorithm.

It can be seen that the contribution of 'conventional' fossil fuels are steadily decreased in the first half of the simulation and replaced with more sustainable alternatives. It can be seen that the market is not yet in equilibrium at the end of the simulation, which is due to the changes in subsidies that happen every 5 years and the market did not have enough time to adapt to the latest change. The output variables are given below positive values are for excises and negative values for subsidies:

Table 5.3: Subsidy/Excise output of the genetic algorithm. Time is in years and the subsidy/excise output is in $/kWh.

Time	**Coal**	**Coal CC**	**Gas**	**Gas CC**	**Nuclear**	**Bio**
0-5	0.0091	0.0178	0.0042	-0.0203	0.0163	0.0192
6-10	0.0091	0.0161	0.0037	-0.0174	0.0162	0.0173
11-15	0.0091	-0.0135	0.0042	-0.0173	0.0163	0.0191
16-20	0.0079	-0.0068	0.0042	-0.0173	0.0146	0.0192
21-25	-0.0311	0.0129	-0.0118	0.0090	-0.0522	-0.0277
26-30	0.0028	-0.0520	0.0022	0.0090	0.0163	-0.0310
Time	**Geo**	**Hydro**	**Onshore**	**Offshore**	**Solar**	
0-5	0.0158	0.0034	-0.0077	0.0126	-0.0073	
6-10	0.0158	0.0034	-0.0064	0.0126	0.0003	
11-15	-0.0350	0.0034	0.0040	0.0122	-0.0016	
16-20	-0.0351	0.0034	0.0027	0.0126	-0.0007	
21-25	0.0158	-0.0308	-0.0117	-0.0503	0.0017	
26-30	0.0158	-0.0299	-0.0211	-0.0499	-0.0053	

The values presented in Table 5.3 are rounded off to the fourth decimal for clarity. It can be seen that not only fossil fuels have excises, but that some sustainable resources also excises. This can be explained due to the fact that some powerplants are still profitable for the market with some excise levied, so it makes sense for the government to tax these sources so it can subsidize others. Not all resources can be used to their full extent (100% power supply) so a combination is the optimal solution.

5.4. Sensitivity Analysis

A sensitivity analysis for all input variables from the government is not feasible nor very interesting, since it would be hard to interpret. Instead a sensitivity analysis was performed by changing the constraints that are put on the scenario. The results that have been presented in the previous section are for a hypothetical country that is based off on the Netherlands. That is why the same genetic algorithm is applied on different hypothetical countries to see what the results would be. Not all countries have equal access to resources which is why four countries are proposed:

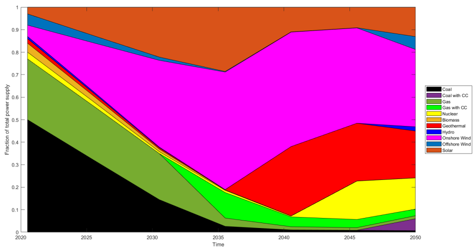
1. A country with 100% access to on-shore wind (Iceland).
2. A country with 100% access to off-shore wind (Scotland).
3. A country with 100% access to hydroelectric power (Norway).
4. A hypothetical country with 100% access to all power sources.

The rest of the constraints remain the same, only 1 constraint is lifted each time.

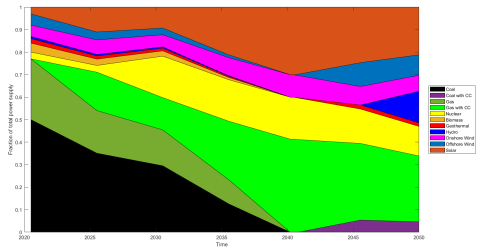
Table 5.4: Overview of runs with changing constraints for power sources.

Country	**Total Cost (billion USD)**
On-shore wind	66.8
Off-shore wind	98.2
Hydroelectric	164.2
100% access	47.2

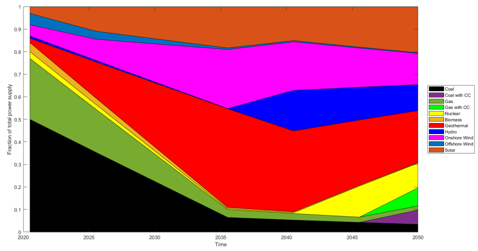
It can be seen that changing the availability of resources by changing the upper bounds in `scenario.m` the cost can significantly change.



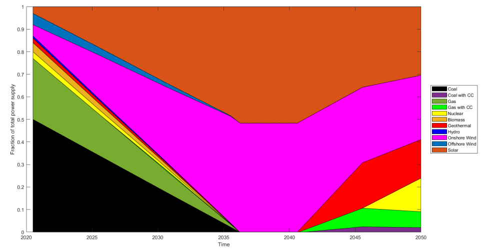
(a) On-shore wind



(b) Off-shore wind



(c) Hydroelectric



(d) Unrestricted acces

Figure 5.4: Overview of changing the access of power resources for countries and the effect it has on their energy transition.

In Figure 5.4 the effect of these changed boundaries for the allowed fractions of energy supply coming from a certain power source can be seen for the energy transition. All scenarios start at the same state, but take vastly different trajectories. Here a problem of this model/optimization comes to light. In the last run a 100% sustainable economy is reached around 2035, but it moves away from that after 5 years, since the government tries to reduce its own costs by levying excise over solar and geothermal energy. In reality this would probably not happen, but this model merely looks at the mathematical aspects of the energy transition, not the most logical choices according to humans.

5.5. Robustness

In the previous section it was shown that changing the settings of a scenario can have a large influence on the outcome of the cost function and the state over the entire energy transition, but how robust and stable are these solutions? In order to analyze how robust the solution is for a given problem, it was decided to model these changes/inconsistencies by adding noise to the input vector. This noise is added, because the solution that has been proposed is a mathematical solution and not one that is likely to be followed perfectly in a world of politics and different stakeholders. This noise represents changes in policy and mistakes that are made by the government. The noise was modelled using Gaussian noise with standard errors comparable to a percentage of the mean absolute values of the inputs. This input vector with noise was run multiple times for different standard deviations to assess how robust the solution was. For every standard deviation 1000 runs were completed.

The values that were given in Table 5.5 are the means of 1000 simulations.

Table 5.5: Overview of the robustness analysis. With the total cost being calculated by taking the mean of 1000 simulations.

Noise (σ)	Total Cost (Billion USD)
1%	136.6
2.5%	145.2
5%	165.4
10%	222.2

In order to give a better sense of how the distribution of the robustness analysis looks like a histogram has been provided of one of the simulations in Figure 5.5

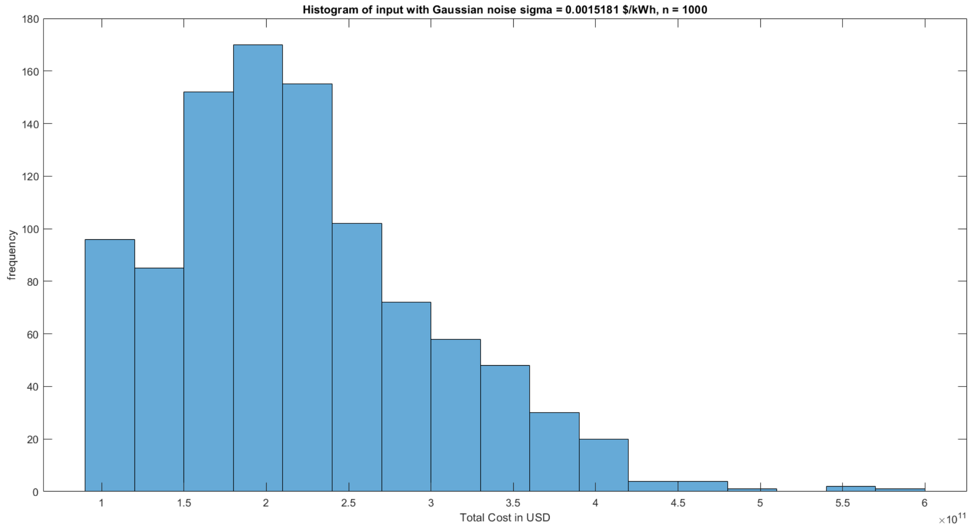


Figure 5.5: Histogram of a robustness simulation run.

It can be seen that the mean of the histogram lies around the value provided in the table, but that the distribution is skewed towards higher cost, with only few of of the simulations being in the same bin as the optimum. This gives reinforcement towards the hypothesis that the calculated optimum is really the optimum, but that care should be given towards deviations from this optimum as small changes in the input can result in significant changes in the output. This is however to be expected as all 66 outputs are changed by the noise, not just a few. For real life applications a more extensive robustness check would be in order.

5.6. Interpretation

The models and results that have been discussed so far show promising results, with regard to the optimization of the energy transition. It seems that with the right intentions/subsidies and political will it is possible and within government and market budgets to transform the energy market, without setting fixed constraints on powerplants. It must be said however that the results and models that have been shown are a simplification of reality and the numbers that are shown are only an indication of what is possible. But the meaning of this research was to show how a framework for this problem could be set up and in that regard it has succeeded.

For this to work for real life applications more variables and conditions need to be taken into account and some assumptions that were used would not be valid. One of the largest obstacles for an optimization like this to work realistically is to model the political will of a country. As seen in the robustness analysis, a small change in strategy can have a large influence on the end result.

That being said it seems that the genetic algorithm is a useful tool to solve this kind of problem. The discontinuities in the problem, combined with the large number of variables that can change make it an algorithm that is able to find a good solution. It is however computationally expensive, so for more complex models it might be necessary to find ways to reduce the number of function evaluations that are necessary.

6
Conclusion & Recommendations

6.1. Conclusion

In the past few years climate change and the energy transition has become a hot topic, but the focus is more often than not on what the ideal situation should be and not on how to get there. The goal of this research was to set up a model framework that could quantify the costs of the energy transition and then find a way to minimize these costs. A combination of two models is used, one that estimates the behavior of the energy market and one that models what the costs of the energy transition would be given a scenario. In the initial investigation it was found that both models show different properties, the market model is relatively straightforward with respect to the scenario model. The properties of the market model favored a 'classical' second order optimization approach, while a zeroth order algorithm was found to be more appropriate for the scenario model.

The 2D optimalization showed that methods such as sequential linear programming and sequential quadratic programming were not sufficient to navigate the energy transition. Thus methods that could handle the dimensionality and the discrete nature of the problem were necessary. It was chosen to implement a genetic algorithm that would evolve and try and find the subsidy/excise inputs that would lead to a minimization of costs for both CO2 as well as direct government costs. After testing multiple runs the model converged to a cost around 100 billion USD for the energy transition given an initial state that was based on the country of the Netherlands. This cost was divided between cost directly paid for by the government as well as costs that would be indirectly paid for by the public because of the emitted CO2 over the 30 years that the scenario would run. Compared to the baseline, where the government was not involved, the cost was 6 times lower.

The sensitivity analysis showed that changing some constraints and values in the model could have a significant effect on the outcome of the model, so great care should be put in validating these constraints for a scenario.

All in all the genetic algorithm is a useful tool in finding the optimum input of the government for the energy transition and the framework that was designed here could be used to make more detailed models. More detailed models will be necessary to take more factors into account. The results and methods that are used here are just a mathematical representation of a much more complex system, but the lessons that have been learned here could be applied in real life applications as a tool to evaluate long term government plans and policies.

6.2. Recommendations for Future Studies

The work that has been done is a firm foundation for future methods and models and throughout the process of making these algorithms, several recommendations and changes were proposed within the team. With regard to the market function and the scenario model, more variables should be included to accurately model how the energy market behaves. As of now, the power requirement of the hypothetical country remains constant, while in reality it will most likely grow and fluctuate over time. Regulations and laws that influence the market can be included depending on the country as constraints and/or in the market function. These models could then be validated by taking historical data and see if the scenario evolves according to the data. This was considered as a validation option in this project, however, not done due to time constraints. One addition which is hard to quantify, but will most likely have a great effect on the model, is modelling the cost of political will. These additions will greatly increase the accuracy and applicability of the model and the results.

Bibliography

[1] M. L. C. . R. G. Newell, *Valuing Climate Damages: Updating Estimation of the Social Cost of Carbon Dioxide* (THE NATIONAL ACADEMIES PRESS, 2017).

[2] Sargunt & Lundy, *Capital Cost and Performance Characteristic Estimates for Utility Scale Electric Power Generating Technologies*, Tech. Rep. (U.S. Energy Information Administration, 2020).

[3] P. Y. Papalambros and D. J. Wilde, *Principles of optimal design: modeling and computation* (Cambridge university press, 2000).

A
MATLAB code

The optimization presented in this report has been executed in MATLAB. The source files can be found attached as a ZIP-file to the assignment in Brightspace. Alternatively via the following link:

https://we.tl/t-btXgL3Q9fU

The MATLAB code contains two folders, with one containing the files regarding the investigation and simplified optimization and one regarding the complete optimization.