Solving the Friedman equation for a Dark Fluid equation of state.

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Introduction

► Cosmological principle:

The universe is homogeneous and isotropic [1].

► General Relativity:

Einstein's field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{1}$$

► Hubble's law:

$$\nu = H_0 r. \tag{2}$$

Expanding universe:

$$r(t) = a(t)\chi. (3)$$

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Friedmann equations

► Fluid equation:

$$\dot{\rho} + 3H(1+\omega)\rho = 0$$
, with $H = \frac{\dot{a}}{a}$ (4)

► Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \rho a^2 - \kappa \frac{c^2}{\chi^2}$$
 (5)

► Raychaudhuri equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + 3P\right) \tag{6}$$

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Concordance model (\Lambda CDM-model)

- Dark Matter.
- ► Accelerated expansion:

Observations of the luminosities of type la supernovae suggest that the universe is undergoing an accelerated expansion [2, 3], which suggests the existence of a Dark energy element.

Assume a perfect fluid equation of state:

$$P = \omega \rho \tag{7}$$

3 Different epochs:

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- Radiation ($\omega = \frac{1}{3}$): $\rho = C_{rad}a^{-4}$
- Matter ($\omega = 0$): $\rho = C_{dust}a^{-3}$
- Dark Energy ($\omega = -1$): $\rho = C_{DE}$

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- ▶ Different Chaplygin gas equations of state [4]:
 - Original Chaplygin gas (OCG):

$$P = -\frac{A_1}{\rho}. (8)$$

Generalised Chaplygin gas (GCG):

$$P = -\frac{A_1}{\rho^{\alpha}}, \quad \alpha > -1. \tag{9}$$

Modified Chaplygin gas (MCG):

$$P = A_2 \rho - \frac{A_1}{\rho^{\alpha}}, \quad \alpha > -1. \tag{10}$$

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Solution to the fluid equation

► Solving the Fluid equation for a MCG equation of state:

$$\rho = \left[\frac{C_2 (1+z)^{3(\alpha+1)(1+A_2)} + A_1}{1+A_2} \right]^{\frac{1}{1+\alpha}}, \tag{11}$$

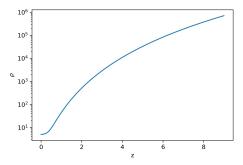


Figure: Here we have taken $A_1 = 50$, $A_2 = C_2 = 1$ and $\alpha = 1$.

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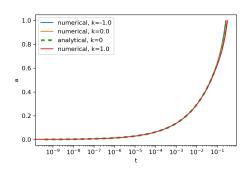
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Assuming a $\kappa = 0$, we have:

$$(t - t_0) = \frac{2}{3A^{\frac{1}{2}}B_2^{\frac{1}{2\beta}}B_1} \left(\frac{B_3}{B_2}a^{-3B_1\beta} + 1\right)^{-\frac{1}{2\beta}} + \frac{1}{2\beta + 1} \left(\frac{B_3}{B_2}a^{-3B_1\beta} + 1\right)^{-1 - \frac{1}{2\beta}} {}_{2}F_1\left(1, 1 + \frac{1}{2\beta}; 2 + \frac{1}{2\beta}; \left(\frac{B_3}{B_2}a^{-3B_1\beta} + 1\right)^{-1}\right),$$
(13)



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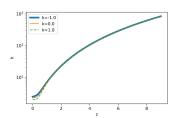
Hubble parameter for MCG case

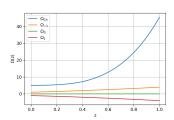
Dimensionless Hubble parameter h

$$h(z) = \frac{1}{H_0} \left[A \left(B_3 (1+z)^{3(\beta)(B_1)} + B_2 \right)^{\frac{1}{\beta}} - \kappa F (1+z)^2 \right]^{\frac{1}{2}}.$$
 (13)

ightharpoonup Fractional energy density Ω

$$\Omega_{Chap}(z) \equiv \frac{A}{H_0^2} \left(B_3 (1+z)^{3(\beta)(B_1)} + B_2 \right)^{\frac{1}{\beta}}
\Omega_{\kappa}(z) \equiv -\frac{\kappa F}{H_0^2} (1+z)^2.$$
(14)





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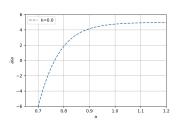
Acceleration of a for MCG case

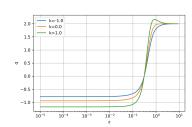
Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2} \left((3B_1 - 2) \left(B_3 a^{-3B_1 \beta} + B_2 \right) \frac{1}{\beta} - 3B_1 B_2 \left(B_3 a^{-3\beta B_1} + B_2 \right) \frac{1 - \beta}{\beta} \right), \quad (15)$$

▶ Deceleration parameter $q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{\frac{A}{2} \left((3B_1 - 2) \left(B_3 (1+z)^{3B_1\beta} + B_2 \right)^{\frac{1}{\beta}} - 3B_1B_2 \left(B_3 (1+z)^{3\beta B_1} + B_2 \right)^{\frac{1-\beta}{\beta}} \right)}{A \left(B_3 (1+z)^{3(\beta)(B_1)} + B_2 \right)^{\frac{1}{\beta}} - \kappa F (1+z)^2}.$$
(16)





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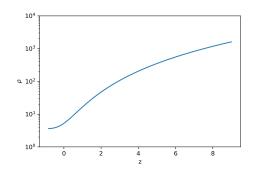
Pressure-Parametrized Unified Dark Fluid (PPUDF)

► The PPUDF equation of state [5]:

$$P = P_a + P_b \left(z + \frac{z}{1+z} \right), \tag{17}$$

Solving the Fluid equation for a PPUDF equation of state:

$$\rho = -P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3,$$
 (18)



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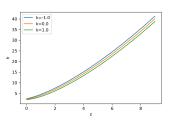
Hubble parameter for PPUDF case

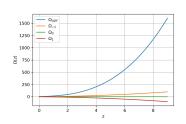
Dimensionless Hubble parameter h

$$h = \frac{1}{H_0} \left[A \left(-P_a + \frac{3}{4} P_b \left[(1+z)^{-1} - 2(1+z) \right] + C (1+z)^3 \right) - \kappa F (1+z)^2 \right]^{\frac{1}{2}}. \tag{19}$$

Fractional energy density Ω

$$\Omega_{PPUDF}(z) \equiv \frac{A}{H_0^2} \left(-P_a + \frac{3}{4} P_b \left[(1+z)^{-1} - 2(1+z) \right] + C(1+z)^3 \right)
\Omega_{\kappa}(z) \equiv -\frac{\kappa F}{H_0^2} (1+z)^2 .$$
(20)





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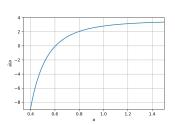
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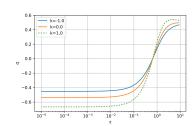
Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2} \left[2P_a - \frac{3}{2} P_b \left(\frac{3}{2} a + a^{-1} \right) + C a^{-3} \right]. \tag{21}$$

▶ Deceleration parameter $q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{A\left[2P_{a} - \frac{3}{2}P_{b}\left(\frac{3}{2}(1+z)^{-1} + (1+z)\right) + C(1+z)^{3}\right]}{2A\left(-P_{a} + \frac{3}{4}P_{b}\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^{3}\right) - \kappa F(1+z)^{2}}.$$
 (22)





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