Solving the Friedman equation for a Dark Fluid equation of state.

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Aim and Motivation

▶ 2 Dark elements

► Is it possible to describe the cosmological behaviour resulting from these elements using a single dark fluid?

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► General Relativity:

Einstein's field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

Hubble's law:

$$\nu = H_0 r \tag{2}$$

► Expanding universe:

$$r(t) = a(t)\chi \tag{3}$$

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► Cosmological principle:

The universe is homogeneous and isotropic [1].

► Fluid equation:

$$\dot{\rho} + 3H(1+\omega)\rho = 0, \text{ with } H = \frac{\dot{a}}{2}$$
 (4)

► Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \rho a^2 - \kappa \frac{c^2}{\chi^2} \tag{5}$$

► Raychaudhuri equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + 3P\right) \tag{6}$$

Concordance model (ACDM-model)

- ► Hot Big Bang
- ► Dark Matter:
 - Galaxy rotation
 - Behaviour of super structure
- ► Accelerated expansion:

Observations of the luminosities of type Ia supernovae suggest that the universe is undergoing an accelerated expansion [2, 3], which suggests the existence of a Dark energy element.

Assume a perfect fluid equation of state:

$$P = \omega \rho \tag{7}$$

▶ 3 Different epochs:

- Radiation (
$$\omega = \frac{1}{3}$$
): $\rho = C_{rad}a^{-4}$

– Matter (
$$\omega = 0$$
): $\rho = C_{dust}a^{-3}$

- Dark Energy (
$$\omega = -1$$
): $\rho = C_{DF}$

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- ► Different Chaplygin gas equations of state [4]:
 - Original Chaplygin gas (OCG):

$$P = -\frac{A_1}{\rho} \tag{8}$$

Generalised Chaplygin gas (GCG):

$$P = -\frac{A_1}{\rho^{\alpha}}, \quad \alpha > -1 \tag{9}$$

Modified Chaplygin gas (MCG):

$$P = A_2 \rho - \frac{A_1}{\rho^{\alpha}}, \quad \alpha > -1 \tag{10}$$

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Solution to the fluid equation

► Solving the Fluid equation for a MCG equation of state:

$$\rho = \left[\frac{C_2 (1+z)^{3(\alpha+1)(1+A_2)} + A_1}{1+A_2} \right]^{\frac{1}{1+\alpha}}$$
(11)

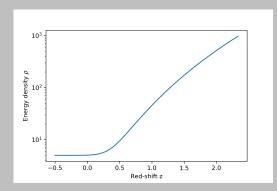


Figure: Here we have taken $A_1=50$, $A_2=C_2=1$ and $\alpha=1$. The figure shows energy density ρ vs red-shift z.

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Solving the Friedmann equation for the MCG equation of state

Assuming a $\kappa = 0$, we have:

$$\begin{split} &(t-t_0) = \frac{2}{3A^{\frac{1}{2}}B_2^{\frac{1}{2\beta}}B_1} \left(\frac{B_3}{B_2}a^{-3B_1\beta} + 1\right)^{-\frac{1}{2\beta}} \\ &+ \frac{1}{2\beta + 1} \left[\left(\frac{B_3}{B_2}a^{-3B_1\beta} + 1\right)^{-1 - \frac{1}{2\beta}} {}_{2}F_1\left(1, 1 + \frac{1}{2\beta}; 2 + \frac{1}{2\beta}; \left(\frac{B_3}{B_2}a^{-3B_1\beta} + 1\right)^{-1}\right) \right] \end{split}$$

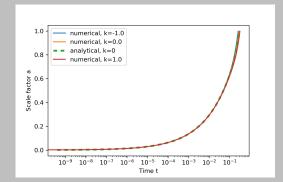


Figure: The figure shows scale factor a vs time t.

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Hubble parameter for MCG case

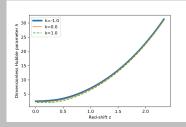
Dimensionless Hubble parameter h

$$h(z) = \frac{1}{H_0} \left[A \left(B_3 (1+z)^{3(\beta)(B_1)} + B_2 \right)^{\frac{1}{\beta}} - \kappa F (1+z)^2 \right]^{\frac{1}{2}}$$
 (13)

ightharpoonup Fractional energy density Ω

$$\Omega_{Chap}(z) \equiv \frac{A}{H_0^2} \left(B_3 \left(1 + z \right)^{3(\beta)(B_1)} + B_2 \right)^{\frac{1}{\beta}}$$

$$\Omega_{\kappa}(z) \equiv -\frac{\kappa F}{H_0^2} \left(1 + z \right)^2$$
(14)



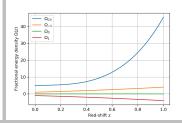


Figure: The figure shows the dimensionless Hubble parameter *h* vs red-shift *z*.

Figure: The figure shows fractional energy density Ω vs red-shift z.

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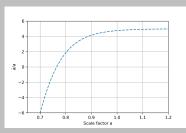
Acceleration of a for MCG case

Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2} \left((3B_1 - 2) \left(B_3 a^{-3B_1\beta} + B_2 \right)^{\frac{1}{\beta}} - 3B_1 B_2 \left(B_3 a^{-3\beta B_1} + B_2 \right)^{\frac{1-\beta}{\beta}} \right)$$
 (15)

ightharpoonup Deceleration parameter $q \equiv -rac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{\frac{A}{2} \left((3B_1 - 2) \left(B_3 (1+z)^{3B_1\beta} + B_2 \right)^{\frac{1}{\beta}} - 3B_1B_2 \left(B_3 (1+z)^{3\beta B_1} + B_2 \right)^{\frac{1-\beta}{\beta}} \right)}{A \left(B_3 (1+z)^{3(\beta)(B_1)} + B_2 \right)^{\frac{1}{\beta}} - \kappa F (1+z)^2}$$
(16)



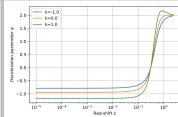


Figure: The figure shows acceleration $\frac{\ddot{a}}{a}$ vs red-shift a.

Figure: The figure shows deceleration parameter *q* vs red-shift *z*.

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Pressure-Parametrized Unified Dark Fluid (PPUDF)

► The PPUDF equation of state [5]:

$$P = P_a + P_b \left(z + \frac{z}{1+z} \right) \tag{17}$$

Solving the Fluid equation for a PPUDF equation of state:

$$\rho = -P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3$$
 (18)

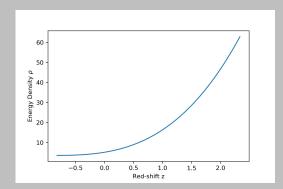


Figure: The figure shows energy density ρ vs red-shift z.

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Pressure-Parametrized Unified Dark Fluid (PPUDF) Hubble parameter for PPUDF case

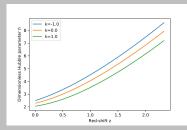
Dimensionless Hubble parameter h

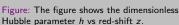
$$h = \frac{1}{H_0} \left[A \left(-P_{a} + \frac{3}{4} P_{b} \left[(1+z)^{-1} - 2 (1+z) \right] + C (1+z)^{3} \right) - \kappa F (1+z)^{2} \right]^{\frac{1}{2}}$$
 (19)

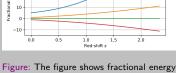
Fractional energy density Ω

$$\Omega_{PPUDF}(z) \equiv \frac{A}{H_0^2} \left(-P_a + \frac{3}{4} P_b \left[(1+z)^{-1} - 2(1+z) \right] + C(1+z)^3 \right)
\Omega_{\kappa}(z) \equiv -\frac{\kappa F}{H_0^2} (1+z)^2$$
(20)

density Ω(z)







density Ω vs red-shift z.

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Acceleration of a:

$$\frac{\ddot{s}}{a} = -\frac{A}{2} \left[2P_a - \frac{3}{2} P_b \left(\frac{3}{2} a + a^{-1} \right) + C a^{-3} \right]$$
 (21)

ightharpoonup Deceleration parameter $q\equiv -rac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{A\left[2P_a - \frac{3}{2}P_b\left(\frac{3}{2}(1+z)^{-1} + (1+z)\right) + C(1+z)^3\right]}{2A\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right) - \kappa F(1+z)^2}$$
(22)

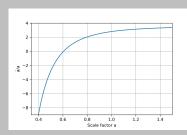


Figure: The figure shows acceleration $\frac{\ddot{a}}{a}$ vs red-shift a.

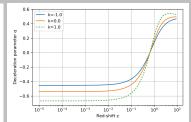


Figure: The figure shows deceleration parameter q vs red-shift z.

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Conclusions

▶ Both Chaplygin gas and the PPUDF equations of state result in behaviour for the energy densities, dimensionless Hubble parameter and acceleration that corresponds with the Concordance model for dust dominated epochs and Dark energy dominated epochs.

It is possible to unify both dark elements into a single dark fluid by parametrizing the equation of state.

► Shortcomings of the Chaplygin gas and PPUDF models.

Future work on this would include constraining the free parameters with observation. Solving the Friedman equation for a Dark Fluid equation of state.

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