5 July 2001

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 511 (2001) 265–268

www.elsevier.nl/locate/npe

An alternative to quintessence

Alexander Kamenshchik [a,b], Ugo Moschella [c], Vincent Pasquier [d]

[a] *L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 2 Kosygina street, 117334 Moscow, Russia*
[b] *Landau Network–Centro Volta, Villa Olmo, via Cantoni 1, 22100 Como, Italy*
[c] *Dipartimento di Scienze Matematiche Fisiche e Chimiche, Università dell Insubria, Via Valleggio 11, 22100 Como and INFN sez. di Milano, Italy*
[d] *Service de Physique Théorique, C.E. Saclay, 91191 Gif-sur-Yvette, France*

Received 26 February 2001; received in revised form 20 April 2001; accepted 27 April 2001
Editor: R. Gatto

Abstract

We consider a FRW cosmological model with an exotic fluid known as Chaplygin gas. We show that the resulting evolution of the universe is not in disagreement with the current observation of cosmic acceleration. The model predict an increasing value for the effective cosmological constant.

The discovery that the expansion of the universe is accelerating [1] has promoted the search for new types of matter that can behave like a cosmological constant [2,3] by combining positive energy density and negative pressure. This type of matter is often called “quintessence”.

Since in a variety of inflationary models scalar fields have been used in describing the transition from the quasi-exponential expansion of the early universe to a power law expansion, it is natural to try to understand the present acceleration of the universe, which has an exponential behaviour too, by constructing models where the matter responsible for such behaviour is also represented by a scalar field [4]. However, now we deal with the opposite task, i.e., we would like to describe the transition from a universe filled with dust-like matter to an exponentially expanding universe, and scalar fields are not the only possibility but there are (of course) alternatives. In particular, one can try to do it by using some perfect fluid but obeying “exotic” equations of state.

In this short Letter we consider an example of this type: the so-called *Chaplygin gas*. Under this name we mean a perfect fluid having the following equation of state:

$$p = -\frac{A}{\rho}, \tag{1}$$

where p and ρ are, respectively, pressure and energy density in a comoving reference frame, with $\rho > 0$; A is a positive constant.

Chaplygin introduced his equation of state [5] as a convenient soluble model to study the lifting force on a plane wing in aerodynamics. Later on, the same equations were rediscovered in [6,7], again in an aerodynamical context. The integrability of the corresponding Euler equations resides in the fact that they have a large symmetry group (see [9] for a modern description).

Chaplygin’s equation of state (1) has raised recently a renewed interest [10] because of its many remarkable and, in some sense, intriguingly unique features.

0370-2693/01/$ – see front matter
PII: S0370-2693(01)00571-8

Indeed it has an amusing connection with string theory: it can be obtained from the Nambu–Goto action for d-branes moving in a $(d+2)$-dimensional spacetime in the light-cone parametrization [11]. Also, the Chaplygin gas is the only fluid which, up to now, admits a supersymmetric generalization [12,13].

The negative pressure following from Chaplygin's equation of state could also be used for the description of effects in deformable solids [8].

Finally, another remarkable feature is that it gives positive and bounded (see below) square of sound velocity $v_s^2 = A/\rho^2$, and this is non-trivial for fluids with negative pressure.

We ourselves came across this fluid [14] when studying the stabilization of branes [15] in black hole bulks [16]. We found that to obtain stabilization it is necessary to add matter on the branes which again obeys the equation of state (1).

For these reasons we have undertaken the simple exercise of stydying a FRW cosmology of a universe filled with a Chaplygin gas. The metric of a homogeneous and isotropic universe is usually written as follows

$$ds^2 = dt^2 - a^2(t)\, dl^2, \tag{2}$$

where dl^2 is the metric of a 3-manifold of constant curvature ($K = 0, \pm 1$), and the expansion factor $a(t)$ evolves according to the Friedman equation

$$\frac{\dot{a}^2}{a^2} = \rho - \frac{K}{a^2}. \tag{3}$$

Energy conservation

$$d\left(\rho a^3\right) = -p\, d\left(a^3\right) \tag{4}$$

together with the equation of state (1) give the following relation:

$$\rho = \sqrt{A + \frac{B}{a^6}}, \tag{5}$$

where B is an integration constant.

By choosing a positive value for B we see that for small a (i.e., $a^6 \ll B/A$) the expression (5) is approximated by

$$\rho \sim \frac{\sqrt{B}}{a^3} \tag{6}$$

that corresponds to a universe dominated by dust-like matter. For large values of the cosmological radius a it follows that

$$\rho \sim \sqrt{A}, \qquad p \sim -\sqrt{A}, \tag{7}$$

which, in turn, corresponds to an empty universe with a cosmological constant $\sqrt{A}$ (i.e., a de-Sitter universe). In the flat case it is possible also to find exact solutions as follows:

$$t = \frac{1}{6\sqrt[4]{A}}\left(\ln \frac{\sqrt[4]{A + B/a^6} + \sqrt[4]{A}}{\sqrt[4]{A + B/a^6} - \sqrt[4]{A}} - 2 \arctan \sqrt[4]{1 + \frac{B}{Aa^6}}\right). \tag{8}$$

Note that $\sqrt{A}$ solves the equation

$$\rho + p = \rho - \frac{A}{\rho} = 0. \tag{9}$$

The circumstance that this equation has a nonzero solution lies at the heart of the possibility of interpreting the model as a "quintessential" model. If this model were realistic we could estimate the constant A by comparing our expressions for pressure and energy with observational data. An indirect and naive way to do it is to consider the nowadays accepted values for the contributions of matter and cosmological constant to the energy density of the universe. To use these data we decompose pressure and energy density as follows:

$$p = p_\Lambda + p_M = -\Lambda, \tag{10}$$

$$\rho = \rho_\Lambda + \rho_M = \Lambda + \rho_M. \tag{11}$$

An application of Eq. (1) gives

$$A = \Lambda(\Lambda + \rho_M). \tag{12}$$

If the cosmological constant contributes seventy percent of the energy we get $\sqrt{A} \approx 1.2\, \Lambda$. We now observe that, in the context of a Chaplygin cosmology, once an expanding universe starts accelerating it cannot decelerate any more. Indeed Eqs. (3) and (4) imply that

$$\frac{\ddot{a}}{a} = -\frac{1}{2}(\rho + 3p). \tag{13}$$

Condition $\ddot{a} > 0$ is equivalent to

$$a^6 > \frac{B}{2A}, \tag{14}$$

which is obviously preserved by time evolution in an expanding universe. It thus follows that the observed value Λ of the (effective) cosmological constant will increase up to $1.2\,\Lambda$.

Considering now the subleading terms in Eq. (5) at large values of a (i.e., $a^6 \gg B/A$), one obtains the following expressions for the energy and pressure:

$$\rho \approx \sqrt{A} + \sqrt{\frac{B}{4\,A}}\, a^{-6}, \tag{15}$$

$$p \approx -\sqrt{A} + \sqrt{\frac{B}{4\,A}}\, a^{-6}. \tag{16}$$

Eqs. (15) and (16) describe the mixture of a cosmological constant $\sqrt{A}$ with a type of matter known as "stiff" matter, and described by the following equation of state:

$$p = \rho. \tag{17}$$

Note that a massless scalar field is a particular instance of stiff matter. Therefore, in a generic situation, a Chaplygin cosmology can be looked at as interpolating between different phases of the universe: from a dust dominated universe to a de Sitter universe passing through an intermediate phase which is the mixture just mentioned above. The interesting point, however, is that such an evolution is accounted by using only one fluid.

In recent series of paper [17] a similar type of evolution has been described, where the universe passes from a dust dominated epoch to a de Sitter phase through an intermediate phase described as mixture of cosmological constant and radiation. In [17] the mechanism responsible for this behaviour is different from ours and is based on the quantum corrections to the effective action of a massive scalar field. However, these corrections lead to a "standard" equation of state in the form of a mixture. We can reproduce this type of evolution by a slight modification to our "exotic" equation of state (1), namely,

$$p = -\frac{C}{\sqrt[3]{\rho}}. \tag{18}$$

The obvious generalization $p = -C\rho^{-\alpha}$ (with $\alpha \geqslant -1$) gives a similar evolution from dust to cosmological constant with an intermediate epoch that can be seen as a mixture of a cosmological constant with a fluid obeying the state equation $p = \alpha\rho$.

For open or flat Chaplygin cosmologies ($K = -1, 0$), the universe always evolves from a decelerating to an accelerating epoch. For the closed Chaplygin cosmological models ($K = 1$), the Friedman equations (3) and (13) say that it is possible to have a static Einstein universe solution $a_0 = (3A)^{-1/4}$ provided the following condition holds:

$$B = \frac{2}{3\sqrt{3\,A}}. \tag{19}$$

When $B > 2/(3\sqrt{3\,A})$ the cosmological radius $a(t)$ can take any value while if

$$B < \frac{2}{3\sqrt{3\,A}},$$

there are two possibilities: either

$$a < a_1 = \frac{1}{\sqrt{3\,A}}\left(\sqrt{3}\,\sin\frac{\varphi}{3} - \cos\frac{\varphi}{3}\right) \tag{20}$$

or

$$a > a_2 = \frac{2}{\sqrt{3\,A}}\cos\frac{\varphi}{3}, \tag{21}$$

where $\varphi = \pi - \arccos 3\sqrt{3A}\,B/2$. The region $a_1 < a < a_2$ is not accessible.

Results connected to those presented in this Letter have been obtained by Barrow [18–21] who has considered cosmologies with fluids admitting a bulk viscosity proportional to a power of the density.

In the flat $K = 0$ case the FRW equations for Chaplygin fits in Barrow's scheme as a special case [19] and indeed a transition from power law to exponential expansion is noticed already in [18,22]. However, since the state equation for the corresponding fluid is different from our Eq. (1) this coincidence of the solutions is destroyed by any small perturbation, for instance by a small spatial curvature or by adding another matter source. We mention also that the role of the Chaplygin-like behaviour in cosmology was also noticed in Refs. [23,24].

Following [20] we now try to find a homogeneous scalar field $\phi(t)$ and a potential $V(\phi)$ to describe the Chaplygin cosmology. Let us consider therefore the Lagrangian

$$L(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{22}$$

and set the energy density of the field equal to that of the Chaplygin gas:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \sqrt{A + \frac{B}{a^6}}. \tag{23}$$

The corresponding "pressure" coincides with the Lagrangian density:

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -\frac{A}{\sqrt{A + B/a^6}}. \tag{24}$$

It immediately follows that

$$\dot{\phi}^2 = \frac{B}{a^6\sqrt{A + B/a^6}} \tag{25}$$

and

$$V(\phi) = \frac{2a^6(A + B/a^6) - B}{2a^6\sqrt{A + B/a^6}}. \tag{26}$$

Now let us restrict ourselves to the flat case $K = 0$. In this case Eq. (25) also implies that

$$\phi' = \frac{\sqrt{B}}{a(Aa^6 + B)^{1/2}}, \tag{27}$$

where prime means differentiation w.r.t. a. This equation can be integrated and it follows that

$$a^6 = \frac{4B\exp(6\phi)}{A(1 - \exp(6\phi))^2}. \tag{28}$$

Finally, by substituting the latter expression for the cosmological radius in Eq. (26) one obtains the following potential, which has a surprisingly simple form:

$$V(\phi) = \frac{1}{2}\sqrt{A}\left(\cosh 3\phi + \frac{1}{\cosh 3\phi}\right). \tag{29}$$

Note that the potential does not depend on the integration constant B and therefore it reflects only the state equation (1) as it should.

In conclusion Chaplygin cosmology provides an interesting possibility to account for current observations about the expansion of the universe. It predicts also that the cosmological constant will increase (or that it was less in the past) and this could in principle be observed. Of course to take this model seriously one should have a good fundamental reason to believe in Eq. (1).

Acknowledgements

We are grateful to Prof. J.D. Barrow who has kindly attracted our attention to his papers [18–21]. The work of A.K. was partially supported by RFBR via grants No 99-02-18409 and No 99-02-16224 and by the Cariplo Science Foundation.

References

[1] N.A. Bachall, J.P. Ostriker, S. Perlmutter, P.J. Steinhardt, Science 284 (1999) 1481.
[2] R.R. Caldwell, R. Dave, P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582, astro-ph/9708069.
[3] V. Sahni, A. Starobinsky, Int. J. Mod. Phys. D 9 (2000) 373, astro-ph/9904398.
[4] A.A. Starobinsky, JETP Lett. 68 (1998) 757;
T.D. Saini, S. Raychaudhury, V. Sahni, A.A. Starobinsky, Phys. Rev. Lett. 85 (2000) 1162.
[5] S. Chaplygin, Sci. Mem. Moscow Univ. Math. Phys. 21 (1904) 1.
[6] H.-S. Tsien, J. Aeron. Sci. 6 (1939) 399.
[7] T. von Karman, J. Aeron. Sci. 8 (1941) 337.
[8] K. Stanyukovich, Unsteady Motion of Continuos Media, Pergamon, Oxford, 1960.
[9] D. Bazeia, R. Jackiw, Ann. Phys. (NY) 270 (1998) 246.
[10] D. Bazeia, Phys. Rev. D 59 (1999) 085007;
R. Jackiw, A.P. Polychronakos, Commun. Math. Phys. 207 (1999) 107;
N. Ogawa, hep-th/0003288.
[11] J. Goldstone, unpublished;
M. Bordemann, J. Hoppe, Phys. Lett. B 317 (1993) 315.
[12] J. Hoppe, hep-th/9311059.
[13] R. Jackiw, A.P. Polychronakos, Phys. Rev. D 62 (2000) 085019.
[14] A. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B 487 (2000) 7.
[15] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690.
[16] M. Banados, M. Henneaux, C. Teitelboim, J. Zanelli, Phys. Rev. D 48 (1993) 1506.
[17] L. Parker, A. Raval, Phys. Rev. D 60 (1999) 063512, gr-qc/9905031;
L. Parker, A. Raval, Phys. Rev. D 60 (1999) 063512, gr-qc/9905031;
L. Parker, A. Raval, Phys. Rev. D 62 (2000) 083503, gr-qc/0003103.
[18] J.D. Barrow, Phys. Lett. B 180 (1986) 335.
[19] J.D. Barrow, Nucl. Phys. B 310 (1988) 743.
[20] J.D. Barrow, Phys. Lett. B 235 (1990) 40.
[21] J.D. Barrow, P. Saich, Phys. Lett. B 249 (1990) 406.
[22] R. Treciokas, G.F.R. Ellis, Commun. Math. Phys. 23 (1971) 1.
[23] R. Klippert, H.C. Rosu, R. Ruffini, gr-qc/0008036.
[24] H.C. Rosu, gr-qc/0003108.