Solving the Friedman equation for a Dark Fluid equation of state.

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Introduction

Aim and Motivation

- 2 Dark elements
- Is it possible to describe the cosmological behaviour resulting from these elements using a single dark fluid?

Introduction

- **General Relativity:**
  Einstein‘s field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

- **Hubble‘s law:**

$$\nu = H_0 r \tag{2}$$

- **Expanding universe:**

$$r(t) = a(t)\chi \tag{3}$$

Friedmann equations

- **Cosmological principle:**
  The universe is homogeneous and isotropic [1].
- **Fluid equation:**

$$\dot{\rho} + 3H(1+\omega)\rho = 0, \text{ with } H = \frac{\dot{a}}{a} \tag{4}$$

- **Friedmann equation:**

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\rho a^2 - \kappa\frac{c^2}{\chi^2} \tag{5}$$

- **Raychaudhuri equation:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho + 3P) \tag{6}$$

Concordance model (ΛCDM-model)

- **Hot Big Bang**
- **Dark Matter:**
  - Galaxy rotation
  - Behaviour of super structure
- **Accelerated expansion:**
  Observations of the luminosities of type Ia supernovae suggest that the universe is undergoing an accelerated expansion [2, 3], which suggests the existence of a Dark energy element.
- **Assume a perfect fluid equation of state:**

$$P = \omega\rho \tag{7}$$

- **3 Different epochs:**
  - Radiation ($\omega = \frac{1}{3}$): $\rho = C_{rad}a^{-4}$
  - Matter ($\omega = 0$): $\rho = C_{dust}a^{-3}$
  - Dark Energy ($\omega = -1$): $\rho = C_{DE}$

Chaplygin gas

- Different Chaplygin gas equations of state [4]:
  - Original Chaplygin gas (OCG):

$$P = -\frac{A_1}{\rho} \tag{8}$$

  - Generalised Chaplygin gas (GCG):

$$P = -\frac{A_1}{\rho^\alpha}, \quad \alpha > -1 \tag{9}$$

  - Modified Chaplygin gas (MCG):

$$P = A_2\rho - \frac{A_1}{\rho^\alpha}, \quad \alpha > -1 \tag{10}$$

Chaplygin gas

Solution to the fluid equation

- **Solving the Fluid equation for a MCG equation of state:**

$$\rho = \left[\frac{C_2 \left(1+z\right)^{3(\alpha+1)(1+A_2)} + A_1}{1+A_2} \right]^{\frac{1}{1+\alpha}} \tag{11}$$

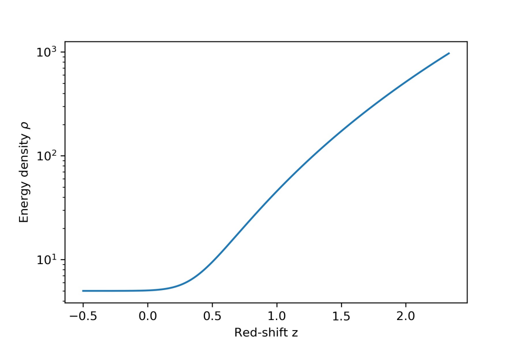


Figure: Here we have taken $A_1 = 50$, $A_2 = C_2 = 1$ and $\alpha = 1$. The figure shows energy density ρ vs red-shift z.

Chaplygin gas

Solving the Friedmann equation for the MCG equation of state

- Assuming a $\kappa = 0$, we have:

$$
\begin{aligned}
(t - t_0) = & \frac{2}{3A^{\frac{1}{2}} B_2^{\frac{1}{2\beta}} B_1} \left(\frac{B_3}{B_2} a^{-3B_1\beta} + 1 \right)^{-\frac{1}{2\beta}} \\
& + \frac{1}{2\beta + 1} \left[\left(\frac{B_3}{B_2} a^{-3B_1\beta} + 1 \right)^{-1-\frac{1}{2\beta}} {}_2F_1 \left(1, 1 + \frac{1}{2\beta}; 2 + \frac{1}{2\beta}; \left(\frac{B_3}{B_2} a^{-3B_1\beta} + 1 \right)^{-1} \right) \right]
\end{aligned}
\tag{12}
$$

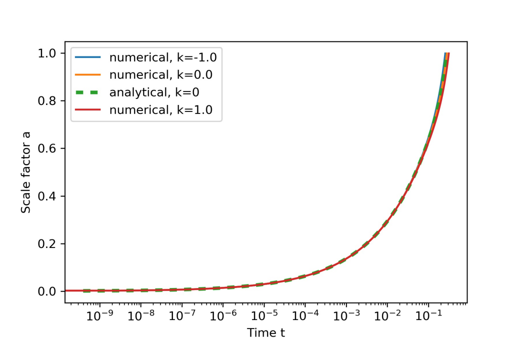


Figure: The figure shows scale factor a vs time t.

Chaplygin gas

Hubble parameter for MCG case

- Dimensionless Hubble parameter h

$$h(z) = \frac{1}{H_0}\left[A\left(B_3\left(1+z\right)^{3(\beta)(B_1)} + B_2\right)^{\frac{1}{\beta}} - \kappa F\left(1+z\right)^2\right]^{\frac{1}{2}} \tag{13}$$

- Fractional energy density Ω

$$\begin{aligned} \Omega_{Chap}(z) &\equiv \frac{A}{H_0^2}\left(B_3\left(1+z\right)^{3(\beta)(B_1)} + B_2\right)^{\frac{1}{\beta}} \\ \Omega_{\kappa}(z) &\equiv -\frac{\kappa F}{H_0^2}\left(1+z\right)^2 \end{aligned} \tag{14}$$

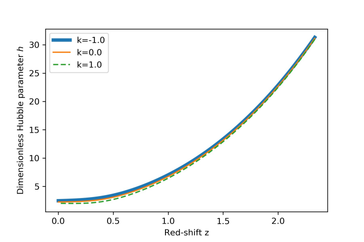


Figure: The figure shows the dimensionless Hubble parameter h vs red-shift z.

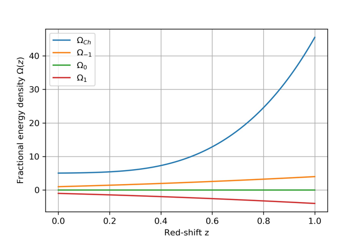


Figure: The figure shows fractional energy density Ω vs red-shift z.

Chaplygin gas

Acceleration of a for MCG case

- Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2}\left((3B_1-2)\left(B_3a^{-3B_1\beta}+B_2\right)^{\frac{1}{\beta}} - 3B_1B_2\left(B_3a^{-3\beta B_1}+B_2\right)^{\frac{1-\beta}{\beta}}\right) \quad (15)$$

- Deceleration parameter $q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{\frac{A}{2}\left((3B_1-2)\left(B_3\left(1+z\right)^{3B_1\beta}+B_2\right)^{\frac{1}{\beta}} - 3B_1B_2\left(B_3\left(1+z\right)^{3\beta B_1}+B_2\right)^{\frac{1-\beta}{\beta}}\right)}{A\left(B_3\left(1+z\right)^{3(\beta)(B_1)}+B_2\right)^{\frac{1}{\beta}} - \kappa F\left(1+z\right)^2} \quad (16)$$

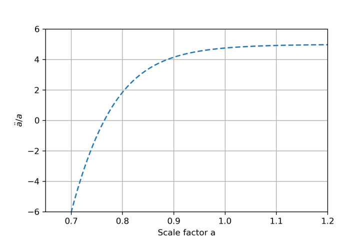


Figure: The figure shows acceleration $\frac{\ddot{a}}{a}$ vs red-shift a.

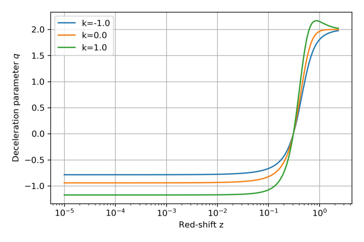


Figure: The figure shows deceleration parameter q vs red-shift z.

Pressure-Parametrized Unified Dark Fluid (PPUDF)

- **The PPUDF equation of state [5]:**

$$P = P_a + P_b \left(z + \frac{z}{1+z} \right) \tag{17}$$

- **Solving the Fluid equation for a PPUDF equation of state:**

$$\rho = -P_a + \frac{3}{4} P_b \left[(1+z)^{-1} - 2(1+z) \right] + C(1+z)^3 \tag{18}$$

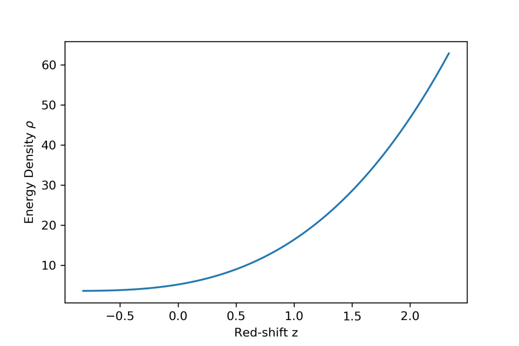


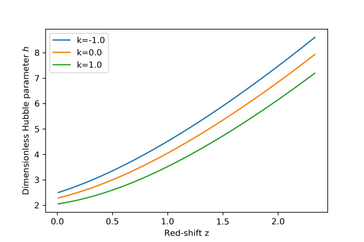
Figure: The figure shows energy density ρ vs red-shift z.

- Dimensionless Hubble parameter h

$$h = \frac{1}{H_0}\left[A\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right) - \kappa F(1+z)^2\right]^{\frac{1}{2}} \quad (19)$$

- Fractional energy density Ω

$$\begin{aligned} \Omega_{PPUDF}(z) &\equiv \frac{A}{H_0^2}\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right) \\ \Omega_\kappa(z) &\equiv -\frac{\kappa F}{H_0^2}(1+z)^2 \end{aligned} \quad (20)$$



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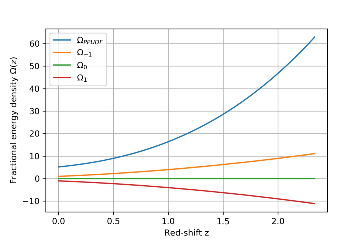


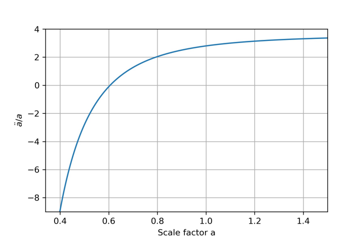
Figure: The figure shows fractional energy density Ω vs red-shift z.

- Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2}\left[2P_a - \frac{3}{2}P_b\left(\frac{3}{2}a + a^{-1}\right) + Ca^{-3}\right] \tag{21}$$

- Deceleration parameter $q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{A\left[2P_a - \frac{3}{2}P_b\left(\frac{3}{2}(1+z)^{-1} + (1+z)\right) + C(1+z)^3\right]}{2A\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right) - \kappa F(1+z)^2} \tag{22}$$



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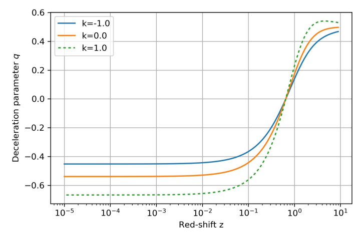

Figure: The figure shows deceleration parameter q vs red-shift z.

Conclusions

- Both Chaplygin gas and the PPUDF equations of state result in behaviour for the energy densities, dimensionless Hubble parameter and acceleration that corresponds with the Concordance model for dust dominated epochs and Dark energy dominated epochs.
- It is possible to unify both dark elements into a single dark fluid by parametrizing the equation of state.
- Shortcomings of the Chaplygin gas and PPUDF models.
- Future work on this would include constraining the free parameters with observation.

Acknowledgements

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[1] M Roos. *Introduction to Cosmology*. John Wiley and Sons Ltd, 2015.

[2] Brian P. Schmidt. Nobel lecture: Accelerating expansion of the universe through observations of distant supernovae. *Rev. Mod. Phys.*, 84:1151–1163, Aug 2012.

[3] Ujjal Debnath, Asit Banerjee, and Subenoy Chakraborty. Role of modified chaplygin gas in accelerated universe. *Classical and Quantum Gravity*, 21(23):5609, 2004.

[4] EO Kahya and B Pourhassan. The universe dominated by the extended chaplygin gas. *Modern Physics Letters A*, 30(13):1550070, 2015.

[5] Deng Wang, Yang-Jie Yan, and Xin-He Meng. A new pressure-parametrization unified dark fluid model. *The European Physical Journal C*, 77(4):263, 2017.