# Solving the Friedman equation for a Dark Fluid equation of state.

Pieter vd Merwe

North-West University Centre for Space Research

October 9, 2018

# Introduction

- **Cosmological principle:**
  The universe is homogeneous and isotropic [1].
- **General Relativity:**
  Einstein's field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{1}$$

- **Hubble's law:**

$$\nu = H_0 r. \tag{2}$$

- **Expanding universe:**

$$r(t) = a(t)\chi. \tag{3}$$

# Friedmann equations

- **Fluid equation:**

$$\dot{\rho} + 3H(1+\omega)\rho = 0, \text{ with } H = \frac{\dot{a}}{a} \tag{4}$$

.

- **Friedmann equation:**

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\rho a^2 - \kappa \frac{c^2}{\chi^2} \tag{5}$$

- **Raychaudhuri equation:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho + 3P) \tag{6}$$

# Concordance model (ΛCDM-model)

- **Dark Matter.**
- **Accelerated expansion:**
  Observations of the luminosities of type Ia supernovae suggest that the universe is undergoing an accelerated expansion [2, 3], which suggests the existence of a Dark energy element.
- **Assume a perfect fluid equation of state:**

$$P = \omega\rho \tag{7}$$

- **3 Different epochs:**
  - Radiation ($\omega = \frac{1}{3}$): $\rho = C_{rad} a^{-4}$
  - Matter ($\omega = 0$): $\rho = C_{dust} a^{-3}$
  - Dark Energy ($\omega = -1$): $\rho = C_{DE}$

# Chaplygin gas

- Different Chaplygin gas equations of state [4]:
  - Original Chaplygin gas (OCG):

$$P = -\frac{A_1}{\rho}. \tag{8}$$

  - Generalised Chaplygin gas (GCG):

$$P = -\frac{A_1}{\rho^{\alpha}}, \quad \alpha > -1. \tag{9}$$

  - Modified Chaplygin gas (MCG):

$$P = A_2\rho - \frac{A_1}{\rho^{\alpha}}, \quad \alpha > -1. \tag{10}$$

# Chaplygin gas

## Solution to the fluid equation

- **Solving the Fluid equation for a MCG equation of state:**

$$\rho = \left[ \frac{C_2 \left(1+z\right)^{3(\alpha+1)(1+A_2)} + A_1}{1+A_2} \right]^{\frac{1}{1+\alpha}}, \tag{11}$$

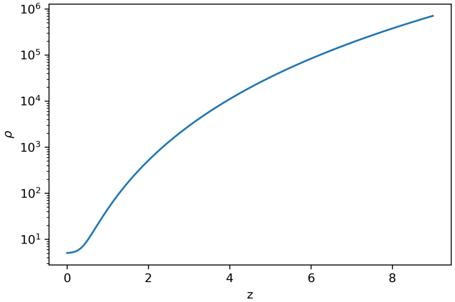


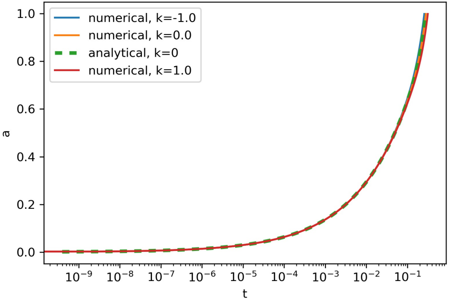

Figure: Here we have taken $A_1 = 50$, $A_2 = C_2 = 1$ and $\alpha = 1$.

# Chaplygin gas

## Solving the Friedmann equation for the MCG equation of state

- Assuming a $\kappa = 0$, we have:

$$(t - t_0) = \frac{2}{3A^{\frac{1}{2}} B_2^{\frac{1}{2\beta}} B_1} \left( \frac{B_3}{B_2} a^{-3B_1\beta} + 1 \right)^{-\frac{1}{2\beta}} + \frac{1}{2\beta + 1} \left( \frac{B_3}{B_2} a^{-3B_1\beta} + 1 \right)^{-1-\frac{1}{2\beta}} {}_2F_1\left(1, 1 + \frac{1}{2\beta}; 2 + \frac{1}{2\beta}; \left( \frac{B_3}{B_2} a^{-3B_1\beta} + 1 \right)^{-1} \right), \tag{12}$$



# Chaplygin gas

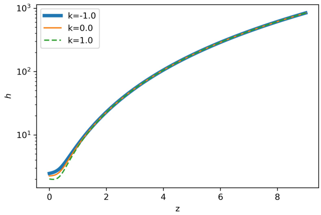
## Hubble parameter for MCG case

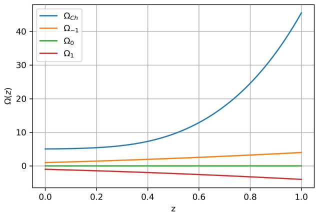
- Dimensionless Hubble parameter $h$

$$h(z) = \frac{1}{H_0}\left[A\left(B_3\left(1+z\right)^{3(\beta)(B_1)} + B_2\right)^{\frac{1}{\beta}} - \kappa F\left(1+z\right)^2\right]^{\frac{1}{2}}. \qquad (13)$$

- Fractional energy density $\Omega$

$$\Omega_{Chap}(z) \equiv \frac{A}{H_0^2}\left(B_3\left(1+z\right)^{3(\beta)(B_1)} + B_2\right)^{\frac{1}{\beta}}$$
$$\Omega_{\kappa}(z) \equiv -\frac{\kappa F}{H_0^2}\left(1+z\right)^2. \qquad (14)$$



# Chaplygin gas

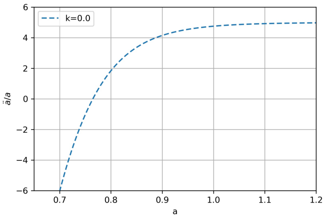
## Acceleration of a for MCG case

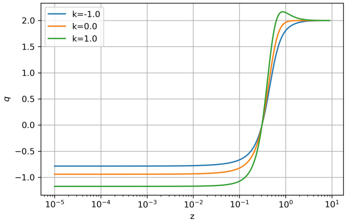
- Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2}\left((3B_1-2)\left(B_3 a^{-3B_1\beta}+B_2\right)^{\frac{1}{\beta}} - 3B_1B_2\left(B_3 a^{-3\beta B_1}+B_2\right)^{\frac{1-\beta}{\beta}}\right), \qquad (15)$$

- Deceleration parameter $q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{\frac{A}{2}\left((3B_1-2)\left(B_3\left(1+z\right)^{3B_1\beta}+B_2\right)^{\frac{1}{\beta}} - 3B_1B_2\left(B_3\left(1+z\right)^{3\beta B_1}+B_2\right)^{\frac{1-\beta}{\beta}}\right)}{A\left(B_3\left(1+z\right)^{3(\beta)(B_1)}+B_2\right)^{\frac{1}{\beta}} - \kappa F\left(1+z\right)^2}. \qquad (16)$$



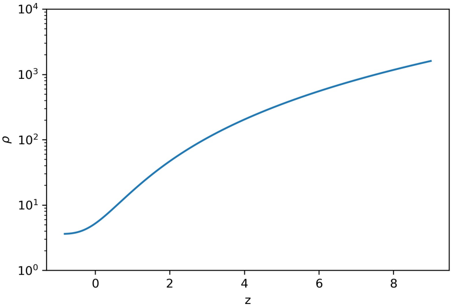
## Pressure-Parametrized Unified Dark Fluid (PPUDF)

- **The PPUDF equation of state [5]:**

$$P = P_a + P_b\left(z + \frac{z}{1+z}\right), \tag{17}$$

- **Solving the Fluid equation for a PPUDF equation of state:**

$$\rho = -P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3, \tag{18}$$



# Pressure-Parametrized Unified Dark Fluid (PPUDF)

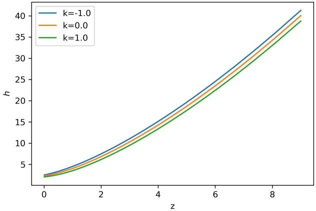
## Hubble parameter for PPUDF case

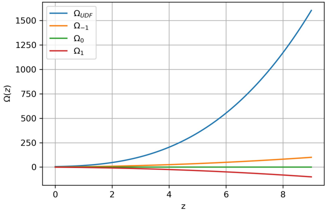
- Dimensionless Hubble parameter $h$

$$h = \frac{1}{H_0}\left[A\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right) - \kappa F(1+z)^2\right]^{\frac{1}{2}}. \tag{19}$$

- Fractional energy density $\Omega$

$$\Omega_{PPUDF}(z) \equiv \frac{A}{H_0^2}\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right)$$
$$\Omega_{\kappa}(z) \equiv -\frac{\kappa F}{H_0^2}(1+z)^2. \tag{20}$$



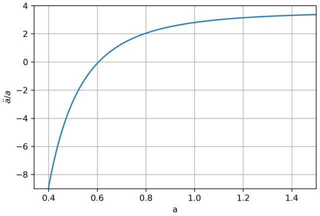
## Acceleration of a for PPUDF case

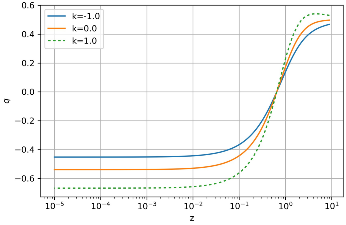
- Acceleration of a:

$$\frac{\ddot{a}}{a} = -\frac{A}{2}\left[2P_a - \frac{3}{2}P_b\left(\frac{3}{2}a + a^{-1}\right) + Ca^{-3}\right]. \tag{21}$$

- Deceleration parameter $q \equiv -\frac{\ddot{a}a}{\dot{a}^2}$

$$q = \frac{A\left[2P_a - \frac{3}{2}P_b\left(\frac{3}{2}(1+z)^{-1} + (1+z)\right) + C(1+z)^3\right]}{2A\left(-P_a + \frac{3}{4}P_b\left[(1+z)^{-1} - 2(1+z)\right] + C(1+z)^3\right) - \kappa F(1+z)^2}. \tag{22}$$



## Conclusions

- stuff

[1] M Roos. *Introduction to Cosmology*. John Wiley and Sons Ltd, 2015.

[2] Brian P. Schmidt. Nobel lecture: Accelerating expansion of the universe through observations of distant supernovae. *Rev. Mod. Phys.*, 84:1151–1163, Aug 2012.

[3] Ujjal Debnath, Asit Banerjee, and Subenoy Chakraborty. Role of modified chaplygin gas in accelerated universe. *Classical and Quantum Gravity*, 21(23):5609, 2004.

[4] EO Kahya and B Pourhassan. The universe dominated by the extended chaplygin gas. *Modern Physics Letters A*, 30(13):1550070, 2015.

[5] Deng Wang, Yang-Jie Yan, and Xin-He Meng. A new pressure-parametrization unified dark fluid model. *The European Physical Journal C*, 77(4):263, 2017.