SS-NN models in fluid dynamics: pitching aerofoil

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Introduction

The goal of this report is to briefly describe the effectiveness of the State-Space Neural Networks (SS-NN) technique for the identification of nonlinear state-space models in fluid dynamics. The test case under consideration consists in the modeling of the unsteady lift force of a pitching NACA 0018 airfoil at a Reynolds number $Re = 3 \times 10^5$. This specific application is particularly interesting and challenging from a modeling perspective, since the underlying data is highly nonlinear. The SS-NN model is trained using swept sines performed at several angle-of-attack ranges, and it is then validated using different sine sweeps as well as single-sine motions. The work demonstrates that the selected SS-NN model represents a powerful tool to accurately predict the unsteady aerodynamic loads of a pitching airfoil, both in pre-stall and post-stall conditions. The accuracy and evaluation speed of this technique make it particularly valuable for all engineering applications involving design optimization and real-time control of systems based on lift.

The SS-NN methodology has been successfully tested on numerical data obtained from Computational Fluid Dynamics (CFD) simulations (see Section 1).

1 SS-NN model applied to CFD data

The considered nonlinear State-Space Neural Network (SS-NN) model is defined as follows:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + W_x\, \sigma\left(\begin{bmatrix} W_{fx} & W_{fu} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + b_f \right) + b_x \\ y(k) &= \begin{bmatrix} C & D \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + W_y\, \sigma\left(\begin{bmatrix} W_{gx} & W_{gu} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + b_g \right) + b_y \end{aligned} \tag{1}$$

In this work, the input $u(t)$ is the time-varying angle of attack of the airfoil $\alpha(t)$, and the output $y(t)$ is its corresponding lift coefficient $C_l(t)$, such that a Single Input Single Output (SISO) model is obtained. The selected model has $n = 3$ states and a neural network with one hidden layer composed of $n_x = n_y = 30$ neurons.

1.1 Model training

Broadband sine-sweep excitations are chosen for model training. The selected sine-sweep motions cover the frequency band $f = [0-2]$ Hz, and they are defined as

$$\alpha(t) = \begin{cases} \alpha_0 + \alpha_1 \sin\big[(a_1 t + b_1)t\big] & \text{for} \quad 0 \le t < T_0 \\ \alpha_0 + \alpha_1 \sin\big[(a_2(t - T_0) + b_2)(t - T_0)\big] & \text{for} \quad T_0 \le t \le 2\,T_0 \end{cases} \tag{2}$$

where α_0 is the mean angle of attack, α_1 is the pitch amplitude, T_0 is the half-sweep period, $a_1 = \pi(f_{\max} - f_{\min})/T_0$, $a_2 = \pi(f_{\min} - f_{\max})/T_0$, $b_1 = 2\pi f_{\min}$, and $b_2 = 2\pi f_{\max}$. The complete training dataset is obtained by concatenating eight different swept sines of equal length $T_0 = 10$ s, characterized by four different mean angles of attack $\alpha_0 = [5°, 10°, 14°, 18°]$ and two different pitch amplitudes $\alpha_1 = [6°, 10°]$. In order to evaluate the accuracy of the model, the relative root-mean-square error (e_{rel}) and the mean absolute error (e_{abs}) are defined as follows

$$e_{\text{rel}} = \sum_{i=1}^{n} e_{\text{rel}\,i} \frac{N_i}{N} \quad \text{where} \quad e_{\text{rel}\,i} = \sqrt{\frac{\sum_{k=1}^{N_i} (y_{\text{CFD}(k)} - y_{\text{model}(k)})^2}{\sum_{k=1}^{N_i} (y_{\text{CFD}(k)} - \bar{y}_{\text{CFD},i})^2}} \tag{3}$$

$$e_{\text{abs}} = \frac{\sum_{k=1}^{N} |y_{\text{CFD}(k)} - y_{\text{model}(k)}|}{N} \tag{4}$$

In Eqs. (3) and (4), N represents the total number of sample points, n is the number of parts in which the data is divided, N_i is the number of sample points of the i^{th} data part and $\bar{y}_{\text{CFD},i}$ is the mean output value associated to the i^{th} data part.

The input signal used for training, $\alpha(t)$, and its corresponding output, $C_l(t)$, are illustrated in Fig. 1 together with a representation of the modeling error calculated as the difference between the training data and the output of the selected NN-SS model at every time instant. The overall model error is very low ($e_{\text{rel}} = 1.38\%$, $e_{\text{abs}} = 0.0031$), and it remains moderate even at the highest angle-of-attack range, where the flow exhibits a high level of unsteadiness and nonlinearity.

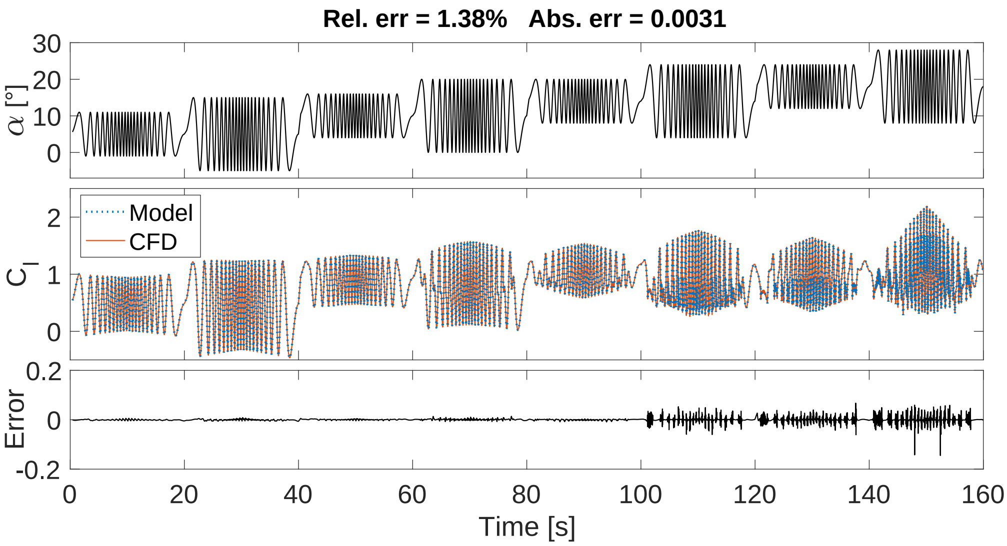
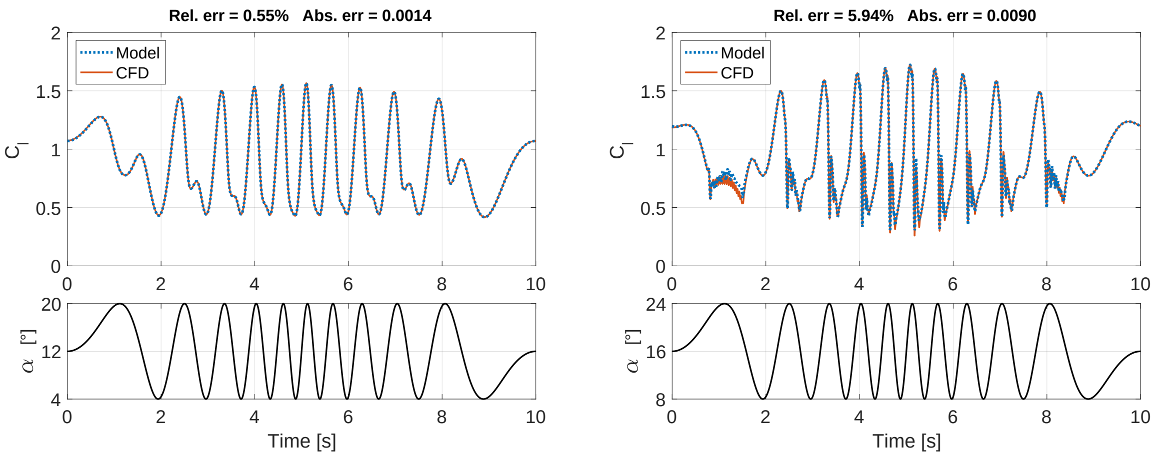


Figure 1: Model training. Top: input of the training data. Middle: output of the training data and SS-NN model prediction. Bottom: SS-NN model error.

1.2 Model validation

Sine-sweep motions

Firstly, the model is validated on two specifically designed sine-sweep motions characterized by a different period, mean angle of attack and pitch amplitude compared to the sine sweeps used for training. More specifically, the period is halved ($T_0 = 5$ s), the pitch amplitude is set to 8°, and the mean angle of attack is set to 12° and 16°, respectively. Figure 2 illustrates the two imposed sweep motions and their corresponding lift coefficient. Overall, a very good agreement is observed between the CFD results and the SS-NN model simulation for both angle-of-attack ranges.

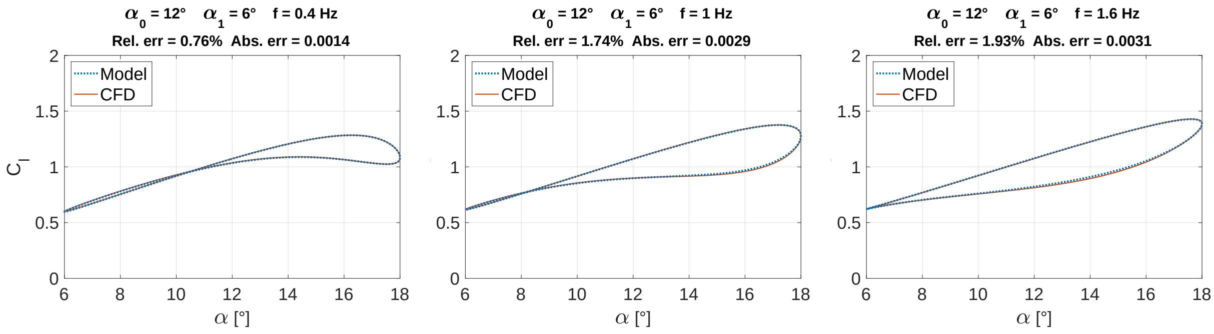


(a) $\alpha_0 = 12°$ (b) $\alpha_0 = 16°$

Figure 2: Model validation using two different sine-sweep motions.

Sinusoidal motions

The model is also validated on sinusoidal motions characterized by different oscillating frequency and/or angle-of-attack range. Figures 3 illustrates the lift coefficient (C_l) as a function of the angle of attack (α) for different values of pitching frequency. Results obtained through CFD are compared against the predictions of the SS-NN model, and an excellent agreement is found between the two techniques.



(a) $f = 0.4$ Hz (b) $f = 1$ Hz (c) $f = 1.6$ Hz

Figure 3: Model validation using sinusoidal motions characterized by different pitching frequencies (TI = 0.3%).

Finally, the selected SS-NN model is further tested on a case outside of the parameter space, i.e a sinusoidal motion where the angle of attack is defined according to the relation $\alpha(t) = 14° + 14° \sin(2\pi t)$. This test case is particularly challenging since it involves a very large amplitude of oscillation and a high-degree of nonlinearity and unsteadiness of the flow. Figure 4 demonstrates that the model succeeds in accurately capturing all load variations, including the abrupt stall behavior.

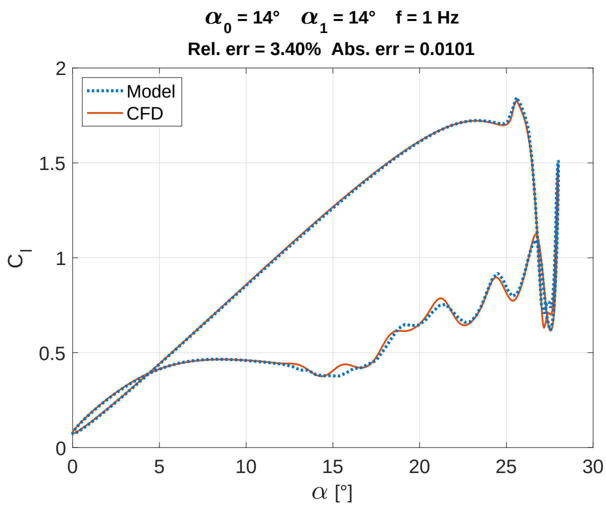


Figure 4: Model validation using a sinusoidal motion characterized by a large amplitude: comparison of the lift coefficient obtained with CFD and model prediction.