Motivation: A formal proof of the Four Color Theorem

Rationale

In 1852 Francis Guthrie observed that the countries of England could be colored with four colors so that neighboring countries were colored differently. Francis Guthrie showed his brother who in turn showed De Morgan the initial 4 color theorem on 10/23/1852. De Morgan was excited while Hamilton was not, and interest died down in the Four Color Theorem. Till 1878, when Alfred Bray Kempe picked up the four colored theorem. He published his work in 1879 in the second edition of the American Journal of Mathematics. In his approach he used the concept of Kempe Chains, which is a connected chain of points on the graph.

The four color theorem is of particular interest because is one of the first results to ever have a computer-assisted proof. This produces an interesting case study reaction amongst the mathematics community as the proof is still not universally accepted, there are documented papers published by "doubters" of the proof. Despite being one of the most contested proofs, it is also notorious for attracting a large number of false proofs and disproofs. As such, this topic is derived from a incredibly interest context, and like most subjects in graph theory, has important applications unrelated to map coloring (which wasn't ever an interest of map cartesians). One of such application is positioning mobile phone masts. The areas covered can be drawn as a map and the different frequencies can be represented as colors. Since the government usually owns all frequencies they want to minimize the cost and thus the number of used frequencies.

Main Result

The four color theorem states that any seperation of a plane into contigous regions, also known as a "Map" can be colored using at most four colors where the regions in our graph G would only be considered adjacent if two regions share border (not just a point).

Defining it in the scope of graph thoery, given any map, where the states/regions represent vertices, there exists a mapping to a planar graph where no two adjacent vertices receive the same color. Such plane graph can be acheived with at most 4 colors and is therefore 4 colorable.

The mapping can be easily done by replacing every region by a vertex and connecting two vertices by an edge exactly when the two regions share a border.

Ultimately, we are trying to illustrate the formal proof of a four color theorem, in short: every planar graph is four colorable. The proof sketch: Let the planar graph with n vertices, n is at least 1, and denoted by Gn . There are 3 cases to discuss. Case 1: When 1 <= n <= 4, the result holds trivially for G1 to G4. Case 2: When n = 5, the maximal planar graph with 5 vertices is the full graph deleting an edge, i.e., the planar graph with 5 vertices and 9 edges, Case 3: various cases for where n => 6.