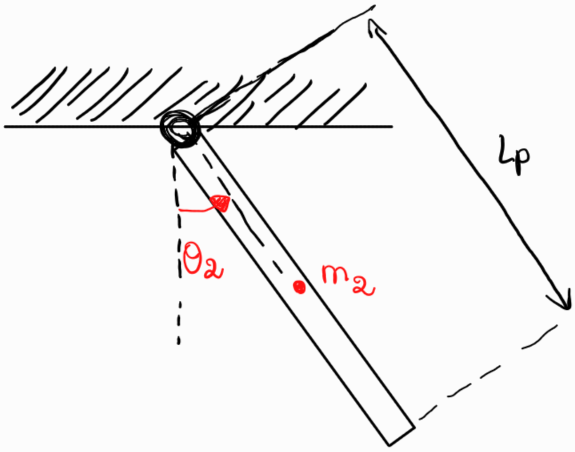


Modello del pendolo

$$(m_2 x_2^2 + I_2)\ddot{\theta}_2 + m_2 g x_2 \sin\theta_2 + k_{f,2}\dot{\theta}_2 = 0$$

Linearizzazione intorno all'equilibrio $\theta_2 = 0$

$$(m_2 x_2^2 + I_2)\ddot{\theta}_2 + m_2 g x_2 \theta_2 + k_{f,2}\dot{\theta}_2 = 0$$

$$\boxed{\ddot{\theta}_2 + \frac{k_{f,2}}{m_2 x_2^2 + I_2}\dot{\theta}_2 + \frac{m_2 g x_2}{m_2 x_2^2 + I_2}\theta_2 = 0}$$

METODO 2 : Dagli autovalori ricavo lo smorzamento

$$\ddot{\theta}_2 + \underbrace{\frac{k_{f,2}}{m_2 x_2^2 + I_2}}_{\alpha}\dot{\theta}_2 + \underbrace{\frac{m_2 g x_2}{m_2 x_2^2 + I_2}}_{\beta}\theta_2 = 0$$

$$\ddot{\theta}_2 + \alpha\dot{\theta}_2 + \beta\theta_2 = 0$$

$x_1 = \theta_2$

$x_2 = \dot{\theta}_2 = \dot{x}_1$

$\dot{x}_2 = \ddot{\theta}_2$

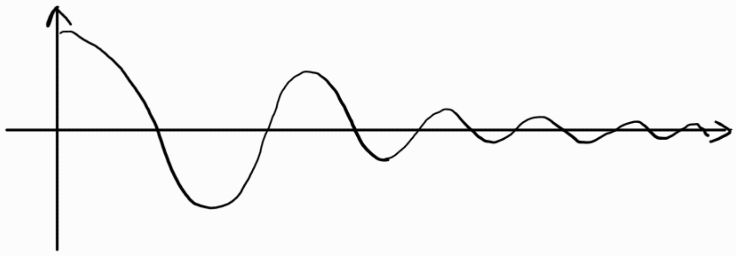
$$\begin{cases} \dot{x}_2 = -\alpha x_2 - \beta x_1 \\ \dot{x}_1 = x_2 \end{cases} \qquad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\beta x_1 - \alpha x_2 \end{cases} \qquad A = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix}$$

autovalori $\det\left(\begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \det\begin{bmatrix} -\lambda & 1 \\ -\beta & -\alpha-\lambda \end{bmatrix} =$

$$= -\lambda(-\alpha-\lambda) + \beta = \lambda^2 + \alpha\lambda + \beta = 0$$

$$\lambda_{1/2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\beta}}{2} \quad \begin{cases} \lambda_1 = -\frac{\alpha}{2} + \frac{\sqrt{\alpha^2-4\beta}}{2} \\ \lambda_2 = -\frac{\alpha}{2} - \frac{\sqrt{\alpha^2-4\beta}}{2} \end{cases}$$

λ_1 e λ_2 sono complessi coniugati



possiamo vedere

$\lambda^2 + \alpha\lambda + \beta = 0$

come

$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$

dove ξ = smorzamento

ω_n = pulsazione naturale

tempo di assestamento $t_a = \frac{5}{\xi\omega_n}$

$$\begin{cases} \alpha = 2\xi\omega_n \\ \beta = \omega_n^2 \end{cases} \quad \begin{cases} \omega_n = \sqrt{\beta} \\ \alpha = 2\xi\sqrt{\beta} \end{cases} \quad \begin{cases} \omega_n = \sqrt{\beta} \\ \xi = \frac{\alpha}{2\sqrt{\beta}} \end{cases}$$

$$t_a = \frac{5}{\xi\omega_n} = \frac{5}{\frac{\alpha}{2\cancel{\sqrt{\beta}}}\cancel{\sqrt{\beta}}} = \frac{10}{\alpha}$$

$\alpha = \frac{k_{f,2}}{m_2 x_2^2 + I_2}$

$$t_a = 10 \cdot \frac{m_2 X_2^2 + I_2}{k_{f,2}}$$

dai test $t_{a(1)} = \frac{25200 - 1870}{500} = 46{,}66\,s$ esperimento 1

$t_{a(2)} = \frac{19790 - 1497}{500} = 36{,}586\,s$ esperimento 2

$m_2 = 0{,}024\ kg$ $\quad x_2 = \frac{0{,}129}{2}\ m = 0{,}0645\ m$ $\quad I_2 = \frac{m_2 \cdot \ell_2^2}{12} = \frac{0{,}024 \cdot 0{,}129^2}{12} = 3{,}33 \cdot 10^{-5}$

esperimento 1 $\quad t_a = 46{,}66\ s$

$$t_a = 10 \cdot \frac{m_2 X_2^2 + I_2}{k_{f,2}}$$

$$k_{f,2} = 10 \cdot \frac{m_2 x_2^2 + I_2}{t_a} = 10 \cdot \frac{0{,}024 \cdot 0{,}0645^2 + 3{,}33 \cdot 10^{-5}}{46{,}66} = 2{,}85 \cdot 10^{-6}\ \frac{kg\,m^2}{s}$$

esperimento 2 $\quad t_a = 36{,}586\ s$

$$t_a = 10 \cdot \frac{m_2 X_2^2 + I_2}{k_{f,2}}$$

$$k_{f,2} = 10 \cdot \frac{m_2 x_2^2 + I_2}{t_a} = 10 \cdot \frac{0{,}024 \cdot 0{,}0645^2 + 3{,}33 \cdot 10^{-5}}{36{,}586} = 3{,}64 \cdot 10^{-6}\ \frac{kg\,m^2}{s}$$

media $\quad k_{f,2} = \frac{k_{f,2(1)} + k_{f,2(2)}}{2} = 3{,}245 \cdot 10^{-6}\ \frac{kg\,m^2}{s}$