DOWNRIGHT LINEARIZED MODEL — WITH SPRING

$K_1 = m_1 x_1^2 + m_2 l_1^2 + I_1$	$K_5 = m_2 l_1 x_2$
$K_2 = m_2 l_1 x_2$	$K_6 = m_2 x_2^2 + I_2$
$K_3 = K_{F,1} + K_m^2 / R_m$	$K_7 = K_{F,2}$
$K_4 = K_m / R_m$	$K_8 = m_2 g x_2$

$$\begin{cases} k_1 \ddot{\theta}_1 + k_2 \ddot{\theta}_2 + k_3 \dot{\theta}_1 + K_S \vartheta_1 = k_4 V_{IN} \\ k_5 \ddot{\theta}_1 + k_6 \ddot{\theta}_2 + k_7 \dot{\theta}_2 + k_8 \theta_2 = 0 \end{cases}$$

$$\dot{\theta}_1 = \omega_1 \qquad \ddot{\theta}_1 = \dot{\omega}_1$$

$$\dot{\theta}_2 = \omega_2 \qquad \ddot{\theta}_2 = \dot{\omega}_2$$

$$\begin{cases} k_1 \dot{\omega}_1 + k_2 \dot{\omega}_2 + k_3 \omega_1 + K_S \vartheta_1 = k_4 V_{IN} \\ k_5 \dot{\omega}_1 + k_6 \dot{\omega}_2 + k_7 \omega_2 + k_8 \theta_2 = 0 \\ \dot{\theta}_1 = \omega_1 \\ \dot{\theta}_2 = \omega_2 \end{cases}$$

$$\begin{cases} \dot{\omega}_1 = -\frac{k_2}{k_1} \dot{\omega}_2 - \frac{k_3}{k_1} \omega_1 + \frac{k_4}{k_1} V_{IN} - \frac{K_S}{K_1} \vartheta_1 \\ k_5 \left(-\frac{k_2}{k_1} \dot{\omega}_2 - \frac{k_3}{k_1} \omega_1 + \frac{k_4}{k_1} V_{IN} - \frac{K_S}{K_1} \vartheta_1 \right) + k_6 \dot{\omega}_2 + K_7 \omega_2 + k_8 \theta_2 = 0 \\ \dot{\theta}_1 = \omega_1 \\ \dot{\theta}_2 = \omega_2 \end{cases}$$

$$-k_5 \frac{k_2}{k_1} \dot{\omega}_2 - k_5 \frac{k_3}{k_1} \omega_1 + k_5 \frac{k_4}{k_1} V_{IN} + k_6 \dot{\omega}_2 + k_7 \omega_2 + k_8 \theta_2 - \frac{K_5 \cdot K_S}{K_1} \vartheta_1 = 0$$

$$\underbrace{\left(k_6 - k_5 \frac{k_2}{k_1} \right)}_{\frac{k_1 k_6 - k_5 k_2}{k_1}} \dot{\omega}_2 = k_5 \frac{k_3}{k_1} \omega_1 - k_7 \omega_2 - k_8 \theta_2 - k_5 \frac{k_4}{k_1} V_{IN} + \frac{K_5 \cdot K_S}{K_1} \vartheta_1$$

$$\dot{\omega}_2 = \frac{\cancel{k_1}}{k_1 k_6 - k_5 k_2} k_5 \frac{k_3}{\cancel{k_1}} \omega_1 - \frac{k_1 k_7}{k_1 k_6 - k_5 k_2} \omega_2 - \frac{k_1 k_8}{k_1 k_6 - k_5 k_2} \theta_2 - \frac{\cancel{k_1}}{k_1 k_6 - k_5 k_2} k_5 \frac{k_4}{\cancel{k_1}} V_{IN} + \frac{K_5 \cdot K_S}{K_1 K_6 - K_2 K_5} \vartheta_1$$

$$
\begin{cases}
\dot{\omega}_2 = \dfrac{k_3k_5}{k_1k_6-k_2k_5}\,\omega_1 - \dfrac{k_1k_7}{k_1k_6-k_2k_5}\,\omega_2 - \dfrac{k_1k_8}{k_1k_6-k_2k_5}\,\theta_2 - \dfrac{k_4k_5}{k_1k_6-k_2k_5}\,V_{IN} + \dfrac{K_5\cdot K_5}{K_1K_6-K_2K_5}\,\vartheta_1 \\
\dot{\omega}_1 = -\dfrac{k_2}{k_1}\left(\dfrac{k_3k_5}{k_1k_6-k_2k_5}\,\omega_1 - \dfrac{k_1k_7}{k_1k_6-k_2k_5}\,\omega_2 - \dfrac{k_1k_8}{k_1k_6-k_2k_5}\,\theta_2 - \dfrac{k_4k_5}{k_1k_6-k_2k_5}\,V_{IN} + \dfrac{K_5\cdot K_5}{K_1K_6-K_2K_5}\,\vartheta_1\right) \\
\qquad - \dfrac{k_3}{k_1}\,\omega_1 + \dfrac{k_4}{k_1}\,V_{IN} - \dfrac{K_5}{K_1}\,\vartheta_1 \\
\dot{\theta}_1 = \omega_1 \\
\dot{\theta}_2 = \omega_2
\end{cases}
$$

$$
\dot{\omega}_1 = -\frac{k_2k_3k_5}{k_1(k_1k_6-k_2k_5)}\,\omega_1 + \frac{k_2k_7}{k_1k_6-k_2k_5}\,\omega_2 + \frac{k_2k_8}{k_1k_6-k_2k_5}\,\theta_2 + \frac{k_2k_4k_5}{k_1(k_1k_6-k_2k_5)}\,V_{IN} - \frac{k_3}{k_1}\,\omega_1 + \frac{k_4}{k_1}\,V_{IN}
$$

$$
- \frac{K_2\cdot K_5\cdot K_5}{K_1(K_1K_6-K_2K_5)}\,\vartheta_1 - \frac{K_5}{K_1}\,\vartheta_1
$$

$$
\dot{\omega}_1 = -\frac{\cancel{k_2k_3k_5} + k_3(k_1k_6-\cancel{k_2k_5})}{k_1(k_1k_6-k_2k_5)}\,\omega_1 + \frac{k_2k_7}{k_1k_6-k_2k_5}\,\omega_2 + \frac{k_2k_8}{k_1k_6-k_2k_5}\,\theta_2 + \left(\frac{k_4(k_1k_6-\cancel{k_2k_5}) + \cancel{k_2k_4k_5}}{k_1(k_1k_6-k_2k_5)}\right)V_{IN}
$$

$$
- \frac{K_5(\cancel{K_2K_5} + K_1K_6 - \cancel{K_2K_5})}{K_1(K_1K_6-K_2K_5)}\,\vartheta_1
$$

$$
\begin{cases}
\dot{\theta}_1 = \omega_1 \\
\dot{\theta}_2 = \omega_2 \\
\dot{\omega}_1 = \dfrac{k_2k_8}{k_1k_6-k_2k_5}\,\theta_2 - \dfrac{k_3k_6}{k_1k_6-k_2k_5}\,\omega_1 + \dfrac{k_2k_7}{k_1k_6-k_2k_5}\,\omega_2 + \dfrac{k_4k_6}{k_1k_6-k_2k_5}\,V_{IN} - \dfrac{K_5K_6}{K_1K_6-K_2K_5}\,\vartheta_1 \\
\dot{\omega}_2 = -\dfrac{k_1k_8}{k_1k_6-k_2k_5}\,\theta_2 + \dfrac{k_3k_5}{k_1k_6-k_2k_5}\,\omega_1 - \dfrac{k_1k_7}{k_1k_6-k_2k_5}\,\omega_2 - \dfrac{k_4k_5}{k_1k_6-k_2k_5}\,V_{IN} + \dfrac{K_5\cdot K_5}{K_1K_6-K_2K_5}\,\vartheta_1 \\
y_1 = \theta_1 \\
y_2 = \theta_2
\end{cases}
$$

input $u = v_{IN}$

output $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $\Big\}$ SIMO system

variabili di stato $x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega_1 \\ \omega_2 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_5K_6}{K_1K_6 - K_2K_5} & \frac{k_2k_8}{k_1k_6 - k_2k_5} & -\frac{k_3k_6}{k_1k_6 - k_2k_5} & \frac{k_2k_7}{k_1k_6 - k_2k_5} \\ \frac{K_5K_5}{K_1K_6 - K_2K_5} & -\frac{k_1k_8}{k_1k_6 - k_2k_5} & \frac{k_3k_5}{k_1k_6 - k_2k_5} & -\frac{k_1k_7}{k_1k_6 - k_2k_5} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{k_4k_6}{k_1k_6 - k_2k_5} \\ -\frac{k_4k_5}{k_1k_6 - k_2k_5} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Experiment #01 : fixed ϑ_2 — WITH SPRING

$$k_1\ddot{\theta}_1 + K_3\dot{\theta}_1 + K_5\vartheta_1 = K_4 V_{IN}$$

$\dot{\theta}_1 = \omega_1$ $\dot{\omega}_1 = \ddot{\theta}_1$

$\Downarrow$

$$k_1\dot{\omega}_1 + K_3\omega_1 + K_5\vartheta_1 = K_4 V_{IN}$$

$\Downarrow$

$$\begin{cases} \dot{\omega}_1 = -\dfrac{K_3}{k_1}\omega_1 + \dfrac{K_4}{k_1}V_{IN} - \dfrac{K_5}{K_1}\vartheta_1 \\ \dot{\theta}_1 = \omega_1 \\ y = \theta_1 \end{cases}$$

INPUT $u = V_{IN}$

OUTPUT $y = \theta_1$

variabili di stato $x = \begin{bmatrix} \theta_1 \\ \omega_1 \end{bmatrix}$

$A = \begin{bmatrix} -\dfrac{K_5}{K_1} & -\dfrac{K_3}{k_1} \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} K_4/k_1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $D = 0$