

1. Downright position linearized model. without spring

$$\begin{cases} J_{eq} + m_2 l_1^2 \\ (m_1 x_1^2 + m_2 l_1^2 + I_1) \ddot{\theta}_1 + m_2 l_1 x_2 \ddot{\theta}_2 + (K_{F,1} + \frac{K_m^2}{R_m}) \dot{\theta}_1 = \frac{K_m V_{IN}}{R_m} \\ m_2 l_1 x_2 \ddot{\theta}_1 + (m_2 x_2^2 + I_2) \ddot{\theta}_2 + K_{F,2} \dot{\theta}_2 + m_2 g x_2 \theta_2 = 0 \end{cases}$$

2. Downright position linearized model. with spring

$$\begin{cases} J_{eq} + m_2 l_1^2 \\ (m_1 x_1^2 + m_2 l_1^2 + I_1) \ddot{\theta}_1 + m_2 l_1 x_2 \ddot{\theta}_2 + (K_{F,1} + \frac{K_m^2}{R_m}) \dot{\theta}_1 + K_S \theta_1 = \frac{K_m V_{IN}}{R_m} \\ m_2 l_1 x_2 \ddot{\theta}_1 + (m_2 x_2^2 + I_2) \ddot{\theta}_2 + K_{F,2} \dot{\theta}_2 + m_2 g x_2 \theta_2 = 0 \end{cases}$$

3. Experiment #1: fixed θ_2 . without spring

$$J_{eq} + m_2 l_1^2 \\ (m_1 x_1^2 + m_2 l_1^2 + I_1) \ddot{\theta}_1 + (K_{F,1} + \frac{K_m^2}{R_m}) \dot{\theta}_1 = \frac{K_m V_{IN}}{R_m}$$

4. Experiment #1: fixed θ_2 . with spring

$$J_{eq} + m_2 l_1^2 \\ (m_1 x_1^2 + m_2 l_1^2 + I_1) \ddot{\theta}_1 + (K_{F,1} + \frac{K_m^2}{R_m}) \dot{\theta}_1 + K_S \theta_1 = \frac{K_m V_{IN}}{R_m}$$

EXTRA. Experiment #2: fixed θ_1

$$(m_2 x_2^2 + I_2) \ddot{\theta}_2 + K_{F,2} \dot{\theta}_2 + m_2 g x_2 \theta_2 = 0$$

5. Upright position linearized model. without spring

$$J_{eq} + m_2 l_1^2$$

$$\begin{cases} (m_1 x_1^2 + m_2 l_1^2 + I_1) \ddot{\theta}_1 - m_2 l_1 x_2 \ddot{\theta}_2 + (K_{F,1} + \frac{K_m^2}{R_m}) \dot{\theta}_1 = \frac{K_m}{R_m} V_{in} \\ -m_2 l_1 x_2 \ddot{\theta}_1 + (m_2 x_2^2 + I_2) \ddot{\theta}_2 + K_{F,2} \dot{\theta}_2 - m g x_2 \theta_2 = 0 \end{cases}$$

6. Upright position linearized model. with spring

$$J_{eq} + m_2 l_1^2$$

$$\begin{cases} (m_1 x_1^2 + m_2 l_1^2 + I_1) \ddot{\theta}_1 - m_2 l_1 x_2 \ddot{\theta}_2 + (K_{F,1} + \frac{K_m^2}{R_m}) \dot{\theta}_1 + K_S \theta_1 = \frac{K_m}{R_m} V_{in} \\ -m_2 l_1 x_2 \ddot{\theta}_1 + (m_2 x_2^2 + I_2) \ddot{\theta}_2 + K_{F,2} \dot{\theta}_2 - m g x_2 \theta_2 = 0 \end{cases}$$

7. Nonlinear model

$$J_{eq}$$

$$\begin{cases} [m_1 x_1^2 + I_1 + m_2 (l_1^2 + x_2^2)] \ddot{\theta}_1 + m_2 l_1 x_2 c_2 \ddot{\theta}_2 + \\ + 2 m_2 x_2^2 s_2 c_2 \dot{\theta}_1 \dot{\theta}_2 - m_2 l_1 x_2 s_2 \dot{\theta}_2^2 + K_S \theta_1 = \\ = K_m / R_m \cdot V_{in} - K_m^2 / R_m \cdot \dot{\theta}_1 - K_{F,1} \cdot \dot{\theta}_1 \\ m_2 l_1 x_2 c_2 \ddot{\theta}_1 + (m_2 x_2^2 + I_2) \ddot{\theta}_2 - m_2 x_2^2 s_2 c_2 \dot{\theta}_1^2 + m_2 g x_2 s_2 = \\ = -K_{F,2} \dot{\theta}_2 \end{cases}$$

8. Motor equation

$$\tau = \frac{K_m}{R_m} (V_{in} - K_m \dot{\theta}_1)$$