DOWNRIGHT LINEARIZED MODEL — NO SPRING

$$K_1 = m_1 x_1^2 + m_2 l_1^2 + I_1 \qquad K_5 = m_2 l_1 x_2$$
$$K_2 = m_2 l_1 x_2 \qquad K_6 = m_2 x_2^2 + I_2$$
$$K_3 = K_{F,1} + K_m^2 / R_m \qquad K_7 = K_{F,2}$$
$$K_4 = K_m / R_m \qquad K_8 = m_2 g x_2$$

$$\begin{cases} k_1 \ddot{\theta}_1 + k_2 \ddot{\theta}_2 + k_3 \dot{\theta}_1 = k_4 V_{IN} \\ k_5 \ddot{\theta}_1 + k_6 \ddot{\theta}_2 + k_7 \dot{\theta}_2 + k_8 \theta_2 = 0 \end{cases}$$

$$\dot{\theta}_1 = \omega_1 \quad \ddot{\theta}_1 = \dot{\omega}_1$$
$$\dot{\theta}_2 = \omega_2 \quad \ddot{\theta}_2 = \dot{\omega}_2$$

$$\begin{cases} k_1 \dot{\omega}_1 + k_2 \dot{\omega}_2 + k_3 \omega_1 = k_4 V_{IN} \\ k_5 \dot{\omega}_1 + k_6 \dot{\omega}_2 + k_7 \omega_2 + k_8 \theta_2 = 0 \\ \dot{\theta}_1 = \omega_1 \\ \dot{\theta}_2 = \omega_2 \end{cases}$$

$$\begin{cases} \dot{\omega}_1 = -\frac{k_2}{k_1} \dot{\omega}_2 - \frac{k_3}{k_1} \omega_1 + \frac{k_4}{k_1} V_{IN} \\ k_5 \left(-\frac{k_2}{k_1} \dot{\omega}_2 - \frac{k_3}{k_1} \omega_1 + \frac{k_4}{k_1} V_{IN} \right) + k_6 \dot{\omega}_2 + k_7 \omega_2 + k_8 \theta_2 = 0 \\ \dot{\theta}_1 = \omega_1 \\ \dot{\theta}_2 = \omega_2 \end{cases}$$

$$-k_5 \frac{k_2}{k_1} \dot{\omega}_2 - k_5 \frac{k_3}{k_1} \omega_1 + k_5 \frac{k_4}{k_1} V_{IN} + k_6 \dot{\omega}_2 + k_7 \omega_2 + k_8 \theta_2 = 0$$

$$\underbrace{\left(k_6 - k_5 \frac{k_2}{k_1} \right)}_{\frac{k_1 k_6 - k_5 k_2}{k_1}} \dot{\omega}_2 = k_5 \frac{k_3}{k_1} \omega_1 - k_7 \omega_2 - k_8 \theta_2 - k_5 \frac{k_4}{k_1} V_{IN}$$

$$\dot{\omega}_2 = \frac{\cancel{k_1}}{k_1 k_6 - k_5 k_2} k_5 \frac{k_3}{\cancel{k_1}} \omega_1 - \frac{k_1 k_7}{k_1 k_6 - k_5 k_2} \omega_2 - \frac{k_1 k_8}{k_1 k_6 - k_5 k_2} \theta_2 - \frac{\cancel{k_1}}{k_1 k_6 - k_5 k_2} k_5 \frac{k_4}{\cancel{k_1}} V_{IN}$$

$$\begin{cases} \dot{\omega}_2 = \frac{k_3 k_5}{k_1 k_6 - k_2 k_5} \omega_1 - \frac{k_1 k_7}{k_1 k_6 - k_2 k_5} \omega_2 - \frac{k_1 k_8}{k_1 k_6 - k_2 k_5} \theta_2 - \frac{k_1 k_5}{k_1 k_6 - k_2 k_5} V_{IN} \\ \dot{\omega}_1 = -\frac{k_2}{k_1} \left(\frac{k_3 k_5}{k_1 k_6 - k_2 k_5} \omega_1 - \frac{k_1 k_7}{k_1 k_6 - k_2 k_5} \omega_2 - \frac{k_1 k_8}{k_1 k_6 - k_2 k_5} \theta_2 - \frac{k_1 k_5}{k_1 k_6 - k_2 k_5} V_{IN} \right) - \frac{k_3}{k_1} \omega_1 + \frac{k_4}{k_1} V_{IN} \\ \dot{\theta}_1 = \omega_1 \\ \dot{\theta}_2 = \omega_2 \end{cases}$$

$$\dot{\omega}_1 = -\frac{k_2 k_3 k_5}{k_1 (k_1 k_6 - k_2 k_5)} \omega_1 + \frac{k_2 k_7}{k_1 k_6 - k_2 k_5} \omega_2 + \frac{k_2 k_8}{k_1 k_6 - k_2 k_5} \theta_2 + \frac{k_2 k_4 k_5}{k_1 (k_1 k_6 - k_2 k_5)} V_{IN} - \frac{k_3}{k_1} \omega_1 + \frac{k_4}{k_1} V_{IN}$$

$$\dot{\omega}_1 = -\frac{\cancel{k_2 k_3 k_5} + k_3 (k_1 k_6 - \cancel{k_2 k_5})}{k_1 (k_1 k_6 - k_2 k_5)} \omega_1 + \frac{k_2 k_7}{k_1 k_6 - k_2 k_5} \omega_2 + \frac{k_2 k_8}{k_1 k_6 - k_2 k_5} \theta_2 + \left(\frac{k_4 (k_1 k_6 - \cancel{k_2 k_5}) + \cancel{k_2 k_4 k_5}}{k_1 (k_1 k_6 - k_2 k_5)} \right) V_{IN}$$

$$
\begin{cases}
\dot{\theta}_1 = \omega_1 \\
\dot{\theta}_2 = \omega_2 \\
\dot{\omega}_1 = \dfrac{k_2 k_8}{k_1 k_6 - k_2 k_5}\theta_2 - \dfrac{k_3 k_6}{k_1 k_6 - k_2 k_5}\omega_1 + \dfrac{k_2 k_7}{k_1 k_6 - k_2 k_5}\omega_2 + \dfrac{k_4 k_6}{k_1 k_6 - k_2 k_5} V_{IN} \\
\dot{\omega}_2 = -\dfrac{k_1 k_8}{k_1 k_6 - k_2 k_5}\theta_2 + \dfrac{k_3 k_5}{k_1 k_6 - k_2 k_5}\omega_1 - \dfrac{k_1 k_7}{k_1 k_6 - k_2 k_5}\omega_2 - \dfrac{k_1 k_5}{k_1 k_6 - k_2 k_5} V_{IN} \\
y_1 = \theta_1 \\
y_2 = \theta_2
\end{cases}
$$

input $u = V_{IN}$

output $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ } SIMO system

variabili di stato $x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \omega_1 \\ \omega_2 \end{bmatrix}$

$$
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & \dfrac{k_2 k_8}{k_1 k_6 - k_2 k_5} & -\dfrac{k_3 k_6}{k_1 k_6 - k_2 k_5} & \dfrac{k_2 k_7}{k_1 k_6 - k_2 k_5} \\
0 & -\dfrac{k_1 k_8}{k_1 k_6 - k_2 k_5} & \dfrac{k_3 k_5}{k_1 k_6 - k_2 k_5} & -\dfrac{k_1 k_7}{k_1 k_6 - k_2 k_5}
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 \\ 0 \\ \dfrac{k_4 k_6}{k_1 k_6 - k_2 k_5} \\ -\dfrac{k_1 k_5}{k_1 k_6 - k_2 k_5}
\end{bmatrix}
$$

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

Experiment #01: fixed ϑ_2 · NO SPRING

$$k_1 \ddot{\theta}_1 + k_3 \dot{\theta}_1 = k_4 V_{IN}$$

$$\dot{\theta}_1 = \omega_1 \qquad \dot{\omega}_1 = \ddot{\theta}_1$$

$$\Downarrow$$

$$k_1 \dot{\omega}_1 + k_3 \omega_1 = k_4 V_{IN}$$

$$\Downarrow$$

$$\begin{cases} \dot{\omega}_1 = -\dfrac{k_3}{k_1}\omega_1 + \dfrac{k_4}{k_1} V_{IN} \\ \dot{\theta}_1 = \omega_1 \\ y = \theta_1 \end{cases}$$

INPUT $u = V_{IN}$

OUTPUT $y = \theta_1$

variabili di stato $x = \begin{bmatrix} \theta_1 \\ \omega_1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & -\dfrac{k_3}{k_1} \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} k_4/k_1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$