Kalman filter in speech and music

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Overview

1. Kalman filter introduction
2. KF in speech - used for speech denoising [Das 2016]
3. KF in music - used for monophonic pitch tracking [Das 2017]

Kalman filter

- Recursive MMSE estimator proposed by R.Kalman in [Kalman 1960]
- Basic idea - Update current state $\hat{x}(n)$ based on present noisy observation $y(n)$ and all past predictions $\hat{x}(0), \hat{x}(1), \ldots, \hat{x}(n-1)$.
- Generalized Model -

$$\begin{aligned} x(n) &= Ax(n-1) + u(n); \mathbf{u} \sim N(0, Q) \\ y(n) &= Hx(n) + w(n); \mathbf{w} \sim N(0, R) \end{aligned} \tag{1}$$

  - $x(n)$ - Current state, $y(n)$ - Current noisy observation
  - $u(n)$ - Process noise, $w(n)$ - Observation noise
  - Q - Process noise covariance matrix, R - Measurement noise covariance matrix
  - A - State transition matrix, H - Observation vector

Kalman filter equations

- Time update equations

$$
\begin{aligned}
\hat{x}_n^- &= A\hat{x}_{n-1} \\
P_n^- &= AP_{n-1}A^T + Q
\end{aligned} \tag{2}
$$

- Measurement update equations

$$
\begin{aligned}
K_n &= P_n^- H^T (HP_n^- H^T + R)^{-1} \\
\hat{x}_n &= \hat{x}_n^- + K_n(y_n - H\hat{x}_n^-) \\
P_n &= (I - K_n H)P_n^-
\end{aligned} \tag{3}
$$

  P is the error covariance matrix and K is the Kalman gain that acts as a weighting factor between observed and predicted values.

- Kalman filter best suited to track model dynamics in the presence of noise, for e.g position of robotic arm with noisy sensors.

Use in speech denoising

- Original paper proposed by Paliwal and Basu in [Paliwal 1987] that made use of autoregressive model of speech.
- Model -

$$
\begin{aligned}
x(n) &= -\sum_{i=1}^{p} \alpha_i x(n-i) + u(n) \\
y(n) &= x(n) + w(n)
\end{aligned}
\tag{4}
$$

- α_i's also known as linear prediction coefficients (LPC) usually calculated by solving the Yule-Walker equations.
- α_i's are all we need to know to form transition matrix, the observation matrix is $H = [0, 0, \dots, 1]$. Remaining KF equations are same.

Kalman filter tuning [Das 2016]

- Tune other filter parameters - Q and R
- R - measurement noise variance can be estimated relatively easily by getting noisy samples from silent regions and calculating their variance.
- Q - process noise variance, part of model. Use robustness and sensitivity metrics to find best compromise value of Q [Saha 2014].

$$\begin{aligned} A_n &= H(AP_n^- A^T)H^T \\ J_1 &= \frac{\sigma_w^2}{\sigma_w^2 + A_n + \sigma_u^2} \\ J_2 &= \frac{\sigma_u^2}{\sigma_u^2 + A_n} \end{aligned} \tag{5}$$

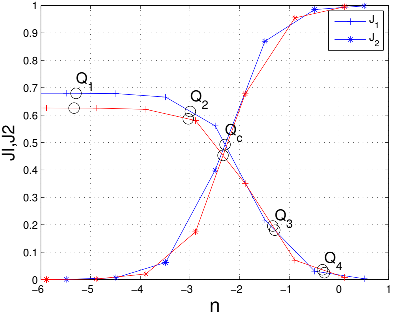


Figure: J_1 - *sensitivity metric,* J_2 - *robustness metric for blue - voiced frames, red- silent frames.*

- Two values of Q used - Q_c for voiced frames for balance between sensitivity and robustness and $Q_2 < Q_c$ for more sensitive performance in silent frames.

Contd. II

- Kalman gain adjustment according to value of Q and frame type - smaller K for silent frames, higher K for voiced frames.

$$\hat{x}_n = (I - HK_n)\hat{x}_n^- + K_n y(n) \tag{6}$$

- Smaller K denotes less weight on noisy observation and more weight on past prediction to predict current state - ideal for silent frames which only have noise, no speech information that needs to be preserved.

Contd. III

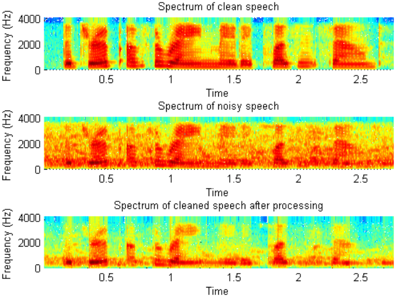


Figure: Example of clean, noisy and enhanced speech at 5dB SNR.

Use in pitch tracking

- Extended complex Kalman filter proposed in [Dash 2000] to track distorted power system signal frequency in presence of noise and harmonics.
- Same model (sines+noise) can be applied to music. ECKF Pitch tracker proposed in [Das 2017].
- Original Kalman filter proposed for linear systems. Extended Kalman filter works on non-linear systems, where the current state estimate is a non-linear function of the previous estimate. EKF linearizes the function by using a first order Taylor series expansion.

ECKF Pitch Tracker I

- Model-

$$y_k = a_1 \cos\left(\omega_1 k T_s + \phi_1\right) + w_k \tag{7}$$

where a_1, w_1 and ϕ_1 are the fundamental amplitude, frequency and phase respectively and T_s is the sampling interval. The state vector is constructed as:

$$x_k = \begin{bmatrix} \alpha \\ u_k \\ u_k^* \end{bmatrix} \tag{8}$$

where

$$\begin{aligned} \alpha &= \exp(j\omega_1 T_s) \\ u_k &= a_1 \exp(j\omega_1 k T_s + j\phi_1) \\ u_k^* &= a_1 \exp(-j\omega_1 k T_s - j\phi_1) \end{aligned} \tag{9}$$

- The state vector estimate update rule x_{k+1} relates to x_k as

$$\begin{aligned}\begin{bmatrix}\alpha \\ u_{k+1} \\ u^*_{k+1}\end{bmatrix} &= \begin{bmatrix}1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \frac{1}{\alpha}\end{bmatrix}\begin{bmatrix}\alpha \\ u_k \\ u^*_k\end{bmatrix} \\ x_{k+1} &= f(x_k) \\ f(x_k) &= \begin{bmatrix}\alpha & \alpha u_k & \frac{u^*_k}{\alpha}\end{bmatrix}^T\end{aligned} \tag{10}$$

- y_k relates to x_k as

$$\begin{aligned}y_k &= Hx_k + w_k \\ H &= \begin{bmatrix}0 & 0.5 & 0.5\end{bmatrix}\end{aligned} \tag{11}$$

ECKF Pitch Tracker III

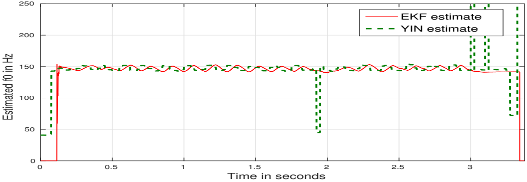
- The transition matrix gets replaced by the Jacobian of $f(x_k)$ w.r.t x_k.

$$F_k = \frac{\partial f(x_k)}{\partial x_k}\bigg|_{x_k = \hat{x}_{k|k}} = \begin{bmatrix} 1 & 0 & 0 \\ \hat{x}_{k|k}(2) & \hat{x}_{k|k}(1) & 0 \\ -\frac{\hat{x}_{k|k}(3)}{\hat{x}^2_{k|k}(1)} & 0 & \frac{1}{\hat{x}_{k|k}(1)} \end{bmatrix} \tag{12}$$

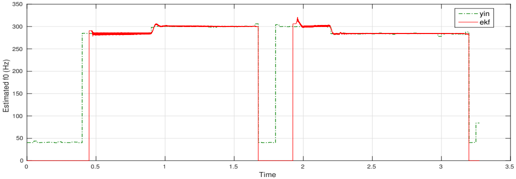
- Detect silent regions (unpitched moments) by dividing signal into frames and looking at spectral flatness. Silent frames will have spectral flatness close to 1.
- Reset covariance matrix and recalculate initial estimates with FFT every time there is a transition from non-silent to silent frame to enable ECKF to track rapidly changing pitch dynamics.
- Adaptive process noise:

$$\log_{10}(\sigma_u^2) = -c + |y_k - H\hat{x}_{k|k}| \tag{13}$$

ECKF Pitch Tracker IV



(a) ECKF tracking vibrato played on the double bass



(b) ECKF tracking two notes hammered on the guitar

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