

The Poisson Process, meaning, properties and simulations

Pietro Colaguori 1936709

November 2023

1 Meaning and Properties

A Poisson process is a specific type of stochastic process that models the occurrence of events in continuous time. It is named after the French mathematician Siméon Denis Poisson, who introduced it in the early 19th century. The key characteristics of a Poisson process are:

1. **Constant rate:** Events in a Poisson process happen at a constant rate λ per unit of time. Mathematically:

$$\lambda(t) = \lambda, \forall t$$

2. **Memoryless property:** The probability of an event happening in the future is independent of the past. Mathematically:

$$P(X > s + t | X > s) = P(X > t); s, t \geq 0$$

3. **Discreteness:** While the Poisson process is continuous in time events are counted within time intervals, the number of events in a fixed time interval follows a Poisson distribution. Mathematically, if $N(t)$ represents the number of events that happened up to time t :

$$P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

4. **Inter-Arrival Times:** The times between consecutive events, called inter-arrival times, follow an exponential distribution, meaning that if T_n is the time between the $(n - 1)$ th event and the n th event, then:

$$P(T_n > t) = e^{-\lambda t}$$

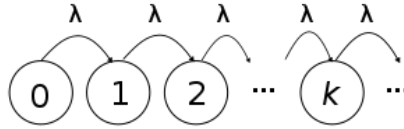


Figure 1: Visual representation of a Poisson process

The Poisson process is a fundamental tool in probability theory and stochastic processes, providing a simple yet powerful model for events occurring in a random and independent manner over time.

2 Simulation

With the following Python script I can select a certain rate λ and a certain number of samples and obtain a graph of the Poisson process Frequency and Numer of Events.

```

1 import streamlit as st
2 import numpy as np
3 import matplotlib.pyplot as plt
4
5 # Simulate the Poisson process
6 lambda_val = 100 # Poisson rate parameter
7 num_samples = 100 # Number of samples to generate
8 samples = np.random.poisson(lambda_val, num_samples)
9
10 # Display the result in Streamlit
11 st.title("Poisson Process Simulation")
12 st.write(f"Lambda (rate parameter): {lambda_val}")
13 st.write(f"Number of samples: {num_samples}")
14 st.write("Generated samples:")
15 st.write(samples)
16
17 # Plot the Poisson process
18 plt.hist(samples, bins=range(max(samples)+1),
19          align='left', rwidth=0.8)
20 plt.xlabel("Number of events")
21 plt.ylabel("Frequency")
22 plt.title("Poisson Process")
23 st.pyplot(plt)

```

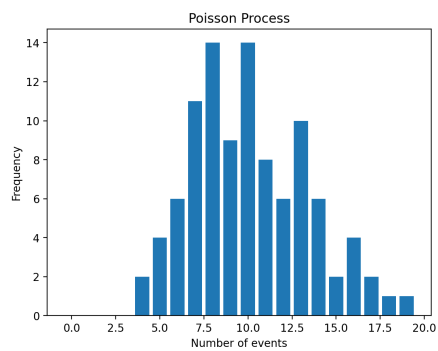


Figure 2: Poisson process with $\lambda = 10$.

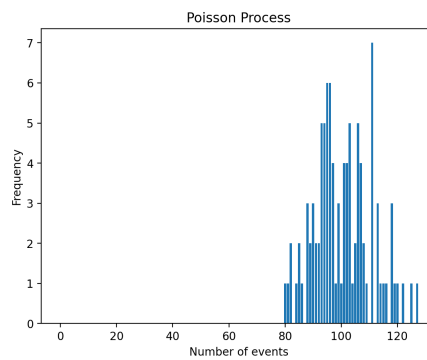


Figure 3: Poisson process with $\lambda = 100$.