Algorithms for Random Variate Generation

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1 Algorithms

A random variate, often referred to as a variate, characterizes the difference between a random variate and the central location of its distribution, typically the mean. This difference is commonly normalized by the standard deviation of the distribution, transforming it into a standard score. In the context of stochastic processes, random variates play a crucial role in simulating phenomena influenced by random factors.

When simulating processes driven by random influences, such as in stochastic processes, modern applications often involve generating random variates corresponding to a specified probability distribution. This task is accomplished using computer procedures designed to produce random variates representing a uniform distribution. These procedures essentially yield values selected from a uniform distribution of pseudorandom numbers.

Procedures dedicated to generating random variates aligned with a specific distribution are known as either procedures for (uniform) random number generation or non-uniform pseudo-random variate generation.

In the realm of probability theory, a random variable is a measurable function mapping from a probability space to a measurable space of possible values. Within this framework, the values taken on by the variable are also termed random variates or random variates. This broader perspective encompasses more than just the concept associated with pseudorandom numbers, expanding the understanding of random variates in a probabilistic context.

Let's proceed by considering some algorithms for random variate generation:

1. **Inverse Transform Method**: Suppose you have F which is the CDF of X. If F is continous and strictly increasing one can prove that $X = F^{-1}(U)$. Where U is a continous uniform distribution between 0 and 1. This means that I can simply generate $u_1, ..., u_N$ independent random observation from a uniform distribution between 0 and 1 and then compute:

$$x_1 = F^{-1}(u_1), ..., x_N = F^{-1}(u_N)$$

Then $x_1, ..., x_N$ are N independent random observations of the variables X.

2. **Convolution Method**: This method is based on "adding things up", as deduced from the name. We start by considering n independent random variables $X_1, ..., X_N$ with known PDFs $f_1(x), ..., f_N(x)$. Let:

$$Y = \sum_{i=1}^{N} X_i$$

Then the PDF of Y is given by the convolution of all the f_i PDFs.

$$f_Y(y) = \int_{-\infty}^{\infty} f_1(y - x_2) f_2(x_2) \dots \int_{-\infty}^{\infty} f_n(y - x_n) f_n(x_n)\, dx_n \dots\, dx_2$$

3. **Acceptance-Rejection Method**: In this case we leverage the idea of acceptance or rejection based on a comparison with another simpler, easily generated distribution. This method is particularly useful when it is challenging to directly sample from a target distribution. Let's see the various steps. First we choose the simple distribution, also known as "envelope distribution", then we generate a random variate x from the envelope distribution. Let $f(x)$ be the PDF of the target's distribution, $g(x)$ the PDF of the envelope distribution and M is a constant s.t. $M \cdot g(x)$ is a valid upper-bound for $f(x), \forall x$; we can define

$$AcceptanceProbability = \frac{f(x)}{M \cdot g(x)}$$

. At this point we generate a uniform random variate u from the interval $[0, 1]$ and proceed as follows: if

$$u \leq AcceptanceProbability$$

then we accept x as a sample from the target distribution, otherwise reject x and try generating a new random sample from the envelope distribution. We keep going until we have obtained the desired number of samples.