Coordination in complex environments

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Coordination and complexity

Coordination motives and uncertainty are common in innovative contexts.

Examples:

1. Interoperability of Electronic Medical Record Systems (Lin '23),
2. Co-op advertising (Jørgensen-Zaccour '14),
3. Technological innovation.

This paper introduces a model of **coordination** in an **informationally complex** environment.

Overview

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(1) A model of coordination in complex environments;

(2) New ***conformity*** phenomenon;

(3) Source of conformity: **correlation** between the outcomes of the decisions of **different players**.

(4) Applications:

1. Oligopoly pricing;
2. Multi-division organization.

Model

n players.

$x_i \in \mathbf{R}$ is player i's **outcome**.

Payoff to player i from the profile of outcomes $\boldsymbol{x}$ is:

$$\pi_i(\boldsymbol{x}) = -\Bigg[\underbrace{(1-\alpha)\delta_i + \alpha \sum_{j \neq i} \gamma^{ij} x_j}_{i\text{'s target}} - x_i\Bigg]^2,$$

in which

$\alpha \geq 0$ captures coordination motives,

$\delta_i \in \mathbf{R}$ is i's favorite outcome,

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[Ballaster *et al.* '06]

Model | Complexity

Players simultaneously choose **policies** from $[\underline{p}, \overline{p}] \subset \mathbf{R}$.

The **outcome function** χ maps every policy p_i to the corresponding outcome $\chi(p_i)$,

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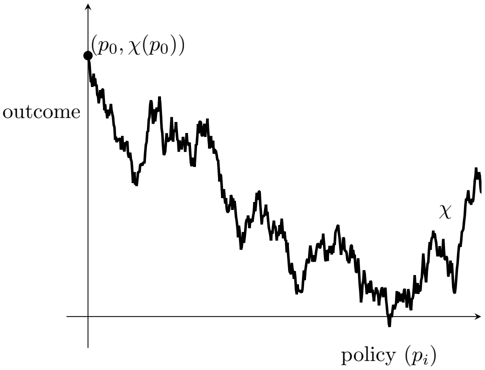
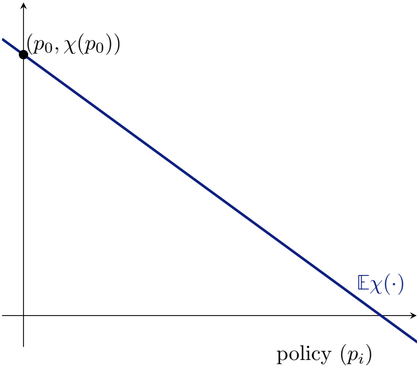
χ is the realization of a Brownian motion with known:

- Drift $\mu < 0$,
- Variance σ^2,
- Initial point $(p_0, \chi(p_0))$.

Model | Complexity



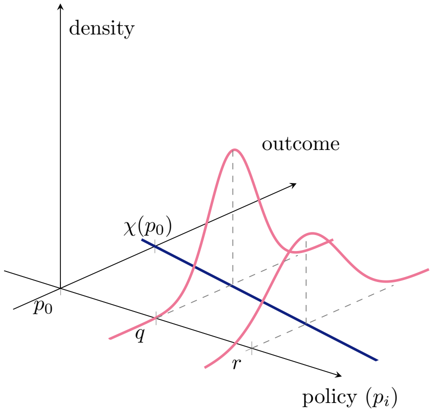
Model | Complexity



Status quo:

$$(p_0, \chi(p_0)).$$

Model | Complexity



Complexity:

$$k = \frac{\sigma^2}{2|\mu|}.$$

Details

Equilibrium

1. Players simultaneously choose policies $p_1, \ldots, p_n$.
2. Player i gets the payoff from the profile of corresponding outcomes:

$$\pi_i(\chi(p_1), \ldots, \chi(p_n)).$$

Equilibrium

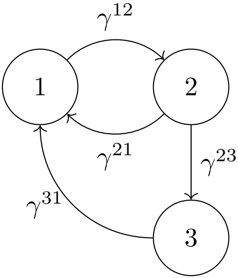
1. Players simultaneously choose policies $p_1, \ldots, p_n$.
2. Player i gets the payoff from the profile of corresponding outcomes:

$$\pi_i(\chi(p_1), \ldots, \chi(p_n)).$$

The policy profile $\boldsymbol{p}$ is an ***equilibrium*** if, for every player i:

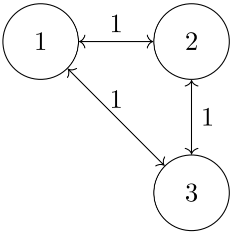
$$\mathbb{E}\pi_i(\boldsymbol{\chi}(\boldsymbol{p})) \geq \mathbb{E}\pi_i(\chi(q_i), \boldsymbol{\chi}(\boldsymbol{p}_{-i})) \text{ for all policies } q_i.$$

Network



$$\mathbf{\Gamma} = (\gamma^{ij}) = \begin{pmatrix} 0 & \gamma^{12} & 0 \\ \gamma^{21} & 0 & \gamma^{23} \\ \gamma^{31} & 0 & 0 \end{pmatrix}$$

Network



$$\mathbf{\Gamma} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

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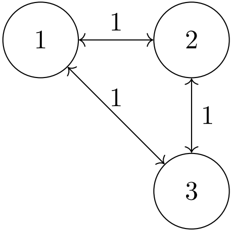

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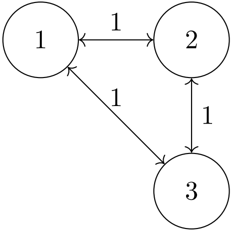
Upper bound on strength of coordination motives:

$$\alpha\lambda(\mathbf{\Gamma}) < 1,$$

in which $\lambda(\mathbf{\Gamma})$ is the largest eigenvalue of the adjacency matrix.

Network



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For this talk: $\gamma^{ij} = \gamma^{ji}$, and:

1. $\underline{p} = p_0$,
2. $\overline{p}$ and $\chi(p_0)$ are sufficiently large.

No complexity

The *centrality of player i* is the ith entry of:

$$\boldsymbol{\beta} = (1 - \alpha)(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}\boldsymbol{\delta}.$$

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β_i counts all 'α-discounted' walks from i and weighs walks to j by $(1-\alpha)\delta_j$, so:

$$\boldsymbol{\beta} \propto \boldsymbol{\delta} + \alpha\boldsymbol{\Gamma}\boldsymbol{\delta} + \alpha^2\boldsymbol{\Gamma}^2\boldsymbol{\delta} + \cdots$$

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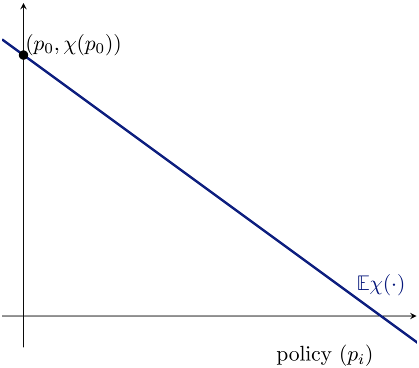

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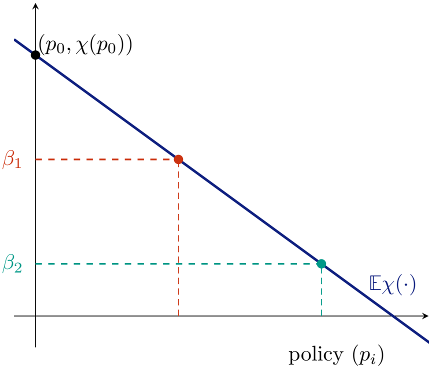
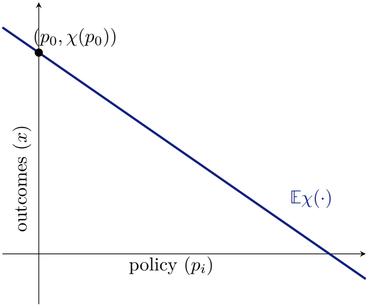


Fact A. (Ballester *et al.* '06)
If $k = 0$, in the unique equilibrium:

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}) = \boldsymbol{\beta}.$$

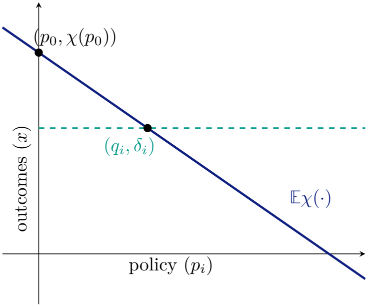
Single player



Fact B. (Callander '11a)
If $\alpha = 0$, player i has a unique optimal policy p_i:

$$\mathbb{E}\chi(p_i) = \delta_i + k.$$

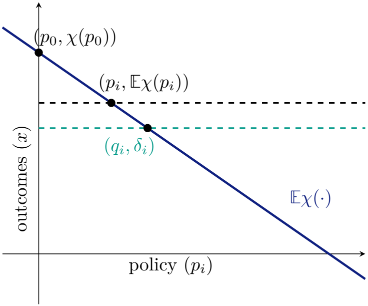
Single player



Fact B. (Callander '11a)
If $\alpha = 0$, player i has a unique optimal policy p_i:

$$\mathbb{E}\chi(p_i) = \delta_i + \underbrace{k}_{\text{status quo bias}}.$$

Single player



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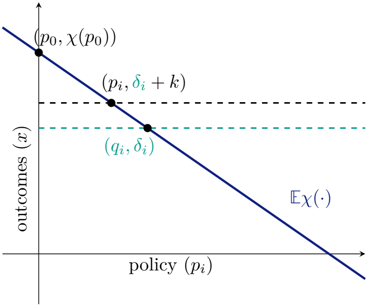

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Two players

$$\begin{tikzcd} 1 \arrow[r, bend left, "\gamma^{12} = 1"] & 2 \arrow[l, bend left, "\gamma^{21} = 1"] \end{tikzcd}$$

$$\boldsymbol{\Gamma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

And: $\delta_1 > \delta_2$

Two players

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$$

And: $\delta_1 > \delta_2 \overbrace{\implies p_1 < p_2}^{\text{no complexity}}$.
Disentangling **pure noise** and **correlation** of players' outcomes.

Two players | Independent outcomes

Player i's outcome of policy p_i is:

$\chi^i(p_i) = \chi(p_0) + \mu p_i + \sigma W^i(p_i),$ for independent standard W^1, W^2.

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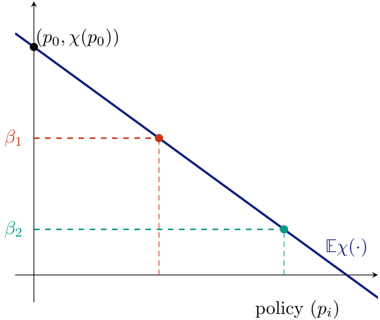

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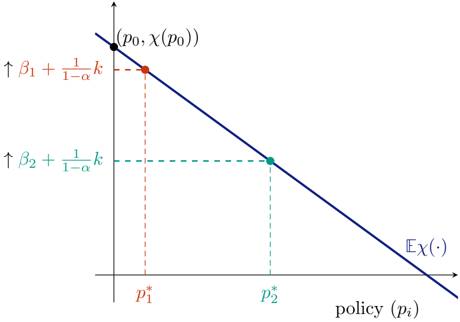

In the unique equilibrium:

$$\mathbb{E}\chi^i(p_i^*) = \beta_i + \underbrace{\overbrace{\frac{1}{1-\alpha}}^{>1}}_{\substack{\text{amplified}\\ \text{s.q. bias}}} k.$$

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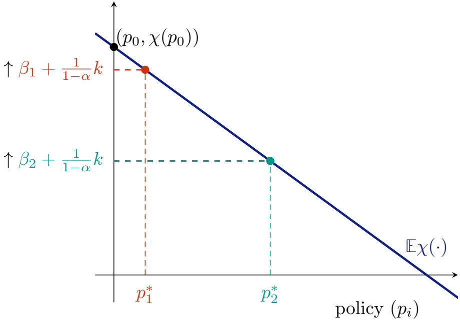

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Conformity? $\mathbb{E}\chi^i(p_i^*) - \mathbb{E}\chi^j(p_j^*) = \beta_i - \beta_j.$

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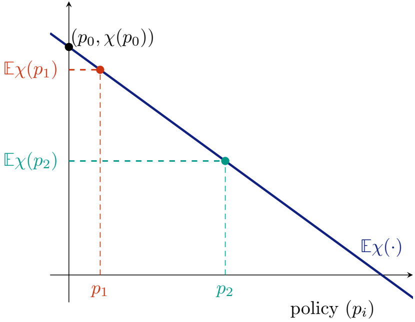

If $p_1 < p_2$, then:
2 is the ***Leader*** and 1 is the ***Follower***,

$$\mathrm{Cov}(\chi(p_1), \chi(p_2)) = \mathrm{Var}\,\chi(p_1).$$

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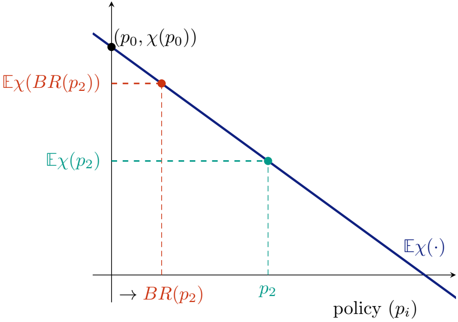

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$\implies$ **Extra Exploration Motive for 1**.

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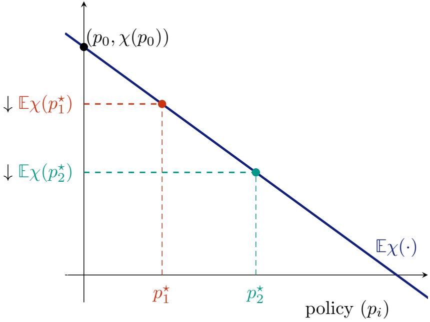

In the unique equilibrium:

$$\mathbb{E}\chi^1(p_1^\star) = \beta_i + k + \frac{1}{1+\alpha}k,$$

$$\mathbb{E}\chi^2(p_2^\star) = \beta_2 + k - \frac{1}{1+\alpha}k,$$

if: $\delta_1 - \delta_2 > 2k\frac{\alpha}{1-\alpha}$.

Conformity: $\mathbb{E}\chi(p_1^\star) - \mathbb{E}\chi(p_2^\star) - (\beta_1 - \beta_2) = \underbrace{-2\frac{\alpha}{1+\alpha}k}_{<0}.$

Two players | Imperfect correlation

Outcomes are given, for $\rho \in [0, 1]$, by:

$$\chi^1(p_1) = \chi(p_0) + \mu p_1 + \sigma W^1(p_1)$$

$$\chi^2(p_2) = \chi(p_0) + \mu p_2 + \rho\sigma W^1(p_2) + \sqrt{1-\rho^2}\sigma W^2(p_2).$$

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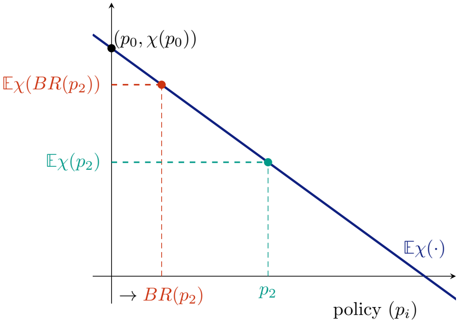

$\Longrightarrow$ ρ-**Weighted Extra Exploration Motive for 1**.

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$\Rightarrow \mathrm{Corr}(\chi^1(p), \chi^2(p)) = \rho$

In equilibrium:

$$\mathbb{E}\chi^1(p_1) - \mathbb{E}\chi^2(p_2) - (\beta_1 - \beta_2) = \rho \underbrace{\left(-2\frac{\alpha}{1+\alpha}k\right)}_{\substack{<0 \\ \text{(perfect correlation)}}}.$$

Strategic complementarities

Lemma 1 (Strategic complementarities)

The expected payoff $\mathbb{E}\pi_i(\boldsymbol{\chi}(\boldsymbol{p}))$ exhibits increasing differences in $(p_i, \boldsymbol{p}_{-i})$, for every player i.

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- Complementarities in outcomes.
- Covariance structure $(\mathrm{Cov}(\chi(p_1), \chi(p_2)) = \mathrm{Var}\,\chi(p_1))$. ▸ More

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Theorem 1 (Existence)

There exist a greatest and least equilibrium.

- Tarski's fixed point theorem.
  (Milgrom-Shannon '90, Vives '90.)

Equilibrium decomposition

Proposition 1 (Decomposition)

The profile of policies $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium if and only if:

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}) = \boldsymbol{\beta} + k\mathbf{1} + \alpha(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} \odot \boldsymbol{A})\mathbf{1}k,$$

for a matrix $\boldsymbol{A} = (a_{ij})$ such that $a_{ij} \in [-1, 1]$ and

$$a_{ij} = \begin{cases} 1 & \text{if } p_i > p_j, \\ -1 & \text{if } p_i < p_j. \end{cases}$$

($\odot$ is element-wise product.)

Equilibrium decomposition

Proposition 1 (Decomposition)

Without complexity, $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium iff:

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Equilibrium decomposition

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Player i's ***conformity effect*** weighs each walk to j by $w_j := \sum_\ell \alpha k \gamma^{j\ell} a_{j\ell}$:

$$\boldsymbol{w} + \alpha\boldsymbol{\Gamma}\boldsymbol{w} + \alpha^2\boldsymbol{\Gamma}^2\boldsymbol{w} + \cdots = \alpha(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} \odot \boldsymbol{A})\mathbf{1}k.$$

($\odot$ is element-wise product.)

Conformity

Suppose the network is complete.

Lemma 2 (Pairwise conformity)

If $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium:

$$\text{If } p_i < p_j \text{, then: } \mathbb{E}\chi(p_i) - \mathbb{E}\chi(p_j) < \beta_i - \beta_j.$$

Conformity

Suppose the network is complete.

Lemma 2 (Conformity in ordered equilibria)

Let $\boldsymbol{p} \in (p_0, \overline{p})^n$ be an equilibrium. If $p_1 < \cdots < p_n$, then:

$$\mathbb{E}\chi(p_i) - \mathbb{E}\chi(p_{i+1}) - (\beta_i - \beta_{i+1}) = \underbrace{-2\frac{\alpha}{1+\alpha}k}_{\downarrow \text{ in } \alpha \text{ \& } k}.$$

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1. If $\uparrow k$, matching a leader's outcome is a more cost effective way of dealing with uncertainty + Conformity 'feeds back' through the network.

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1. If $\uparrow k$, matching a leader's outcome is a more cost effective way of dealing with uncertainty + Conformity 'feeds back' through the network.
2. "*Yielding is far greater on* ***difficult*** *items than on easy ones.*" (Asch '51; difficulty elicited as "*certainty of judgement*".)

▸ counterformity

Extra slides

Literature

Complexity à la Callander '11a

- Decision problems, players interacting over time: Jovanovic-Rob '90, Callander-Hummel '14, Garfagnini-Strulovici '16, Callander-Matouschek '19.
- Competitive elections: Callander '11b.
- Principal-Agent models: Callander '08, Callander *et al.* '21, Aybas-Callander '23.

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Coordination games with quadratic payoffs

- Complete information: Ballester *et al.* '06, Bramoullé *et al.* '14, Galeotti *et al.* '20, oligopoly (Amir *et al.* '17), ...
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Team & potential games Radner '62, Monderer-Shapley '96, ...

Order structure of the equilibrium set

Let $n = 2$ and $\delta_1 = \delta_2 = 0$.

Every equilibrium $\boldsymbol{p}$ is symmetric: $p_1 = p_2$.

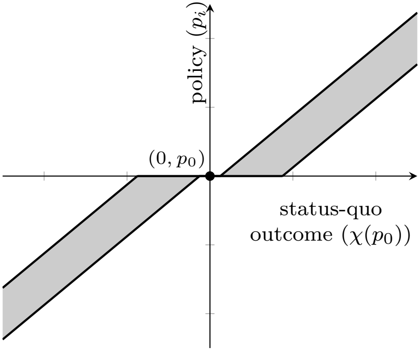

Figure: The equilibrium set, represented by player i's policy, for every status-quo outcome.

Extensions

(1) The outcome of policy p to player i is:

$$\chi^i(p) = \chi(p_0) + \mu p + \sigma W^i(p),$$

with $dW^i(p)dW^j(p) = \rho c_{ij} dt$.

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In equilibrium, if $\mathbf{\Gamma}$ is irreducible:

$$\mathbb{E}\chi^i(p_i) = \beta_i + \overbrace{\underbrace{a_i}_{>1} k}^{\substack{\text{amplified} \\ \text{s.q. bias}}} + \overbrace{\rho \underbrace{b_i}_{\leq 0} k}^{\substack{\text{exploration} \\ \text{motive}}},$$

[(c_{ij}) symm. pos.-def., $c_{ij}\rho \in [0,1]$.]

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---

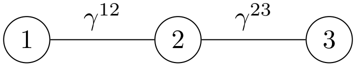
(2) Player i believes that the initial point is:

$$(p_0^i, \underbrace{\chi(p_0^i)}_{\text{private information.}}).$$

More

Single Crossing.
The expected payoff $\mathbb{E}^i \pi_i(\chi(p_i), \boldsymbol{\chi}(\boldsymbol{\sigma}_{-i}))$ has strictly increasing differences in $(p_i, \chi(p_0^i))$, if strategies in $\boldsymbol{\sigma}_{-i}$ are nondecreasing.

Counterformity



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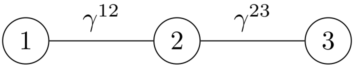
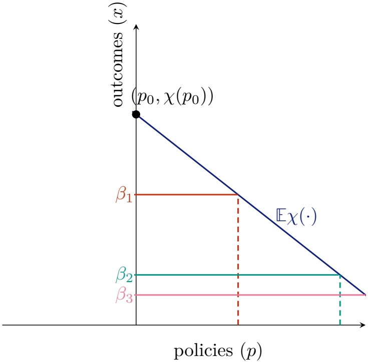


Without complexity:



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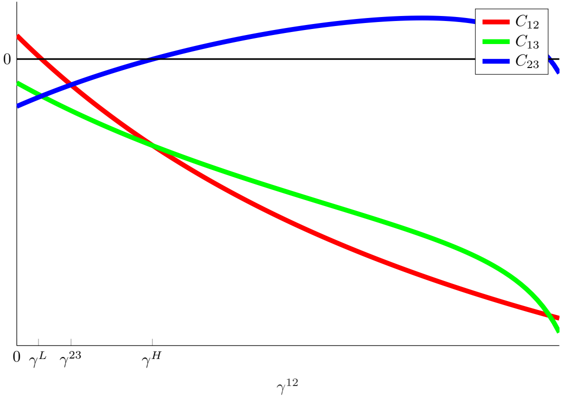

$$C_{ij} = \mathbb{E}\chi(p_i^\star) - \mathbb{E}\chi(p_j^\star) - \beta_i + \beta_j.$$

conformity

Distribution

For $p_0 < p < q$:

$$\begin{aligned}
\mathbb{E}\chi(p) &= \chi(p_0) + \mu(p - p_0) \\
\operatorname{Var}\chi(p) &= (p - p_0)\sigma^2 \\
\operatorname{Cov}(\chi(p), \chi(q)) &= \operatorname{Var}\chi(p). \\
&= \min\{p - p_0, q - p_0\}\sigma^2
\end{aligned}$$

Back

Coordination and complexity

If $\omega > 0$ and $\alpha > 0$, 'kinked' mean-variance decomposition.

Coordination and complexity

With $n = 2$ and $\delta_1 = \delta_2 = 0$, player i's loss given $p_i \geq p_j \geq p_0$ is

$$\begin{aligned}\mathbb{E}(\chi(p_i) - \alpha\chi(p_j))^2 = (\mathbb{E}\chi(p_i) - \alpha\mathbb{E}\chi(p_j))^2 + \mathbb{V}\chi(p_i) \\ \underbrace{-2\alpha\mathbb{C}(\chi(p_i), \chi(p_j))}_{k>0\ \&\ \alpha>0} + \cdots ,\end{aligned}$$

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in which:

$$\begin{aligned}\mathbb{C}(\chi(p_i), \chi(p_j)) &= \mathbb{C}(\chi(p_j) + \underbrace{\chi(p_i) - \chi(p_j)}_{\text{increment from } \chi(p_j)}, \chi(p_j)) \\ &= \min\{\mathbb{V}\chi(p_i), \mathbb{V}\chi(p_j)\}.\end{aligned}$$

(Independent increments = 'maximum ignorance', Jovanovic-Rob '90.)

Endogenous location of the kink: p_j.

Coordination and complexity

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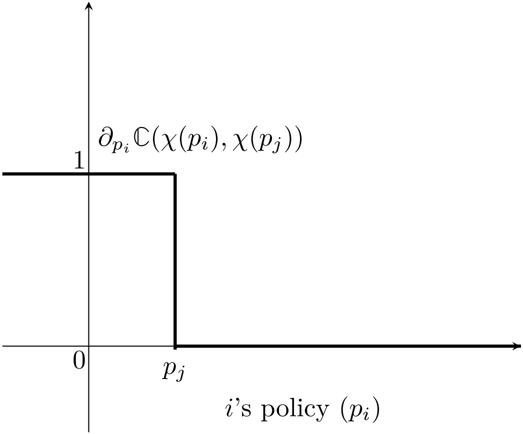
Coordination and complexity

Covariance ($\min\{\mathbb{V}\chi(p_i), \mathbb{V}\chi(p_j)\}$) is supermodular in (p_i, p_j).



Back

Coordination and complexity

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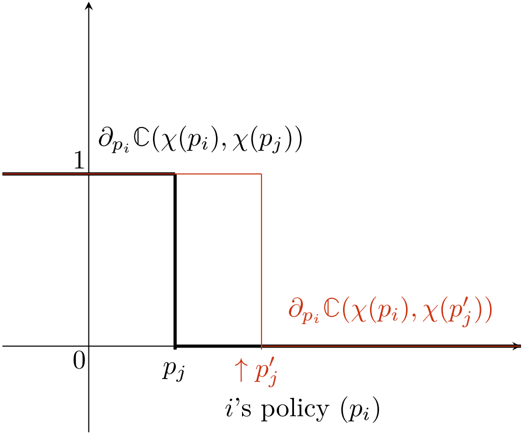

Back

Covariance

$f(p_1, p_2)$ has ***strictly increasing differences*** in p_1 and p_2 if:

$$p_1' > p_1 \text{ and } p_2' > p_2 \implies f(p_1', p_2') - f(p_1, p_2') > f(p_1', p_2) - f(p_1, p_2).$$

Covariance

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$\mathrm{Cov}(\chi(p), \chi(p'))$, for $p_0 = 0$ and $p, p' > 0$, can be:

- Brownian:

$$\min\{p, p'\}\sigma^2; \qquad \checkmark$$

- Ornstein-Uhlenbeck:

$$e^{-\frac{|p-p'|}{\ell}}, \ \ell > 0; \qquad \text{X}$$

- Squared exponential:

$$e^{-\left(\frac{p-p'}{\ell}\right)^2}, \ \ell > 0. \qquad \text{X}$$

▸ Back

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