

# **Screening in digital monopolies**

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## Free damaging and replication

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2. Free damaging.

↑ taste heterogeneity

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This paper studies monopoly provision of goods whose production features free replication and free damaging.

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Examples of **digital goods**:

1. Software goods;
2. Digital content;
3. Data.

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Maximum number of variables <small> ⓘ</small>					
Up to 2,048 variables	✓	✓	✓	✓	✓
Up to 32,767 variables	-	✓	✓	✓	✓
Up to 120,000 variables	-	-	✓	✓	✓
Maximum number of observations <small> ⓘ</small>					
Up to 2.14 billion	✓	✓	✓	✓	✓
Up to 20 billion	-	-	✓	✓	✓

Equifax offers three standard Business Credit Reports: BCR Complete View, BCR Risk View and BCR Profile View. The difference between reports is the number of sections that are available in the report:

REPORT SECTION TITLE	COMPLETE VIEW	RISK VIEW	PROFILE VIEW
Business Information & Firmographics	✓	✓	✓
Report Highlights & Alerts	✓	✓	✓
Index Values	✓	✓	✓
Quarterly Index Value & Payment History	✓	✓	✓
Credit Risk Scores	✓	✓	
Trade Details (Industry & Financial)	✓		
Negative Occurrences (Bankruptcy, Collections, Returned Cheques)	✓		
Legal Data (Suits, Judgments, Liens)	✓		
Inquiries	✓		
Other	✓		

## Preview

This paper studies screening when production features free replication and free damaging.

The **monopoly** provision features two interdependent inefficiencies:

1. **Productive inefficiency**: the best quality is lower than its efficient level;
2. **Distributional inefficiency**: some buyers purchase a damaged good.

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1. **Productive inefficiency**: the best quality is lower than its efficient level;
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Extensions:

- ▶ A no-damaging constraint makes productive inefficiency stronger;
- ▶ **Competition** is harmful for productive efficiency and beneficial for damaging inefficiency.

# Plan

1. Model;
2. Efficiency benchmark;
3. Monopoly allocation and inefficiencies;
4. No-damaging constraint;
5. Single buyer;
6. Competition.

# **Model**

# Model

A unit mass of buyers, each drawing a **type**  $\theta \in [0, 1] = \Theta$ , interacts with a seller.

Type  $\theta$  is privately informed about  $\theta \sim F$ , for twice diff.  $F$ ;

→  $F$  is regular except in dedicated slide,  $\mathbb{E}\{\cdot\}$  refers to the r.v.  $\theta$ .

Type  $\theta$  has payoff from **quality**  $q \in \mathbb{R}_+$  and transfer  $t \in \mathbb{R}$ :

$$\underbrace{g(q) + \theta q - t}_{\text{utility } u(q, \theta)}$$

for a strictly concave, increasing, and twice diff.  $g$  with  $g(0) = 0$ .

An **allocation** is a measurable  $q: \Theta \rightarrow \mathbb{R}_+$ .

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An **allocation** is a measurable  $\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ .

The cost of allocation  $\mathbf{q}$  is

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)),$$

for a **production cost**  $c$ , increasing, strictly convex, twice diff., with  $c'(0) = 0$  and  $\lim_{q \rightarrow \infty} c'(q) = \infty$ .

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An **allocation** is a measurable  $\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ .

With *separable* costs, the cost of  $\mathbf{q}$  is

$$C(\mathbf{q}) = \mathbb{E}\{k(\mathbf{q}(\theta))\},$$

for some  $k$  (Mussa-Rosen '78.)

# Model

1. The cost of allocation  $\mathbf{q}$  is

$$\begin{aligned} C(\mathbf{q}) &= c(\sup \mathbf{q}(\Theta)) \\ &\neq \int_{\Theta} c(\text{what } \theta \text{ gets}) dF(\theta), \end{aligned}$$

for a **production cost**  $c$ , increasing, strictly convex, twice diff., with  $c'(0) = 0$  and  $\lim_{q \rightarrow \infty} c'(q) = \infty$ .

2. The main results hold with increasing-differences  $u$ ,
  - 2.1 Basic task (text editing):  $g(q)$ ;
  - 2.2 Advanced task (computing):  $\theta q$ .

# Model

1. In general,

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \int_{\Theta} k(\mathbf{q}(\theta)) dF(\theta),$$

for some  $k$ .

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# Efficiency

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The *surplus* induced by allocation  $\mathbf{q}$  is

$$\mathbb{E}\{u(\mathbf{q}(\theta), \theta)\} - c(\sup \mathbf{q}(\Theta)).$$

The *efficient* allocation  $\mathbf{q}^*$  maximizes surplus.

## Proposition 1

The efficient allocation is given by  $\mathbf{q}^*(\theta) = q^*$  for all  $\theta$ , in which  $q^*$  is the unique quality  $q$  such that

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The *surplus* induced by allocation  $\theta \mapsto q$  is

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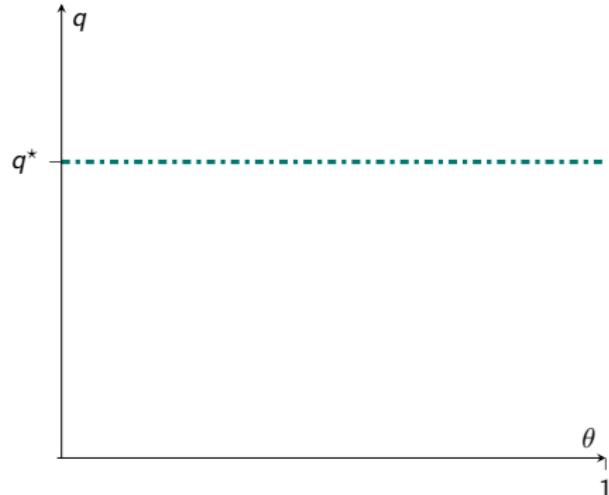
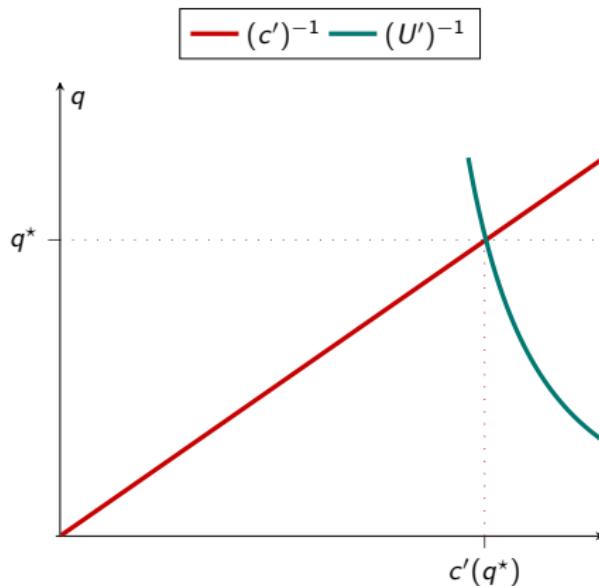
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The utility induced by allocation  $\theta \mapsto q$  is  $U(q) = g(q) + \mathbb{E}\{\theta\}q$ .

# Monopoly

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The monopolist problem is:

$$(\mathcal{P}^M) \quad \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) - c(\sup \mathbf{q}(\Theta)) \text{ subject to:}$$

$$\begin{aligned} u(\mathbf{q}(\theta), \theta) - t(\theta) &\geq u(\hat{\mathbf{q}}(\hat{\theta}), \theta) - t(\hat{\theta}), \text{ for all } (\theta, \hat{\theta}), \\ u(\mathbf{q}(\theta), \theta) - t(\theta) &\geq 0, \text{ for all } \theta. \end{aligned}$$

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- ▶ The *monopolist* allocation  $\mathbf{q}^M$  solves  $\mathcal{P}^M$  for some  $t(\cdot)$ .
- ▶ Without separable costs: the monopolist problem cannot be solved via “pointwise maximization”.

# Monopoly

The  $q$ -constrained problem is:

$$(\mathcal{P}(q)) \quad V(q) := \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) - \cancel{c(\sup_{\Theta} \mathbf{q}(\Theta))} \text{ subject to:}$$

$$\mathbf{q}(\theta) \leq q, \text{ for all } \theta,$$

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## Lemma 1 (Invest then distribute)

The allocation  $\mathbf{q}$  solves  $\mathcal{P}^M$  if and only if:

1.  $\mathbf{q}$  solves  $\mathcal{P}(q^M)$ ,
2.  $q^M$  solves  $\max_q V(q) - c(q)$ .

# Monopoly

The  $q$ -constrained problem is:

$$(\mathcal{P}(q)) \quad V(q) := \max_{\mathbf{q}} \int_{\Theta} \underbrace{g(\mathbf{q}(\theta)) + \varphi(\theta)\mathbf{q}(\theta)}_{\text{Virtual surplus}} dF(\theta) \text{ subject to:}$$

$$\mathbf{q}(\theta) \leq q, \text{ for all } \theta,$$

$\mathbf{q}$  is nondecreasing;

in which  $\varphi(\theta) := \theta - \frac{1-F(\theta)}{F'(\theta)}$ .

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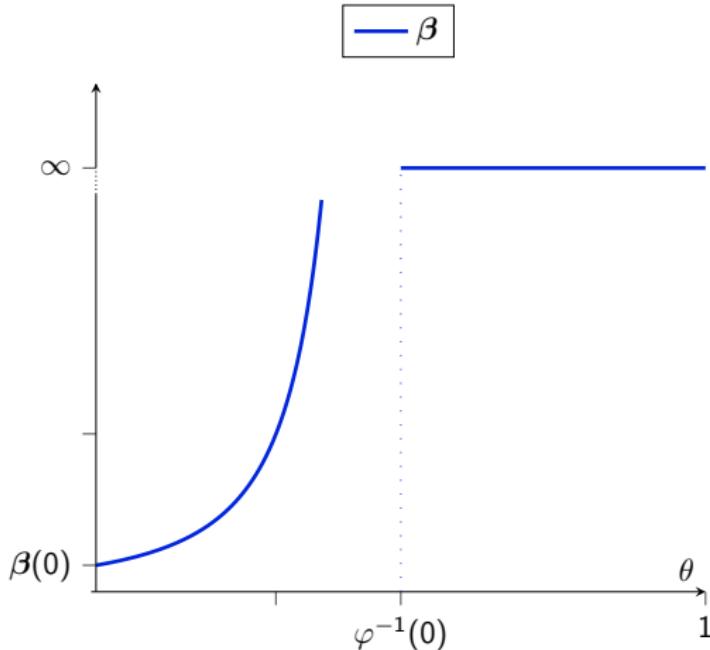
# Virtual surplus maximization

The *virtual-surplus maximizer*

$$\beta(\theta) \in \operatorname{Argmax}_q g(q) + \varphi(\theta)q$$

satisfies:

1.  $\beta(\theta) = \infty$  if  $\theta \geq \varphi^{-1}(0)$ ;
2.  $\beta$  is increasing;
3.  $\beta(0) > 0$  ("Inada"  $g$ ).



# Virtual surplus maximization

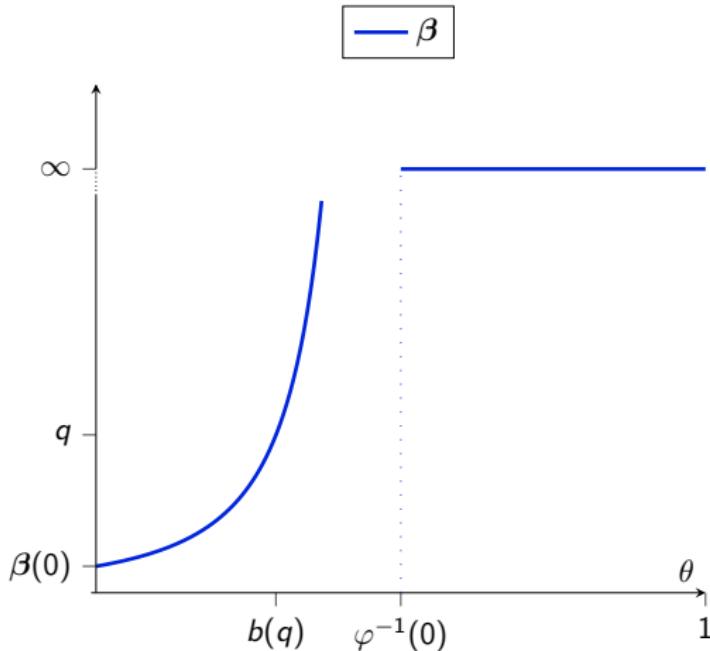
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$b$  is the inverse of  $\beta$ .

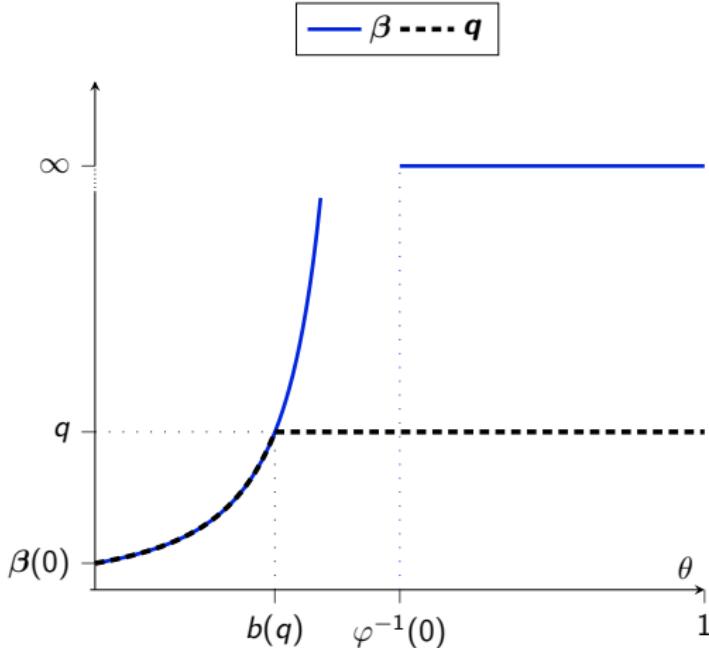


# Virtual surplus maximization

## Lemma 2

Allocation  $\mathbf{q}$  solves  $\mathcal{P}(\mathbf{q})$  iff:

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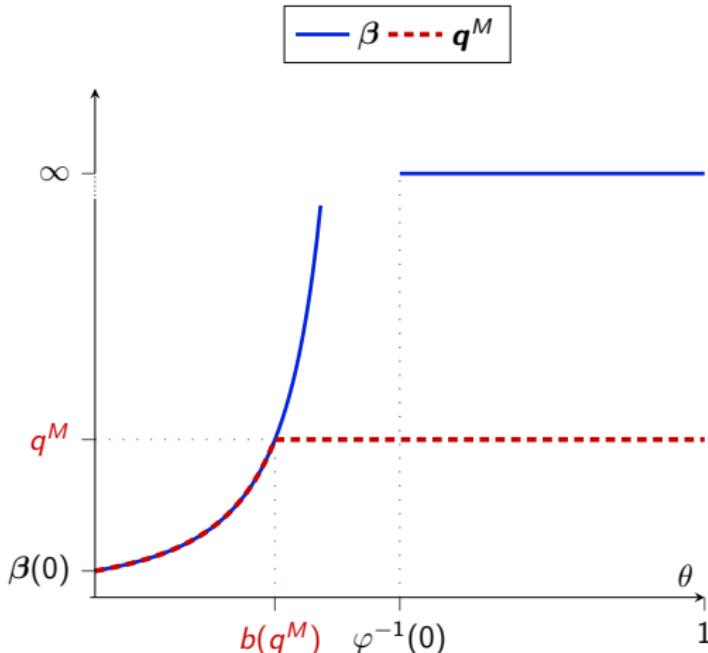
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Distributive properties of  $\mathbf{q}^M$ :

1. Bunching at the top;
2. Distributional inefficiency at the bottom or full bunching;
3. No exclusion (if  $q^M > 0$ .)



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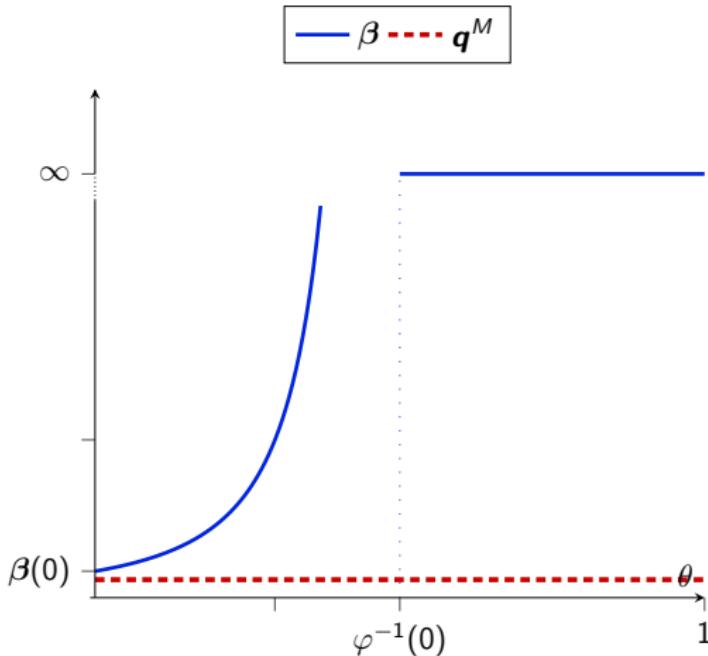
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# Linear preferences

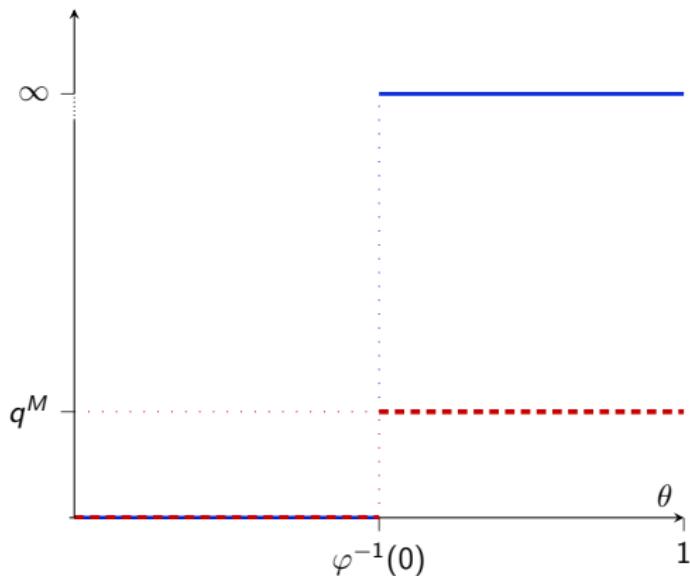
Distributive properties if  
 $g(q) = 0$ :

1. Bunching at the top;  
 $\beta(\theta) = \infty$  for  $\theta \geq \varphi^{-1}(0)$
2. Exclusion at the bottom;  
 $\beta(\theta) = 0$  for  $\theta < \varphi^{-1}(0)$

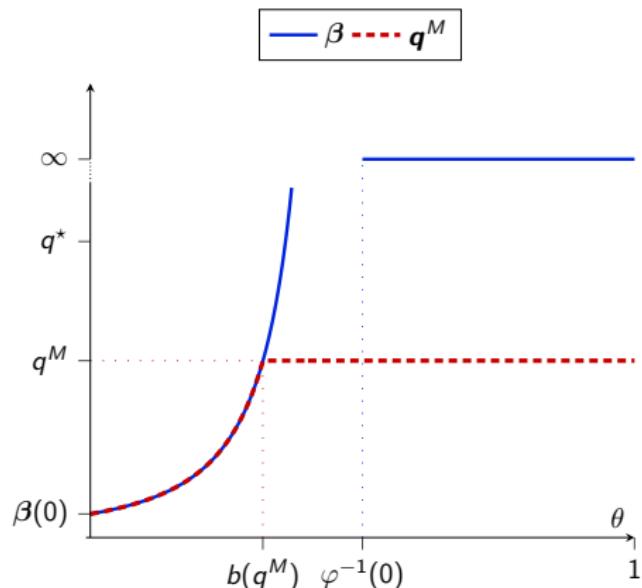
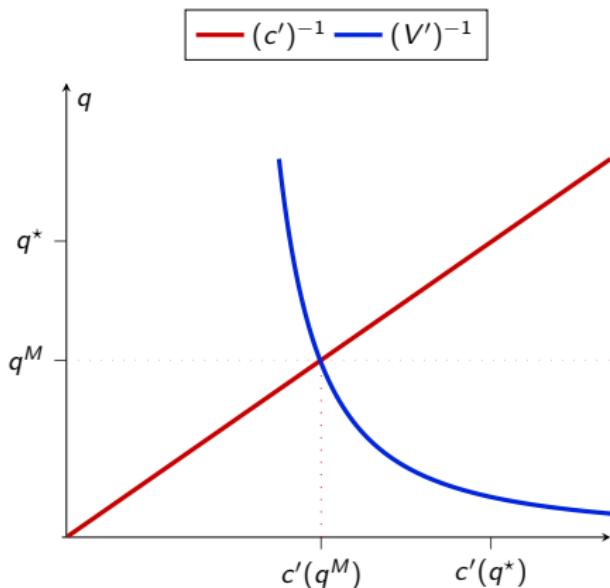
$\implies$  **single version.**

Richness in digital markets is  
due solely to preferences.

$$\begin{array}{c} \text{---} \beta \\ \text{---} \cdot \cdot \cdot q^M \end{array}$$



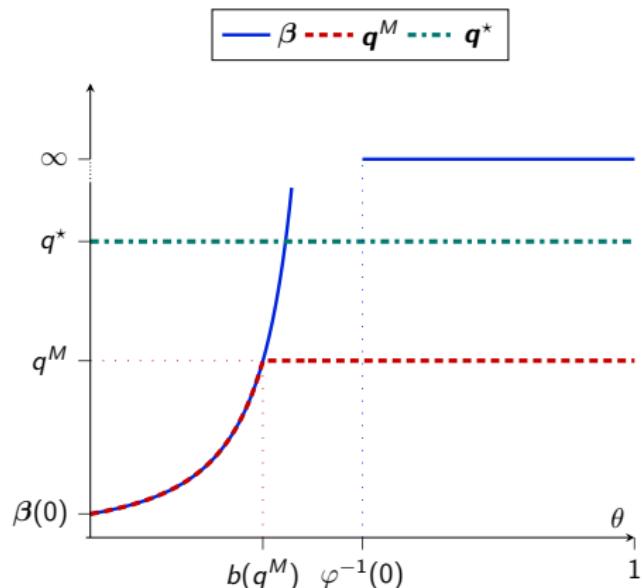
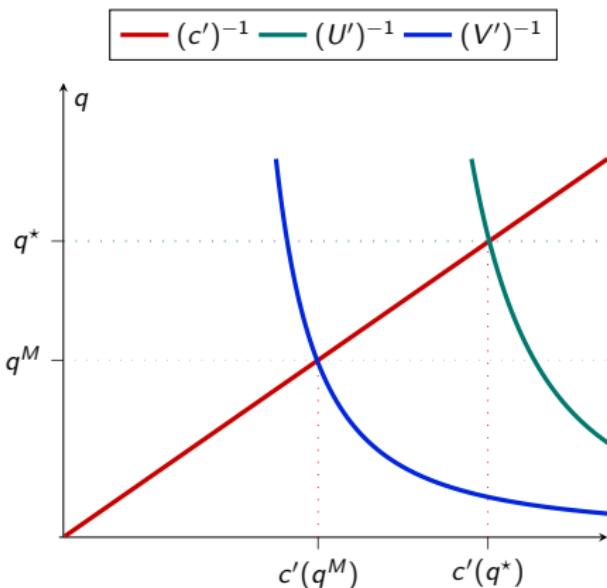
# The monopolist allocation



**Acquisition**

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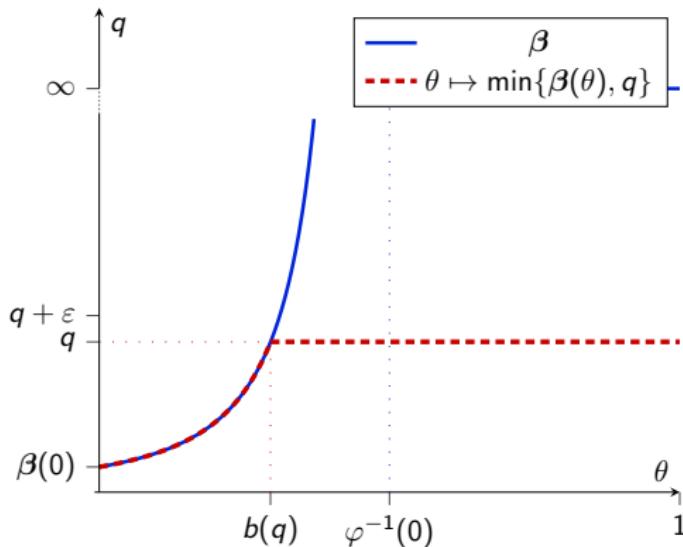
## Marginal revenues

The return from increasing the cap of the  $q$ -constrained allocation is:

$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{bunched types}} \underbrace{(g'(q) + b(q))}_{u_q \text{ of } b(q)}.$$

The change from  $q$  to  $q + \varepsilon$  leads to:

1. same revenues from  $q' < q$ :  
 $q'$  sold to the same  $\theta$ , and  
 $\theta$  gets the same **rent**;
2. higher quality for bunched types;
3. higher price by  $u_q(q, b(q))$ .



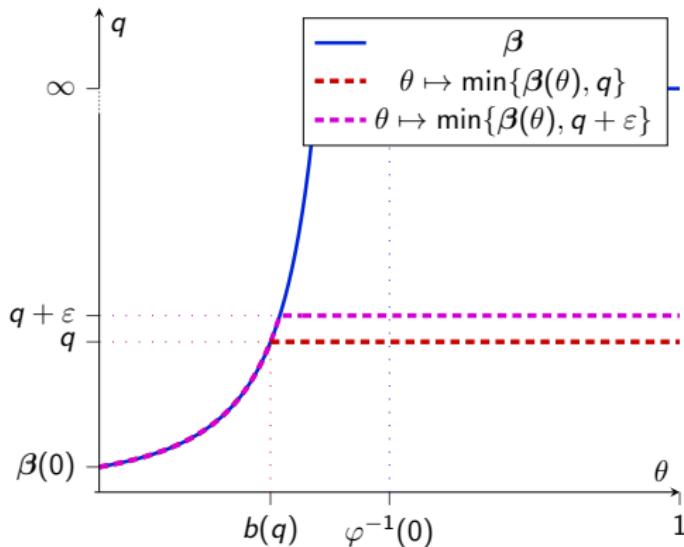
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1. By the distributive properties

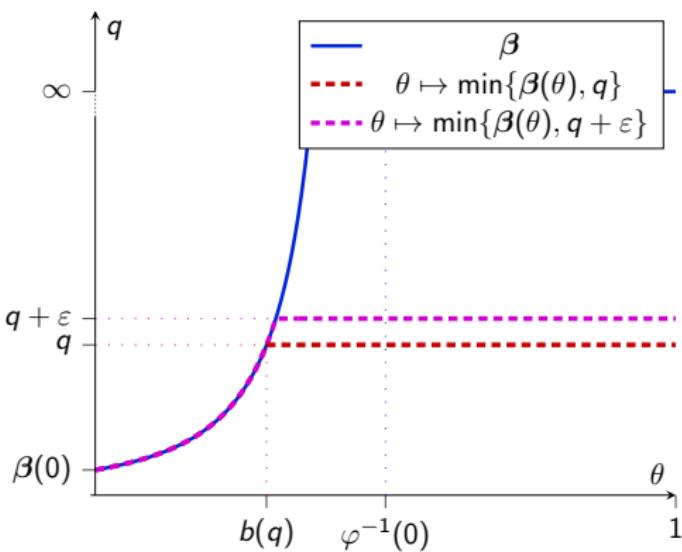
$$1 - F(b(q)) < 1;$$

2. By Markov's inequality:

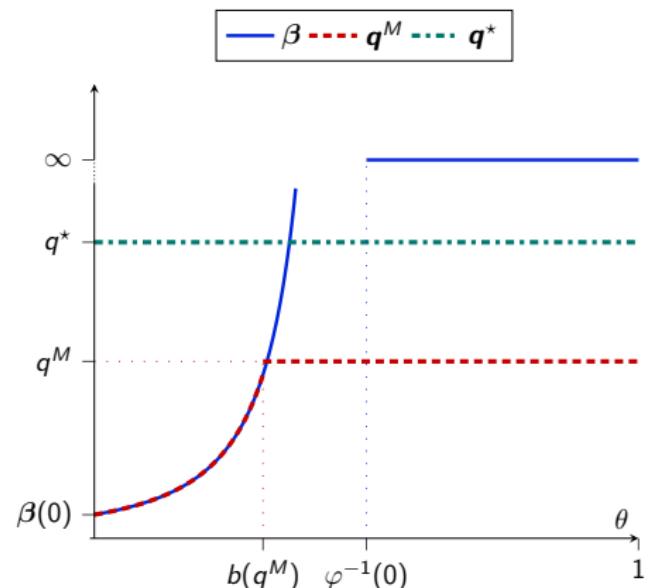
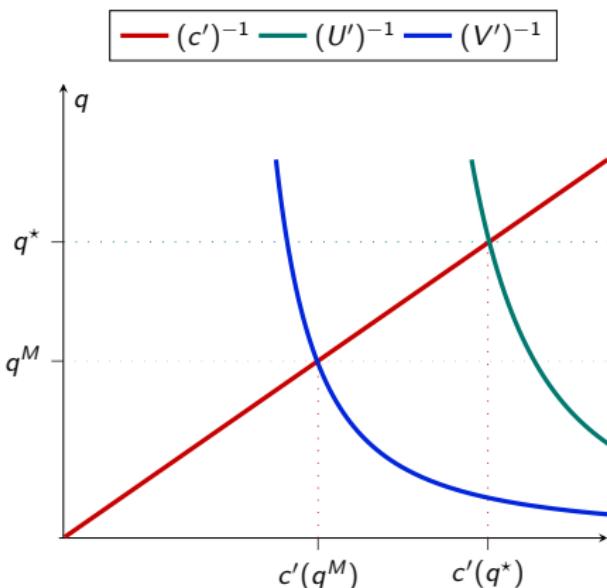
$$(1 - F(b(q)))b(q) \leq \mathbb{E}\{\theta\};$$

So:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$



# Productive inefficiency



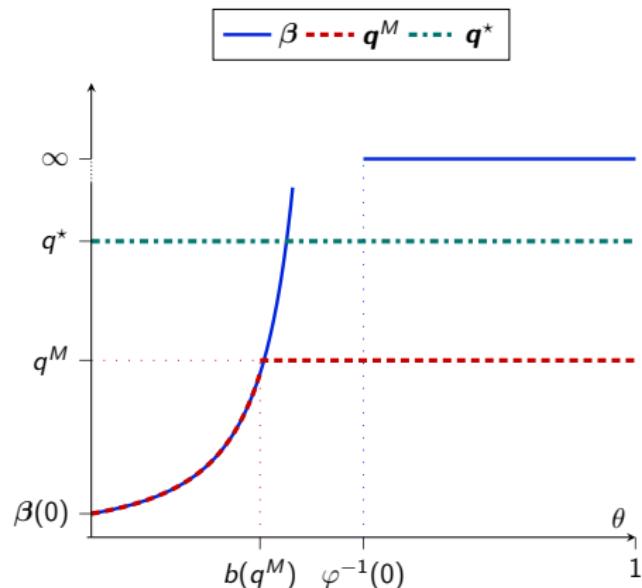
# Productive inefficiency

## Proposition 2

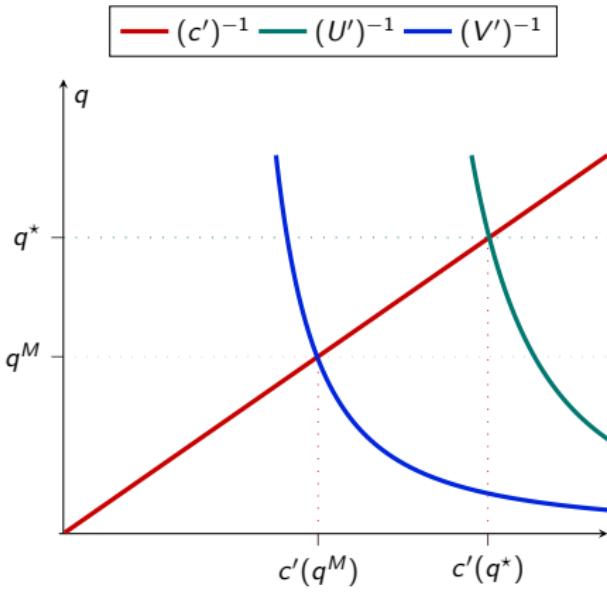
The monopolist allocation is given by  $q^M(\theta) = \min\{\beta(\theta), q\}$  for all  $\theta$ , in which  $q^M$  is the unique  $q$  solving

$$V'(q) = c'(q).$$

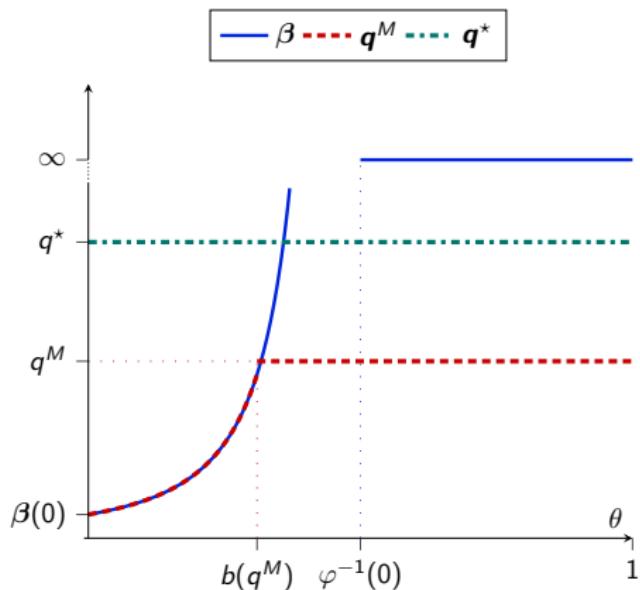
Moreover, it holds that:  $q^M < q^*$ .



# Productive inefficiency

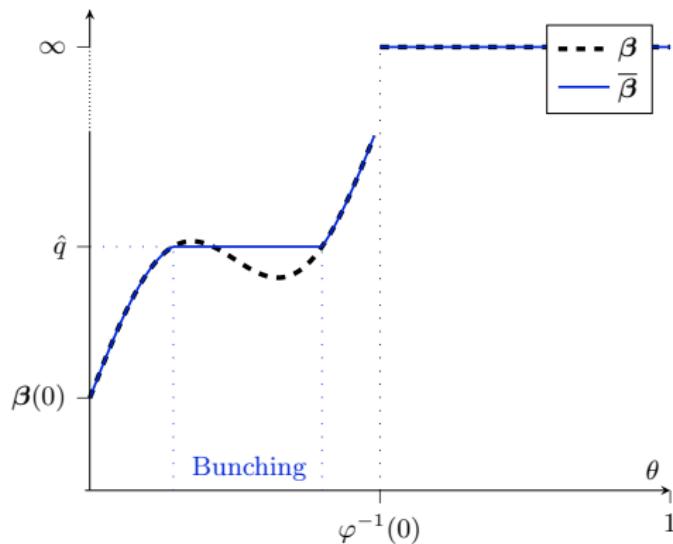


**Acquisition:**  $q^M$  is the quality  $q$  solving:  
$$(1 - F(b(q)))u_q(q, b(q)) = c'(q).$$



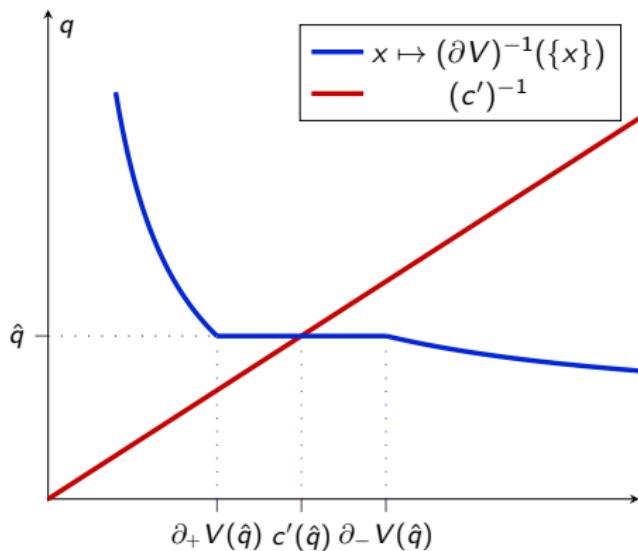
**Distribution:**  
$$q^M(\theta) = \min\{\beta(\theta), q^M\}.$$

# Non-regular distribution



- ▶  $\beta$  is ironed to obtain  $\bar{\beta}$ ;
- ▶ By Lemma 1,  
 $\theta \mapsto \min\{\bar{\beta}(\theta), q\}$  solves  $\mathcal{P}(q)$ ;

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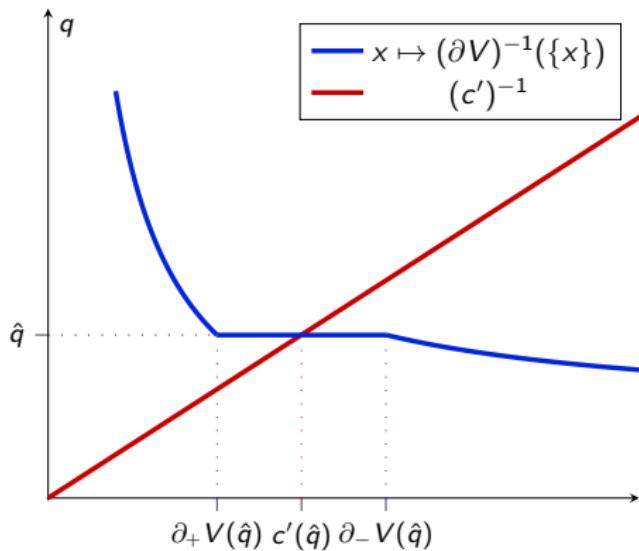


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- ▶ If types in  $(\theta', \theta'')$  are bunched “at”  $\hat{q} \in (0, q)$ ,

$$\partial_- V(\hat{q}) > \partial_+ V(\hat{q}),$$

the extra revenues from  $\hat{q} + \varepsilon$  come from types higher than  $\theta''$ .

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## Proposition 3

The monopolist allocation is  $q^M(\theta) = \min\{\bar{\beta}(\theta), q^M\}$ , in which  $q^M$  is the unique  $q$  with  $c'(q) \in \partial V(q)$ . Moreover, it holds that  $q^M < q^*$ .

**No damaging constraint**

## The highest quality increment

- ▶ **Taxation principle:** The problem  $\mathcal{P}(q)$  can be stated as the choice of a **tariff**  $T: \mathbb{R}_+ \rightarrow \mathbb{R}$ .
- ▶ The **price increment**  $T'(q)$  of the optimal tariff  $T$  satisfies

$$T'(q) \text{ solves } \max_{p \in \mathbb{R}_+} p \Pr(\{\theta \mid g'(q) + \theta \geq p\})$$

(Wilson, 1993).

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(Wilson, 1993).

- ▶ The price increment at  $q$  is  $g'(q) + b(q)$ .

$$\implies g'(q) + b(q) \text{ solves } \max_{p \in \mathbb{R}_+} p \Pr(\{\theta \mid g'(q) + \theta \geq p\}).$$

## Fact 1

The type  $b(q)$  solves  $\max_{\theta \in \Theta} (g'(q) + \theta)(1 - F(\theta))$ .

## No damaging

Without damaging, the  $q$ -constrained problem is:

$$V_N(q) := \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) \text{ subject to:}$$

IC, IR,  $\mathbf{q}(\theta) \in \{0, q\}$ , for all  $\theta$ .

The constraint is irrelevant under:

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- ▶ so productive inefficiency is worse:

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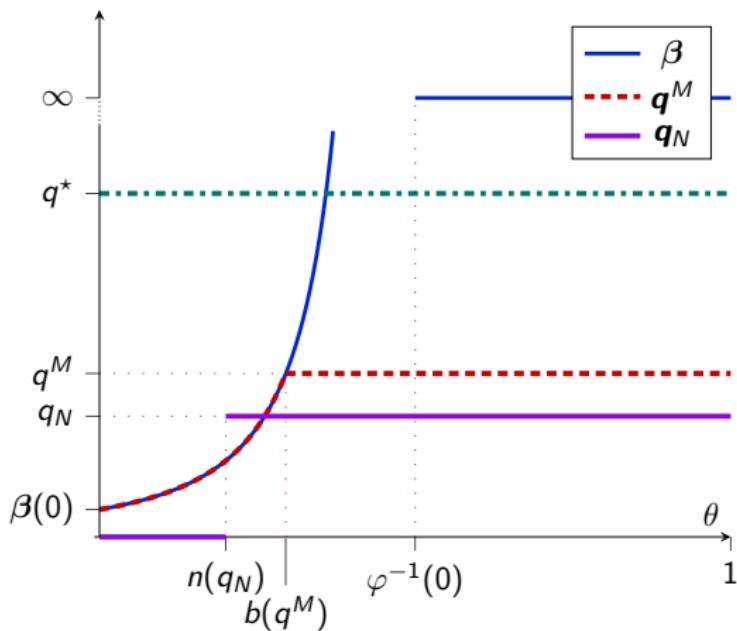
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The no-damaging allocation  $q_N$  features:

- ▶ Less production;
- ▶ No damaging;
- ▶ (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.

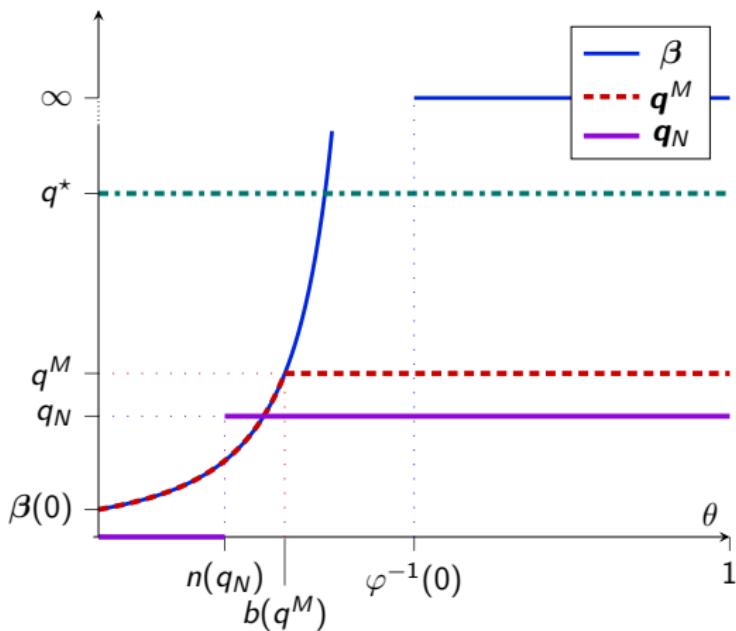


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## Proposition 4

Without damaging, the monopolist allocation is  $q_N(\theta) = \mathbf{1}_{[b_N(q_N), 1]}(\theta)q_N$ , in which  $q_N$  is the unique  $q$  solving  $V'_N(q) = c'(q)$ . Moreover, we have  $q_N \leq q^M$ , strictly if  $b(q^M) > 0$ .

## **Separable costs**

## Cost interpretation

For digital goods:  $\Pi(\mathbf{q}) := \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{per-agent revenues}} - \underbrace{c(\sup \mathbf{q}(\Theta))}_{\text{per-agent unit cost}}$ ,

1. Payment  $t(\theta)$  and production cost  $c(q^M)$  have different size; (Shapiro and Varian, 1998).

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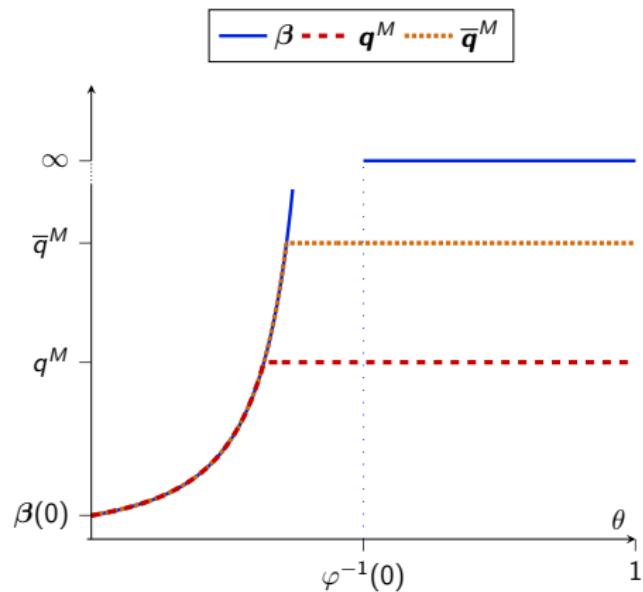
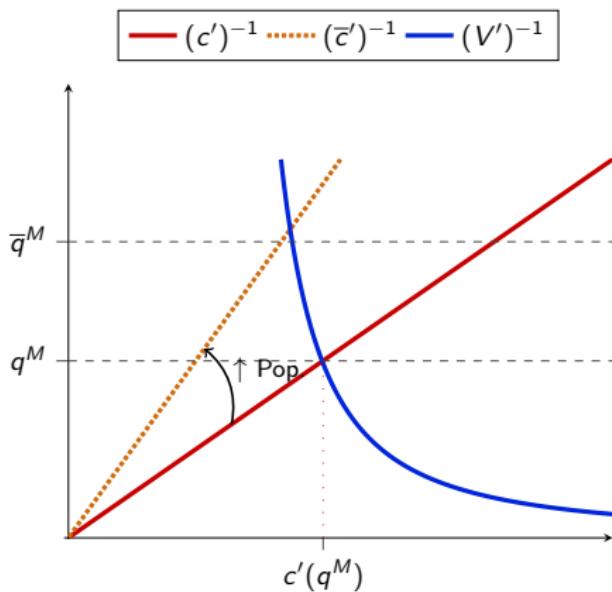
1. Payment  $t(\theta)$  and production cost  $c(q^M)$  have different size; (Shapiro and Varian, 1998).
2. Population size impacts  $q^M$ ;

For separable costs:  $\Pi^{M-R}(\mathbf{q}) := \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{per-agent revenues}} - \underbrace{\int_{\Theta} k(\mathbf{q}(\theta)) dF(\theta)}_{\text{per-agent costs}},$

1. Payment  $t(\theta)$  and production cost  $k(\mathbf{q}(\theta))$  are comparable;
2. Population size only scales profits;

In general:  $C(\mathbf{q}) = \int_{\Theta} k(\mathbf{q}(\theta)) dF(\theta) + c(\sup \mathbf{q}(\Theta)).$

# Population size



## Single buyer

$$\Pi^{\text{M-R}}(\mathbf{q}) = \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{expected revenues}} - \underbrace{\int_{\Theta} c(\mathbf{q}(\theta)) dF(\theta)}_{\text{expected costs}},$$

1. Payment  $t(\theta)$  and production cost  $c(\mathbf{q}(\theta))$  are comparable;
2. Production occurs **after** eliciting the buyer's type;
3. Free damaging and replication are irrelevant.

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3. Free damaging matters, replication is irrelevant.

# Single buyer

$$\mathbf{M-R:} \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) - c(\mathbf{q}(\theta)) dF(\theta) \quad \mathbf{D-S:} \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) - c(\sup \mathbf{q}(\Theta)) dF(\theta)$$

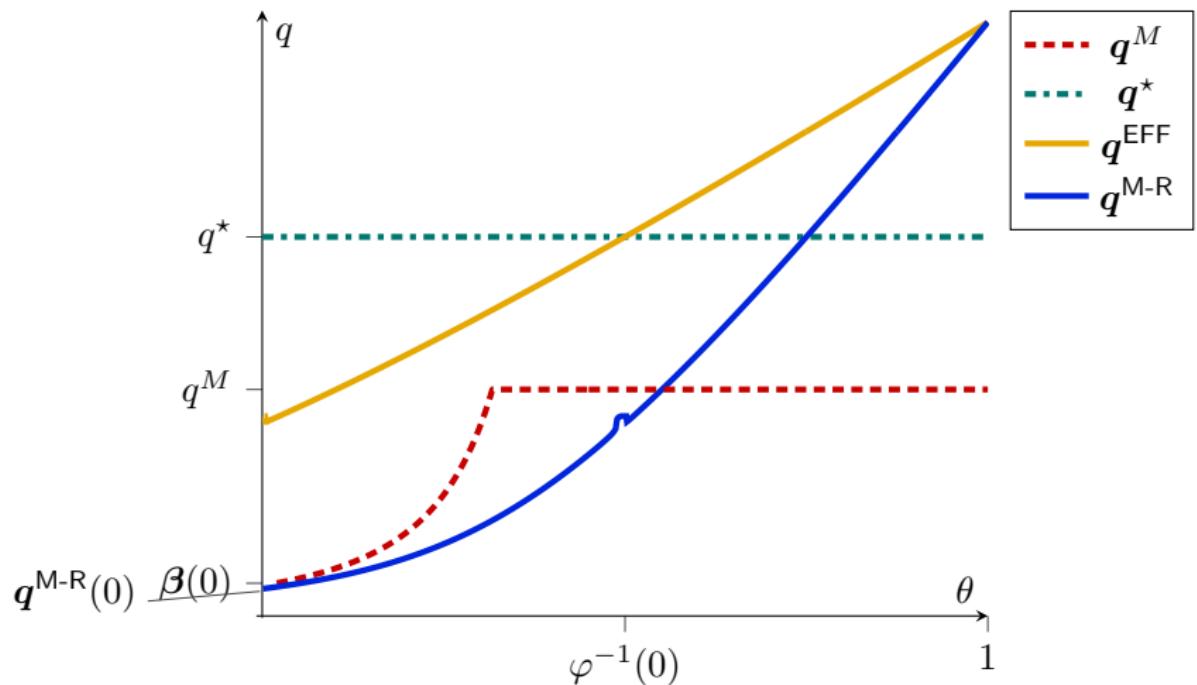
subject to: IC, IR.

elicit  $\rightarrow$  produce ( $\rightarrow$  damage)

produce  $\rightarrow$  elicit  $\rightarrow$  damage

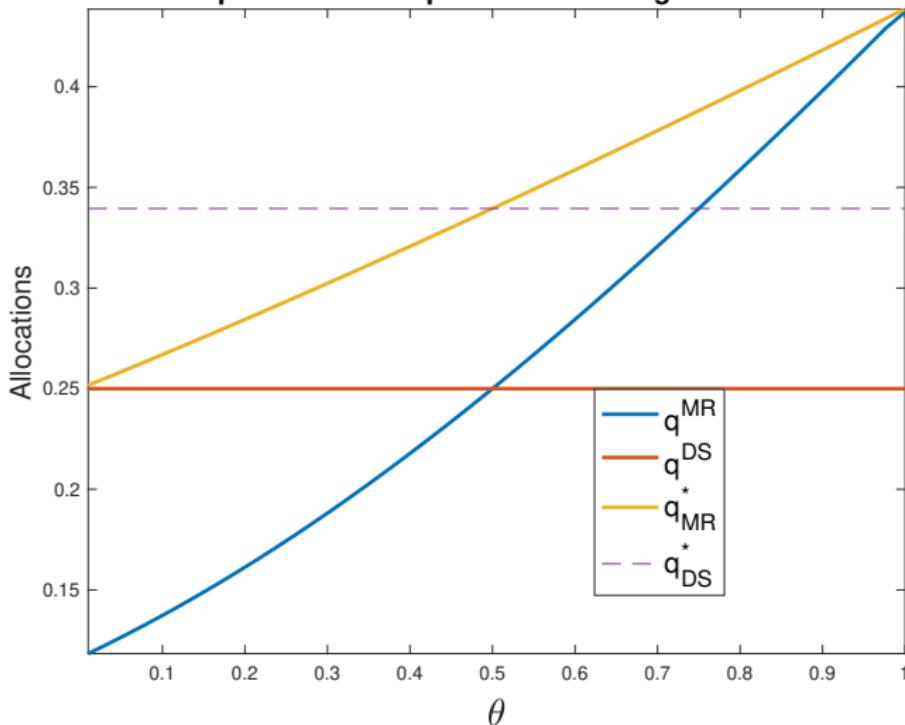
- ▶ “Invest-then-distribute” Lemma becomes a timing assumption:
  - ▶ seller produces in advance and, after learning the type, can only damage.
  - ▶ Free damaging is irrelevant in M-R.
  - ▶ No-damaging seller: produce  $\rightarrow$  elicit  $\cancel{\rightarrow \text{damage}}$ .
  - ▶ Payment  $t(\theta)$  and production cost  $c(\sup \mathbf{q}(\Theta))$  are comparable;
  - ▶ Efficiency benchmark:  $u_q(q^{\text{EFF}}(\theta), \theta) = c'(q^{\text{EFF}}(\theta))$ ,  $\uparrow$  in  $\theta$ .

## Single buyer



# Single buyer

Comparison with Separable Screening: Allocations



▶ More

# Literature

## Monopolistic screening

Mussa and Rosen (1978); Maskin and Riley (1984); Wilson (1993) ...  
Costs are separable.

## Damaged goods

Oren, Smith, and Wilson (1985); Deneckere and McAfee (1996); Grubb (2009);  
Corrao, Flynn, and Sastry (2023).  
Costs are separable, and consumers can damage the good.

## Pricing of information with buyer's private information

Bergemann, Bonatti, and Smolin (2018); Bimpikis, Crapis, and Tahbaz-Salehi (2019); Bergemann and Ottaviani (2021); Yang (2022); Bergemann, Cai, Velegkas, and Zhao (2022); Rodríguez Olivera (2024); Bonatti, Dahleh, Horel, and Nouripour (2024) ...

Information is sold to strategic buyers, without production.

Kolotilin, Mylovanov, Zapecelnyuk, and Li (2017) ...

Information is allocated without production.

## Mechanism & information design

Mensch and Ravid (2025); Thereze (2025).

# **Competition**

# Competition

$N$  firms play the two-stage game of perfect information with timing:

1. Every firm  $i$  simultaneously chooses a distribution  $H_i$  of her quality, **paying**  $c(q_i)$  for the realization  $q_i$ ;
2. Every firm  $i$  simultaneously chooses a pricing function  $p_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ , with  $p_i(q) = \infty$  if  $q > q_i$ , **observing** realized qualities  $q_1, \dots, q_n$ .

Then, every type  $\theta$  buys one good from a firm, or does not buy any good for a payoff of 0.  
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Then, every type  $\theta$  buys one good from a firm, or does not buy any good for a payoff of 0.  
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The “market” price of  $q$  is  $\min_i p_i(q)$ .

Firms play a subgame-perfect Nash equilibrium.

## Definition 1

An  $n$  equilibrium is an equilibrium in which exactly  $n$  firms are active; an  $n$  equilibrium is symmetric if active firms play the same strategy.

# Literature

Forms of “market power” that guarantee equilibrium existence (Rothschild and Stiglitz (1976), . . . , Stole (2007)):

1. Incumbent-entrant timing (Borenstein and Rose, 1994; Johnson and Myatt, 2003; Gerardi and Shapiro, 2009; Crawford et al., 2019; Boik and Takahashi, 2020);
2. Rich buyer heterogeneity (Rochet and Stole, 2002; Lehmann et al., 2014; Garrett et al., 2019);
3. “Relevance” assumption (Chade and Swinkels, 2021): every firm has a cost advantage over a quality range.

Related models:

1. Kreps and Scheinkman (1983);  
Quantity-and-price competition, no private information, and Cournot outcome;
2. Champsaur and Rochet (1989);  
Quality range-and-price competition, costless quality ranges, which are disjoint in equilibrium.

## Second stage

Let's order qualities  $(q_1, \dots, q_N)$  so that:  $x \geq y \geq \dots$

**Bertrand force:** Every quality below  $y$  comes at zero price.

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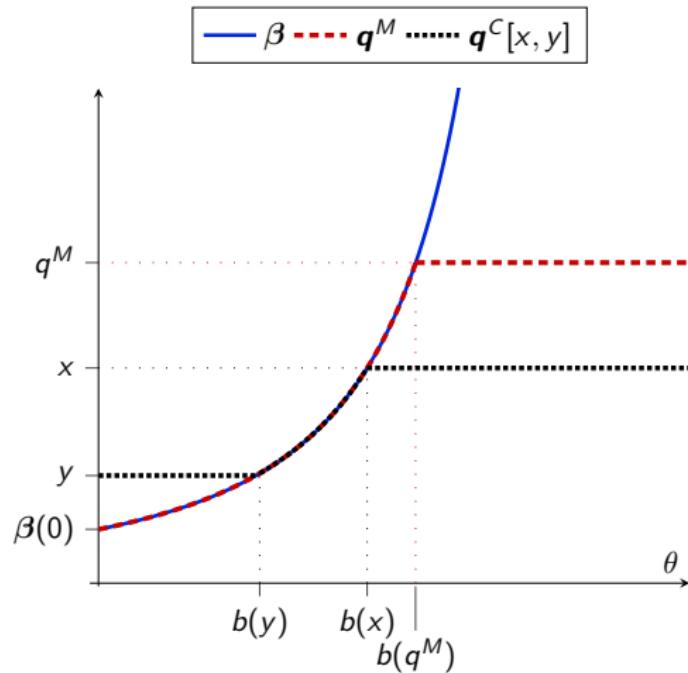
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**Bertrand force:** Every quality below  $y$  comes at zero price.

The allocation in the subgame is  $\mathbf{q}^C[x, y]$ , given by:

$$\theta \mapsto \begin{cases} y, & \text{if } \theta \leq b(y), \\ \beta(\theta), & \text{if } \theta \in (b(y), b(x)), \\ x, & \text{if } \theta \geq b(x). \end{cases}$$

**Competitive constraint:** if  $q$  is feasible for some competitor, then  $q$  can be purchased for free. The “monopolist” earns  $V(x) - V(y) - c(x)$



# Competitive equilibria

## Lemma 3

In every pure-strategy equilibrium: one firm produces  $q^M$  and other firms are idle.

Intuition:

1.  $q^M$  is the best response to  $(0, \dots, 0)$ , and  $q \neq q^M$  is **not** a best response to  $(q^M, 0, \dots, 0)$ .
2. In any  $(x, y, \dots)$ ,  $y > 0$  is not a best response, and  $(0, \dots, 0)$  is not an equilibrium.

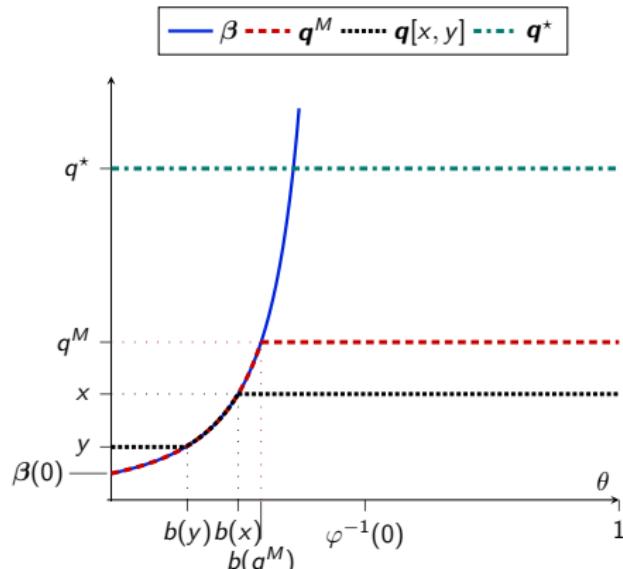
⇒ Every symmetric  $n$  equilibrium is mixed if  $n \geq 2$  (*competitive*.)

# Competitive equilibria

## Proposition 5

1. For all  $n \leq N$ , there exists a symmetric  $n$  equilibrium.
2. Every symmetric and competitive  $n$  equilibrium induces the random allocation  $\mathbf{q}^C[\hat{x}, \hat{y}]$ , where  $\hat{x}$  and  $\hat{y}$  are the first and second order statistics of the  $n$  i.i.d. draws from the atomless CDF on  $[0, q^M]$  given by

$$H_n(q) = \left( \frac{c'(q)}{V'(q)} \right)^{\frac{1}{n-1}}.$$

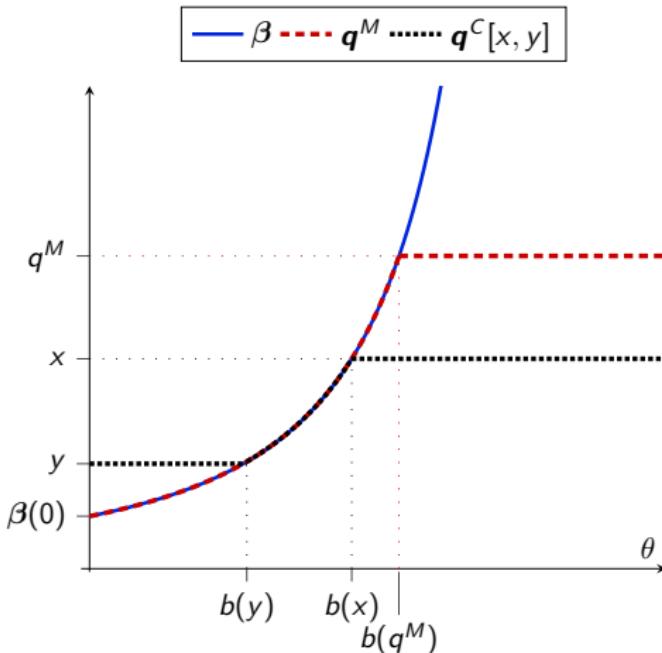


# Competitive equilibria

## Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. The lowest quality is free and  $\geq \beta(0)$ ;
2. The highest quality is strictly lower than  $q^M$ ;
3. The marginally bunched type is lower than  $b(q^M)$ .



## Properties of competitive equilibria

- ▶ Productive inefficiency is stronger;
- ▶ Distributional inefficiency is more prevalent.

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In the paper:

1. Equilibrium welfare with  $n \geq 2$  active firms decreases in  $n$ .
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

# Conclusion

1. With digital goods, two interdependent inefficiencies arise:  
**productive and damaging.**
2. The *efficiency at the top* insight is revisited:  
**'distributional efficiency for high types.'**

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5. **Competition** is beneficial for damaging inefficiency and harmful for productive inefficiency.

▶ More

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▶ More

Thanks!

## **Extra slides**

## Hybrid costs

With more general costs:  $C(\mathbf{q}) = \int_{\Theta} \hat{c}(\mathbf{q}(\theta), \sup \mathbf{q}(\Theta)) dF(\theta)$ ,  
the seller pays:

1. Development / production costs:  $\sup \mathbf{q}(\Theta)$ ;
2. Distribution / replication / damaging costs:  $\mathbf{q}(\theta)$ .

Lemma 1 holds, but the characterization of  $\mathbf{q}^M$  has two complications:

1. Distribution: the solution to  $\mathcal{P}(q)$  does not depend on  $q$  solely through capping;
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If  $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \kappa \log \left( \frac{\sup \mathbf{q}(\Theta)}{\mathbf{q}(\theta)} \right)$ , then 1. is turned off.

## Damaging costs

If  $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \kappa \log\left(\frac{\sup \mathbf{q}(\Theta)}{\mathbf{q}(\bar{\theta})}\right)$ , then:

- ▶ Production costs + pure-damaging replication / distribution costs;

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- ▶ Production costs + pure-damaging replication / distribution costs;
- ▶ The efficient allocation is flat: damaging decreases utility and increases costs;
- ▶ The solution to  $\mathcal{P}(q)$  is  $\theta \mapsto \min\{\beta_\kappa(\theta), q\}$ .

1.  $\kappa > 0$  acts as a preference shift ( $\uparrow g$ ) at the distribution stage:

- ▶  $\uparrow \beta_\kappa$  and  $\downarrow b_\kappa(q)$ ;

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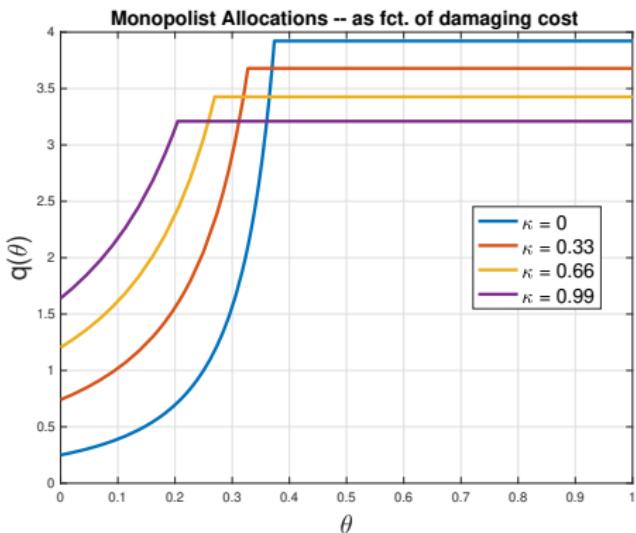
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1.  $\kappa > 0$  acts as a preference shift ( $\uparrow g$ ) at the distribution stage:
  - ▶  $\uparrow \beta_\kappa$  and  $\downarrow b_\kappa(q)$ ;
2.  $\kappa > 0$  impacts production directly:
  - ▶  $V'(q) = (1 - F(b_\kappa(q)))(g'(q) + b_\kappa(q)) - \kappa \frac{b_\kappa(q)}{q}$ .

# Damaging costs

$\kappa > 0$  implies

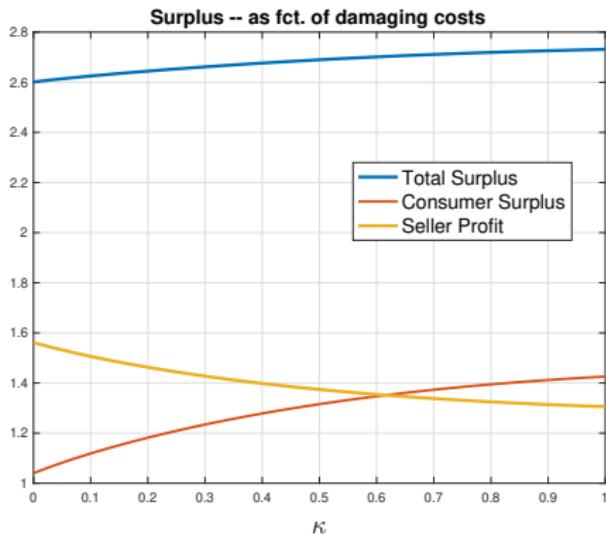
1. Less damaging;
2. Lower production.



# Damaging costs

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# Efficiency with general $u$ and $k$

## Proposition 6

The allocation  $\mathbf{q}^*$  is efficient iff  $\mathbf{q}^*(\theta) = \min\{\gamma(\theta), q^*\}$  for all  $\theta$ , in which:  $q^*$  is the unique  $q$  such that  $\int_{[a(q), 1]} u_1(q, \theta) - k'(q) dF(\theta) = c'(q)$ , and  $\gamma$  is an allocation such that  $\gamma(\theta) = \alpha(\theta)$  almost everywhere.

In general,  $q \in [0, \bar{q}]$ , and:

$$J(q, \theta) := u(q, \theta) - \frac{1}{h(\theta)} u_2(q, \theta) - k(q),$$

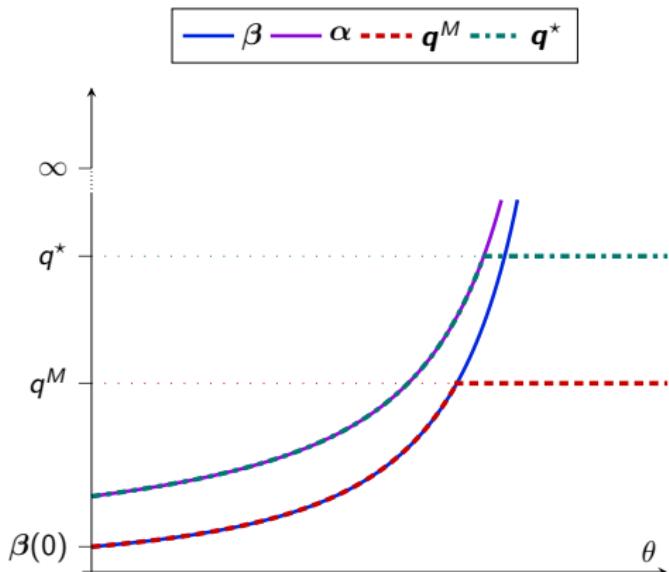
$\beta(\theta)$  is the largest element of

$$\operatorname{Argmax}_q J(q, \theta),$$

$\alpha(\theta)$  is the largest element of

$$\operatorname{Argmax}_q u(q, \theta) - k(q),$$

$u$  and  $J$  satisfy incr. differences, and are: twice diff., concave in  $q$  for all  $\theta$ , str. quasiconcave in  $q$  a.e. on  $\Theta$ ;  $k$  is Inada.



# Monopoly with general $u$ and $k$

## Proposition 7

The allocation  $\mathbf{q}^M$  is monopolist iff  $\mathbf{q}^M(\theta) = \min\{\gamma(\theta), q^M\}$  for all  $\theta$ , in which:  $q^M$  is the unique  $q$  such that  $\int_{[b(q), 1]} J_1(q, \theta) dF(\theta) = c'(q)$ , and  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere. Moreover,  $0 < q^M < q^*$ .

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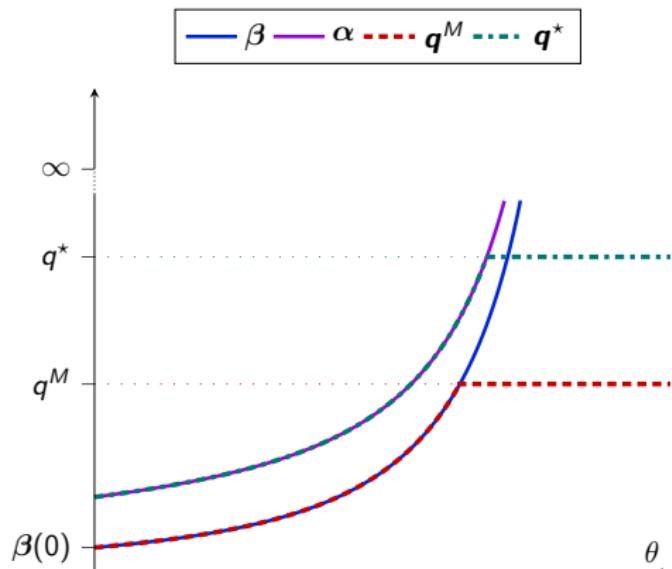
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$\alpha(\theta)$  is the largest element of

$$\operatorname{Argmax}_q u(q, \theta) - k(q),$$

$u$  and  $J$  satisfy incr. differences, and are: twice diff., concave in  $q$  for all  $\theta$ , str. quasiconcave in  $q$  a.e. on  $\Theta$ ;  $k$  is Inada.



## No-damaging monopoly with general $u$ and $k$

Assumption:  $J(0, \theta) = 0$  for all  $\theta$  and  $J(q, \cdot)$  is increasing for all  $q > 0$ .

### Proposition 8

The allocation  $\mathbf{q}_N$  is no screening iff  $\mathbf{q}_N(\theta) = [\theta \geq b_N(q_N)]q_N$  for all  $\theta \neq b_N(q_N)$  and  $\mathbf{q}_N(b_N(q_N)) \in \{0, q_N\}$ , in which  $q_N$  is the unique  $q$  such that:  $\int_{[b_N(q), 1]} J_1(q, \theta) dF(\theta) = c'(q)$ . Moreover, it holds that:

1.  $0 < q_N \leq q^M$ ;
2.  $q_N < q^M$  if  $b(q^M) > b_N(q^M)$ .

We use Iverson brackets:  $[P] = 1$  if  $P$  is true, and  $[P] = 0$  otherwise.

# Productive inefficiency addendum 1/3

Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\substack{\text{Marginal revenues} \\ \text{given } \theta \mapsto \min\{\beta(\theta), q\}}} < \underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\substack{\text{Marginal total utility} \\ \text{given } \theta \mapsto q}} .$$

1. The  $q$  constrained allocation  $\theta \mapsto \min\{\beta(\theta), q\}$  induces total utility

$$U(q) := \mathbb{E}\{u(\min\{\beta(\theta), q\}, \theta)\};$$

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$$U(q) := \mathbb{E}\{u(\min\{\beta(\theta), q\}, \theta)\};$$

2.  $U'(q) <$  marginal total utility given  $\theta \mapsto q$ , because

$$U'(q) = \int_{[b(q), 1]} g'(\theta) + \theta dF(\theta) \leq g'(q) + \mathbb{E}\{\theta\};$$

3. It holds that

$$V'(q) < V'(q) + \text{Rents}'(q) = U'(q).$$

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3. It holds that

$$V'(q) < V'(q) + \text{Rents}'(q) = U'(q).$$

## Productive inefficiency addendum 2/3

Productive inefficiency arises because:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$

The monopoly that produces quality  $q$  implies total surplus

$$V(q) + U(q) - c(q),$$

with  $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \bar{q}\} d\theta' dF(\theta)$  (Envelope Theorem).

The marginal surplus is  $V'(q) + U'(q)$  and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.

## Productive inefficiency addendum 3/3

WTS:  $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$ .

1.  $U'(q) = \int_{[b(q),1]} \theta - b(q) dF(\theta)$ ,

because the marginal  $u(q(\theta), \theta)$  increases at rate  $g'(q) + \theta$  and the marginal transfer at rate  $g'(q) + b(q)$ , for  $\theta > b(q)$  and  $q(\cdot) = \min\{\beta(\cdot), q\}$ ;

2. Using  $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$ , we have

$$V'(q) + U'(q) = (1 - F(b(q)))(g'(q) + b(q)) + \int_{[b(q),1]} \theta dF(\theta).$$

Note that  $U'(q) > 0$  for all  $q > 0$ , because  $b(q) \leq \varphi^{-1}(0) < 1$  for all  $q \geq 0$ .)

## The game

Type  $\theta$  buys quality  $D_p(\theta) \in Q := \mathbb{R}_+$  from firm  $\iota_p(\theta)$ , given the pricing functions in  $(p_1, \dots, p_N) = p$ . The set of firms is  $\mathcal{N} = (1, \dots, N)$ .

The revenues of  $i$  given the pricing functions in  $(p_1, \dots, p_N) = p$  are

$$R_i(p_1, \dots, p_N) := \int_{\{\theta | \iota_p(\theta) = i\}} p_i(D_p(\theta)) dF(\theta).$$

The set of pure strategies for firm  $i$  is  $S_i := Q \times \mathbf{P}_i$ , letting  $\mathbf{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$  be the set of “conditional” pricing functions of firm  $i$ .

The *payoff* of firm  $i$  from the profile  $s := (\dots, (\bar{q}_i^s, P_i^s), \dots) \in \times_{i=1}^N S_i$  is

$$\Pi_i(s) := R_i(P_1^s[\bar{q}^s], \dots, P_N^s[\bar{q}^s]) - c(\bar{q}_i^s).$$

# The equilibrium

A strategy for player  $i$  is a pair of a cap distribution and conditional pricing function,  $(H_i, P_i)$ , in which  $H_i$  is a distribution function with support contained in  $Q$  and  $P_i \in \mathbf{P}_i$  for all  $i$ .

A strategy profile  $((H_1, P_1), \dots, (H_N, P_N))$  is an *equilibrium* if it is a subgame-perfect Nash equilibrium of the game:

1. The conditional pricing functions maximize conditional profits, that is, for all cap profiles  $\bar{q} = (\bar{q}_1, \dots, \bar{q}_N)$ ,

$$R_i(P_i[\bar{q}], P_{-i}[\bar{q}]) \geq R_i(p'_i, P_{-i}[\bar{q}]) \quad \text{for all } p'_i \in \mathbb{R}^Q, \ i.$$

2. The cap distributions maximize expected profits, that is, for all  $\bar{q}_i$  in the support of  $H_i$  we have

$$\int_{Q^{N-1}} R_i(P_i[\bar{q}], P_{-i}[\bar{q}]) dH_{-i}(\bar{q}_{-i}) - c(\bar{q}_i) \geq \\ \int_{Q^{N-1}} R_i(P_i[\bar{q}'_i, \bar{q}_{-i}], P_{-i}[\bar{q}'_i, \bar{q}_{-i}]) dH_{-i}(\bar{q}_{-i}) - c(\bar{q}'_i)$$

for all  $\bar{q}'_i \in Q$ , letting  $H_{-i}$  denote the joint distribution of the caps of  $i$ 's opponents under  $((H_1, P_1), \dots, (H_N, P_N))$ , and for all  $i$ .

## Auxiliary monopoly problem

The allocation  $\mathbf{q}$  is  $x$ - $y$  second best, for qualities  $x, y$  with  $x \leq y$ , if there exists a transfer function  $t(\cdot)$  such that:  $(\mathbf{q}, t(\cdot))$  solves

$$\sup_{\mathbf{q}, t(\cdot)} \int t(\theta) dF(\theta) \text{ subject to: } y \leq \mathbf{q}(\theta) \leq x \text{ for all } \theta \in \Theta.$$

subject to: IC, IR.

### Lemma 4

The allocation  $\mathbf{q}$  is  $x$ - $y$  second best iff  $\mathbf{q}(\theta) = \max\{\min\{\gamma(\theta), x\}, y\}$  for all  $\theta$ , in which  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere.

## The pricing subgame

The pricing game given the quality profile  $(q_1, \dots, q_N)$  is the strategic-form game  $\Gamma(q_1, \dots, q_N) = (\mathcal{N}, (\mathbb{R}^{[0, q_i]}, R_i(\cdot))_{i \in \mathcal{N}})$ .

The profile of pricing functions  $(p_1, \dots, p_N)$  is a  $(q_1, \dots, q_N)$  equilibrium if  $(p_1, \dots, p_N)$  is a Nash equilibrium of  $\Gamma(q_1, \dots, q_N)$ .

We study the subgame starting at the given production profile  $(q_1, \dots, q_N) \in Q^N$  with  $x := \max\{q_1, \dots, q_N\}$  and  $y := \max\{q_1, \dots, q_N\} \setminus \{x\}$ .

### Lemma 5

For every  $(q_1, \dots, q_N)$  equilibrium  $(p_1, \dots, p_N)$ , the following formulas hold:

$$R_i(p_1, \dots, p_N) = (V(q_i) - V(y))_+,$$

and

$$D_{(p_1, \dots, p_N)}(\theta) = \max\{\min\{\gamma(\theta), x\}, y\},$$

for all  $\theta$ , in which  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere.

# The production game

Every Nash equilibrium of  $\Gamma(q_1, \dots, q_N)$  induces the same revenues, i.e.,

$$R_i(p_1^*, \dots, p_N^*) = R_i(p_1^{**}, \dots, p_N^{**})$$

for all  $(q_1, \dots, q_N)$  equilibria  $(p_1^*, \dots, p_N^*), (p_1^{**}, \dots, p_N^{**})$ . We call  $\bar{R}_i(q_1, \dots, q_N)$  the unique equilibrium revenues of  $i$  in the pricing game  $\Gamma(q_1, \dots, q_N)$ , that is:

$$\bar{R}_i(q_1, \dots, q_N) = (V(q_i) - V(y))_+.$$

The *production game* is the strategic-form game  $\Gamma = (\mathcal{N}, (Q, \bar{R}_i(\cdot) - c(\cdot))_{i \in \mathcal{N}})$ . By mixed strategies, we refer to probability distributions over  $Q$  identified by their distribution functions, and we extend payoffs of  $\Gamma$  to mixed-strategy profiles defining

$$\Pi_i: (\Delta Q)^N \rightarrow \mathbb{R}$$

$$(H_1, \dots, H_N) \mapsto \int_Q \cdots \int_Q \bar{R}_i(q_1, \dots, q_N) - c(q_i) dH_1(q_1) \cdots dH_N(q_N).$$

## Auxiliary result for the production game

Necessary conditions for mixed-strategy equilibria:

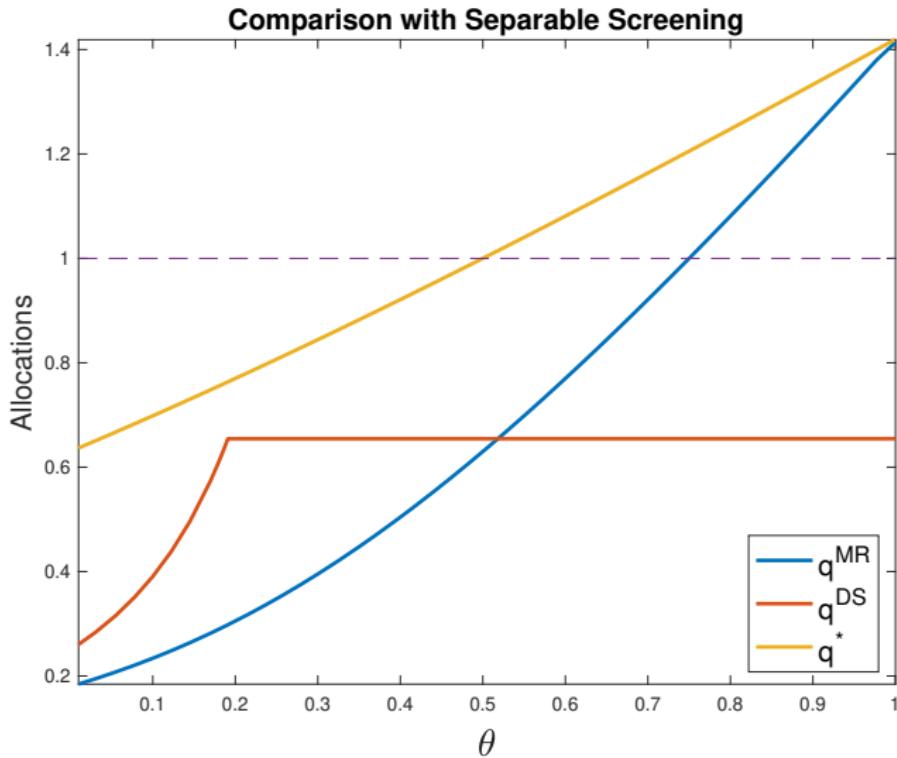
### Lemma 6

If  $(H_1, \dots, H_N)$  is a Nash equilibrium in mixed strategies with multiple active players, then for all  $i$  with  $H_i(q) > 0$  for some  $q > 0$  we have:

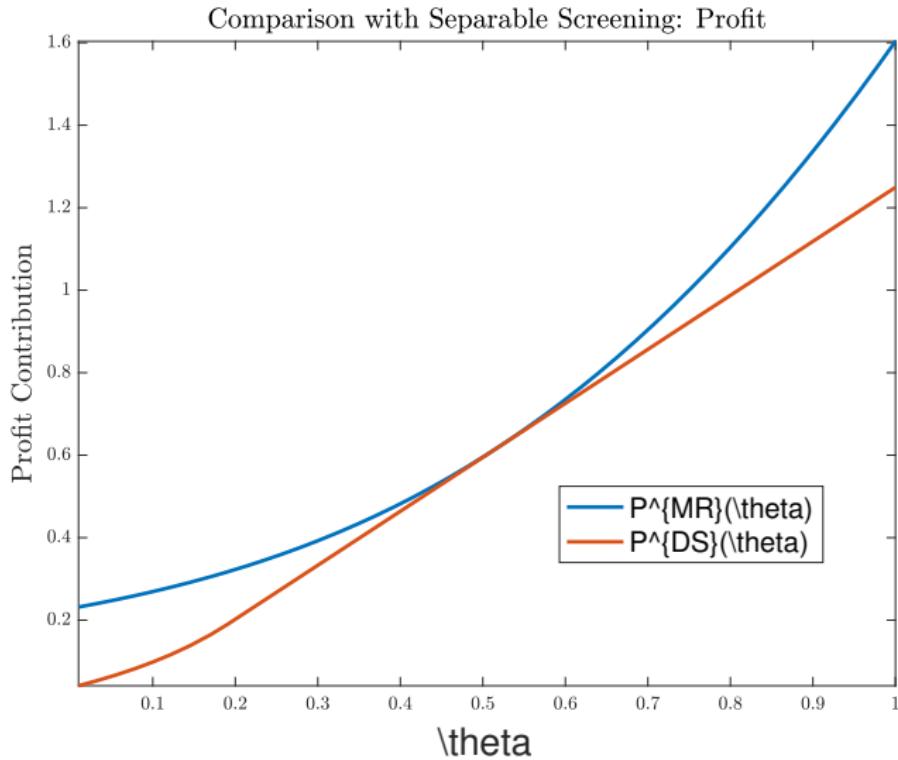
$$\prod_{j \in N \setminus \{i\}} H_j(q) = \begin{cases} \frac{c'(q)}{V'(q)}, & \text{if } q \in (0, \bar{q}^M), \\ 1, & \text{if } q \geq \bar{q}^M. \end{cases}$$

Note: There are only two kinds of equilibria in mixed strategies that are symmetric among active players: those with at least 2 active players and the monopolistic equilibria.

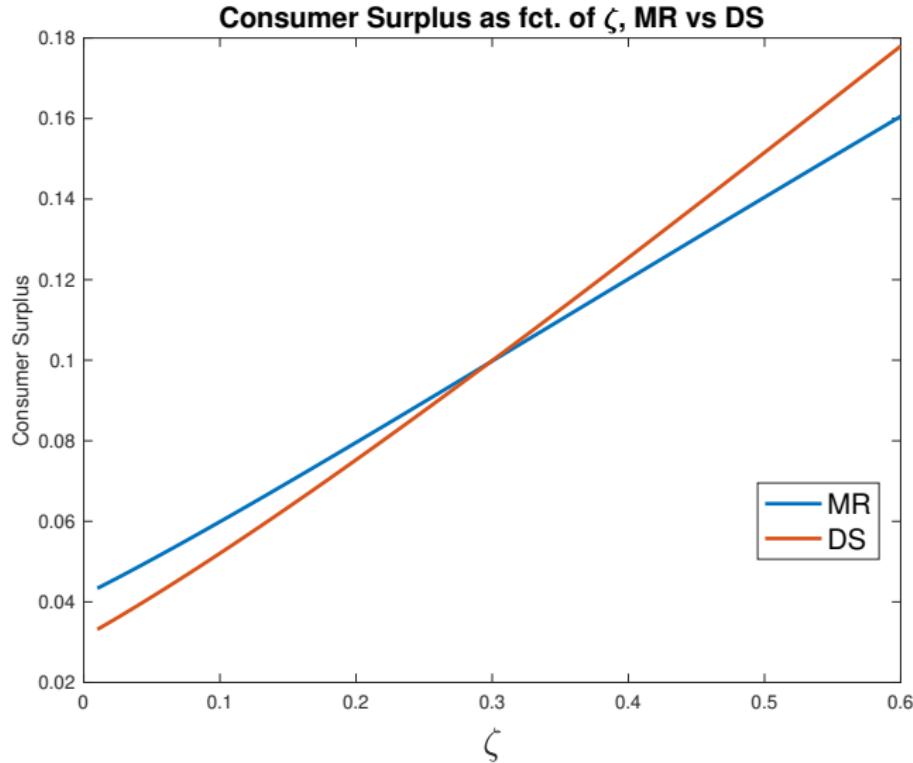
# Single buyer



# Single buyer

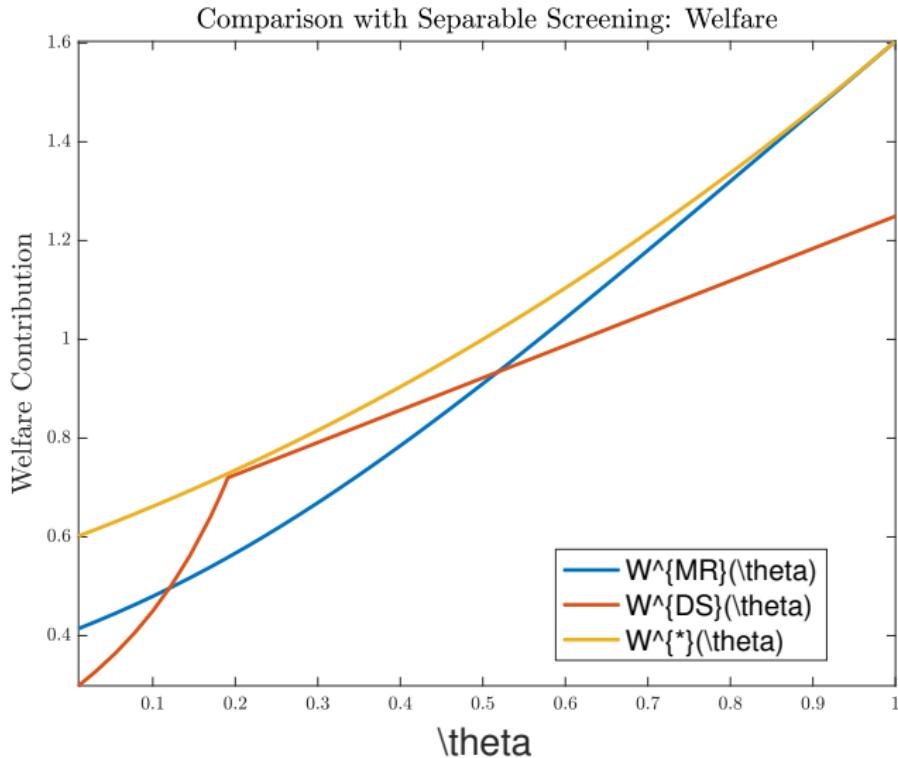


# Single buyer



Consumer surplus of  $q$  is  $\int_{\Theta} q(\theta)(1 - F(\theta)) d\theta$ , with  
 $u(q, \theta) = \zeta g(q) + \theta q$ .

# Single buyer



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