

# Screening in digital monopolies

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# Free damaging and replication

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This paper studies monopoly provision of goods whose production structure exhibits free replication and free damaging.

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Examples of **digital goods**:

1. Software goods;
2. Digital audio content;
3. Data.

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			2-core	4-core	6+
Maximum number of variables <sup>i</sup>					
Up to 2,048 variables	✓	✓	✓	✓	✓
Up to 32,767 variables	-	✓	✓	✓	✓
Up to 120,000 variables	-	-	✓	✓	✓
Maximum number of observations <sup>i</sup>					
Up to 2.14 billion	✓	✓	✓	✓	✓
Up to 20 billion	-	-	✓	✓	✓

# Plan

1. Model;
2. Efficiency benchmark;
3. Monopoly allocation and inefficiencies;
4. No-damaging constraint, extensions, and interpretations.



# Model

A unit mass of buyers, each drawing a **type**  $\theta \in [0, 1] = \Theta$ , interacts with a seller.

Type  $\theta$  is privately informed about  $\theta \sim F$ , for twice diff.  $F$  on  $(0, 1)$ ;  
 $\hookrightarrow F$  is regular in these slides,  $\mathbb{E}\{\cdot\}$  refers to the r.v.  $\theta$ .

Type  $\theta$  has payoff from **quality**  $q \in \mathbb{R}_+$  and transfer  $t \in \mathbb{R}$ :

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q, \theta)} - t,$$

for a strictly concave, increasing, and twice diff.  $g$  (Chade-Swinkels '21.)

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The cost of **allocation**  $\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$  is

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)),$$

for a **production cost**  $c$ , increasing, strictly convex, twice diff., with  $c'(0) = 0$  and  $\lim_{q \rightarrow \infty} c'(q) = \infty$ .

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for a strictly concave, increasing, and twice diff.  $g$  (Chade-Swinkels '21.)

With *separable* costs, the cost of  $q$  is

$$C(q) = \mathbb{E}\{k(q(\theta))\},$$

for some  $k$  (Mussa-Rosen '78.)

# Efficiency

The *surplus* induced by allocation  $\mathbf{q}$  is

$$\mathbb{E}\{u(\mathbf{q}(\theta), \theta)\} - c(\sup \mathbf{q}(\Theta)).$$

The *efficient* quality allocation  $\mathbf{q}^*$  maximizes surplus.

## Proposition 1

The efficient allocation is given by  $\mathbf{q}^*(\theta) = q^*$  for all  $\theta$ , in which  $q^*$  is the unique quality  $q$  such that

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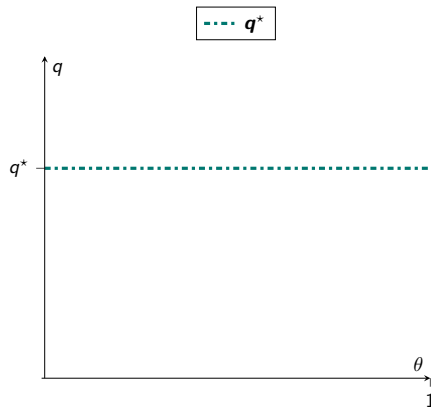
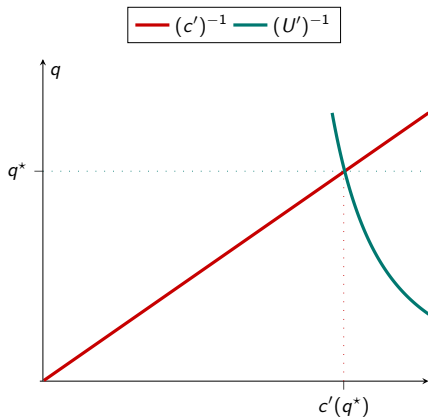
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# Efficiency



Define  $U(q) = g(q) + \int_{\Theta} \theta dF(\theta)q$ .

# Monopoly

The monopolist problem is:

$$(\mathcal{P}^M) \quad \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) \, dF(\theta) - c(\sup \mathbf{q}(\Theta)) \text{ subject to:}$$

$$\begin{aligned} u(\mathbf{q}(\theta), \theta) - t(\theta) &\geq u(\mathbf{q}(\hat{\theta}), \theta) - t(\hat{\theta}), \text{ for all } (\theta, \hat{\theta}), \\ u(\mathbf{q}(\theta), \theta) - t(\theta) &\geq 0, \text{ for all } \theta. \end{aligned}$$

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- ▶ The *monopolist* allocation  $\mathbf{q}^M$  solves  $\mathcal{P}^M$  for some  $t(\cdot)$ .
- ▶ Without separable costs: the monopolist problem cannot be solved via “pointwise maximization”.

# Monopoly

The  $q$  constrained problem and its value  $V(q)$  are:

$$(\mathcal{P}(q)) \quad V(q) := \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) - \cancel{c(\sup_{\theta} \mathbf{q}(\theta))} \text{ subject to:}$$

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## Lemma 1 (Invest then distribute)

The allocation  $\mathbf{q}$  solves  $\mathcal{P}^M$  if and only if:

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$$(\mathcal{P}(q)) \quad V(q) := \max_{\mathbf{q}} \int_{[0,1]} \underbrace{g(\mathbf{q}(\theta)) + \varphi(\theta)\mathbf{q}(\theta)}_{\text{Virtual surplus}} dF(\theta) \text{ subject to:}$$
$$\mathbf{q}(\theta) \leq q, \text{ for all } \theta \in \Theta,$$
$$\mathbf{q} \text{ is nondecreasing;}$$

$$\text{in which } \varphi(\theta) := \theta - \frac{1-F(\theta)}{F'(\theta)}.$$

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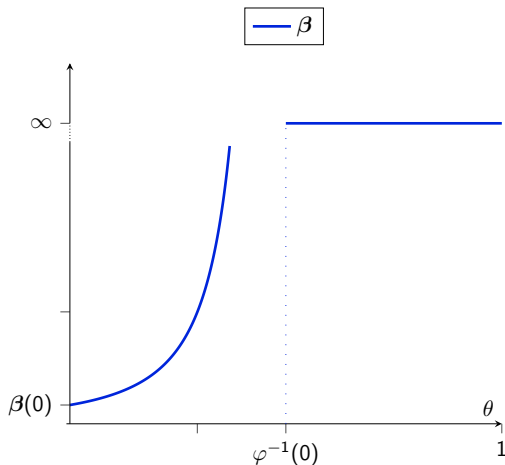
# Virtual surplus maximization

The *virtual-surplus maximizer*

$$\beta(\theta) \in \operatorname{Argmax}_q g(q) + \varphi(\theta)q$$

is such that:

1.  $\beta(\theta) = \infty$  if  $\theta \geq \varphi^{-1}(0)$ ;
2.  $\beta$  is increasing;
3.  $\beta(0) > 0$  ("lnada"  $g$ ).



# Virtual surplus maximization

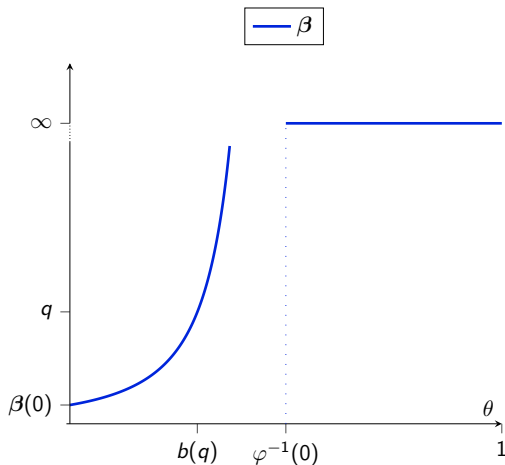
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$b$  is the inverse of  $\beta$ .

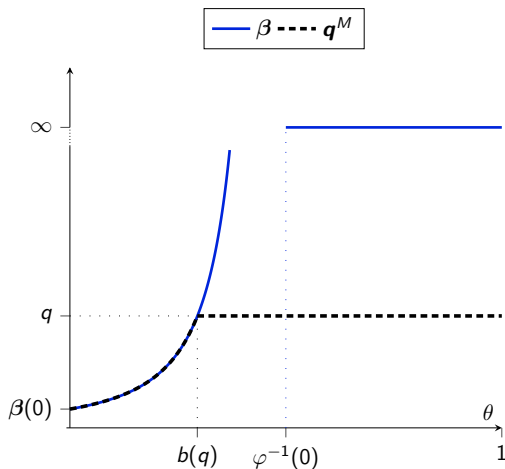


# Virtual surplus maximization

## Lemma 2

Allocation  $\mathbf{q}$  solves  $\mathcal{P}(\mathbf{q})$  iff:

$$\mathbf{q}(\theta) = \min\{\beta(\theta), \mathbf{q}\}, \text{ for all } \theta.$$



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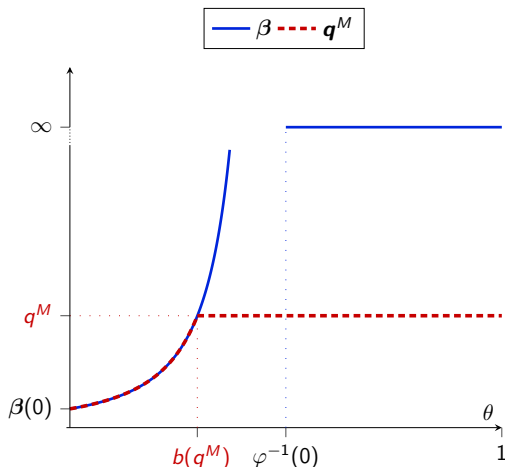
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Allocation  $\mathbf{q}$  solves  $\mathcal{P}(\mathbf{q})$  iff:

$$\mathbf{q}(\theta) = \min\{\beta(\theta), q^M\}, \text{ for all } \theta.$$

Distributive properties of  $\mathbf{q}^M$ :

1. Bunching at the top;
2. Distributional inefficiency at the bottom or full bunching;
3. No exclusion.





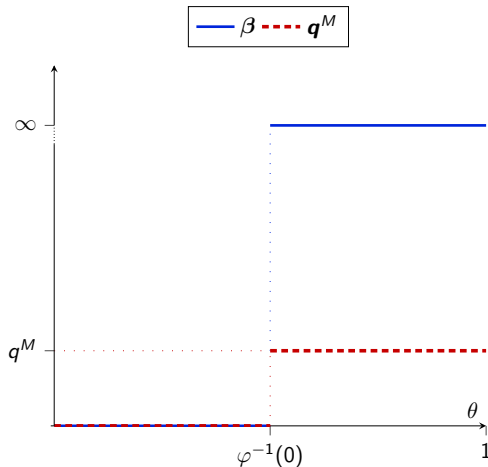
# Linear preferences

Distributive properties if  $g(q) = 0$ :

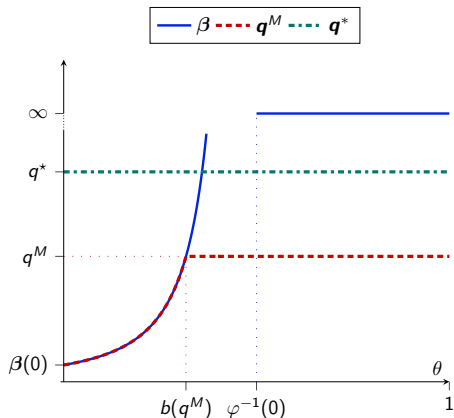
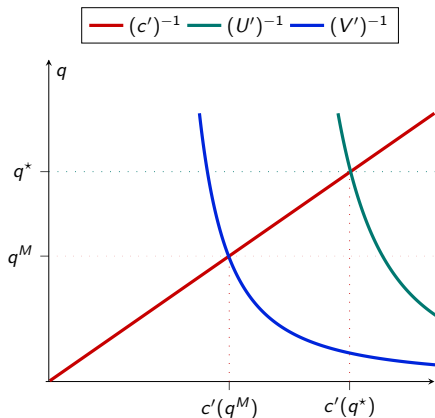
1. Bunching at the top;  
 $\beta(\theta) = \infty$  for  $\theta \geq \varphi^{-1}(0)$
2. Exclusion at the bottom;  
 $\beta(\theta) = 0$  for  $\theta < \varphi^{-1}(0)$

⇒ **Single-quality menu.**

Richness in digital markets is due solely to preferences.



# The monopolist allocation



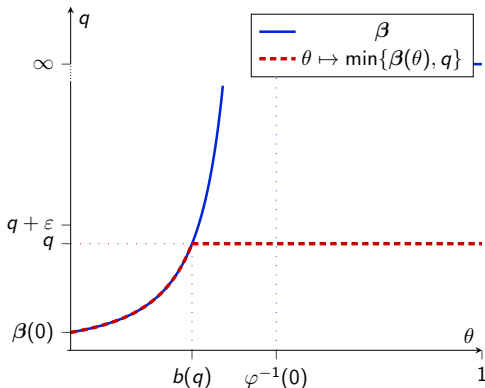
# Marginal revenues

$V'(q)$  is the marginal return from increasing the cap of the  $q$ -constrained allocation  $\theta \mapsto \min\{\beta(\theta), q\}$ :

$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{bunched types}} \underbrace{(g'(q) + b(q))}_{u_q \text{ of } b(q)}.$$

The change from  $q$  to  $q + \varepsilon$  leads to:

1. same revenues from  $q' < q$ :  
 $q'$  sold to the same  $\theta$ , and  $\theta$  gets the same **rent**;
2. higher quality for bunched types;
3. higher price by  $u_q(q, b(q))\varepsilon$ .



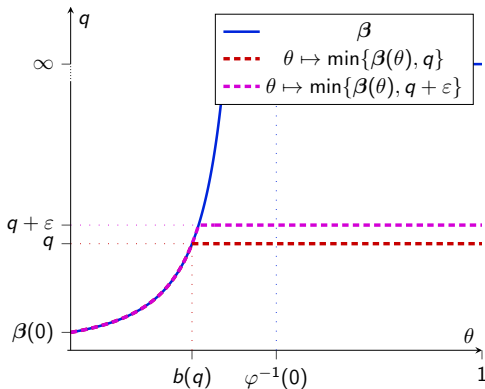
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1. By Markov's inequality:

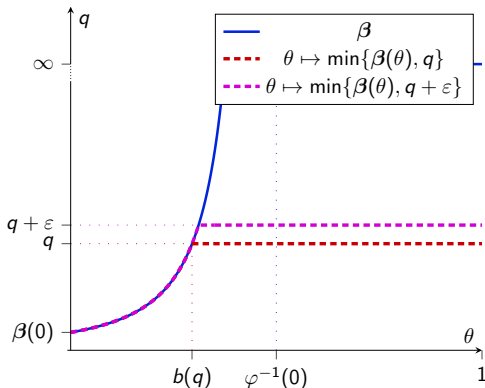
$$(1 - F(b(q)))b(q) \leq \mathbb{E}\{\theta\};$$

2. By the distributive properties

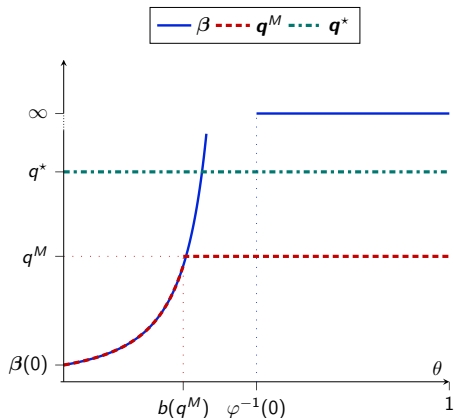
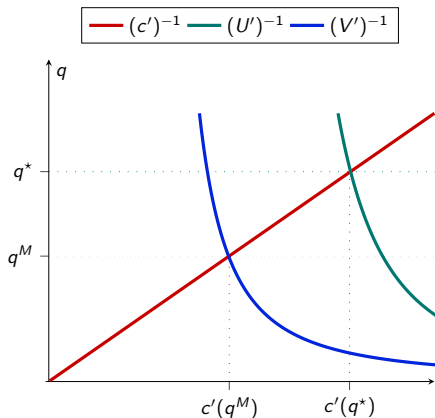
$$b(q) < 1,$$

So:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$



# Productive inefficiency



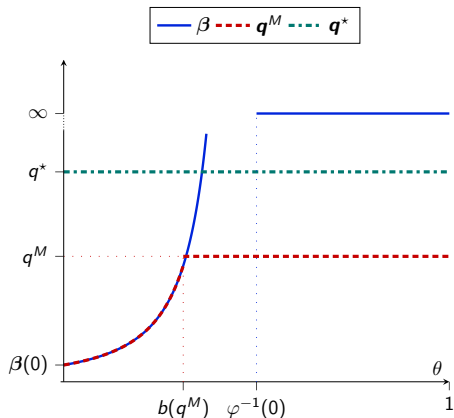
# Productive inefficiency

## Proposition 2

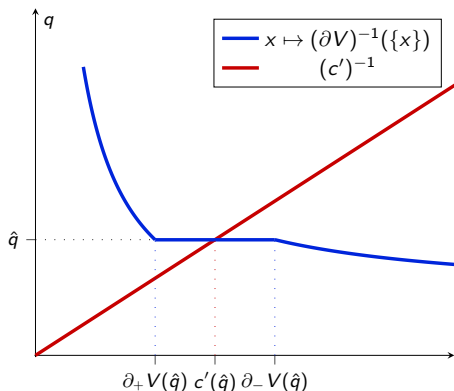
The monopolist allocation is given by  $q^M(\theta) = \min\{\beta(\theta), q\}$  for all  $\theta$ , in which  $q^M$  is the unique  $q$  solving

$$V'(q) = c'(q).$$

Moreover, it holds that:  $q^M < q^*$ .



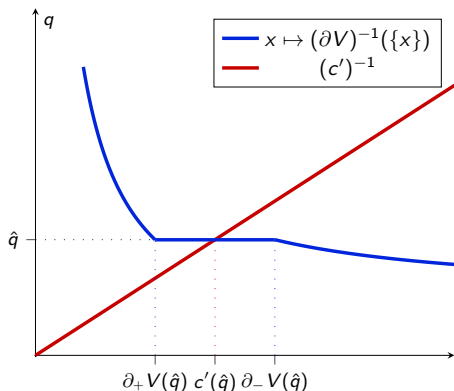
# Non-regular distribution



- $\beta$  is ironed to obtain  $\bar{\beta}$ ;
- By Lemma 1,  
 $\theta \mapsto \min\{\bar{\beta}(\theta), q\}$  solves  $\mathcal{P}(q)$ ;



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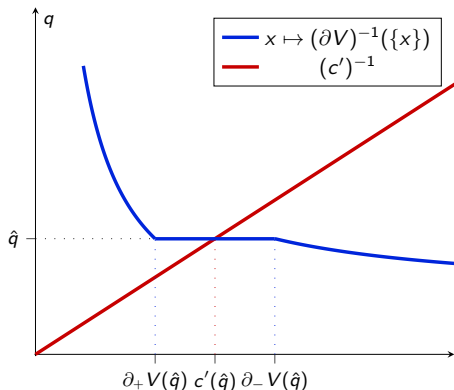


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- If types in  $(\theta', \theta'')$  are bunched  
 “at”  $\hat{q} \in (0, q)$ ,

$$\partial_- V(\hat{q}) > \partial_+ V(\hat{q}),$$

the extra revenues from  $\hat{q} + \varepsilon$   
 come from types higher than  $\theta''$ .

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$V$  is concave by concavity of  $u$  in  $q$ , and productive inefficiency holds.

## Proposition 3

Without regularity, the monopolist allocation is  $\mathbf{q}^M(\theta) = \min\{\bar{\beta}(\theta), q^M\}$ , in which  $q^M$  is the unique  $q$  with  $c'(q) \in \partial V(q)$ . Moreover, it holds that  $q^M < q^*$ .

# No damaging

Without damaging, the  $q$  constrained problem is:

$$V_N(q) := \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) \text{ subject to:}$$

IC, IR,  $\mathbf{q}(\theta) \in \{0, q\}$ , for all  $\theta$ .

The constraint is irrelevant under:

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The monopolist chooses a **marginally excluded** type  $n(q)$ , so

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- ▶ Intuitively: damaging ban  $\implies n(q) \leq b(q)$ , strictly if  $b(q) > 0$ ,
- ▶ so productive inefficiency is worse:

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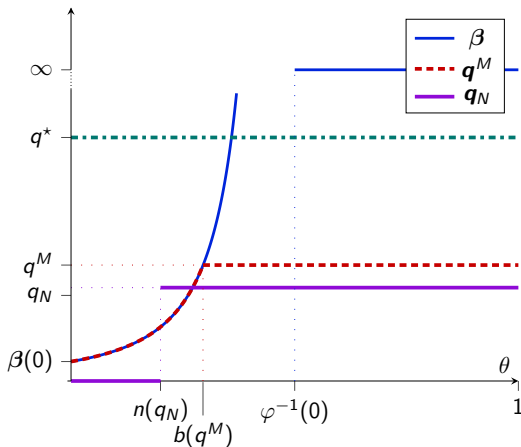


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The no-damaging allocation  $q_N$  features:

- ▶ Less production;
- ▶ No damaging;
- ▶ (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.

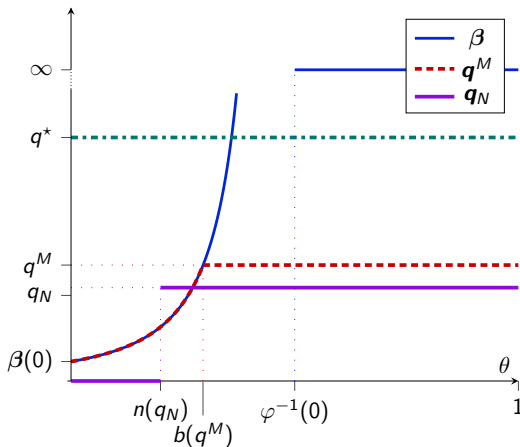


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## Proposition 4

Without damaging, the monopolist allocation is  $q_N(\theta) = \mathbf{1}_{[b_N(q_N), 1]}(\theta)q_N$ , in which  $q_N$  is the unique  $q$  solving  $V'_N(q) = c'(q)$ . Moreover, we have  $q_N \leq q^M$ , strictly if  $b(q^M) > 0$ .

# Cost interpretation

For separable costs:  $\Pi^{\text{M-R}}(\mathbf{q}) = \underbrace{\int_{\Theta} t(\theta) \, dF(\theta)}_{\text{per-agent revenues}} - \underbrace{\int_{\Theta} k(\mathbf{q}(\theta)) \, dF(\theta)}_{\text{per-agent costs}},$

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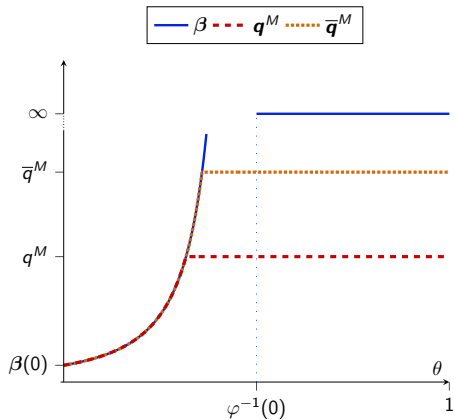
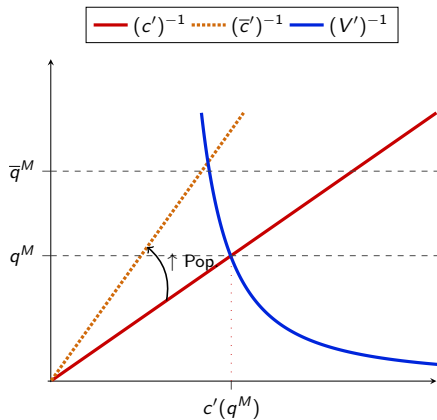
1. Payment  $t(\theta)$  and production cost  $k(\mathbf{q}(\theta))$  are comparable;
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For digital goods:  $\Pi(\mathbf{q}) = \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{per-agent revenues}} - \underbrace{c(\sup \mathbf{q}(\Theta))}_{\text{per-agent costs}},$

1. Payment  $t(\theta)$  and production cost  $c(q^M)$  are of different sizes;
2. Population size impacts  $q^M$ ;

In general:  $C(\mathbf{q}) = \int_{\Theta} k(\mathbf{q}(\theta)) dF(\theta) + c(\sup \mathbf{q}(\Theta)).$

# Population size



# Separable interpretation

For a separable interpretation (with a continuum of buyers:)

$$\Pi(\mathbf{q}) = \int_{\Theta} t(\theta) - \underbrace{c(\sup \mathbf{q}(\Theta))}_{\substack{\text{same magnitude} \\ \text{as } t(\theta)}} dF(\theta),$$

under which production exhibits:

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In the damaged-goods model of Deneckere and McAfee (1996):

1. Costs are purely-separable **damaging** costs  $k$ , with  $k'(q) \leq 0$ ;  
 $\hookrightarrow$  identify gains from screening
2. The cost of the undamaged good is fixed.  
 $\hookrightarrow$  if damaging is free, our separable interpretation

# Single buyer

$$\Pi^{\text{M-R}}(\mathbf{q}) = \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{expected revenues}} - \underbrace{\int_{\Theta} c(\mathbf{q}(\theta)) dF(\theta)}_{\text{expected costs}},$$

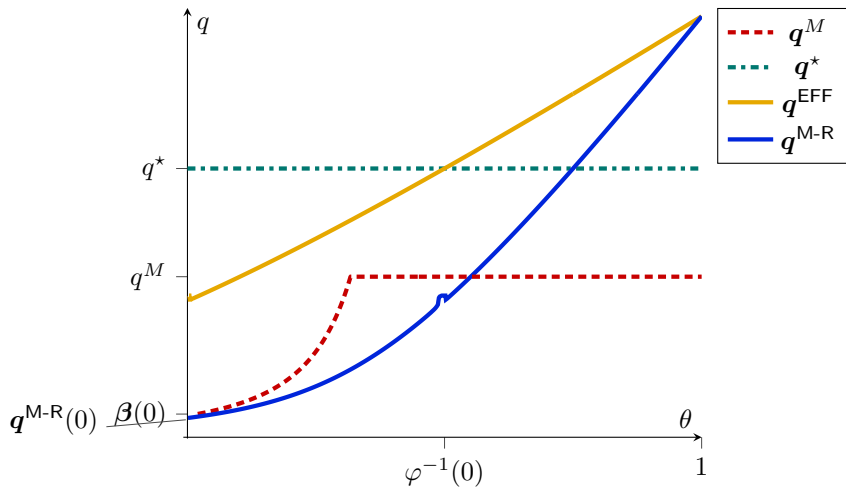
1. Payment  $t(\theta)$  and production cost  $k(\mathbf{q}(\theta))$  are comparable;
2. Production occurs after type is elicited;
3. Damaging and replication costs do not bite.

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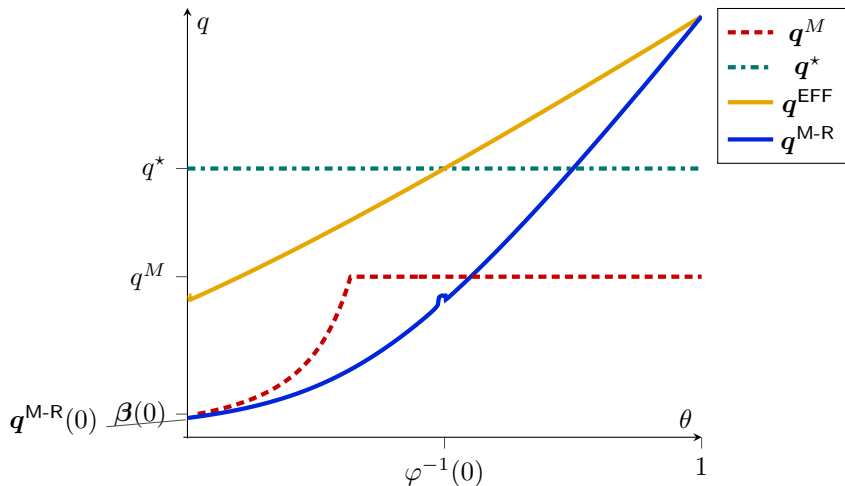
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# Single buyer



# Single buyer



$\Pi^{\text{M-R}}(q^{\text{M-R}}) - \Pi(q^M)$  = gains from “interim” damaging wrt ex-ante damaging.

# Hybrid costs

With more general costs:  $C(\mathbf{q}) = \int_{\Theta} \hat{c}(\mathbf{q}(\theta), \sup \mathbf{q}(\Theta)) dF(\theta)$ ,  
the seller pays:

1. Development / production costs:  $c(\sup \mathbf{q}(\Theta))$ ;
2. Distribution / replication costs:  $k(\mathbf{q}(\theta))$ ;
3. Damaging costs.

Lemma 1 holds, but the characterization of  $\mathbf{q}^M$  has two complications:

1. Distribution: the solution to  $\mathcal{P}(q)$  does not depend on  $q$  solely through capping;
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If  $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \kappa \log\left(\frac{\sup \mathbf{q}(\Theta)}{q(\theta)}\right)$ , then **1.** is turned off.

# Damaging costs

If  $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \kappa \log\left(\frac{\sup \mathbf{q}(\Theta)}{\mathbf{q}(\theta)}\right)$ , then:

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- ▶ The efficient allocation is flat: damaging decreases utility and increases costs;
- ▶ The solution to  $\mathcal{P}(q)$  is  $\theta \mapsto \min\{\beta_\kappa(\theta), q\}$ .

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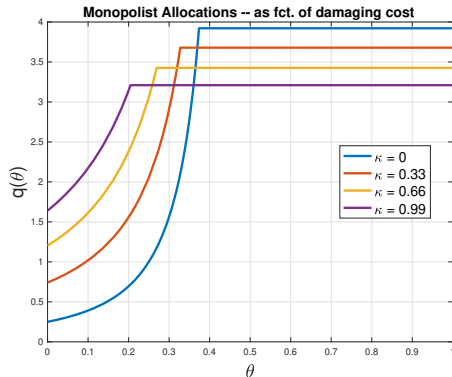
2.  $\kappa > 0$  impacts production directly:

- ▶  $V'(q) = (1 - F(b_\kappa(q)))(g'(q) + b_\kappa(q)) - \kappa \frac{b_\kappa(q)}{q}$ .

# Damaging costs

$\kappa > 0$  implies

1. Less damaging;
2. Lower production.

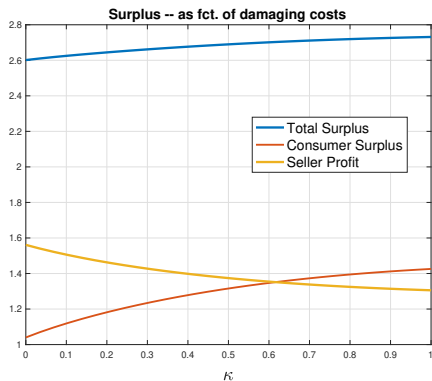




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# Conclusion

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Thanks!

Extra slides

# Literature

## **Multi-product monopoly**

Mussa and Rosen (1978); Maskin and Riley (1984); Wilson (1993) ...  
Costs are separable.

## **Damaged goods**

Deneckere and McAfee (1996); McAfee (2007); Grubb (2009); Corrao, Flynn, and Sastry (2023).  
Costs are separable, and consumers can damage the good.

## **Pricing of information with buyer's private information**

Bergemann, Bonatti, and Smolin (2018); Bergemann and Ottaviani (2021); Yang (2022); Bergemann, Cai, Velegkas, and Zhao (2022); Rodríguez Olivera (2024); Bonatti, Dahleh, Horel, and Nouripour (2024) ...  
Information is sold to strategic buyers, without production.

Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) ...  
Information is allocated without production.

## **Mechanism & information design**

Bergemann, Heumann, and Morris (2025); Mensch and Ravid (2025); Thereze (2025).

# Efficiency with general $u$ and $k$

## Proposition 5

The allocation  $\mathbf{q}^*$  is efficient iff  $\mathbf{q}^*(\theta) = \min\{\gamma(\theta), q^*\}$  for all  $\theta$ , in which:  $q^*$  is the unique quality  $q$  such that

$\int_{[\alpha(q), 1]} u_1(q, \theta) - k'(q) dF(\theta) = c'(q)$ , and  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \alpha(\theta)$  almost everywhere.

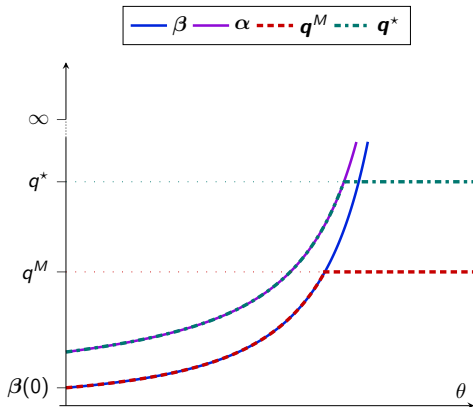
In general,  $q \in [0, \bar{q}]$ , and:

$$J(q, \theta) := u(q, \theta) - \frac{1}{h(\theta)} u_2(q, \theta) - k(q),$$

$\beta(\theta)$  is the largest element of  
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$u$  and  $J$  satisfy incr. differences, and are: twice diff., concave in  $q$  for all  $\theta$ , str. quasiconcave in  $q$  a.e. on  $\Theta$ ;  $k$  is lnada.





# Monopoly with general $u$ and $k$

## Proposition 6

The allocation  $\mathbf{q}^M$  is monopolist iff  $\mathbf{q}^M(\theta) = \min\{\gamma(\theta), q^M\}$  for all  $\theta$ , in which:  $q^M$  is the unique quality  $q$  such that

$\int_{[b(q), 1]} J_1(q, \theta) dF(\theta) = c'(q)$ , and  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere. Moreover,  $0 < q^M < q^*$ .

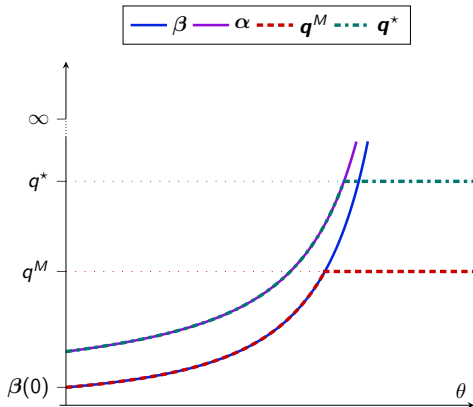
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# No-damaging monopoly with general $u$ and $k$

Assumption:  $J(0, \theta) = 0$  for all  $\theta$  and  $J(q, \cdot)$  is increasing for all  $q > 0$ .

## Proposition 7

The allocation  $\mathbf{q}_N$  is no screening iff  $\mathbf{q}_N(\theta) = [\theta \geq b_N(q_N)]q_N$  for all  $\theta \in \Theta \setminus \{b_N(q)\}$  and  $\mathbf{q}_N(b_N(q_N)) \in \{0, q_N\}$ , in which  $q_N$  is the unique quality  $q$  such that:  $\int_{[b_N(q), 1]} J_1(q, \theta) dF(\theta) = c'(q)$ . Moreover, it holds that:

1.  $0 < q_N \leq q^M$ ;
2.  $q_N < q^M$  if  $b(q^M) > b_N(q^M)$ .

We use Iverson brackets:  $[P] = 1$  if  $P$  is true, and  $[P] = 0$  otherwise.

# Productive inefficiency addendum 1/3

Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\substack{\text{Marginal revenues} \\ \text{given } \theta \mapsto \min\{\beta(\theta), q\}}} < \underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\substack{\text{Marginal total utility} \\ \text{given } \theta \mapsto q}} .$$

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# Productive inefficiency addendum 2/3

Productive inefficiency arises because:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$

The monopoly that produces quality  $q$  implies total surplus

$$V(q) + U(q) - c(q),$$

with  $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \bar{q}\} d\theta' dF(\theta)$  (Envelope Theorem).

The marginal surplus is  $V'(q) + U'(q)$  and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.

# Productive inefficiency addendum 3/3

WTS:  $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$ .

1.  $U'(q) = \int_{[b(q), 1]} \theta - b(q) \, dF(\theta)$ ,  
because the marginal  $u(\mathbf{q}(\theta), \theta)$  increases at rate  $g'(q) + \theta$  and the marginal transfer at rate  $g'(q) + b(q)$ , for  $\theta > b(q)$  and  $\mathbf{q}(\cdot) = \min\{\beta(\cdot), q\}$ ;
2. Using  $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$ , we have

$$V'(q) + U'(q) = (1 - F(b(q)))(g'(q) + b(q)) + \int_{[b(q), 1]} \theta \, dF(\theta).$$

Note that  $U'(q) > 0$  for all  $q > 0$ , because  $b(q) \leq \varphi^{-1}(0) < 1$  for all  $q \geq 0$ .)

# Competition

The game among  $N$  firms has two stages:

1. Every firm  $i$  simultaneously chooses a quality  $q_i$ .
2. Every firm  $i$ , observing all stage-1 qualities, simultaneously chooses a pricing function  $p_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ , with  $p_i(q) = \infty$  if  $q > q_i$ .

Then: each type buys a good from a firm  $i$ , or does not buy any good for a payoff of 0.

Firms play a subgame-perfect Nash equilibrium.

## Definition 1

An  $n$  *equilibrium* is an equilibrium in which exactly  $n$  firms are active; an  $n$  equilibrium is *symmetric* if active firms play the same strategy.



# The game

Type  $\theta$  buys quality  $D_p(\theta)$  from firm  $\iota_p(\theta)$ , given the pricing functions in  $(p_1, \dots, p_N) = p$ .

The revenues of  $i$  given the pricing functions in  $(p_1, \dots, p_N) = p$  are

$$R_i(p_1, \dots, p_N) := \int_{\{\theta | \iota_p(\theta) = i\}} p_i(D_p(\theta)) dF(\theta).$$

The set of strategies for firm  $i$  is  $S_i := Q \times \mathbf{P}_i$ , letting  $\mathbf{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$  be the set of “conditional” pricing functions of firm  $i$ .

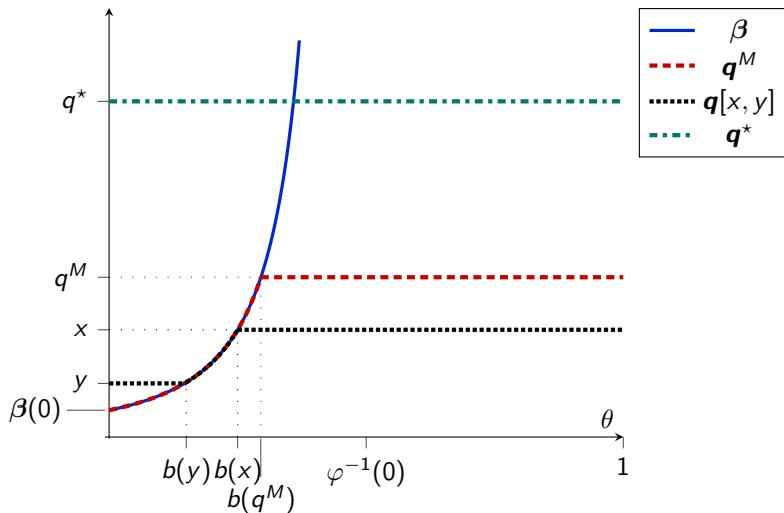
The *payoff* of firm  $i$  from the profile  $s := (\dots, (\bar{q}_i^s, P_i^s), \dots) \in \times_{i=1}^N S_i$  is

$$\Pi_i(s) := R_i(P_1^s[\bar{q}^s], \dots, P_N^s[\bar{q}^s]) - c(\bar{q}_i^s).$$

# Competitive allocations

Let's order qualities ( $q_1, \dots, q_N$ ) so that:  $x > y > \dots$

Every quality below  $y$  comes at zero price.



# Competitive equilibria

## Lemma 3

In any pure-strategy equilibrium: one firm produces  $q^M$  and other firms are idle.

$\implies$  Every symmetric  $n$  equilibrium is mixed if  $n \geq 2$  (*competitive*.)

## Proposition 8

1. For all  $n \leq N$ , there exists a symmetric  $n$  equilibrium.
2. Every symmetric and competitive  $n$  equilibrium induces the random allocation  $\mathbf{q}[\hat{x}, \hat{y}]$ , letting  $\hat{x}$  and  $\hat{y}$  be, resp., the first and second order statistics of the  $n$  i.i.d. draws  $[0, q^M]$  with CDF

$$H_n(q) = \sqrt[n-1]{\frac{c'(q)}{V'(q)}}.$$

# Properties of competitive equilibria

## Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. The lowest quality is positive and free;
2. The highest quality is strictly lower than  $q^M$ .

In the paper:

1. Equilibrium welfare with  $n \geq 2$  active firms decreases in  $n$ .
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

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