

Screening in digital monopolies

Pietro Dall'Ara Elia Sartori

SAET

Free damaging and replication

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This paper studies monopoly provision of goods exhibiting free replication and free damaging.

Free damaging and replication

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Examples of **digital goods**:

1. Software goods;
2. Digital audio content.

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Up to 32,767 variables	-	✓	✓	✓	✓
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Maximum number of observations ⓘ					
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Up to 20 billion	-	-	✓	✓	✓

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<u>Play songs in any order</u>	—	✓
<u>High audio quality</u>	—	✓
<u>Listen with friends in real time</u>	—	✓
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Spotify's HiFi streaming could finally arrive this year



Image: Cath Virginia / The Verge

/ A \$6/ month "Music Pro" tier could include lossless audio and perks like discounted concert tickets.

by Quentyn Karsmeiser
Feb 17, 2025, 10:18 PM GMT+1



Quentyn Karsmeiser is a writer who helps The Verge's readers save money by surfacing the best tech deals and presenting the latest product recommendations from our experts. He has covered tech and gaming for all of his 15-plus-year career for publications like Forbes, Business Insider, and more.

Plan

1. Model;
2. Monopoly;
3. Monopoly inefficiencies;
4. No screening case;
5. Conclusion.

Model

A continuum of buyer **types**, $\theta \in [0, 1] = \Theta$, interact with a seller.

Type θ is privately informed about $\theta \sim F$, for twice diff. F .

F is regular for these slides and $\mathbb{E}\{\cdot\}$ refers to the r.v. θ .

Type θ has payoff from **quality** $q \in \mathbb{R}_+$ and transfer $t \in \mathbb{R}$:

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q, \theta)} - t,$$

for a concave, nondecreasing, and twice diff. g (Chade-Swinkels '21.)

$q: \Theta \rightarrow \mathbb{R}_+$ is a **quality allocation**.

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$\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ is a **quality allocation**.

The cost of a quality allocation \mathbf{q} is

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)),$$

for a **production cost** c , increasing, strictly convex, twice diff. with $c(0) = c'(0) = 0$ and $\lim_{q \rightarrow \infty} c'(q) = \infty$.

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$\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ is a **quality allocation**.

With *separable* costs, the cost of quality allocation \mathbf{q} is

$$K(\mathbf{q}) = \mathbb{E}\{k(\mathbf{q}(\theta))\},$$

for some k (Mussa-Rosen '78.)

Efficiency

The *total surplus* induced by the allocation \mathbf{q} is

$$\mathbb{E}\{u(\mathbf{q}(\theta), \theta)\} - c(\sup \mathbf{q}(\Theta)).$$

The *efficient* quality allocation \mathbf{q}^* maximizes total surplus.

Proposition 1

The efficient quality allocation satisfies $\mathbf{q}^(\theta) = q^*$ for all θ , in which q^* is the unique quality q such that*

$$\underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\text{Marginal total utility}} = c'(q).$$

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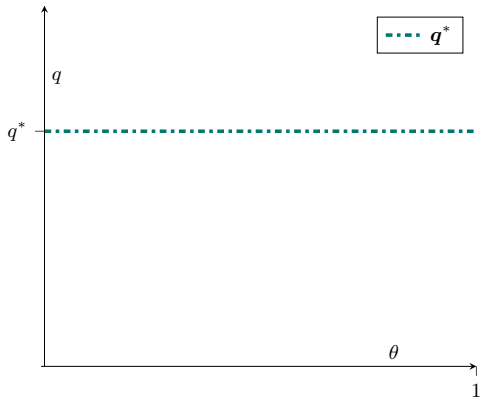
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Efficiency

- Versioning is inefficient.



Monopoly

The monopolist problem is:

$$\mathcal{P}^M \quad \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) \, dF(\theta) - c(\sup \mathbf{q}(\Theta)) \text{ subject to:}$$

$$u(\mathbf{q}(\theta), \theta) - t(\theta) \geq u(\mathbf{q}(\hat{\theta}), \theta) - t(\hat{\theta}),$$

$$u(\mathbf{q}(\theta), \theta) - t(\theta) \geq 0, \text{ for all } (\theta, \hat{\theta}) \in \Theta^2.$$

The *monopolist* quality allocation \mathbf{q}^M solves \mathcal{P}^M .

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With separable costs: the monopolist problem can be solved via “pointwise maximization”.

Monopoly

The q *contingent problem* and its value $V(q)$ are:

$$\begin{aligned} \mathcal{P}(q) \quad V(q) &:= \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) \, dF(\theta) - \cancel{c(\sup_{\Theta} \mathbf{q}(\Theta))} \text{ subject to:} \\ &\quad \mathbf{q}(\theta) \leq q, \\ &\quad u(\mathbf{q}(\theta), \theta) - t(\theta) \geq u(\mathbf{q}(\hat{\theta}), \theta) - t(\hat{\theta}), \\ &\quad u(\mathbf{q}(\theta), \theta) - t(\theta) \geq 0, \text{ for all } (\theta, \hat{\theta}) \in \Theta^2. \end{aligned}$$

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Lemma 1 (Invest then distribute)

The quality allocation \mathbf{q} solves \mathcal{P}^M if and only if:

- (1) \mathbf{q} solves $\mathcal{P}(q^M)$, in which
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The *q* contingent problem and its value, $V(q)$, are:

$$\begin{aligned} \mathcal{P}(q) \quad V(q) := \max_{\mathbf{q}, t(\cdot)} \int_{[0,1]} \underbrace{g(\mathbf{q}(\theta)) + \varphi(\theta)\mathbf{q}(\theta)}_{\text{Virtual surplus}} dF(\theta) \text{ subject to:} \\ \mathbf{q}(\theta) \leq q \text{ for all } \theta \in \Theta, \\ \mathbf{q} \text{ is nondecreasing;} \end{aligned}$$

in which $\varphi(\theta) := \theta - \frac{1-F(\theta)}{F'(\theta)}$.

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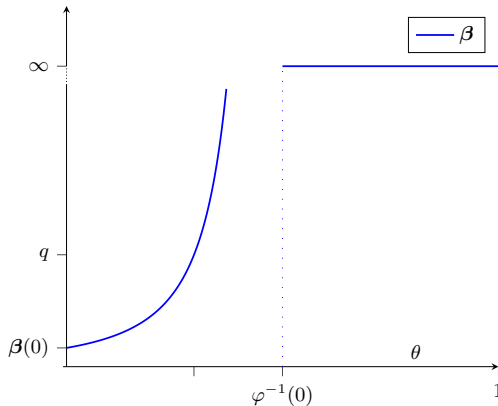
Virtual surplus maximization

The *virtual-surplus maximizer* β is

$$\beta(\theta) \in \operatorname{argmax}_q g(q) + \varphi(\theta)q,$$

and is such that:

1. $\beta(\theta) = \infty$ if $\theta > \varphi^{-1}(0)$;



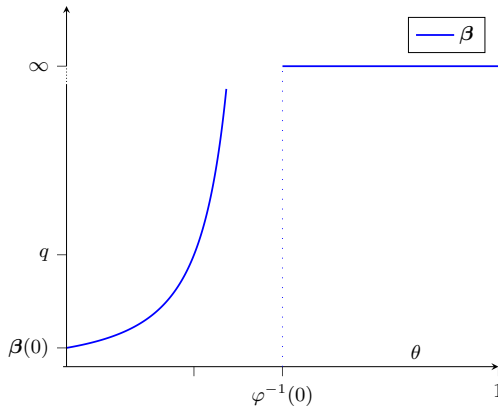
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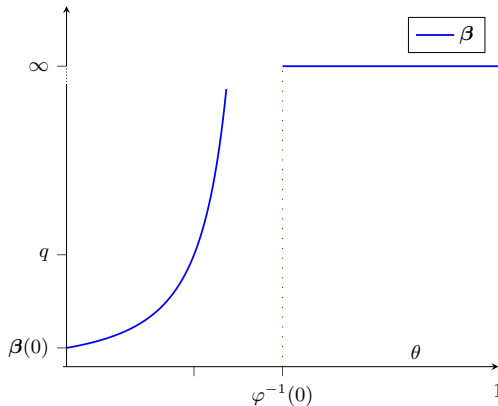
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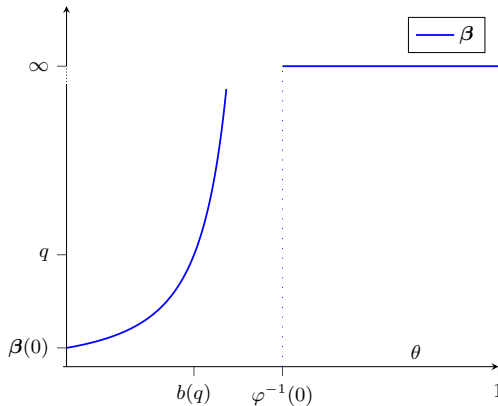
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b is the inverse of β .



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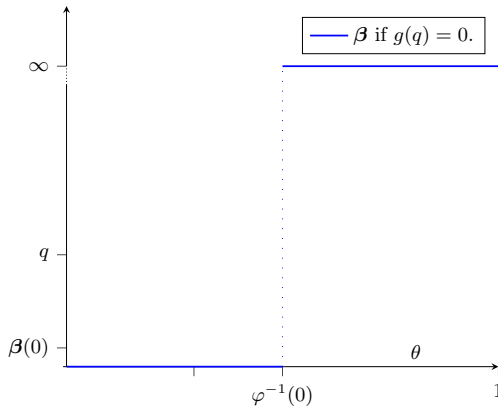
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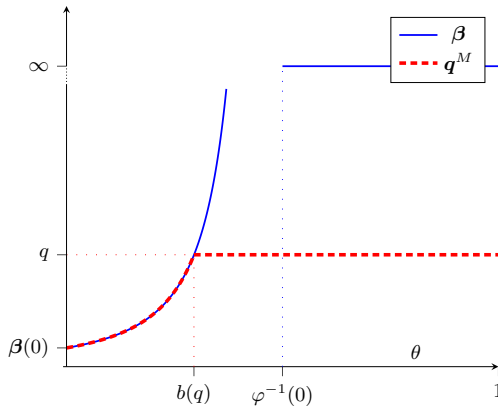
Virtual surplus maximization

Lemma 2

The quality allocation \mathbf{q} solves $\mathcal{P}(q)$ iff:

$$\mathbf{q}(\theta) = \min\{\beta(\theta), q\},$$

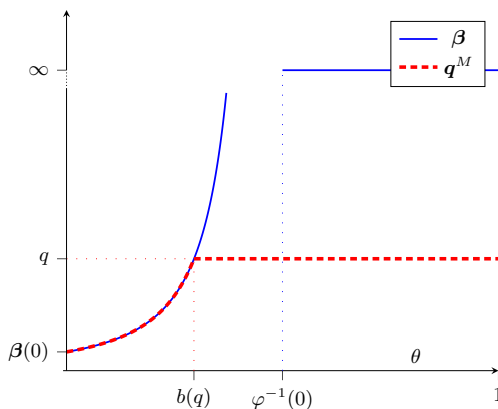
for all θ .



Virtual surplus maximization

Distributive properties of the monopolist allocation for Inada g :

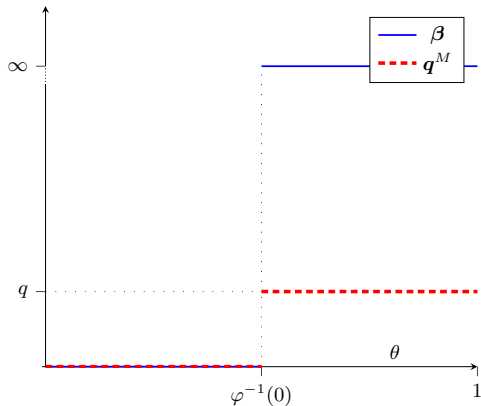
1. Bunching at the top;
 $\beta(\theta) = \infty$ for $\theta > \varphi^{-1}(0)$
2. Distributional inefficiency at the bottom or full bunching;
 β increasing on $[0, \varphi^{-1}(0))$
3. No exclusion.
 $\beta(0) > 0$



Linear preferences

Distributive properties of the monopolist allocation for $g(q) = 0$:

1. Bunching at the top;
 $\beta(\theta) = \infty$ for $\theta > \varphi^{-1}(0)$
2. Exclusion at the bottom;
 β is 0 on $[0, \varphi^{-1}(0))$
3. **Single-quality menu.**

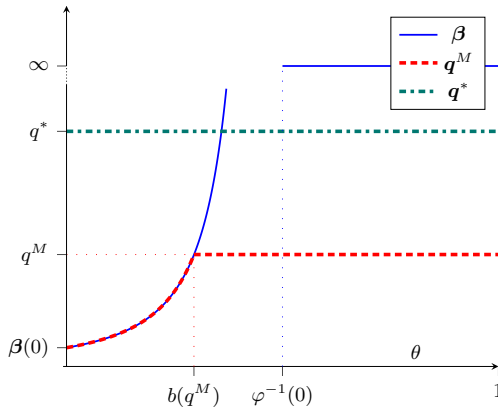


The monopolist allocation

Proposition 2

The monopolist allocation q^M is such that:

- (1) $q^M : \theta \mapsto \min\{\beta(\theta), q^M\}$,
for
- (2) q^M such that
 $0 < q^M < q^*$.



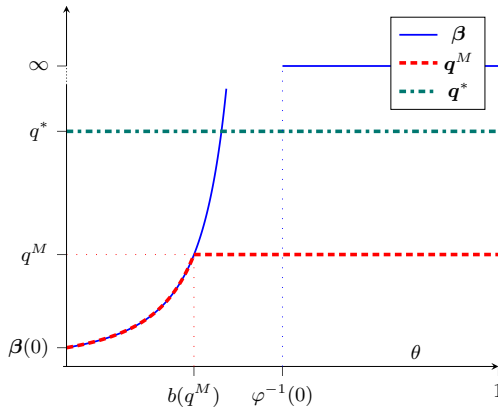
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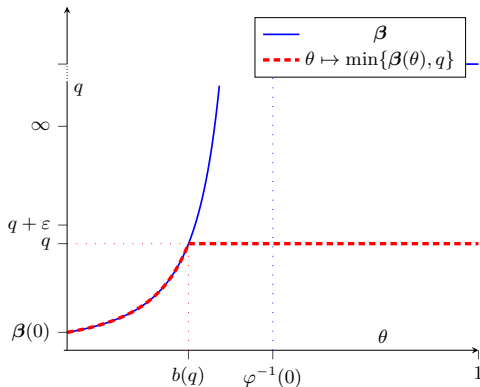
- (1) $q^M : \theta \mapsto \min\{\beta(\theta), q^M\}$,
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► Productive inefficiency occurs.



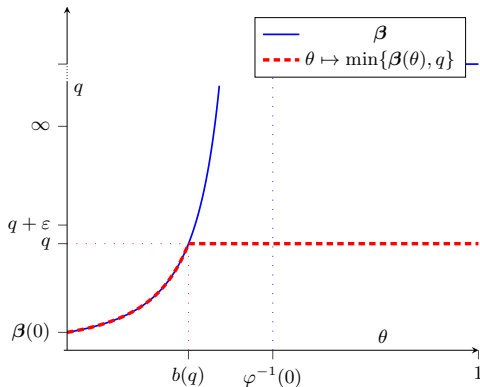
Marginal revenues

$V'(q)$ is the marginal revenue given the q constrained allocation $\theta \mapsto \min\{\beta(\theta), q\}$,



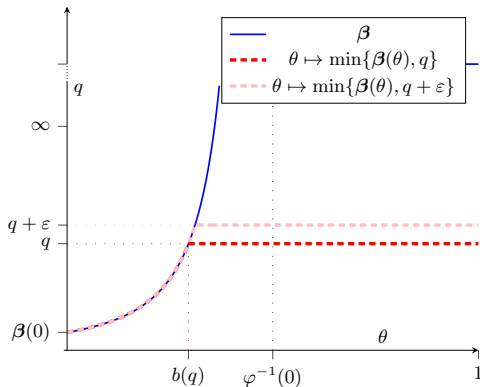
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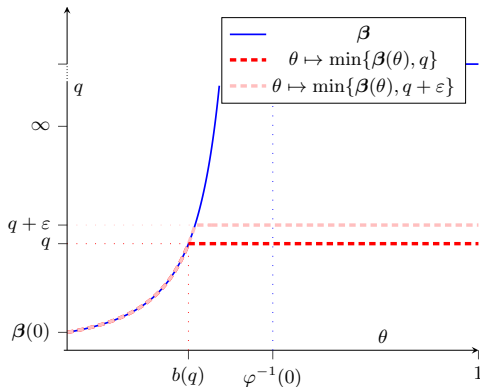
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Marginal revenues

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$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{Mass of bunched types}} \underbrace{(g'(q) + b(q))}_{\text{Marginal } u(\cdot, b(q))}.$$

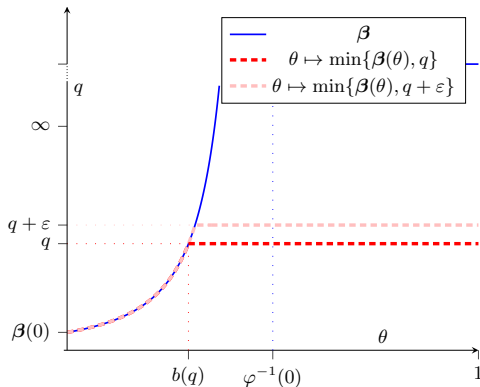


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- V is not differentiable if F is not regular: a kink occurs if β “bunches” at q .



Productive inefficiency

Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\substack{\text{Marginal revenues} \\ \text{given } \theta \mapsto \min\{\beta(\theta), q\}}} < \underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\substack{\text{Marginal total utility} \\ \text{given } \theta \mapsto q}} .$$

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$$U(q) := \mathbb{E}\{u(\min\{\beta(\theta), q\}, \theta)\};$$

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2. $U'(q) < \text{marginal total utility given } \theta \mapsto q$, because

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No damaging

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- The q *contingent no-damaging problem* and its value, $V_N(q)$, are:

$$V_N(q) := \max_{\mathbf{q}(\cdot), t(\cdot)} \int_{\Theta} t(\theta) \, dF(\theta) \text{ subject to:}$$
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- The monopolist chooses the **marginally excluded** type $n(q)$ and the revenues are

$$V_N(q) = (1 - F(n(q)))(g(q) + n(q)q) \quad \text{for} \quad g(q) + \varphi(n(q))q = 0.$$

- With damaging:

$$V'(q) = (1 - F(b(q)))(g'(q) + b(q)), \quad \text{for} \quad g'(q) + \varphi(b(q)) = 0.$$

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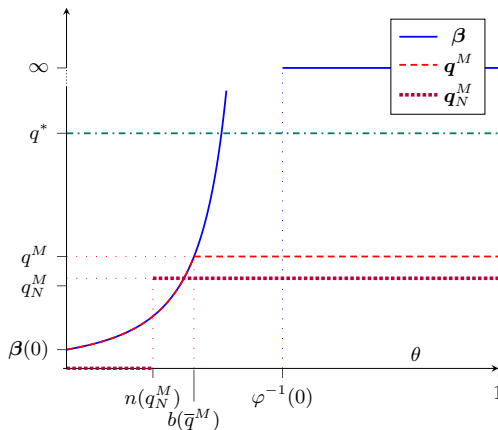
Productive inefficiency is worse: $q_N^M < q^M$, because

$$b(q) \in \operatorname{argmax}_{\theta} (1 - F(\theta))(g'(\theta) + \theta).$$

No damaging

Compared to q^M , the no-damaging quality allocation q_N^M features:

- Less production;
- Less damaging;
- Potential exclusion for “Inada” g .



Conclusion

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$$\underbrace{\int_{\Theta} t(\theta) \, dF(\theta) - c(\sup \mathbf{q}(\Theta))}_{\text{Magnitude gap}} \quad \text{v.} \quad \underbrace{\int_{\Theta} t(\theta) - c(\sup \mathbf{q}(\Theta)) \, dF(\theta)}_{\text{Costly replication}}.$$

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4. In general, $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \int_{\Theta} k(\mathbf{q}(\theta)) \, dF(\theta)$, for some k ,

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4. In general, $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \int_{\Theta} k(\mathbf{q}(\theta)) \, dF(\theta)$, for some k , or, also, $C(\mathbf{q}) = \int_{\Theta} k(\mathbf{q}(\theta); \mathbf{q}) \, dF(\theta)$ (Bergemann et al. (2025).)

Conclusion

1. With digital goods: the ‘**efficiency at the top**’ insight changes, because of the damaging and productive inefficiencies.
2. **Richness** of versions is necessarily due to curvature in preferences.
3. The profit expression admits two **interpretations**:

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5. **Competition** is beneficial for damaging inefficiency and harmful for productive efficiency.

► More details

Literature

Multi-product monopoly

Mussa and Rosen (1978); Maskin and Riley (1984); Deneckere and McAfee (1996) ... Grubb (2009); Corrao et al. (2023).

Mechanism & information design

Bergemann et al. (2025); Mensch and Ravid (2025); Thereze (2025).

Pricing of information

Bergemann et al. (2018); Bergemann and Ottaviani (2021); Yang (2022); Bergemann et al. (2022) ...

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Thanks!

Extra slides

Interpretation

The profit expression admits two **interpretations**:

$$\underbrace{\int_{\Theta} t(\theta) dF(\theta) - c(\sup \mathbf{q}(\Theta))}_{\text{Magnitude gap}} \quad \text{v.} \quad \underbrace{\int_{\Theta} t(\theta) - c(\sup \mathbf{q}(\Theta)) dF(\theta)}_{\text{Costly replication}}.$$

1. Maintained interpretation: $c(\sup \mathbf{q}(\Theta))$ is paid once and is “incomparable” with the utility of a single buyer.
 - ▶ Population \uparrow by $\alpha \iff$ Costs \downarrow by α .
 - ▶ q^M and $b(q^M)$ increase as population scales up.
2. Alternative interpretation: $c(\sup \mathbf{q}(\Theta))$ is paid for every buyer.
 - ▶ Free replication does not hold: the cheapest way to produce $q \leq \sup \mathbf{q}(\Theta)$ is to damage $\sup \mathbf{q}(\Theta)$.
 - ▶ Profits scale up by α as population scales up by α .

Competition

The game among N firms has two stages:

1. Every firm i simultaneously chooses a quality q_i .
2. Every firm i , observing all stage-1 qualities, simultaneously chooses a pricing function $p_i: \mathbb{R}_+ \rightarrow \mathbb{R}$, with $p_i(q) = \infty$ if $q > q_i$.

Then: each type buys a good from a firm i , or does not buy any good for a payoff of 0.

Firms play a subgame-perfect Nash equilibrium.

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Definition 1

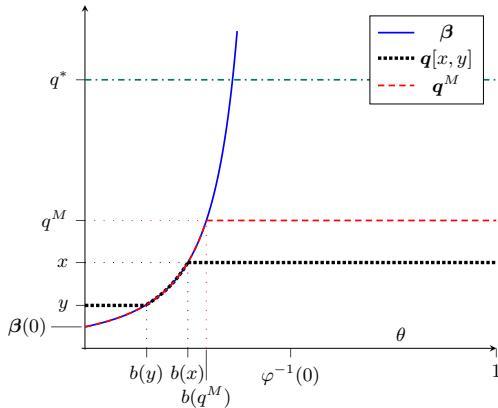
An n equilibrium is an equilibrium in which exactly n firms are active; an n equilibrium is symmetric if active firms play the same strategy.

Competitive allocations

Let's order qualities
 (q_1, \dots, q_N) so that:

$$x > y > \dots$$

Every quality below y comes
 at zero price.



Competitive equilibria

Lemma 2

In any pure-strategy equilibrium: one firm produces q^M and other firms are idle.

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\implies Every symmetric n equilibrium is mixed if $n \geq 2$ (*competitive.*)

Competitive equilibria

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In any pure-strategy equilibrium: one firm produces q^M and other firms are idle.

\implies Every symmetric n equilibrium is mixed if $n \geq 2$ (competitive.)

Proposition 3

1. *For all $n \leq N$, there exists a symmetric n equilibrium.*
2. *Every symmetric and competitive n equilibrium induces the random allocation $\mathbf{q}[\hat{x}, \hat{y}]$, letting \hat{x} and \hat{y} be, resp., the first and second order statistics of the n i.i.d. draws from the CDF*

$$H_n(q) = \left(\frac{c'(q)}{V'(q)} \right)^{\frac{1}{n-1}}, \quad \text{for } q \in [0, q^M].$$

Properties of competitive equilibria

Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. *The lowest quality is positive and free;*
2. *The highest quality is strictly lower than q^M .*

In the paper:

1. Equilibrium welfare with $n \geq 2$ active firms decreases in n .
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

Productive inefficiency from damaging 1

Productive inefficiency arises because:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$

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The monopoly that produces quality q implies total surplus

$$V(q) + U(q) - c(q),$$

with $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \bar{q}\} d\theta' dF(\theta)$ (Envelope Theorem).

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The marginal surplus is $V'(q) + U'(q)$ and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.

Productive inefficiency from damaging 2

WTS: $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$.

1. $U'(q) = \int_{[b(q), 1]} \theta - b(q) \, dF(\theta)$,
because the marginal $u(\mathbf{q}(\theta), \theta)$ increases at rate $g'(q) + \theta$ and the marginal transfer at rate $g'(q) + b(q)$, for $\theta > b(q)$ and $\mathbf{q}(\cdot) = \min\{\beta(\cdot), q\}$;
2. Using $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$, we have

$$V'(q) + U'(q) = (1 - F(b(q)))(g'(q) + b(q)) + \int_{[b(q), 1]} \theta \, dF(\theta).$$

(Note that $U'(q) > 0$ for all $q > 0$, because $b(q) \leq \varphi^{-1}(0) < 1$ for all $q \geq 0$.)

The game

Type θ buys quality $D_{(p_1, \dots, p_N)}(\theta)$ from firm $\iota_{(p_1, \dots, p_N)}(\theta)$, given the pricing functions p_1, \dots, p_N .

The revenues of i given pricing functions p_1, \dots, p_N are

$$R_i(p_1, \dots, p_N) := \int_{\{\theta | \iota_{(p_1, \dots, p_N)}(\theta) = i\}} p_i(D_{(p_1, \dots, p_N)}(\theta)) \, dF(\theta).$$

The set of strategies for firm i is $S_i := Q \times \mathbf{P}_i$, letting $\mathbf{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$ be the set of “conditional” pricing functions of firm i .

The *payoff* of firm i from the profile $s := (\dots, (\bar{q}_i^s, P_i^s), \dots) \in \times_{i=1}^N S_i$ is

$$\Pi_i(s) := R_i(P_1^s[\bar{q}^s], \dots, P_N^s[\bar{q}^s]) - c(\bar{q}_i^s).$$

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