Coordination in Complex Environments

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[Paper]

Coordination & Complexity

Coordination motives and uncertainty are common in innovative contexts.

Examples:

1. Interoperability of Electronic Medical Record Systems (Lin '23),
2. Co-Op advertising (Jørgensen-Zaccour '14),
3. Technological innovation.

This paper introduces a model of **coordination** in an **informationally complex** environment.

Overview

Complexity: the more innovative a decision, the more uncertain its outcome (Callander '11).

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(4) Applications:

1. Oligopoly pricing;
2. Multi-Division organization.

Outline

Model

Conformity

Characterization of Equilibria, Multiplicity, Selection

Applications

1. Oligopoly
2. Organization
3. Network Game

Extensions

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(1) Preferences over profiles of individual outcomes.
$\hookrightarrow$ **Coordination**.

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Model | Preferences

n players.

Player i's outcome is $x_i \in \mathbf{R}$.

Payoff to player i from the profile of outcomes $\boldsymbol{x}$:

$$\pi_i(\boldsymbol{x}) = -\Big[\underbrace{(1-\alpha)\delta_i + \alpha \sum_{j \neq i} \gamma^{ij} x_j}_{i\text{'s target}} - x_i\Big]^2,$$

in which:

- $\alpha \in [0, 1)$ captures coordination motives.
- $\delta_i \in \mathbf{R}$ is i's favorite outcome.
- $\gamma^{ij} \geq 0$ weighs the link from j to i.

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[Ballaster *et al.* '06]

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Players simultaneously choose **policies** from $[\underline{p}, \overline{p}] \subset \mathbf{R}$.

The **outcome function** χ maps every policy, p_i, to the corresponding outcome, $\chi(p_i)$,

$$\chi \colon \mathbf{R} \to \mathbf{R}.$$

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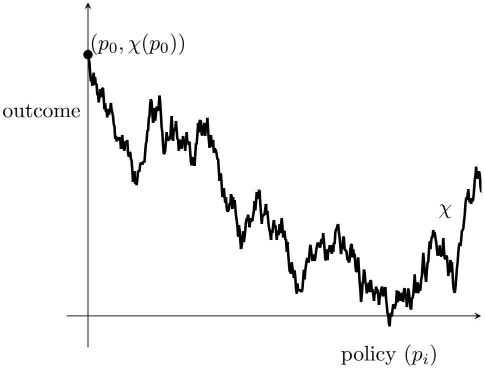
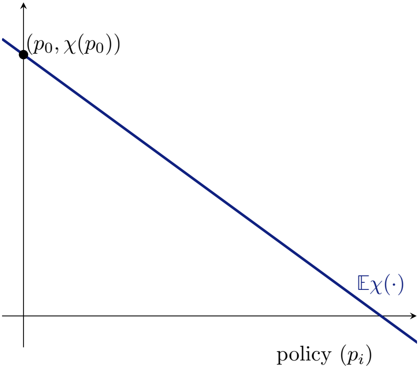
χ is the realization of a Brownian motion with known:

- Drift $\mu < 0$,
- Variance σ^2,
- Initial point $(p_0, \chi(p_0))$.

Model | Complexity

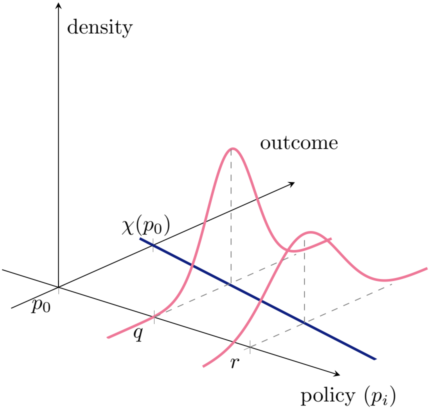




Status quo:

$$(p_0, \chi(p_0)).$$

Model | Complexity



Complexity:

$$k = \frac{\sigma^2}{2|\mu|}.$$

▸ Details

Equilibrium

1. Players simultaneously choose policies $p_1, \ldots, p_n$.
2. Player i gets the payoff from the corresponding outcomes:

$$\pi_i(\chi(p_1), \ldots, \chi(p_n)).$$

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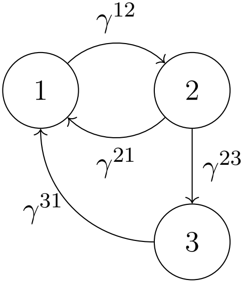
1. Players simultaneously choose policies $p_1, \ldots, p_n$.
2. Player i gets the payoff from the corresponding outcomes:

$$\pi_i(\chi(p_1), \ldots, \chi(p_n)).$$

The policy profile $\boldsymbol{p}$ is an ***equilibrium*** if, for every player i:

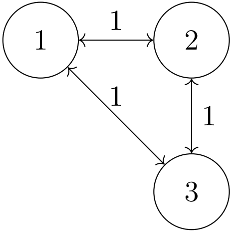
$$\mathbb{E}\pi_i(\boldsymbol{\chi}(\boldsymbol{p})) \geq \mathbb{E}\pi_i(\chi(q_i), \boldsymbol{\chi}(\boldsymbol{p}_{-i})) \text{ for all policies } q_i.$$

Network



$$\mathbf{\Gamma} = (\gamma^{ij}) = \begin{pmatrix} 0 & \gamma^{12} & 0 \\ \gamma^{21} & 0 & \gamma^{23} \\ \gamma^{31} & 0 & 0 \end{pmatrix}$$

Network



$$\mathbf{\Gamma} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

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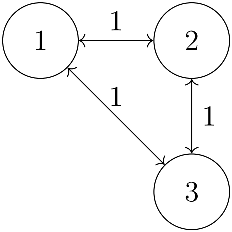

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The upper bound on strength of coordination motives is:

$$\alpha\lambda(\mathbf{\Gamma}) < 1,$$

in which $\lambda(\mathbf{\Gamma})$ is the largest eigenvalue of the adjacency matrix.

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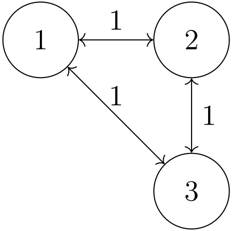

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For this talk: $\gamma^{ij} = \gamma^{ji}$, and:

1. $\underline{p} = p_0$,
2. $\overline{p}$ and $\chi(p_0)$ are large enough.

No Complexity

The *centrality of player i* is the ith entry of:

$$\boldsymbol{\beta} = (1 - \alpha)(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}\boldsymbol{\delta}.$$

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β_i counts all 'α-discounted' walks from i and weighs walks to j by $(1-\alpha)\delta_j$, so:

$$\boldsymbol{\beta} \propto \boldsymbol{\delta} + \alpha\boldsymbol{\Gamma}\boldsymbol{\delta} + \alpha^2\boldsymbol{\Gamma}^2\boldsymbol{\delta} + \cdots$$

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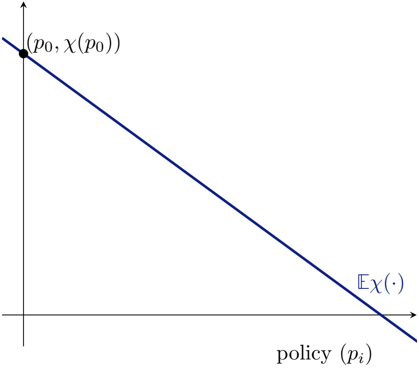

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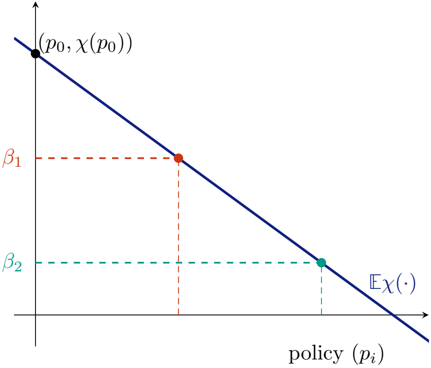
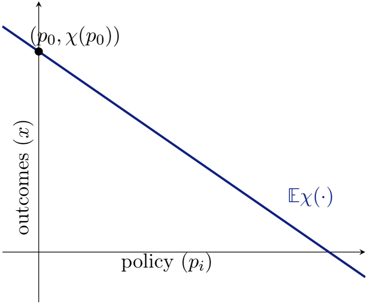


Fact A. (Ballester *et al.* '06)
If $k = 0$, in the unique equilibrium:

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}^{\star}) = \boldsymbol{\beta}.$$

Single Player



Fact B. (Callander '11a) If $\alpha = 0$, player i has a unique optimal policy p_i:

$$\mathbb{E}\chi(p_i) = \delta_i + k.$$

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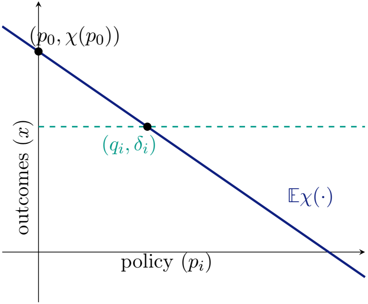
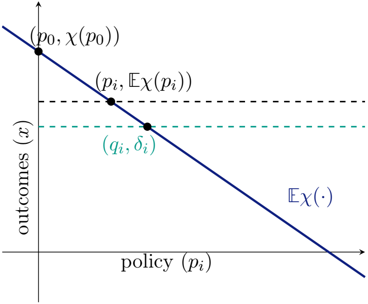


Fact B. (Callander '11a) If $\alpha = 0$, player i has a unique optimal policy p_i:

$$\mathbb{E}\chi(p_i) = \delta_i + \underbrace{k}_{\text{status quo bias}} .$$

Single Player



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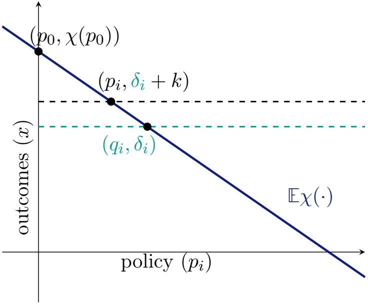

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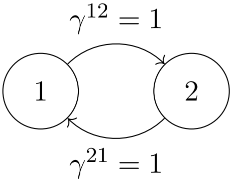
Two Players

$$
\begin{tikzcd}
1 \arrow[r, bend left, "\gamma^{12} = 1"] & 2 \arrow[l, bend left, "\gamma^{21} = 1"]
\end{tikzcd}
$$

$$\mathbf{\Gamma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

And: $\delta_1 > \delta_2$

Two Players



$$\mathbf{\Gamma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

And: $\delta_1 > \delta_2 \underbrace{\implies p_1 < p_2}_{\text{no complexity}}$.

Disentangling Puree noise and Correlation between players.

Two Players | Independent Outcomes

Player i's outcome of policy p_i is given by:

$\chi^i(p_i) = \chi(p_0) + \mu p_i + \sigma W^i(p_i),$ for independent standard W^1, W^2.

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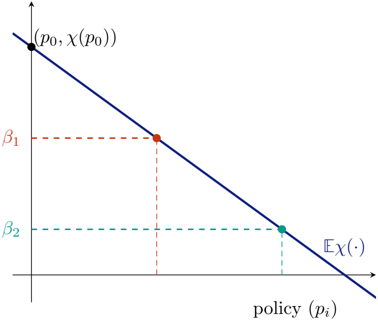

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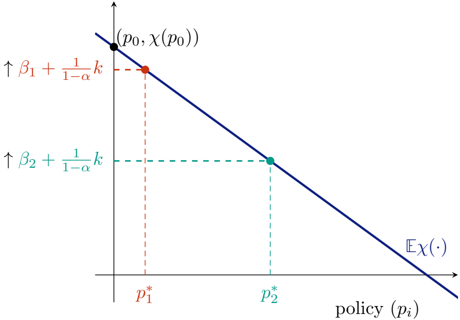

In the unique equilibrium:

$$\mathbb{E}\chi^i(p_i^*) = \beta_i + \underbrace{\overbrace{\frac{1}{1-\alpha}}^{>1}}_{\substack{\text{amplified}\\ \text{s.q. bias}}} k.$$

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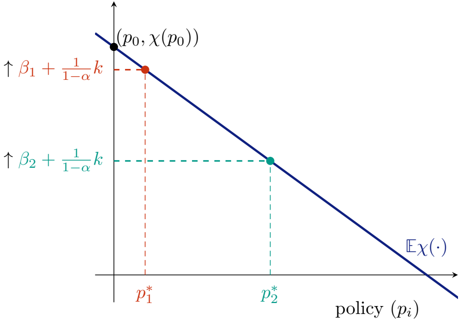

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Conformity? $\mathbb{E}\chi^i(p_i^*) - \mathbb{E}\chi^j(p_j^*) = \beta_i - \beta_j.$

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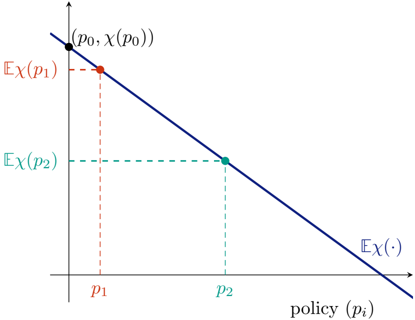

If $p_1 < p_2$, then:
2 is the ***Leader*** and 1 is the ***Follower***,

$\mathrm{Cov}(\chi(p_1), \chi(p_2)) = \mathrm{Var}\,\chi(p_1)$.

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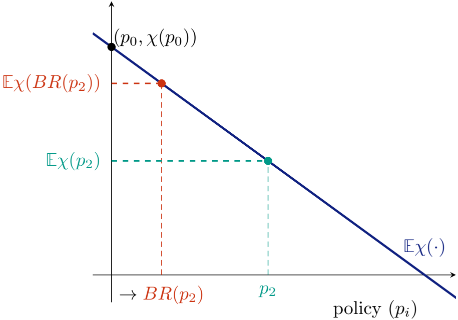

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$\Longrightarrow$ **Extra Exploration Motive for 1**.

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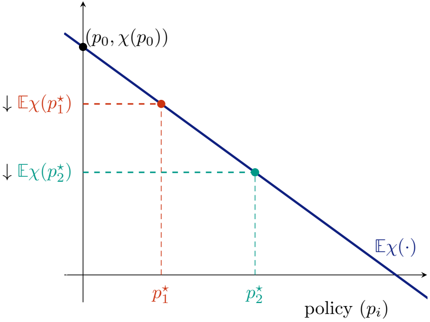

In the unique equilibrium:

$$\mathbb{E}\chi^1(p_1^\star) = \beta_i + k + \frac{1}{1+\alpha}k,$$

$$\mathbb{E}\chi^2(p_2^\star) = \beta_2 + k - \frac{1}{1+\alpha}k,$$

if: $\delta_1 - \delta_2 > 2k\frac{\alpha}{1-\alpha}$.

Conformity: $\mathbb{E}\chi(p_1^\star) - \mathbb{E}\chi(p_2^\star) - (\beta_1 - \beta_2) = \underbrace{-2\frac{\alpha}{1+\alpha}k}_{<0}.$

2 Players | Imperfect Correlation

Outcomes are given, for $\rho \in [0, 1]$, by:

$$\chi^1(p_1) = \chi(p_0) + \mu p_1 + \sigma W^1(p_1)$$

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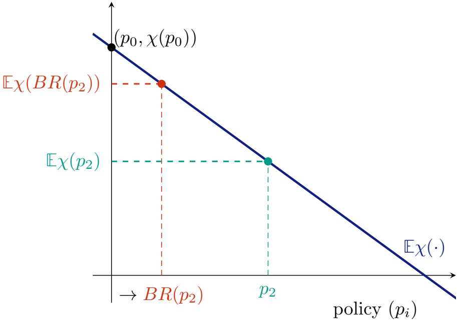

$\Longrightarrow$ **ρ-Weighted Extra Exploration Motive for 1**.

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In equilibrium:

$$\mathbb{E}\chi^1(p_1) - \mathbb{E}\chi^2(p_2) - (\beta_1 - \beta_2) = \rho \underbrace{\left(-2\frac{\alpha}{1+\alpha}k\right)}_{\substack{<0 \\ \text{(perfect correlation)}}} .$$

Lemma 1 (Strategic Complementarities)

The expected payoff $\mathbb{E}\pi_i(\boldsymbol{\chi}(\boldsymbol{p}))$ exhibits strictly increasing differences in $(p_i, \boldsymbol{p}_{-i})$, for every player i.

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Theorem 1 (Existence)

There exist a greatest and least equilibrium.

- Tarski's fixed point theorem.
  $([\underline{p}, \overline{p}]^n$ endowed with product order; Milgrom-Shannon '90, Vives '90.)

Order Structure of the Equilibrium Set

Let $n = 2$ and $\delta_1 = \delta_2 = 0$.

Every equilibrium $\boldsymbol{p}$ is symmetric: $p_1 = p_2$.

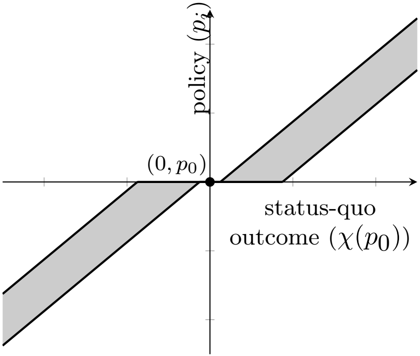

Figure: The equilibrium set, represented by player i's policy, for every status-quo outcome. The equilibrium without complexity is $(p^\star, p^\star)$.

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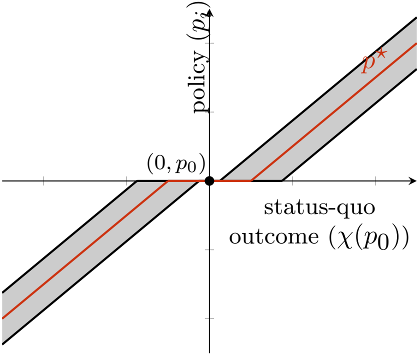

Figure: The equilibrium set, represented by player i's policy, for every status-quo outcome. The equilibrium without complexity is $(p^\star, p^\star)$.

Equilibrium Decomposition

Proposition 1 (Decomposition)

The profile of policies $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium if and only if:

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}) = \boldsymbol{\beta} + k\mathbf{1} + \alpha(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} \odot \boldsymbol{A})\mathbf{1}k,$$

for a matrix $\boldsymbol{A} = (a_{ij})$ such that $a_{ij} \in [-1, 1]$ and

$$a_{ij} = \begin{cases} 1 & \text{if } p_i > p_j, \\ -1 & \text{if } p_i < p_j. \end{cases}$$

($\odot$ denotes element-wise product.)

Equilibrium Decomposition

Proposition 1 (Decomposition)

Without complexity, $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium iff:

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}) = \underbrace{\boldsymbol{\beta}}_{k=0}$$

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Proposition 1 (Decomposition)

Without coordination, $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium iff:

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Player i's ***conformity effect*** weighs each walk to j by $w_j := \sum_\ell \alpha k \gamma^{j\ell} a_{j\ell}$:

$$\boldsymbol{w} + \alpha\boldsymbol{\Gamma}\boldsymbol{w} + \alpha^2\boldsymbol{\Gamma}^2\boldsymbol{w} + \cdots = \alpha(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} \odot \boldsymbol{A})\mathbf{1}k.$$

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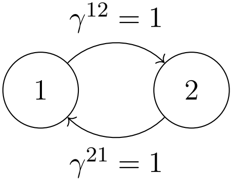
Equilibrium Decomposition | Dyad

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}) = \boldsymbol{\beta} + k\mathbf{1} + \alpha(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} \odot \boldsymbol{A})\mathbf{1}k,$$

for a dyad:



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Equilibrium Decomposition | Dyad

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for a dyad:

$$\begin{tikzcd} 1 \arrow[r, bend left, "\gamma^{12} = 1"] & 2 \arrow[l, bend left, "\gamma^{21} = 1"] \end{tikzcd} \quad , \ \boldsymbol{\Gamma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

If 1 is the follower:

$$\boldsymbol{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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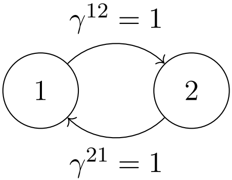

$$, \ \boldsymbol{\Gamma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

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Suppose the network is complete.

Lemma 2 (Pairwise Conformity)

If $\boldsymbol{p} \in (p_0, \overline{p})^n$ is an equilibrium:

$$\text{If } p_i < p_j\text{, then: } \mathbb{E}\chi(p_i) - \mathbb{E}\chi(p_j) < \beta_i - \beta_j.$$

Suppose the network is complete.

Lemma 2 (Conformity in Ordered Equilibria)

Let $\boldsymbol{p} \in (p_0, \overline{p})^n$ be an equilibrium. If $p_1 < \cdots < p_n$, then:

$$\mathbb{E}\chi(p_i) - \mathbb{E}\chi(p_{i+1}) - (\beta_i - \beta_{i+1}) = \underbrace{-2\frac{\alpha\gamma}{1+\alpha\gamma}k}_{\downarrow \text{ in } \alpha \text{ \& } k}.$$

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1. If $\uparrow k$, matching a leader's outcome is a more cost effective way of dealing with uncertainty + Conformity 'feeds back' through the network.

Conformity

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1. If $\uparrow k$, matching a leader's outcome is a more cost effective way of dealing with uncertainty + Conformity 'feeds back' through the network.
2. "*Yielding is far greater on **difficult** items than on easy ones.*" (Asch '51; difficulty elicited as "*certainty of judgement*".)

▸ counterformity

Potential Maximizer

A game *admits a potential* if it is 'best-response equivalent' to a common-interest game.
$\hookrightarrow$ The common payoff is called ***potential***. (Monderer-Shapley '96.)

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Proposition 2

The game admits a potential. Moreover, the potential is unique up to a constant and has a unique maximizer.

More Details

Equilibrium Selection

Suppose 2 players and $\delta_1 = \delta_2 = 0$.

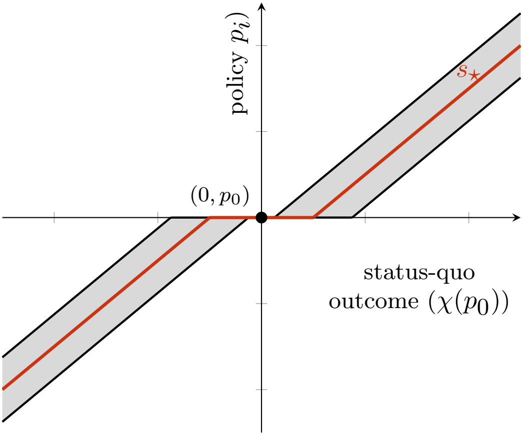

Figure: Equilibrium set and potential maximizer equilibrium $s_\star(\chi(p_0))$, for every status quo outcome.

Coordination and Complexity

If $\omega > 0$ and $\alpha > 0$, 'kinked' mean-variance decomposition.

Coordination and Complexity

With $n = 2$ and $\delta_1 = \delta_2 = 0$, player i's loss given $p_i \geq p_j \geq p_0$ is

$$\mathbb{E}(\chi(p_i) - \alpha\chi(p_j))^2 = (\mathbb{E}\chi(p_i) - \alpha\mathbb{E}\chi(p_j))^2 + \mathbb{V}\chi(p_i) \underbrace{-2\alpha\mathbb{C}(\chi(p_i), \chi(p_j))}_{k > 0 \;\&\; \alpha > 0} + \cdots,$$

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Decomposition of the Potential Maximizer

Proposition 3

The profile $\boldsymbol{p} \in (p_0, \overline{p})^n$ is the unique potential maximizer if, and only if:

$$\mathbb{E}\boldsymbol{\chi}(\boldsymbol{p}) = \underbrace{\boldsymbol{\beta}}_{\omega=0} + \underbrace{k\mathbf{1}}_{\substack{\text{status quo}\\ \text{bias}}} + \underbrace{\alpha k(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})^{-1}(\boldsymbol{\Gamma} \odot \boldsymbol{A})\mathbf{1}}_{\text{strat. uncertainty effect}},$$

for a <u>skew-symmetric</u> matrix $\boldsymbol{A} = (a_{ij})$ such that $a_{ij} \in [-1, 1]$ and $a_{ij} = 1$, if $p_i > p_j$.

(Skew-Symmetry: $a_{ij} = -a_{ji}$.)

Model

Conformity

Characterization of Equilibria, Multiplicity, Selection

Applications

Extensions

A representative consumer has preferences represented by

$$U(\boldsymbol{q}, m) = \sum_i a_i q_i - \frac{1}{2} b \sum_i q_i^2 - \frac{1}{2} c \sum_{i,j:j \neq i} q_i q_j + m,$$

in which $b > c \geq 0$.

Oligopoly Pricing

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in which $b > c \geq 0$.

Demand for good i, given price vector $\boldsymbol{x}$:

$$h_i(\boldsymbol{x}) = a_i - x_i + \zeta \sum_{j \neq i} x_j,$$

in which $\zeta = \frac{1-(b-c)}{b-c}$ captures 'strategic complementarities'.

$\zeta \in [0, \frac{2}{n-1})$ because we normalize own-price coefficient.

Oligopoly Pricing

In the ***pricing game***:

- n firms choose **pricing policies** $p_1, \ldots, p_n$.
- Profits of firm i are:

$$\pi_i^B(\boldsymbol{\chi}(\boldsymbol{p})) = [\chi(p_i) - \underbrace{m_i}_{\text{M.C.}}]h_i(\boldsymbol{\chi}(\boldsymbol{p})).$$

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If $a_i - m_i > a_{i+1} - m_{i+1}$ for all $i \in \{1, \ldots, n-1\}$, then every equilibrium is 'ordered' without complexity:

$$x_1 > x_2 > \cdots > x_n.$$

Let $\boldsymbol{\beta}^B$ be equilibrium prices without complexity.

Proposition 4

Let $a_i - m_i - (a_{i+1} - m_{i+1}) > 2\zeta k$ for all $i \in \{1, \ldots, n-1\}$. The equilibrium $\boldsymbol{p}$ is unique and, if $\boldsymbol{p}$ is interior:

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1. Joint profit = potential of an auxiliary game, with twice coordination motives.

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Joint profits?

1. Joint profit = potential of an auxiliary game, with twice coordination motives.
2. Potential characterization.

Centralization in Organizations

The cost of division $i \in \{1, 2\}$ when j produces quantity x_j is

$$mx_i - gx_1x_2,$$

in which $g > 0$ measures cost externalities.

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In the ***production game***:

- Division managers choose **production policies** $p_1, \ldots, p_n$.
- Profits of division i are:

$$\pi_i^O(\boldsymbol{\chi}(\boldsymbol{p})) = \left[\underbrace{a - \frac{1}{b}\chi(p_i)}_{\text{inv. demand}} - m + g\chi(p_j)\right]\chi(p_i),$$

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The firm's ***profits*** are $\pi_i^O + \pi_j^O$.

Proposition 5

There exists a unique policy profile $\boldsymbol{p}^O$ that maximizes expected total profits. Moreover, $\boldsymbol{p}^O$ is an equilibrium of the production game if, and only if:

$$2k \geq (a-m)\frac{b}{1-bg}.$$

Proposition 5

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Proof:

- Expected firm's profits = utilitarian welfare.
- Utilitarian welfare = potential with twice the coordination motives.

To maximize total profits, CEO can leverage a sufficiently high complexity.

Extensions

(1) The outcome of policy p to player i is:

$$\chi^i(p) = \chi(p_0) + \mu p + \sigma W^i(p),$$

with $dW^i(p)dW^j(p) = \rho c_{ij}dt$.

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---

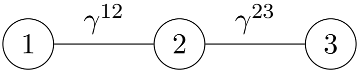
(2) Player i believes that the initial point is:

$$(p_0^i, \ \underbrace{\chi(p_0^i)}_{\substack{\text{private} \\ \text{information.}}}).$$

Single Crossing.
The expected payoff $\mathbb{E}^i \pi_i(\chi(p_i), \boldsymbol{\chi}(\boldsymbol{\sigma}_{-i}))$ has strictly increasing differences in $(p_i, \chi(p_0^i))$, if strategies in $\boldsymbol{\sigma}_{-i}$ are nondecreasing.

◂ More

Counterformity



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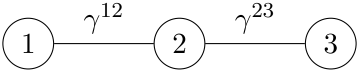
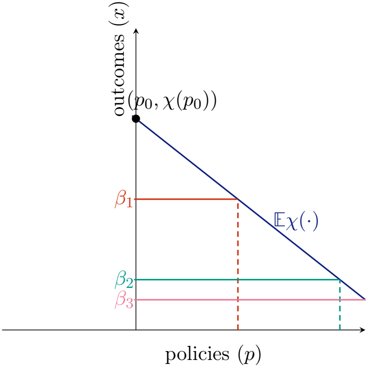


Without complexity:



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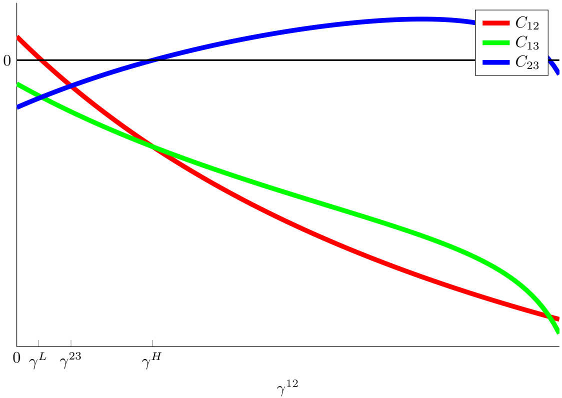

$$C_{ij} = \mathbb{E}\chi(p_i^\star) - \mathbb{E}\chi(p_j^\star) - \beta_i + \beta_j.$$

▸ conformity

Literature

Complexity à la Callander '11a

- Decision problems, players interacting over time: Jovanovic-Rob '90, Callander-Hummel '14, Garfagnini-Strulovici '16, Callander-Matouschek '19.
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Team & potential games Radner '62, Monderer-Shapley '96, ...

Takeaway

- This paper uncovers a new conformity phenomenon.
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- Incomplete information about $\boldsymbol{\delta}$.
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Thanks!

Distribution

For $p_0 < p < q$:

$$\begin{aligned}
\mathbb{E}\chi(p) &= \chi(p_0) + \mu(p - p_0) \\
\operatorname{Var}\chi(p) &= (p - p_0)\sigma^2 \\
\operatorname{Cov}(\chi(p), \chi(q)) &= \operatorname{Var}\chi(p). \\
&= \min\{p - p_0, q - p_0\}\sigma^2
\end{aligned}$$

▸ Back

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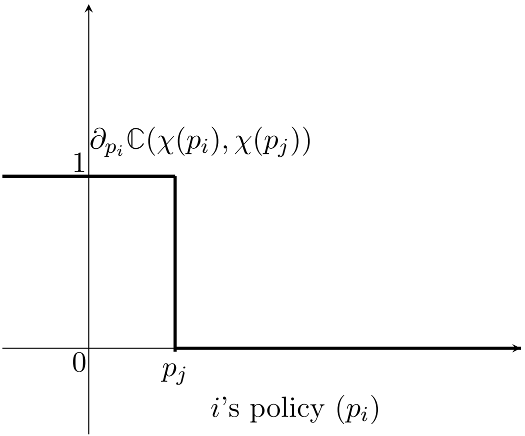
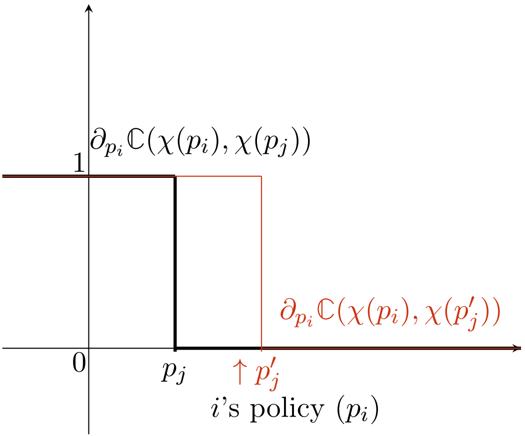


Back

Coordination and Complexity

Covariance ($\min\{\mathbb{V}\chi(p_i), \mathbb{V}\chi(p_j)\}$) is supermodular in (p_i, p_j).



Back

Covariance Structure

$f(p_1, p_2)$ has ***strictly increasing differences*** in p_1 and p_2 if:

$p_1' > p_1$ and $p_2' > p_2 \implies f(p_1', p_2') - f(p_1, p_2') > f(p_1', p_2) - f(p_1, p_2)$.

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$\mathrm{Cov}(\chi(p), \chi(p'))$, for $p_0 = 0$ and $p, p' > 0$, can be:

- Brownian:

$$\min\{p, p'\}\sigma^2; \qquad \checkmark$$

- Ornstein-Uhlenbeck:

$$e^{-\frac{|p-p'|}{\ell}}, \ell > 0; \qquad \text{X}$$

- Squared exponential:

$$e^{-\left(\frac{p-p'}{\ell}\right)^2}, \ell > 0. \qquad \text{X}$$

Back

Potential

The *potential* is $V\colon [p_0, \overline{p}]^n \to \mathbf{R}$, given by

$$V(\boldsymbol{p}) = \mathbb{E}v(\boldsymbol{\chi}(\boldsymbol{p})),$$

in which $v\colon \boldsymbol{x} \mapsto 2(1-\alpha)\boldsymbol{\delta}^\top \boldsymbol{x} - \boldsymbol{x}^\top(\boldsymbol{I} - \alpha\boldsymbol{\Gamma})\boldsymbol{x}$ is the potential in the game without complexity.

Lemma 3 (Potential Game)

For every player i, there exists $g_i\colon [p_0, \overline{p}]^{n-1}$ such that:

$$\mathbb{E}\pi_i(\boldsymbol{\chi}(\boldsymbol{p})) = V(\boldsymbol{p}) + g_i(\boldsymbol{p}_{-i}), \text{ for all } \boldsymbol{p} \in [p_0, \overline{p}]^n.$$

(Monderer-Shapley '96; Morris-Ui '04) .

Potential Maximizer

A *potential maximizer* is profile $\boldsymbol{p}^\star$ that maximizes the potential:

$$\boldsymbol{p}^\star \in \arg\max_{\boldsymbol{p}\in[p_0,\overline{p}]^n} V(\boldsymbol{p}).$$

Proposition 6 (Potential Maximizer)

(1) If the policy profile $\boldsymbol{p}$ is a potential maximizer, then $\boldsymbol{p}$ is an equilibrium.
(2) There exists a unique potential maximizer.

◂ Back

Incomplete Information

Player i believes that the outcome function χ is the path of a Brownian motion with:

- Drift $\mu < 0$,
- Variance $\sigma^2 > 0$,
- Initial point $(p_0^i, \chi(p_0^i))$.

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The *status quo outcome of player* i, $\chi(p_0^i)$, is known only to i. Players know the status quo policies $(p_0^1, \ldots, p_0^n)$, with $p_0^i \neq p_0^j$ for distinct players i, j.

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A strategy for player i is a measurable function $\sigma_i \colon \mathbf{R} \to P_i$.

Equilibrium

BNE: The strategy profile σ is an *equilibrium of* $\mathcal{G}(\boldsymbol{p}_0)$ if, for every player i:

$$\sigma_i(x_0^i) \in \arg\max_{p_i \in P_i} \mathbb{E}^i[\pi_i(\chi(p_i), \boldsymbol{\chi}(\sigma_{-i}))], \quad \text{for all } x_0^i \in \mathbf{R}.$$

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Lemma 4 (FOSD Monotonicity)

Player i's belief about outcome $\chi(q)$ is nondecreasing in $\chi(p_0^i)$ according to FOSD.

Intuition: $\mathbb{E}^i\chi(p) = \chi(p_0^i) + \mu(p - p_0^i)$

Beliefs

Assumption: $\alpha \sum_j \gamma^{ij} < 1$.

Lemma 5 (Single Crossing)

The expected payoff $\mathbb{E}^i[\pi_i(\chi(p_i), \boldsymbol{\chi}(\sigma_{-i}))]$ has strictly increasing differences in $(p_i, \chi(p_0^i))$, $i \in N$, if strategies in σ_{-i} are nondecreasing.

Proof:

- By FOSD, i's optimal policy increases in $\chi(p_0^i)$ when $\alpha = 0$;
- Bound on coordination motives.

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Proof:

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- Bound on coordination motives.

Proposition 7 (Existence)

There exist a greatest and a least equilibrium, $\overline{\sigma}$ and $\underline{\sigma}$, resp., and they are in nondecreasing strategies.

Back

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