# Screening in digital monopolies

Pietro Dall'Ara     Elia Sartori
*CSEF & University of Naples Federico II*

1st European Economic Theory Conference

# Free damaging and replication

Several goods exhibit:

1. Free replication;

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taste heterogeneity

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This paper studies monopoly provision of goods whose production structure exhibits free replication and free damaging.

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Examples of **digital goods**:

1. Software goods;
2. Digital audio content;
3. Data.

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| Up to 20 billion | - | - | ✔ | ✔ | ✔ |

# Plan

1. Model;
2. Efficiency benchmark;
3. Monopoly allocation and inefficiencies;
4. No-damaging constraint, extensions, and interpretations.

# Model

A unit mass of buyers, each drawing a **type** $\theta \in [0,1] = \Theta$, interacts with a seller.

Type $\theta$ is privately informed about $\theta \sim F$, for twice diff. $F$ on $(0,1)$;
$\hookrightarrow$ $F$ is regular in these slides, $\mathbb{E}\{\cdot\}$ refers to the r.v. $\theta$.

Type $\theta$ has payoff from **quality** $q \in \mathbb{R}_+$ and transfer $t \in \mathbb{R}$:

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q,\theta)} \quad -t,$$

for a strictly concave, increasing, and twice diff. $g$ (Chade-Swinkels '21.)

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for a strictly concave, increasing, and twice diff. $g$ (Chade-Swinkels '21.)

The cost of **allocation** $\boldsymbol{q} \colon \Theta \to \mathbb{R}_+$ is

$$C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)),$$

for a **production cost** $c$, increasing, strictly convex, twice diff., with $c'(0) = 0$ and $\lim_{q \to \infty} c'(q) = \infty$.

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for a strictly concave, increasing, and twice diff. $g$ (Chade-Swinkels '21.)

With *separable* costs, the cost of $\boldsymbol{q}$ is

$$C(\boldsymbol{q}) = \mathbb{E}\{k(\boldsymbol{q}(\theta))\},$$

for some $k$ (Mussa-Rosen '78.)

# Efficiency

The *surplus* induced by allocation $\boldsymbol{q}$ is

$$\mathbb{E}\{u(\boldsymbol{q}(\theta), \theta)\} - c(\sup \boldsymbol{q}(\Theta)).$$

The *efficient* quality allocation $\boldsymbol{q}^\star$ maximizes surplus.

**Proposition 1**
The efficient allocation is given by $\boldsymbol{q}^\star(\theta) = q^\star$ for all $\theta$, in which $q^\star$ is the unique quality $q$ such that

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1. Damaging is inefficient: $\mathbb{E}\{u(\sup \boldsymbol{q}(\Theta), \theta)\} \geq \mathbb{E}\{u(\boldsymbol{q}(\theta), \theta)\}$;
2. Average marginal utility equals marginal production costs.

# Efficiency

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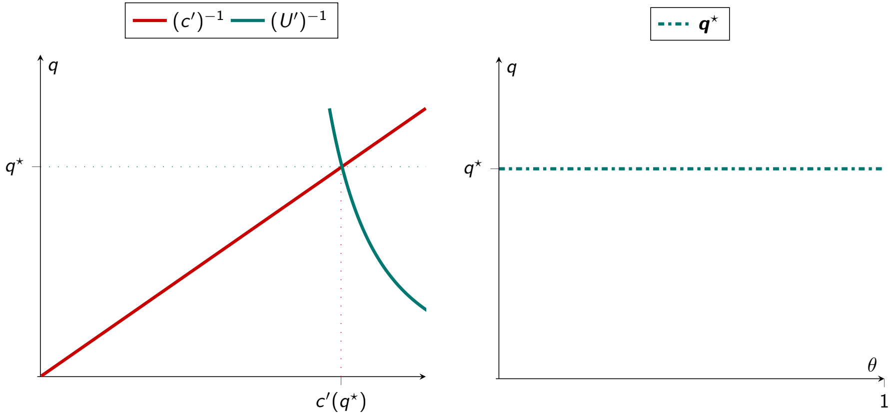


Define $U(q) = g(q) + \int_\Theta \theta \,\mathrm{d}F(\theta) q$.

# Monopoly

The monopolist problem is:

$$(\mathcal{P}^M) \qquad \max_{\boldsymbol{q},\, t(\cdot)} \int_{\Theta} t(\theta)\,\mathrm{d}F(\theta) - c(\sup \boldsymbol{q}(\Theta)) \text{ subject to:}$$

$$\begin{aligned} &u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq u(\boldsymbol{q}(\hat{\theta}),\theta) - t(\hat{\theta}), \text{ for all } (\theta,\hat{\theta}), \\ &u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq 0, \text{ for all } \theta. \end{aligned}$$

- The *monopolist* allocation $\boldsymbol{q}^M$ solves $\mathcal{P}^M$ for some $t(\cdot)$.

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- The *monopolist* allocation $\boldsymbol{q}^M$ solves $\mathcal{P}^M$ for some $t(\cdot)$.
- Without separable costs: the monopolist problem cannot be solved via "pointwise maximization".

# Monopoly

The $q$ *constrained problem* and its value $V(q)$ are:

$$(\mathcal{P}(q)) \qquad V(q) \coloneqq \max_{\boldsymbol{q},\, t(\cdot)} \int_{\Theta} t(\theta)\,\mathrm{d}F(\theta) - \cancel{c(\sup \boldsymbol{q}(\Theta))} \text{ subject to:}$$

$$\boldsymbol{q}(\theta) \leq q, \text{ for all } \theta,$$

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**Lemma 1 (Invest then distribute)**

The allocation $\boldsymbol{q}$ solves $\mathcal{P}^M$ if and only if:

1. $\boldsymbol{q}$ solves $\mathcal{P}(q^M)$,
2. $q^M$ solves $\max_q V(q) - c(q)$.

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$$(\mathcal{P}(q)) \qquad V(q) \coloneqq \max_{\boldsymbol{q}} \int_{[0,1]} \underbrace{g(\boldsymbol{q}(\theta)) + \varphi(\theta)\boldsymbol{q}(\theta)}_{\text{Virtual surplus}} \mathrm{d}F(\theta) \text{ subject to:}$$

$$\boldsymbol{q}(\theta) \leq q, \text{ for all } \theta \in \Theta,$$

$$\boldsymbol{q} \text{ is nondecreasing;}$$

in which $\varphi(\theta) \coloneqq \theta - \frac{1-F(\theta)}{F'(\theta)}$.

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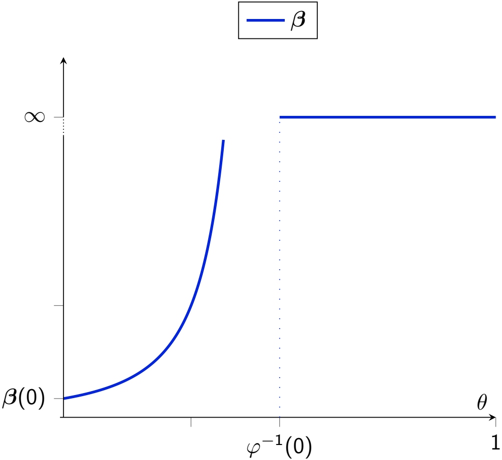
# Virtual surplus maximization

The *virtual-surplus maximizer*

$$\boldsymbol{\beta}(\theta) \in \operatorname*{Argmax}_{\boldsymbol{q}} g(\boldsymbol{q}) + \varphi(\theta)\boldsymbol{q}$$

is such that:

1. $\boldsymbol{\beta}(\theta) = \infty$ if $\theta \geq \varphi^{-1}(0)$;
2. $\boldsymbol{\beta}$ is increasing;
3. $\boldsymbol{\beta}(0) > 0$ ("Inada" $g$).



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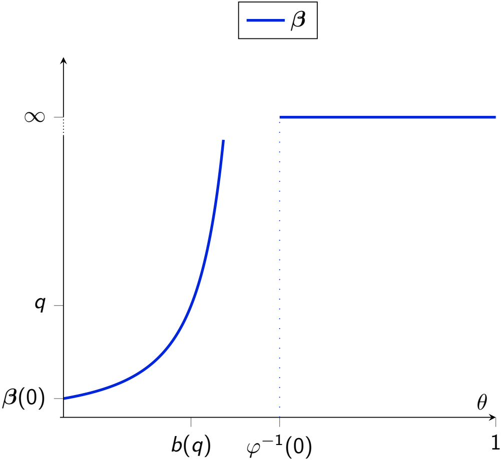
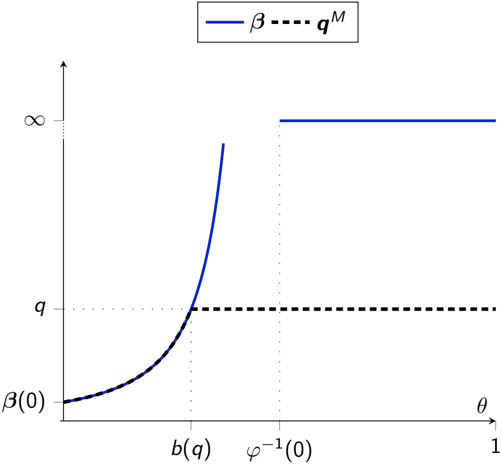
$b$ is the inverse of $\boldsymbol{\beta}$.



# Virtual surplus maximization

**Lemma 2**
Allocation $\boldsymbol{q}$ solves $\mathcal{P}(q)$ iff:

$$\boldsymbol{q}(\theta) = \min\{\boldsymbol{\beta}(\theta), q\}, \text{ for all } \theta.$$



# Virtual surplus maximization

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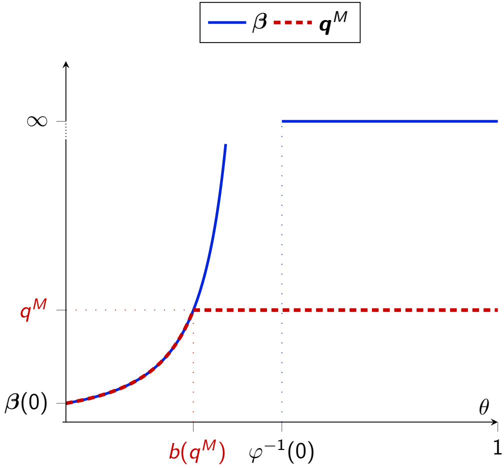
Distributive properties of $\boldsymbol{q}^M$:

1. Bunching at the top;
2. Distributional inefficiency at the bottom or full bunching;
3. No exclusion.



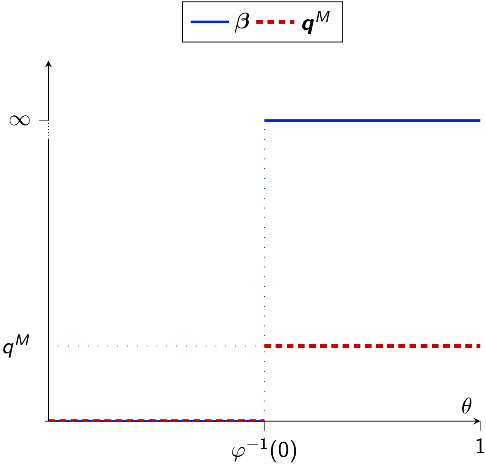
# Linear preferences

Distributive properties if $g(q) = 0$:

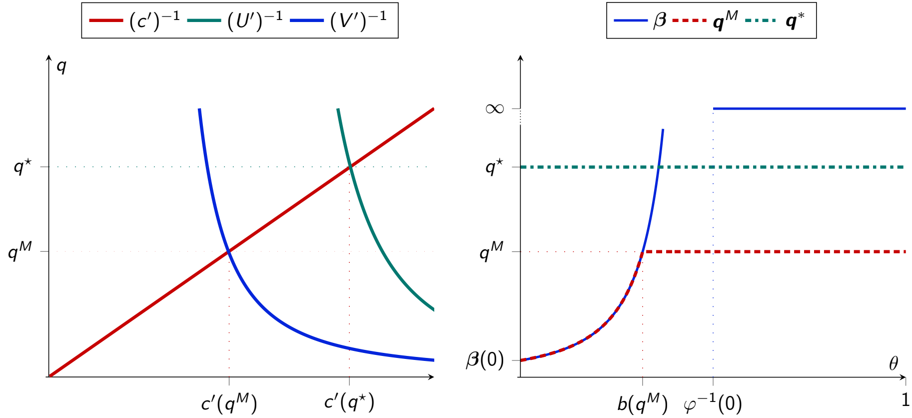
1. Bunching at the top; $\beta(\theta) = \infty$ for $\theta \geq \varphi^{-1}(0)$
2. Exclusion at the bottom; $\beta(\theta) = 0$ for $\theta < \varphi^{-1}(0)$

$\Longrightarrow$ **Single-quality menu**.

Richness in digital markets is due solely to preferences.



# The monopolist allocation



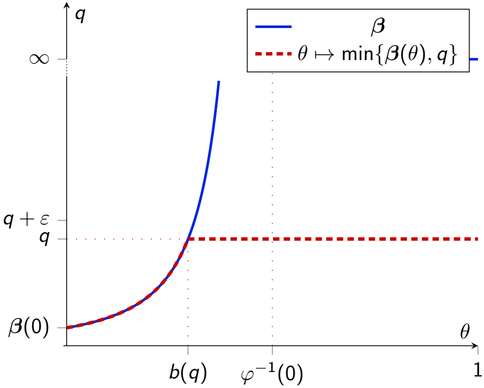
# Marginal revenues

$V'(q)$ is the marginal return from increasing the cap of the $q$-constrained allocation $\theta \mapsto \min\{\boldsymbol{\beta}(\theta), q\}$:

$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{bunched types}} \underbrace{(g'(q) + b(q))}_{u_q \text{ of } b(q)}.$$

The change from $q$ to $q + \varepsilon$ leads to:

1. same revenues from $q' < q$: $q'$ sold to the same $\theta$, and $\theta$ gets the same **rent**;
2. higher quality for bunched types;
3. higher price by $u_q(q, b(q))\varepsilon$.



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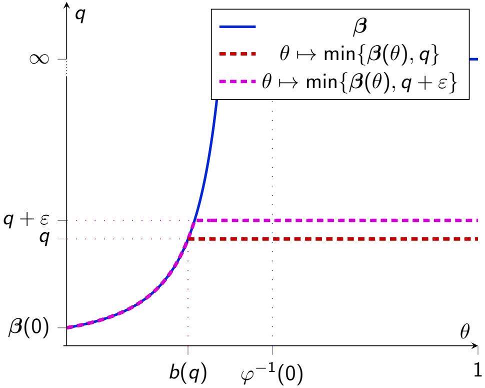

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1. By Markov's inequality:

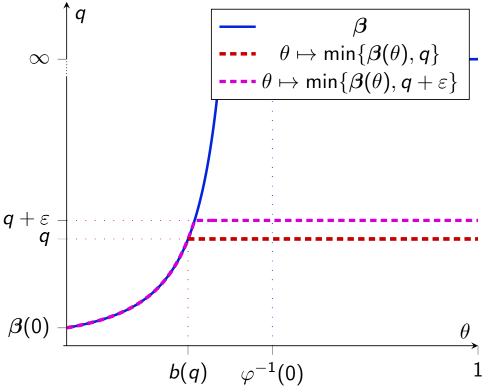
   $$(1 - F(b(q)))b(q) \leq \mathbb{E}\{\theta\};$$

2. By the distributive properties

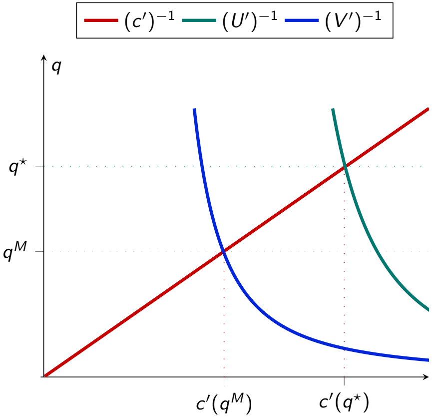
   $$b(q) < 1,$$

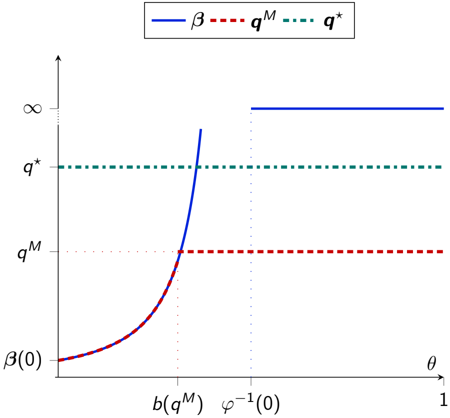
So:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$



# Productive inefficiency



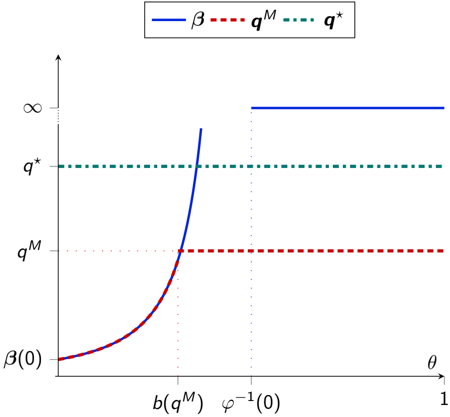
# Productive inefficiency

**Proposition 2**

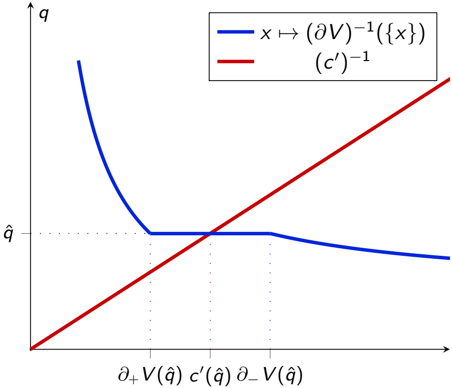
The monopolist allocation is given by $\boldsymbol{q}^M(\theta) = \min\{\boldsymbol{\beta}(\theta), q\}$ for all $\theta$, in which $q^M$ is the unique $q$ solving

$$V'(q) = c'(q).$$

Moreover, it holds that: $q^M < q^\star$.



# Non-regular distribution




- $\boldsymbol{\beta}$ is ironed to obtain $\overline{\boldsymbol{\beta}}$;
- By Lemma 1, $\theta \mapsto \min\{\overline{\beta}(\theta), q\}$ solves $\mathcal{P}(q)$;

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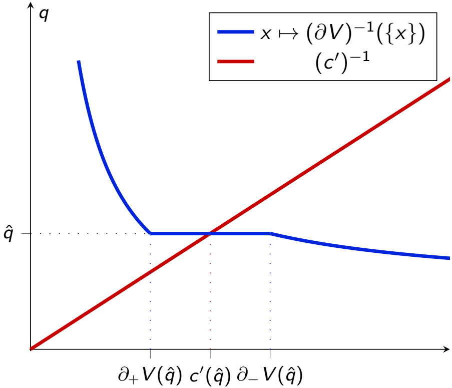


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- By Lemma 1,
  $\theta \mapsto \min\{\overline{\beta}(\theta), q\}$ solves $\mathcal{P}(q)$;
- If types in $(\theta', \theta'')$ are bunched "at" $\hat{q} \in (0, q)$,

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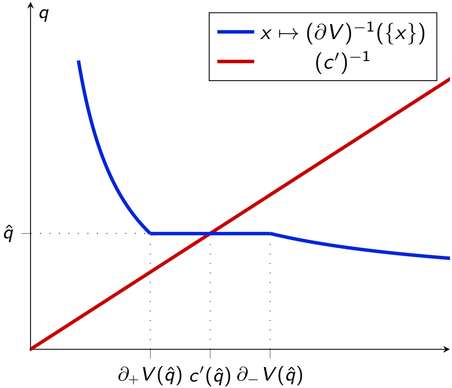


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$V$ is concave by concavity of $u$ in $q$, and productive inefficiency holds.

**Proposition 3**

Without regularity, the monopolist allocation is $\boldsymbol{q}^M(\theta) = \min\{\overline{\boldsymbol{\beta}}(\theta), q^M\}$, in which $q^M$ is the unique $q$ with $c'(q) \in \partial V(q)$. Moreover, it holds that $q^M < q^\star$.

# No damaging

Without damaging, the $q$ constrained problem is:

$$V_N(q) := \max_{\boldsymbol{q},\, t(\cdot)} \int_\Theta t(\theta)\,\mathrm{d}F(\theta) \text{ subject to:}$$
$$\text{IC, IR, } \boldsymbol{q}(\theta) \in \{0, q\}, \text{ for all } \theta.$$

The constraint is irrelevant under:
1. Full bunching by $\boldsymbol{q}^M$ ;
2. Linear preferences.

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The monopolist chooses a **marginally excluded** type $n(q)$, so

$$V_N(q) = (1 - F(n(q)))(g(q) + n(q)q), \quad \text{for } g(q) + \varphi(n(q))q = 0.$$

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$$\text{(Recall:} \quad V'(q) = (1 - F(b(q)))(g'(q) + b(q)), \quad \text{for} \quad g'(q) + \varphi(b(q)) = 0.)$$

- Intuitively: damaging ban $\implies$ $n(q) \leq b(q)$, strictly if $b(q) > 0$,
- so productive inefficiency is worse:

$$V_N'(q) - V'(q) = \int_{[n(q),b(q)]} (1 - F(\theta))(g'(q) + \theta)\,\mathrm{d}\theta \geq 0,$$

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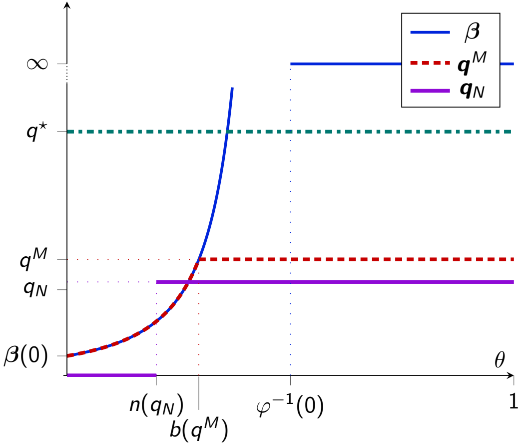
strictly if $b(q) > 0$.

# No damaging

The no-damaging allocation $\boldsymbol{q}_N$ features:

- Less production;
- No damaging;
- (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.



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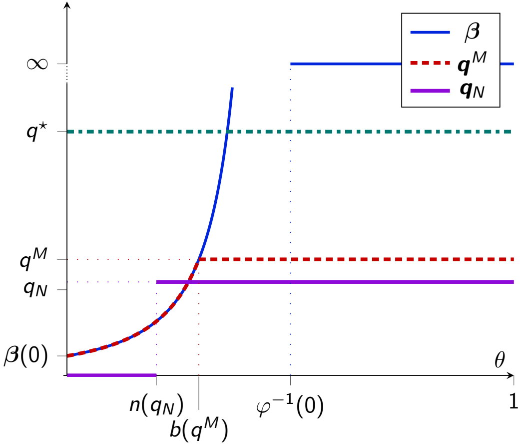
- Less production;
- No damaging;
- (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.




**Proposition 4**

Without damaging, the monopolist allocation is $\boldsymbol{q}_N(\theta) = \mathbf{1}_{[b_N(q_N),1]}(\theta) q_N$, in which $q_N$ is the unique $q$ solving $V_N'(q) = c'(q)$. Moreover, we have $q_N \leq q^M$, strictly if $b(q^M) > 0$.

# Cost interpretation

For separable costs: $\Pi^{\text{M-R}}(\boldsymbol{q}) = \underbrace{\int_\Theta t(\theta)\,\mathrm{d}F(\theta)}_{\text{per-agent revenues}} - \underbrace{\int_\Theta k(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta)}_{\text{per-agent costs}},$

**1.** Payment $t(\theta)$ and production cost $k(\boldsymbol{q}(\theta))$ are comparable;

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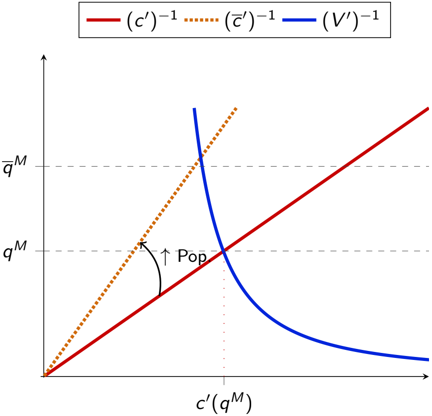
For digital goods: $\Pi(\boldsymbol{q}) = \underbrace{\int_\Theta t(\theta)\,\mathrm{d}F(\theta)}_{\text{per-agent revenues}} - \underbrace{c(\sup \boldsymbol{q}(\Theta))}_{\text{per-agent costs}},$

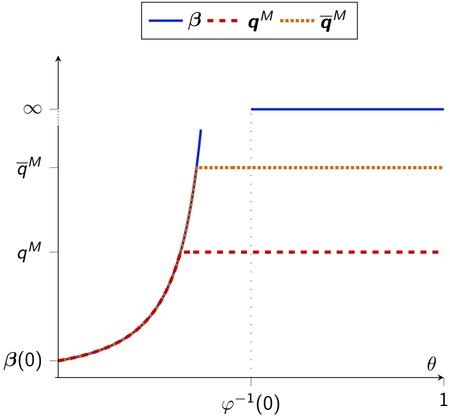
**1.** Payment $t(\theta)$ and production cost $c(q^M)$ are of different sizes;

**2.** Population size impacts $q^M$;

In general: $C(\boldsymbol{q}) = \int_\Theta k(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta) + c(\sup \boldsymbol{q}(\Theta)).$

# Population size



# Separable interpretation

For a separable interpretation (with a continuum of buyers:)

$$\Pi(\boldsymbol{q}) = \int_{\Theta} t(\theta) - \underbrace{c(\sup \boldsymbol{q}(\Theta))}_{\substack{\text{same magnitude} \\ \text{as } t(\theta)}} \mathrm{d}F(\theta),$$

under which production exhibits:

1. costly replication;
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1. costly replication;
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3. impossibility or high costs of producing $q' < \sup \boldsymbol{q}(\Theta)$.

In the damaged-goods model of Deneckere and McAfee (1996):

1. Costs are purely-separable **damaging** costs $k$, with $k'(q) \leq 0$;

   $\hookrightarrow$ identify gains from screening
2. The cost of the undamaged good is fixed.

   $\hookrightarrow$ if damaging is free, our separable interpretation

# Single buyer

$$\Pi^{\text{M-R}}(\boldsymbol{q}) = \underbrace{\int_\Theta t(\theta)\,\mathrm{d}F(\theta)}_{\text{expected revenues}} - \underbrace{\int_\Theta c(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta)}_{\text{expected costs}},$$

**1.** Payment $t(\theta)$ and production cost $k(\boldsymbol{q}(\theta))$ are comparable;

**2.** Production occurs after type is elicited;

**3.** Damaging and replication costs do not bite.

$$\Pi(\boldsymbol{q}) = \underbrace{\int_\Theta t(\theta)\,\mathrm{d}F(\theta)}_{\text{expected revenues}} - \underbrace{c(\sup \boldsymbol{q}(\Theta))}_{\text{costs}}.$$

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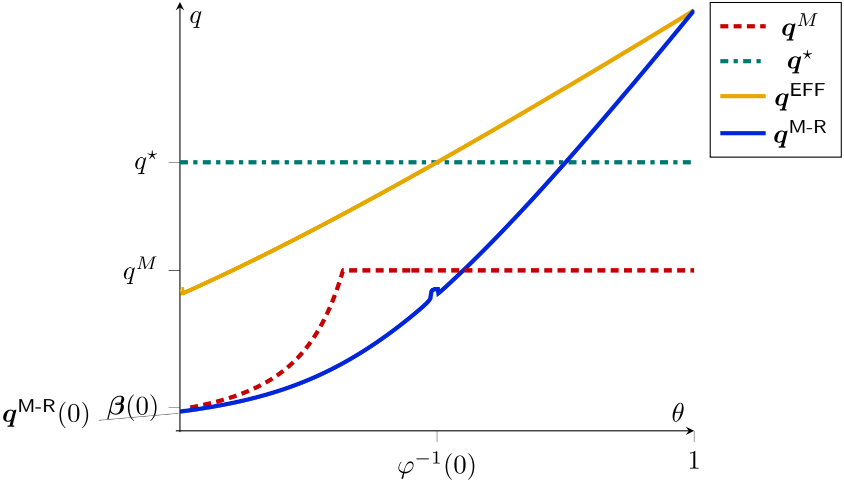
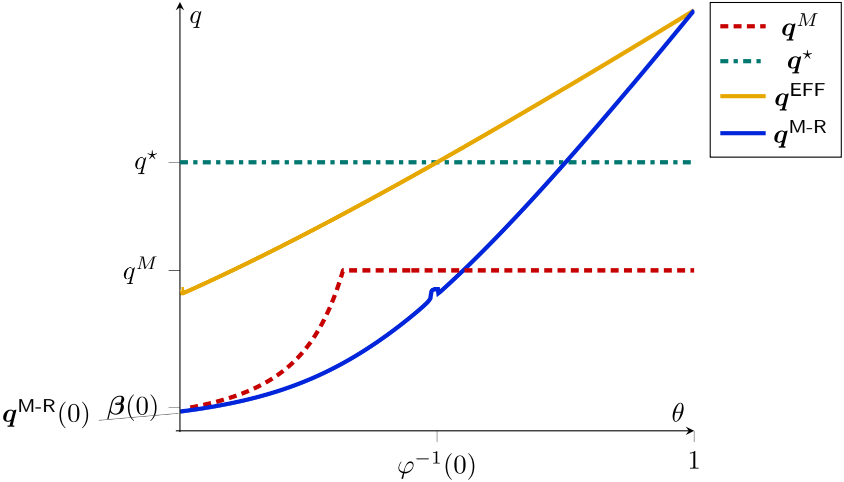
# Single buyer



# Single buyer




$\Pi^{\text{M-R}}(\boldsymbol{q}^{\text{M-R}}) - \Pi(\boldsymbol{q}^M) =$ gains from "interim" damaging wrt ex-ante damaging.

# Hybrid costs

With more general costs: $C(\boldsymbol{q}) = \int_{\Theta} \hat{c}(\boldsymbol{q}(\theta), \sup \boldsymbol{q}(\Theta)) \, \mathrm{d}F(\theta)$, the seller pays:

1. Development / production costs: $c(\sup \boldsymbol{q}(\Theta))$;
2. Distribution / replication costs: $k(\boldsymbol{q}(\theta))$;
3. Damaging costs.

Lemma 1 holds, but the characterization of $\boldsymbol{q}^M$ has two complications:

1. Distribution: the solution to $\mathcal{P}(q)$ does not depends on $q$ solely through capping;
2. Production: the marginal return $V'(q)$ depends on: (i) bunching region, and (ii) damaging.

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If $C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) + \kappa \log\left(\frac{\sup \boldsymbol{q}(\Theta)}{\boldsymbol{q}(\theta)}\right)$, then **1.** is turned off.

# Damaging costs

If $C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) + \kappa \log\left(\frac{\sup \boldsymbol{q}(\Theta)}{\boldsymbol{q}(\theta)}\right)$, then:

- Production costs + pure-damaging replication / distribution costs;

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- Production costs + pure-damaging replication / distribution costs;
- The efficient allocation is flat: damaging decreases utility and increases costs;
- The solution to $\mathcal{P}(q)$ is $\theta \mapsto \min\{\boldsymbol{\beta}_\kappa(\theta), q\}$.

**1.** $\kappa > 0$ acts as a preference shift ($\uparrow g$) at the distribution stage:
   - $\uparrow \boldsymbol{\beta}_\kappa$ and $\downarrow b_\kappa(q)$;

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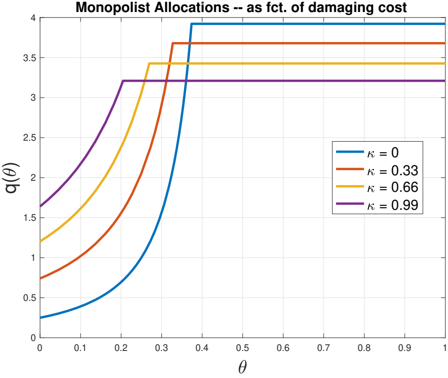
1. $\kappa > 0$ acts as a preference shift ($\uparrow g$) at the distribution stage:
   - $\uparrow \boldsymbol{\beta}_\kappa$ and $\downarrow b_\kappa(q)$;
2. $\kappa > 0$ impacts production directly:
   - $V'(q) = (1 - F(b_\kappa(q))(g'(q) + b_\kappa(q)) - \kappa \frac{b_\kappa(q)}{q}$.

# Damaging costs

$\kappa > 0$ implies

1. Less damaging;
2. Lower production.



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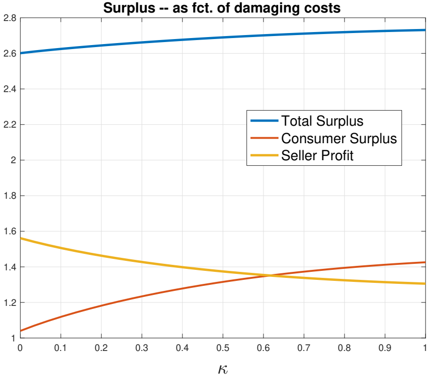

# Conclusion

1. With digital goods, two interdependent inefficiencies arise: **productive and damaging**.
2. The *efficiency at the top* insight is revisited: '**distributional efficiency** for **high types**.'

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3. The profit expression admits two **interpretations**:

$$\underbrace{\int_{\Theta} t(\theta)\,\mathrm{d}F(\theta) - c(\sup \boldsymbol{q}(\Theta))}_{\text{Magnitude gap}} \quad \text{v.} \quad \underbrace{\int_{\Theta} t(\theta) - c(\sup \boldsymbol{q}(\Theta))\,\mathrm{d}F(\theta)}_{\text{Costly replication}}.$$

- Free-replication–and-damaging technology for mass of buyers; (plus a timing mismatch for a single buyer;)
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4. The results extend to: $C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) + \int_\Theta k(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta)$, general $u$ (damaging costs), and nonregular $F$.

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Thanks!

# Extra slides

# Literature

**Multi-product monopoly**
Mussa and Rosen (1978); Maskin and Riley (1984); Wilson (1993) ...
Costs are separable.

**Damaged goods**
Deneckere and McAfee (1996); McAfee (2007); Grubb (2009); Corrao, Flynn, and Sastry (2023).
Costs are separable, and consumers can damage the good.

**Pricing of information with buyer's private information**
Bergemann, Bonatti, and Smolin (2018); Bergemann and Ottaviani (2021); Yang (2022); Bergemann, Cai, Velegkas, and Zhao (2022); Rodríguez Olivera (2024); Bonatti, Dahleh, Horel, and Nouripour (2024) ...
Information is sold to strategic buyers, without production.

Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) ...
Information is allocated without production.

**Mechanism & information design**
Bergemann, Heumann, and Morris (2025); Mensch and Ravid (2025); Thereze (2025).

# Efficiency with general $u$ and $k$

**Proposition 5**

The allocation $\boldsymbol{q}^\star$ is efficient iff $\boldsymbol{q}^\star(\theta) = \min\{\boldsymbol{\alpha}(\theta), q^\star\}$ a.e., in which $q^\star$ is the unique quality $q$ such that $\int_{[a(q),1]} u_1(q,\theta) - k'(q)\,\mathrm{d}F(\theta) = c'(q)$.

In general, $q \in [0, \overline{q}]$, and:

$$J(q,\theta) := u(q,\theta) - \frac{1}{h(\theta)} u_2(q,\theta) - k(q),$$

$\boldsymbol{\beta}(\theta)$ is the largest element of

$$\operatorname*{Argmax}_q J(q,\theta),$$

$\boldsymbol{\alpha}(\theta)$ is the largest element of

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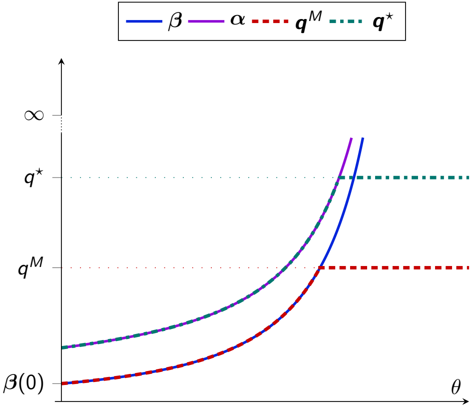
$u$ and $J$ satisfy incr. differences, and are: twice diff., concave in $q$ for all $\theta$, str. quasiconcave in $q$ a.e. on $\Theta$; $k$ is Inada.



# Monopoly with general $u$ and $k$

**Proposition 6**

The allocation $\boldsymbol{q}^M$ is monopolist iff $\boldsymbol{q}^M(\theta) = \min\{\boldsymbol{\beta}(\theta), q^M\}$ a.e., in which $q^M$ is the unique quality $q$ such that: $\int_{[b(q),1]} J_1(q,\theta)\,\mathrm{d}F(\theta) = c'(q)$. Moreover, it holds that $0 < q^M < q^\star$.

In general, $q \in [0, \overline{q}]$, and:

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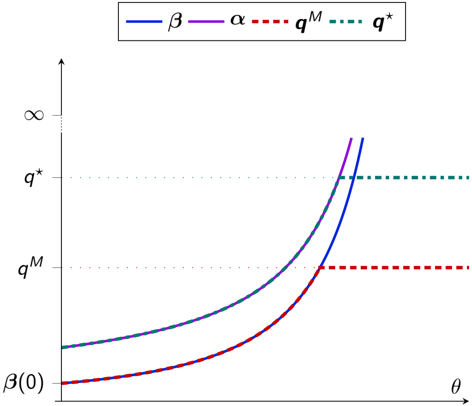

# No-damaging monopoly with general $u$ and $k$

Assumption: $J(0, \theta) = 0$ for all $\theta$ and $J(q, \cdot)$ is increasing for all $q > 0$.

**Proposition 7**

The allocation $\boldsymbol{q}_N^M$ is no screening iff $\boldsymbol{q}_N^M(\theta) = [\theta \geq b_N(q_N^M)] q_N^M$ a.e., in which $q_N^M$ is the unique quality $q$ such that:
$\int_{[b_N(q),1]} J_1(q, \theta) \, \mathrm{d}F(\theta) = c'(q)$. Moreover, it holds that:

**1.** $0 < q_N^M \leq q^M$;

**2.** $q_N^M < q^M$ if $b(q^M) > b_N(q^M)$.

We use Iverson brackets: $[P] = 1$ if $P$ is true, and $[P] = 0$ otherwise.

# Productive inefficiency addendum 1/3

Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\substack{\text{Marginal revenues} \\ \text{given } \theta \mapsto \min\{\boldsymbol{\beta}(\theta), q\}}} \quad < \quad \underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\substack{\text{Marginal total utility} \\ \text{given } \theta \mapsto q}} \; .$$

**1.** The $q$ constrained allocation $\theta \mapsto \min\{\boldsymbol{\beta}(\theta), q\}$ induces total utility

$$U(q) := \mathbb{E}\{u(\min\{\boldsymbol{\beta}(\theta), q\}, \theta)\};$$

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$$U(q) \coloneqq \mathbb{E}\{u(\min\{\boldsymbol{\beta}(\theta), q\}, \theta)\};$$

**2.** $U'(q) <$ marginal total utility given $\theta \mapsto q$, because

$$U'(q) = \int_{[b(q),1]} g'(q) + \theta \, \mathrm{d}F(\theta) \leq g'(q) + \mathbb{E}\{\theta\};$$

**3.** It holds that

$$V'(q) < V'(q) + \mathrm{Rents}'(q) = U'(q).$$

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# Productive inefficiency addendum 2/3

Productive inefficiency arises because:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$

The monopoly that produces quality $q$ implies total surplus

$$V(q) + U(q) - c(q),$$

with $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \overline{q}\} \, \mathrm{d}\theta' \, \mathrm{d}F(\theta)$ (Envelope Theorem).

The marginal surplus is $V'(q) + U'(q)$ and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.

# Productive inefficiency addendum 3/3

WTS: $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$.

1. $U'(q) = \int_{[b(q),1]} \theta - b(q)\,\mathrm{d}F(\theta)$,
because the marginal $u(\boldsymbol{q}(\theta), \theta)$ increases at rate $g'(q) + \theta$ and the marginal transfer at rate $g'(q) + b(q)$, for $\theta > b(q)$ and $\boldsymbol{q}(\cdot) = \min\{\boldsymbol{\beta}(\cdot), q\}$;
2. Using $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$, we have

$$V'(q) + U'(q) = (1 - F(b(q)))g'(q) + \int_{[b(q),1]} \theta\,\mathrm{d}F(\theta).$$

Note that $U'(q) > 0$ for all $q > 0$, because $b(q) \leq \varphi^{-1}(0) < 1$ for all $q \geq 0$.)

# Competition

The game among $N$ firms has two stages:

1. Every firm $i$ simultaneously chooses a quality $q_i$.
2. Every firm $i$, observing all stage-1 qualities, simultaneously chooses a pricing function $p_i \colon \mathbb{R}_+ \to \mathbb{R}$, with $p_i(q) = \infty$ if $q > q_i$.

Then: each type buys a good from a firm $i$, or does not buy any good for a payoff of 0.

Firms play a subgame-perfect Nash equilibrium.

**Definition 1**
An *$n$ equilibrium* is an equilibrium in which exactly $n$ firms are active; an $n$ equilibrium is *symmetric* if active firms play the same strategy.

# The game

Type $\theta$ buys quality $D_p(\theta)$ from firm $\iota_p(\theta)$, given the pricing functions in $(p_1, \ldots, p_N) = p$.

The revenues of $i$ given the pricing functions in $(p_1, \ldots, p_N) = p$ are

$$R_i(p_1, \ldots, p_N) \coloneqq \int_{\{\theta | \iota_p(\theta) = i\}} p_i(D_p(\theta)) \, \mathrm{d}F(\theta).$$

The set of strategies for firm $i$ is $S_i \coloneqq Q \times \boldsymbol{P}_i$, letting $\boldsymbol{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$ be the set of "conditional" pricing functions of firm $i$.

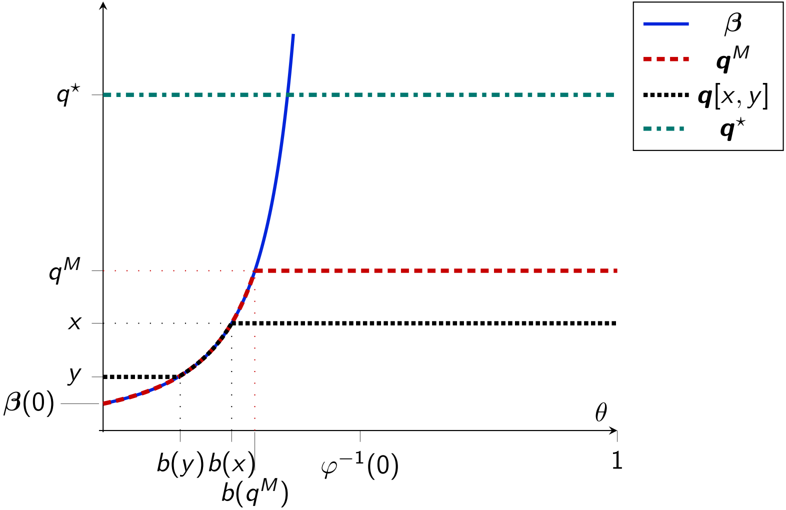
The *payoff* of firm $i$ from the profile $s \coloneqq (\ldots, (\overline{q}_i^s, P_i^s), \ldots) \in \times_{i=1}^N S_i$ is

$$\Pi_i(s) \coloneqq R_i(P_1^s[\overline{q}^s], \ldots, P_N^s[\overline{q}^s]) - c(\overline{q}_i^s).$$

# Competitive allocations

Let's order qualities $(q_1, \ldots, q_N)$ so that: $x > y > \cdots$

Every quality below $y$ comes at zero price.



# Competitive equilibria

**Lemma 3**
In any pure-strategy equilibrium: one firm produces $q^M$ and other firms are idle.

$\implies$ Every symmetric $n$ equilibrium is mixed if $n \geq 2$ (*competitive*.)

**Proposition 8**

1. For all $n \leq N$, there exists a symmetric $n$ equilibrium.
2. Every symmetric and competitive $n$ equilibrium induces the random allocation $\boldsymbol{q}[\hat{x}, \hat{y}]$, letting $\hat{x}$ and $\hat{y}$ be, resp., the first and second order statistics of the $n$ i.i.d. draws $[0, q^M]$ with CDF

$$H_n(q) = \sqrt[n-1]{\frac{c'(q)}{V'(q)}}.$$

# Properties of competitive equilibria

**Corollary 1**

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. The lowest quality is positive and free;
2. The highest quality is strictly lower than $q^M$.

In the paper:

1. Equilibrium welfare with $n \geq 2$ active firms decreases in $n$.
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

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