Screening in digital monopolies

Pietro Dall'Ara
CSEF
University of Naples Federico II

Elia Sartori
CSEF
University of Naples Federico II

Economic Theory Seminar
University of Manchester

Free damaging and replication

Several goods exhibit:

1. Free replication;
2. Free damaging.

taste heterogeneity

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This paper studies monopoly provision of goods whose production features free replication and free damaging.

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Examples of **digital goods**:

1. Software goods;
2. Digital content;
3. Data.

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Up to 2,048 variables	✔	✔	✔	✔	✔
Up to 32,767 variables	-	✔	✔	✔	✔
Up to 120,000 variables	-	-	✔	✔	✔
Maximum number of observations					
Up to 2.14 billion	✔	✔	✔	✔	✔
Up to 20 billion	-	-	✔	✔	✔

Equifax offers three standard Business Credit Reports: BCR Complete View, BCR Risk View and BCR Profile View. The difference between reports is the number of sections that are available in the report:

REPORT SECTION TITLE	COMPLETE VIEW	RISK VIEW	PROFILE VIEW
Business Information & Firmographics	✓	✓	✓
Report Highlights & Alerts	✓	✓	✓
Index Values	✓	✓	✓
Quarterly Index Value & Payment History	✓	✓	✓
Credit Risk Scores	✓	✓	
Trade Details (Industry & Financial)	✓		
Negative Occurrences (Bankruptcy, Collections, Returned Cheques)	✓		
Legal Data (Suits, Judgments, Liens)	✓		
Inquiries	✓		
Other	✓		

Preview

This paper studies screening when production features free replication and free damaging.

The **monopoly** provision features two interdependent inefficiencies:

1. **Productive inefficiency**: the best quality is lower than its efficient level.
2. **Distributional inefficiency**: some buyers purchase a damaged good;

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2. **Distributional inefficiency**: some buyers purchase a damaged good;

Extensions:

- A no-damaging constraint makes productive inefficiency stronger;
- **Competition** is harmful for productive efficiency and beneficial for damaging inefficiency.

Plan

1. Model;
2. Efficiency benchmark;
3. Monopoly allocation and inefficiencies;
4. No-damaging constraint;
5. Single buyer;
6. Competition.

Model

Model

A unit mass of buyers, each drawing a **type** $\theta \in [0,1] = \Theta$, interacts with a seller.

Type θ is privately informed about $\theta \sim F$, for twice diff. F;

$\hookrightarrow$ F is regular in these slides, $\mathbb{E}\{\cdot\}$ refers to the r.v. θ.

Type θ has payoff from **quality** $q \in \mathbb{R}_+$ and transfer $t \in \mathbb{R}$:

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q,\theta)} \; -t,$$

for a strictly concave, increasing, and twice diff. g with $g(0) = 0$.

An **allocation** is a measurable $\boldsymbol{q} \colon \Theta \to \mathbb{R}_+$.

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An **allocation** is a measurable $\boldsymbol{q} \colon \Theta \to \mathbb{R}_+$.

The cost of allocation $\boldsymbol{q}$ is

$$C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)),$$

for a **production cost** c, increasing, strictly convex, twice diff., with $c'(0) = 0$ and $\lim_{q \to \infty} c'(q) = \infty$.

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An **allocation** is a measurable $\boldsymbol{q} \colon \Theta \to \mathbb{R}_+$.

With *separable* costs, the cost of $\boldsymbol{q}$ is

$$C(\boldsymbol{q}) = \mathbb{E}\{k(\boldsymbol{q}(\theta))\},$$

for some k (Mussa-Rosen '78.)

Model

1. Main results extend to increasing-differences u,
   1.1 Basic task (text editing): $g(q)$;
   1.2 Advanced task (computing): θq.
2. The cost of allocation $\boldsymbol{q}$ is

$$C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) \neq \int_{\Theta} c(\text{what } \theta \text{ gets}) \, \mathrm{d}F(\theta),$$

   for a **production cost** c, increasing, strictly convex, twice diff., with $c'(0) = 0$ and $\lim_{q \to \infty} c'(q) = \infty$.

Model

1. Main results extend to increasing-differences u,
   1.1 Basic task (text editing): $g(q)$;
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2. In general,

$$C(\boldsymbol{q}) = \int_{\Theta} k(\boldsymbol{q}(\theta), \boldsymbol{q}) \, \mathrm{d}F(\theta),$$

   for some k.

Efficiency

Efficiency

The *surplus* induced by allocation $\boldsymbol{q}$ is

$$\mathbb{E}\{u(\boldsymbol{q}(\theta), \theta)\} - c(\sup \boldsymbol{q}(\Theta)).$$

The *efficient* allocation $\boldsymbol{q}^\star$ maximizes surplus.

Proposition 1

The efficient allocation is given by $\boldsymbol{q}^\star(\theta) = q^\star$ for all θ, in which $q^\star$ is the unique quality q such that

$$g'(q) + \mathbb{E}\{\theta\} = c'(q).$$

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1. Damaging is inefficient: $\mathbb{E}\{u(\sup \boldsymbol{q}(\Theta), \theta)\} \geq \mathbb{E}\{u(\boldsymbol{q}(\theta), \theta)\}$;
2. Average marginal utility equals marginal production costs.

Efficiency

The *surplus* induced by allocation $\theta \mapsto q$ is

$$g(q) + \mathbb{E}\{\theta\}q - c(q).$$

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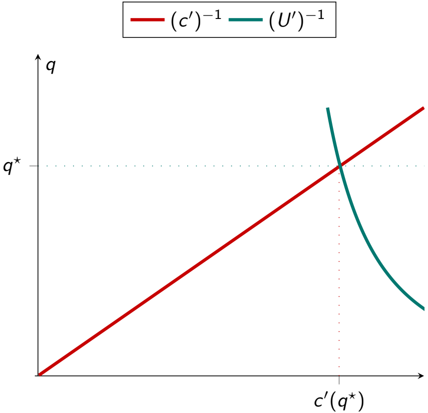

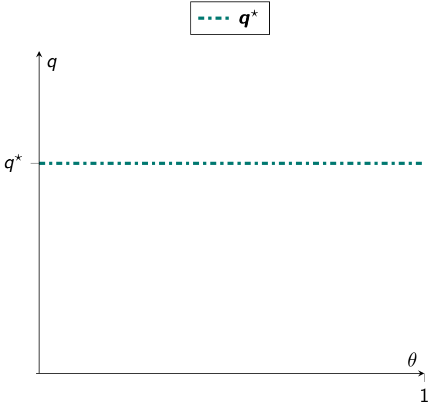

The utility induced by allocation $\theta \mapsto q$ is $U(q) = g(q) + \mathbb{E}\{\theta\}q$.

Monopoly

Monopoly

The monopolist problem is:

$$(\mathcal{P}^M) \qquad \max_{\boldsymbol{q},\, t(\cdot)} \int_\Theta t(\theta)\,\mathrm{d}F(\theta) - c(\sup \boldsymbol{q}(\Theta)) \text{ subject to:}$$

$$u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq u(\boldsymbol{q}(\hat{\theta}),\theta) - t(\hat{\theta}), \text{ for all } (\theta,\hat{\theta}),$$
$$u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq 0, \text{ for all } \theta.$$

- The *monopolist* allocation $\boldsymbol{q}^M$ solves $\mathcal{P}^M$ for some $t(\cdot)$.

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- The *monopolist* allocation $\boldsymbol{q}^M$ solves $\mathcal{P}^M$ for some $t(\cdot)$.
- Without separable costs: the monopolist problem cannot be solved via "pointwise maximization".

Monopoly

The *q-constrained problem* is:

$$(\mathcal{P}(q)) \qquad V(q) \coloneqq \max_{\boldsymbol{q},\, t(\cdot)} \int_\Theta t(\theta)\,\mathrm{d}F(\theta) - \cancel{c(\sup \boldsymbol{q}(\Theta))} \text{ subject to:}$$

$$\boldsymbol{q}(\theta) \le q, \ \text{for all } \theta,$$

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Lemma 1 (Invest then distribute)

The allocation $\boldsymbol{q}$ solves $\mathcal{P}^M$ if and only if:

1. $\boldsymbol{q}$ solves $\mathcal{P}(q^M)$,
2. q^M solves $\max_q V(q) - c(q)$.

Monopoly

The *q-constrained problem* is:

$$(\mathcal{P}(q)) \qquad V(q) := \max_{\boldsymbol{q}} \int_{[0,1]} \underbrace{g(\boldsymbol{q}(\theta)) + \varphi(\theta)\boldsymbol{q}(\theta)}_{\text{Virtual surplus}} \mathrm{d}F(\theta) \text{ subject to:}$$

$$\boldsymbol{q}(\theta) \leq q, \text{ for all } \theta,$$

$$\boldsymbol{q} \text{ is nondecreasing;}$$

in which $\varphi(\theta) := \theta - \frac{1-F(\theta)}{F'(\theta)}$.

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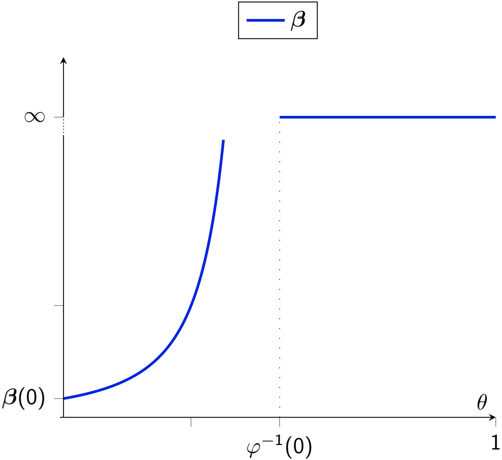
Virtual surplus maximization

The *virtual-surplus maximizer*

$$\boldsymbol{\beta}(\theta) \in \operatorname*{Argmax}_{q} g(q) + \varphi(\theta) q$$

satisfies:

1. $\boldsymbol{\beta}(\theta) = \infty$ if $\theta \geq \varphi^{-1}(0)$;
2. $\boldsymbol{\beta}$ is increasing;
3. $\boldsymbol{\beta}(0) > 0$ ("Inada" g).



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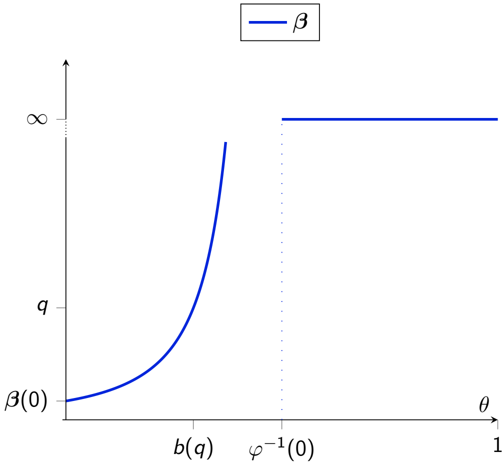
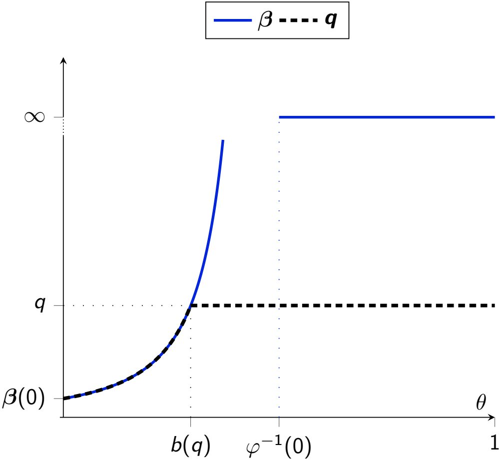
b is the inverse of $\boldsymbol{\beta}$.



Virtual surplus maximization

Lemma 2
Allocation $\boldsymbol{q}$ solves $\mathcal{P}(q)$ iff:

$$\boldsymbol{q}(\theta) = \min\{\boldsymbol{\beta}(\theta), q\}, \text{ for all } \theta.$$



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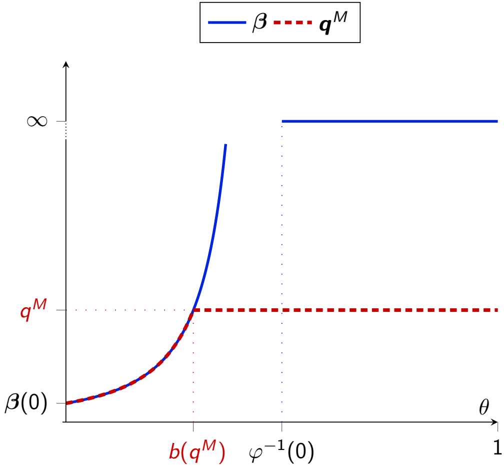
Distributive properties of $\boldsymbol{q}^M$:

1. Bunching at the top;
2. Distributional inefficiency at the bottom or full bunching;
3. No exclusion (if $q^M > 0$.)



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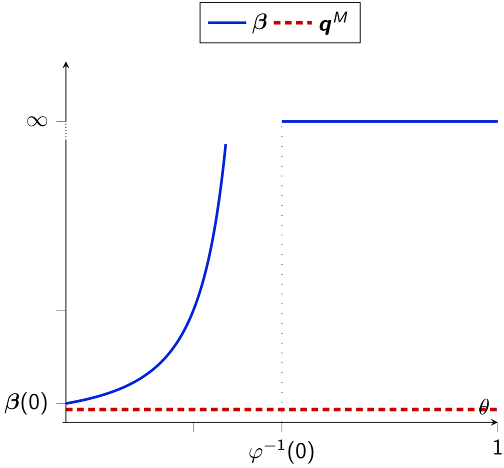


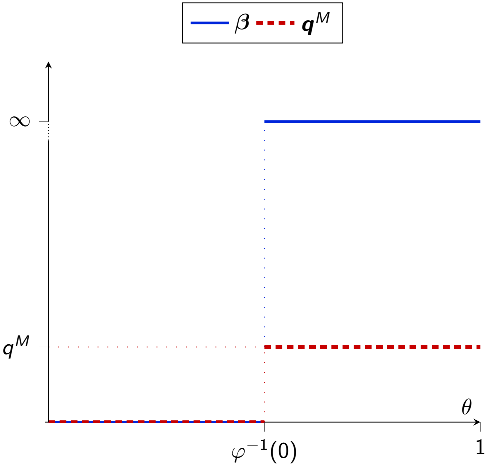
Linear preferences

Distributive properties if $g(q) = 0$:

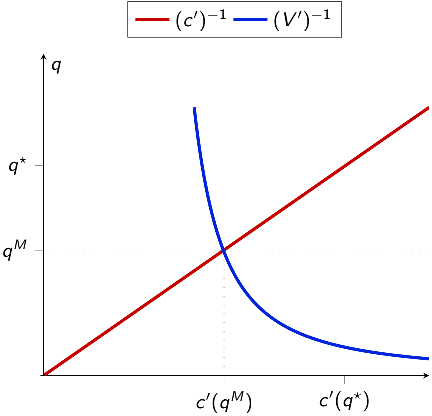
1. Bunching at the top; $\beta(\theta) = \infty$ for $\theta \geq \varphi^{-1}(0)$
2. Exclusion at the bottom; $\beta(\theta) = 0$ for $\theta < \varphi^{-1}(0)$

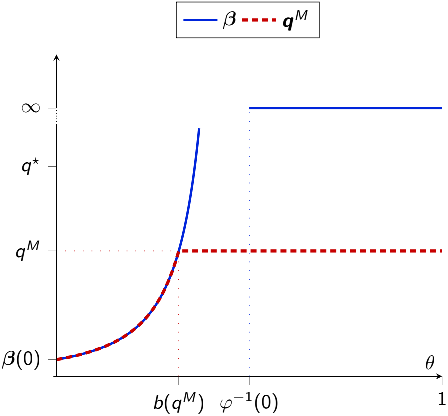
$\Longrightarrow$ **single version**.

Richness in digital markets is due solely to preferences.



The monopolist allocation

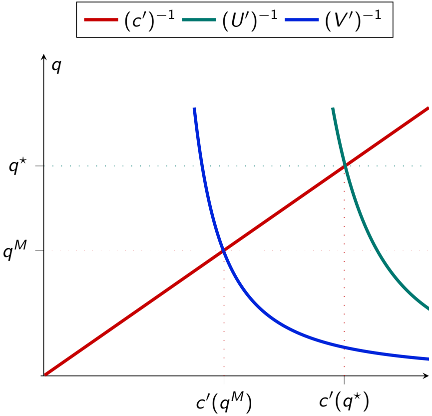


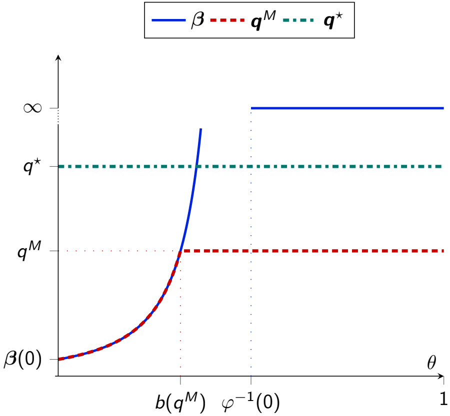


Acquisition

Distribution:
$\boldsymbol{q}^M(\theta) = \min\{\boldsymbol{\beta}(\theta), q^M\}$.

The monopolist allocation





Acquisition

Distribution:
$\boldsymbol{q}^M(\theta) = \min\{\beta(\theta), q^M\}$.

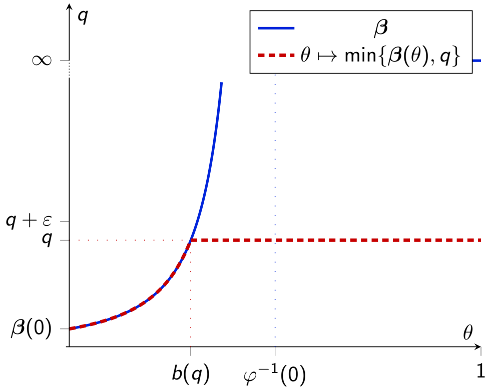
Marginal revenues

The return from increasing the cap of the q-constrained allocation is:

$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{bunched types}} \underbrace{(g'(q) + b(q))}_{u_q \text{ of } b(q)}.$$

The change from q to $q + \varepsilon$ leads to:

1. same revenues from $q' < q$: q' sold to the same θ, and θ gets the same **rent**;
2. higher quality for bunched types;
3. higher price by $u_q(q, b(q))$.



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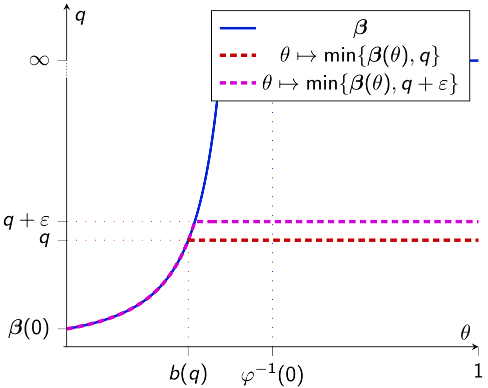

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1. By Markov's inequality:

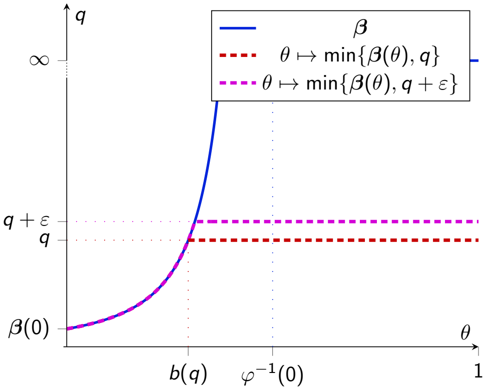
   $$(1 - F(b(q)))b(q) \leq \mathbb{E}\{\theta\};$$

2. By the distributive properties

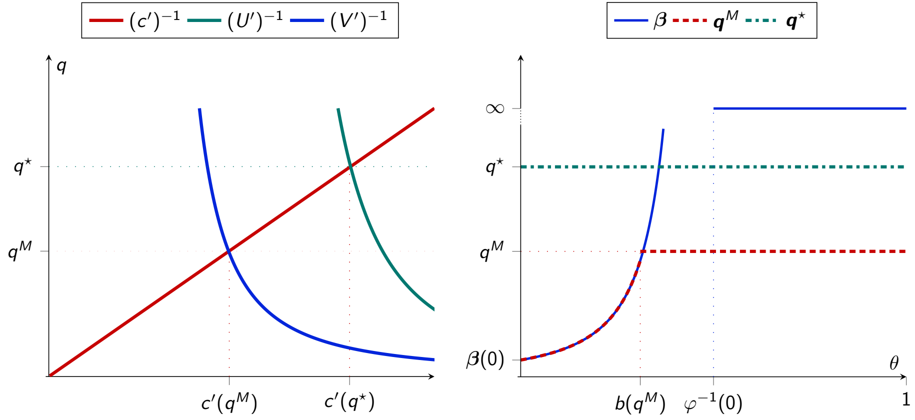
   $$b(q) < 1,$$

So:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$



Productive inefficiency



Productive inefficiency

Proposition 2

The monopolist allocation is given by $\boldsymbol{q}^M(\theta) = \min\{\boldsymbol{\beta}(\theta), q\}$ for all θ, in which q^M is the unique q solving

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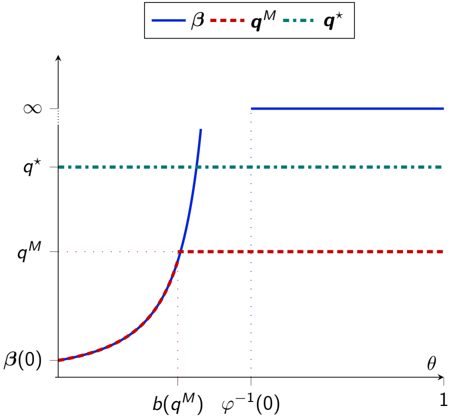
Moreover, it holds that: $q^M < q^\star$.



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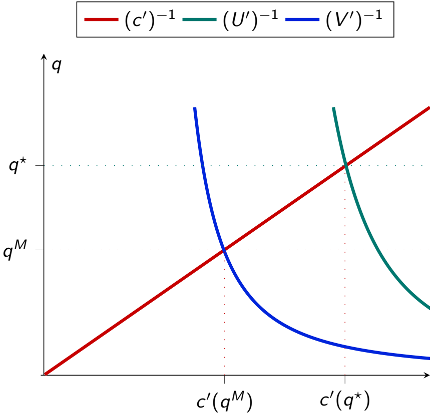
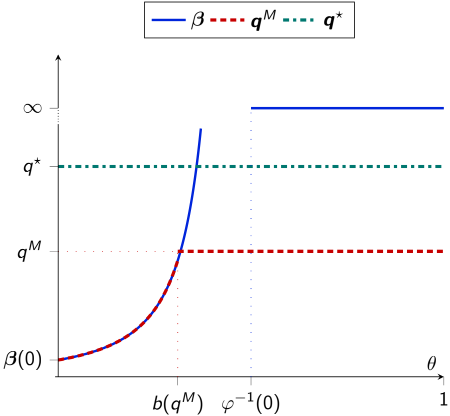
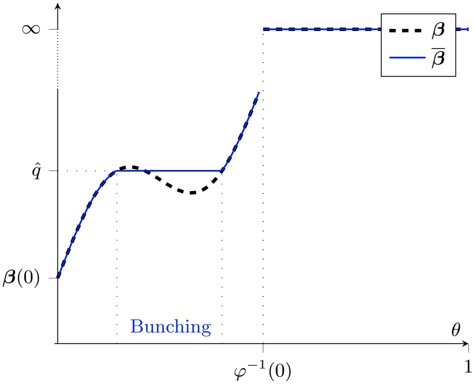




Acquisition: q^M is the quality q solving:
$(1 - F(b(q)))u_q(q, b(q)) = c'(q)$.

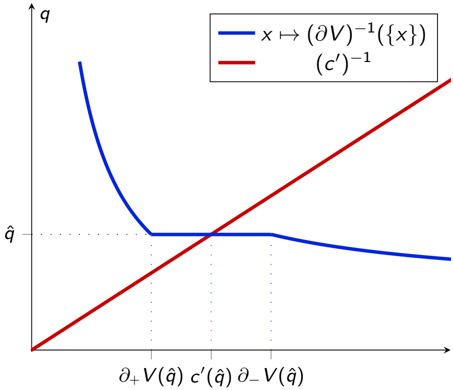
Distribution:
$\boldsymbol{q}^M(\theta) = \min\{\boldsymbol{\beta}(\theta), q^M\}$.

Non-regular distribution



- $\boldsymbol{\beta}$ is ironed to obtain $\overline{\boldsymbol{\beta}}$;
- By Lemma 1,
  $\theta \mapsto \min\{\overline{\boldsymbol{\beta}}(\theta), q\}$ solves $\mathcal{P}(q)$;

Non-regular distribution



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- If types in (θ', θ'') are bunched "at" $\hat{q} \in (0, q)$,

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the extra revenues from $\hat{q} + \varepsilon$ come from types higher than θ''.

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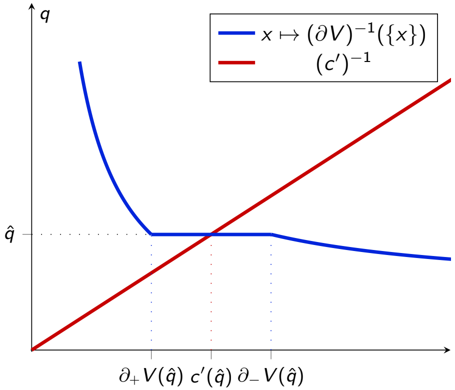

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Proposition 3

Without regularity, the monopolist allocation is $\boldsymbol{q}^M(\theta) = \min\{\overline{\beta}(\theta), q^M\}$, in which q^M is the unique q with $c'(q) \in \partial V(q)$. Moreover, it holds that $q^M < q^\star$.

No damaging constraint

The highest quality increment

The problem $\mathcal{P}(q)$ can be stated as the choice of a **tariff** $T : \mathbb{R}_+ \to \mathbb{R}$ (Taxation principle).
The optimal tariff T satisfies

$$T'(q) \text{ solves } \max_{p \in \mathbb{R}_+} p \Pr(\{\theta : u_q(q, \theta) \geq p\})$$

(Wilson, 1993).

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(Wilson, 1993).

- The **price increment at** q, $T'(q)$, is given by $g'(q) + b(q)$ (Envelope theorem).

$\implies$ The value $g'(q) + b(q)$ is such that

$$g'(q) + b(q) \text{ solves } \max_{p \in \mathbb{R}_+} p \Pr(\{\theta : g'(q) + \theta \geq p\}).$$

Fact 1
The type $b(q)$ is the value of θ that solves $\max_{\theta \in \Theta}(g'(q) + \theta)(1 - F(\theta))$.

No damaging

Without damaging, the q-constrained problem is:

$$V_N(q) := \max_{\boldsymbol{q},\, t(\cdot)} \int_{\Theta} t(\theta)\, \mathrm{d}F(\theta) \text{ subject to:}$$

$$\text{IC, IR, } \boldsymbol{q}(\theta) \in \{0, q\}, \text{ for all } \theta.$$

The constraint is irrelevant under:
1. Full bunching by $\boldsymbol{q}^M$;
2. Linear preferences.

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$$\text{IC, IR, } \boldsymbol{q}(\theta) \in \{0, q\}, \text{ for all } \theta.$$

The constraint is irrelevant under:
1. Full bunching by $\boldsymbol{q}^M$;
2. Linear preferences.

The monopolist chooses a **marginally excluded** type $n(q)$, so

$$V_N(q) = (1 - F(n(q)))(g(q) + n(q)q), \quad \text{for } g(q) + \varphi(n(q))q = 0.$$

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- Intuitively: damaging ban $\implies$ $n(q) \leq b(q)$, strictly if $b(q) > 0$,
- so productive inefficiency is worse:

$$V_N'(q) - V'(q) = \int_{[n(q), b(q)]} (1 - F(\theta))(g'(q) + \theta)\, \mathrm{d}\theta \geq 0,$$

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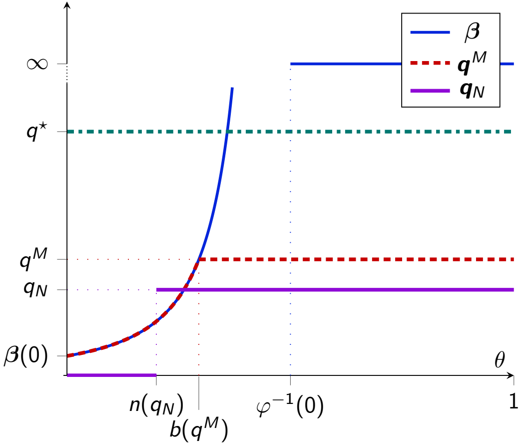
No damaging

The no-damaging allocation $\boldsymbol{q}_N$ features:

- Less production;
- No damaging;
- (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.



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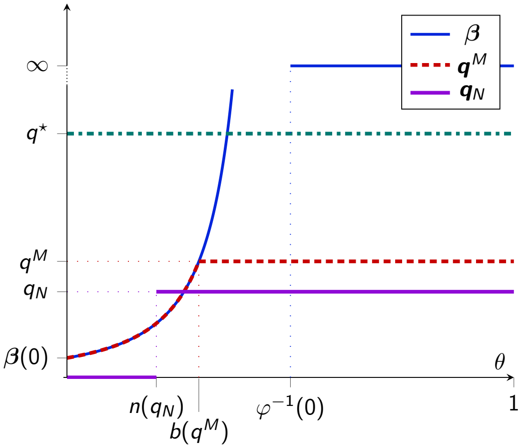

Proposition 4

Without damaging, the monopolist allocation is $\boldsymbol{q}_N(\theta) = \mathbf{1}_{[b_N(q_N),1]}(\theta)q_N$, in which q_N is the unique q solving $V_N'(q) = c'(q)$. Moreover, we have $q_N \leq q^M$, strictly if $b(q^M) > 0$.

Separable costs

Cost interpretation

For digital goods: $\Pi(\boldsymbol{q}) := \underbrace{\int_\Theta t(\theta)\,\mathrm{d}F(\theta)}_{\text{per-agent revenues}} - \underbrace{c(\sup \boldsymbol{q}(\Theta))}_{\text{per-agent unit cost}}$,

1. Payment $t(\theta)$ and production cost $c(q^M)$ have different size; (Shapiro and Varian, 1998).

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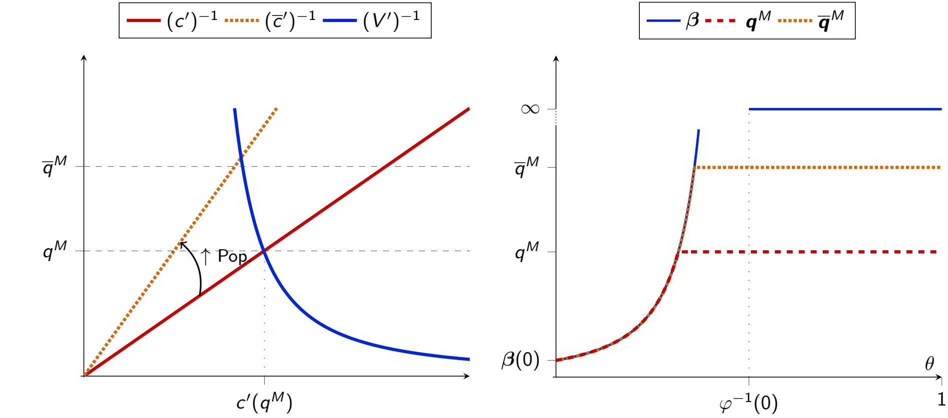
2. Population size impacts q^M;

For separable costs: $\Pi^{\text{M-R}}(\boldsymbol{q}) := \underbrace{\int_\Theta t(\theta)\,\mathrm{d}F(\theta)}_{\text{per-agent revenues}} - \underbrace{\int_\Theta k(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta)}_{\text{per-agent costs}},$

1. Payment $t(\theta)$ and production cost $k(\boldsymbol{q}(\theta))$ are comparable;
2. Population size only scales profits;

In general: $C(\boldsymbol{q}) = \int_\Theta k(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta) + c(\sup \boldsymbol{q}(\Theta))$.

Population size



Separable interpretation

For a separable interpretation (with a continuum of buyers:)

$$\Pi(\boldsymbol{q}) = \int_{\Theta} t(\theta) - \underbrace{c(\sup \boldsymbol{q}(\Theta))}_{\substack{\text{same magnitude} \\ \text{as } t(\theta)}} \,\mathrm{d}F(\theta),$$

under which production exhibits:

1. costly replication;
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In the damaged-goods model of Deneckere and McAfee (1996):

1. Quality space is $\{0, L, H\}$.
2. Costs are separable **production & damaging** costs k, with $k(H) < k(L)$;
3. Sufficient conditions for no-damaging $\boldsymbol{q}_N$ to be Pareto worse than $\boldsymbol{q}^M$.

Single buyer

$$\Pi^{\text{M-R}}(\boldsymbol{q}) = \underbrace{\int_{\Theta} t(\theta)\,\mathrm{d}F(\theta)}_{\text{expected revenues}} - \underbrace{\int_{\Theta} c(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta)}_{\text{expected costs}},$$

1. Payment $t(\theta)$ and production cost $c(\boldsymbol{q}(\theta))$ are comparable;
2. Production occurs **after** eliciting the buyer's type;
3. Free damaging and replication are irrelevant.

$$\Pi(\boldsymbol{q}) = \underbrace{\int_{\Theta} t(\theta)\,\mathrm{d}F(\theta)}_{\text{expected revenues}} - \underbrace{c(\sup \boldsymbol{q}(\Theta))}_{\text{costs}}.$$

1. Payment $t(\theta)$ and production cost $c(\sup \boldsymbol{q}(\Theta))$ are comparable;
2. Production occurs **before** eliciting the buyer's type;
3. Free damaging matters, replication is irrelevant.

Single buyer

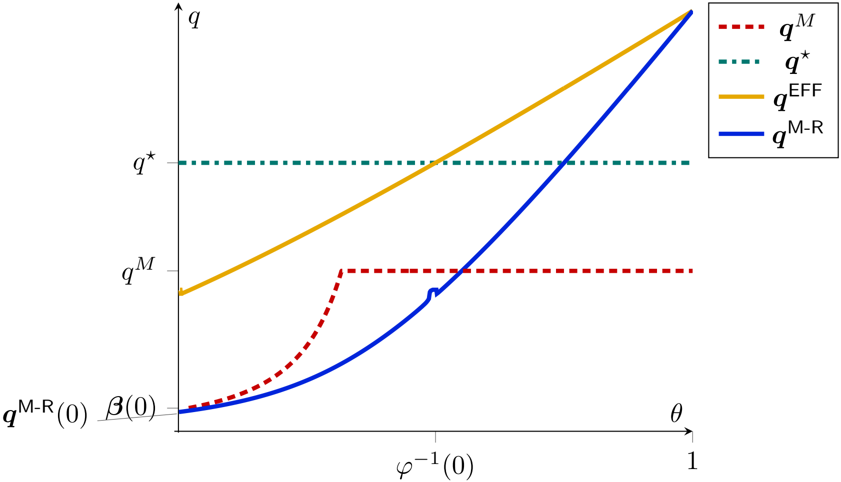
MR: $\max_{\boldsymbol{q},\, t(\cdot)} \int_\Theta t(\theta) - c(\boldsymbol{q}(\theta))\,\mathrm{d}F(\theta)$ **DS:** $\max_{\boldsymbol{q},\, t(\cdot)} \int_\Theta t(\theta) - c(\sup \boldsymbol{q}(\Theta))\,\mathrm{d}F(\theta)$

subject to: IC, IR.

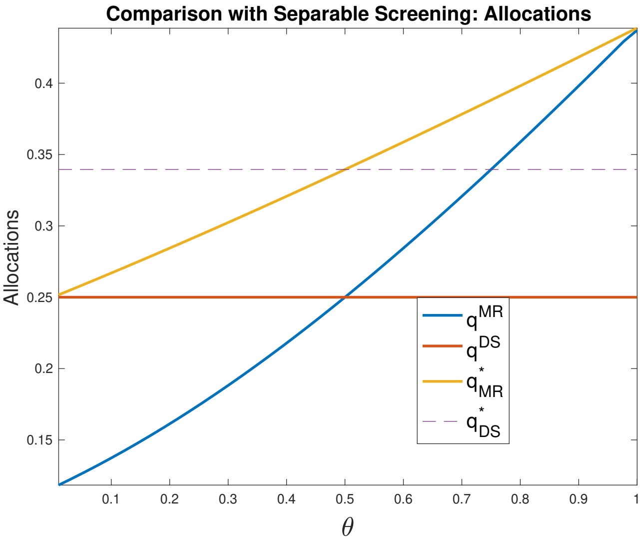
elicit $\to$ produce ($\to$ damage) produce $\to$ elicit $\to$ damage

- "Invest-then-distribute" Lemma becomes a timing assumption:
  - seller produces in advance and, after learning the type, can only damage.
- Free damaging is irrelevant in MR.
- No-damaging seller: produce $\to$ elicit ~~$\to$ damage~~.
- Payment $t(\theta)$ and production cost $c(\sup \boldsymbol{q}(\Theta))$ are comparable;
- Efficiency benchmark: $u_q(q^{\mathrm{EFF}}(\theta), \theta) = c'(q^{\mathrm{EFF}}(\theta))$, $\uparrow$ in θ.

Single buyer



Single buyer



▸ More

Literature

Monopolistic screening
Mussa and Rosen (1978); Maskin and Riley (1984); Wilson (1993) ...
Costs are separable.

Damaged goods
Oren, Smith, and Wilson (1985); Deneckere and McAfee (1996); Grubb (2009); Corrao, Flynn, and Sastry (2023).
Costs are separable, and consumers can damage the good.

Pricing of information with buyer's private information
Bergemann, Bonatti, and Smolin (2018); Bimpikis, Crapis, and Tahbaz-Salehi (2019); Bergemann and Ottaviani (2021); Yang (2022); Bergemann, Cai, Velegkas, and Zhao (2022); Rodríguez Olivera (2024); Bonatti, Dahleh, Horel, and Nouripour (2024) ...
Information is sold to strategic buyers, without production.

Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) ...
Information is allocated without production.

Mechanism & information design
Mensch and Ravid (2025); Thereze (2025).

Competition

Competition

N firms play the two-stage game of perfect information with timing:

1. Every firm i simultaneously chooses a distribution H_i of her quality, **paying** $c(q_i)$ for the realization q_i;
2. Every firm i simultaneously chooses a pricing function $p_i : \mathbb{R}_+ \to \mathbb{R}$, with $p_i(q) = \infty$ if $q > q_i$, **observing** realized qualities $q_1, \dots q_n$.

Then, every type θ buys one good from a firm, or does not buy any good for a payoff of 0. (tie-breaker favors lowest index.)

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Then, every type θ buys one good from a firm, or does not buy any good for a payoff of 0. (tie-breaker favors lowest index.)

The "market" price of q is $\min_i p_i(q)$.

Firms play a subgame-perfect Nash equilibrium.

Definition 1

An *n equilibrium* is an equilibrium in which exactly n firms are active; an n equilibrium is *symmetric* if active firms play the same strategy.

Literature

Forms of "market power" that guarantee equilibrium existence (Rothschild and Stiglitz (1976), . . . , Stole (2007)):

1. Incumbent-entrant timing (Borenstein and Rose, 1994; Johnson and Myatt, 2003; Gerardi and Shapiro, 2009; Crawford et al., 2019; Boik and Takahashi, 2020);
2. Rich buyer heterogeneity (Rochet and Stole, 2002; Lehmann et al., 2014; Garrett et al., 2019);
3. "Relevance" assumption (Chade and Swinkels, 2021): every firm has a cost advantage over a quality range.

Related models:

1. Kreps and Scheinkman (1983);
   Quantity-and-price competition, no private information, and Cournot outcome;
2. Champsaur and Rochet (1989);
   Quality range-and-price competition, costless quality ranges, which are disjoint in equilibrium.

Second stage

Let's order qualities $(q_1, \ldots, q_N)$ so that: $x \geq y \geq \cdots$

Bertrand force: Every quality below y comes at zero price.

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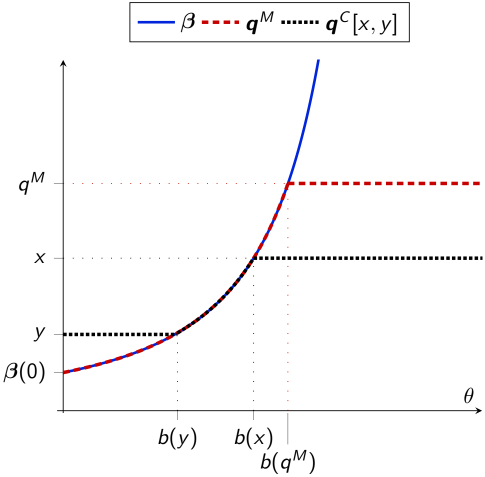
Bertrand force: Every quality below y comes at zero price.

The allocation in the subgame is $\boldsymbol{q}^C[x, y]$, given by:

$$\theta \mapsto \begin{cases} y, & \text{if } \theta \leq b(y), \\ \beta(\theta), & \text{if } \theta \in (b(y), b(x)), \\ x, & \text{if } \theta \geq b(x). \end{cases}$$

Competitive constraint: if q is feasible for some competitor, then q can be purchased for free. The "monopolist" earns $V(x) - V(y) - c(x)$



Competitive equilibria

Lemma 3

In every pure-strategy equilibrium: one firm produces q^M and other firms are idle.

Intuition:

1. q^M is the best response to $(0, \ldots, 0)$, and $q \neq q^M$ is **not** a best response to $(q^M, 0, \ldots, 0)$.
2. In any $(x, y, \ldots)$, $y > 0$ is not a best response, and $(0, \ldots, 0)$ is not an equilibrium.

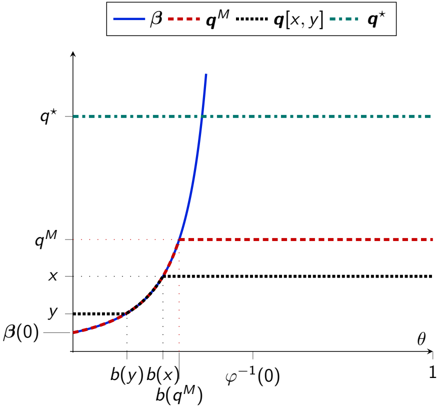
$\Longrightarrow$ Every symmetric n equilibrium is mixed if $n \geq 2$ (*competitive.*)

Competitive equilibria

Proposition 5

1. For all $n \leq N$, there exists a symmetric n equilibrium.
2. Every symmetric and competitive n equilibrium induces the random allocation $\boldsymbol{q}^C[\hat{x}, \hat{y}]$, where $\hat{x}$ and $\hat{y}$ are the first and second order statistics of the n i.i.d. draws from the atomless CDF on $[0, q^M]$ given by

$$H_n(q) = \left(\frac{c'(q)}{V'(q)}\right)^{\frac{1}{n-1}}.$$

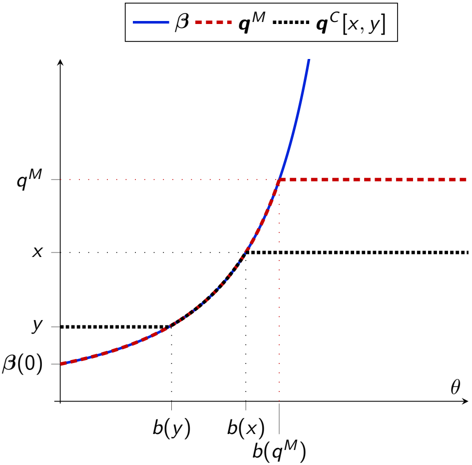


Competitive equilibria

Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. The lowest quality is positive and free;
2. The highest quality is strictly lower than q^M;
3. The marginally bunched type is lower than $b(q^M)$.



Properties of competitive equilibria

- Productive inefficiency is stronger;
- Distributional inefficiency is more prevalent.

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In the paper:

1. Equilibrium welfare with $n \geq 2$ active firms decreases in n.
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

Conclusion

1. With digital goods, two interdependent inefficiencies arise: **productive and damaging**.
2. The *efficiency at the top* insight is revisited: '**distributional efficiency** for **high types**.'

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▸ More

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More

Thanks!

Extra slides

Hybrid costs

With more general costs: $C(\boldsymbol{q}) = \int_\Theta \hat{c}(\boldsymbol{q}(\theta), \sup \boldsymbol{q}(\Theta))\, \mathrm{d}F(\theta)$,
the seller pays:

1. Development / production costs: $\sup \boldsymbol{q}(\Theta)$;
2. Distribution / replication / damaging costs: $\boldsymbol{q}(\theta)$.

Lemma 1 holds, but the characterization of $\boldsymbol{q}^M$ has two complications:

1. Distribution: the solution to $\mathcal{P}(q)$ does not depends on q solely through capping;
2. Production: the marginal return $V'(q)$ depends on: (i) bunching region, and (ii) damaging.

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If $C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) + \kappa \log\left(\frac{\sup \boldsymbol{q}(\Theta)}{\boldsymbol{q}(\theta)}\right)$, then **1.** is turned off.

Damaging costs

If $C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) + \kappa \log\left(\frac{\sup \boldsymbol{q}(\Theta)}{\boldsymbol{q}(\theta)}\right)$, then:

- Production costs + pure-damaging replication / distribution costs;

Damaging costs

If $C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)) + \kappa \log\left(\frac{\sup \boldsymbol{q}(\Theta)}{\boldsymbol{q}(\theta)}\right)$, then:

- Production costs + pure-damaging replication / distribution costs;
- The efficient allocation is flat: damaging decreases utility and increases costs;
- The solution to $\mathcal{P}(q)$ is $\theta \mapsto \min\{\boldsymbol{\beta}_\kappa(\theta), q\}$.

1. $\kappa > 0$ acts as a preference shift ($\uparrow g$) at the distribution stage:
    - $\uparrow \boldsymbol{\beta}_\kappa$ and $\downarrow b_\kappa(q)$;

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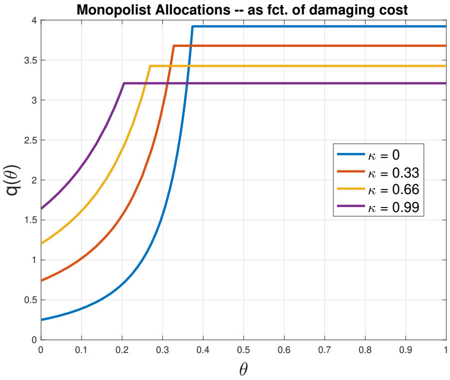
1. $\kappa > 0$ acts as a preference shift ($\uparrow g$) at the distribution stage:
   - $\uparrow \boldsymbol{\beta}_\kappa$ and $\downarrow b_\kappa(q)$;
2. $\kappa > 0$ impacts production directly:
   - $V'(q) = (1 - F(b_\kappa(q))(g'(q) + b_\kappa(q)) - \kappa \frac{b_\kappa(q)}{q}$.

Damaging costs

$\kappa > 0$ implies

1. Less damaging;
2. Lower production.



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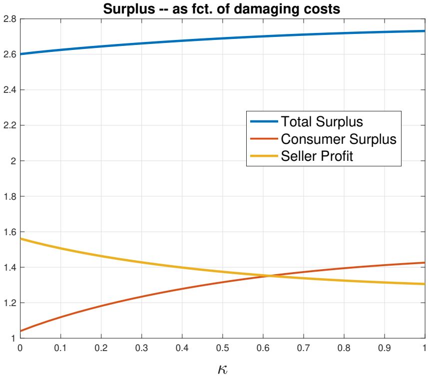


Efficiency with general u and k

Proposition 6

The allocation $\boldsymbol{q}^\star$ is efficient iff $\boldsymbol{q}^\star(\theta) = \min\{\boldsymbol{\gamma}(\theta), q^\star\}$ for all θ, in which: $q^\star$ is the unique q such that $\int_{[a(q),1]} u_1(q,\theta) - k'(q)\,\mathrm{d}F(\theta) = c'(q)$, and $\boldsymbol{\gamma}$ is an allocation such that $\boldsymbol{\gamma}(\theta) = \boldsymbol{\alpha}(\theta)$ almost everywhere.

In general, $q \in [0, \overline{q}]$, and:

$$J(q,\theta) := u(q,\theta) - \frac{1}{h(\theta)} u_2(q,\theta) - k(q),$$

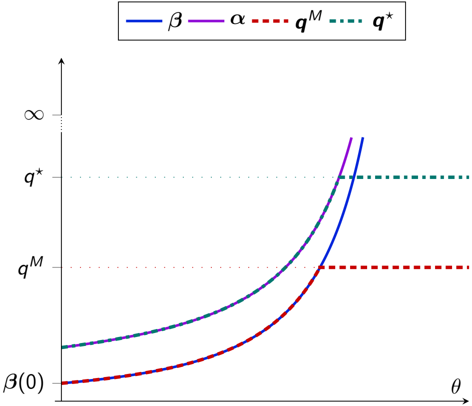
$\boldsymbol{\beta}(\theta)$ is the largest element of

$$\operatorname*{Argmax}_q J(q,\theta),$$

$\boldsymbol{\alpha}(\theta)$ is the largest element of

$$\operatorname*{Argmax}_q u(q,\theta) - k(q),$$

u and J satisfy incr. differences, and are: twice diff., concave in q for all θ, str. quasiconcave in q a.e. on Θ; k is Inada.



Monopoly with general u and k

Proposition 7

The allocation $\boldsymbol{q}^M$ is monopolist iff $\boldsymbol{q}^M(\theta) = \min\{\boldsymbol{\gamma}(\theta), q^M\}$ for all θ, in which: q^M is the unique q such that $\int_{[b(q),1]} J_1(q,\theta)\,\mathrm{d}F(\theta) = c'(q)$, and $\boldsymbol{\gamma}$ is a nondecreasing allocation such that $\boldsymbol{\gamma}(\theta) = \boldsymbol{\beta}(\theta)$ almost everywhere. Moreover, $0 < q^M < q^\star$.

In general, $q \in [0, \overline{q}]$, and:

$$J(q,\theta) := u(q,\theta) - \frac{1}{h(\theta)} u_2(q,\theta) - k(q),$$

$\boldsymbol{\beta}(\theta)$ is the largest element of

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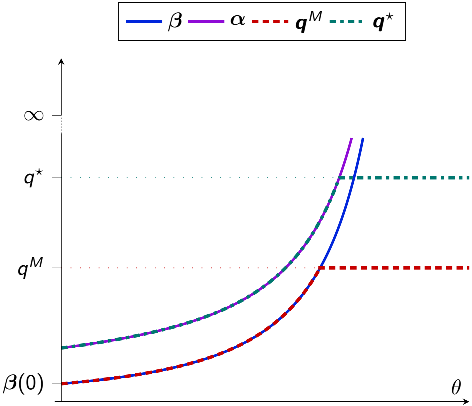

Back

No-damaging monopoly with general u and k

Assumption: $J(0, \theta) = 0$ for all θ and $J(q, \cdot)$ is increasing for all $q > 0$.

Proposition 8

The allocation $\boldsymbol{q}_N$ is no screening iff $\boldsymbol{q}_N(\theta) = [\theta \geq b_N(q_N)]q_N$ for all $\theta \neq b_N(q_N)$ and $\boldsymbol{q}_N(b_N(q_N)) \in \{0, q_N\}$, in which q_N is the unique q such that: $\int_{[b_N(q),1]} J_1(q, \theta)\,\mathrm{d}F(\theta) = c'(q)$. Moreover, it holds that:

1. $0 < q_N \leq q^M$;
2. $q_N < q^M$ if $b(q^M) > b_N(q^M)$.

We use Iverson brackets: $[P] = 1$ if P is true, and $[P] = 0$ otherwise.

Productive inefficiency addendum 1/3

Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\substack{\text{Marginal revenues} \\ \text{given } \theta \mapsto \min\{\beta(\theta), q\}}} \quad < \quad \underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\substack{\text{Marginal total utility} \\ \text{given } \theta \mapsto q}} \; .$$

1. The q constrained allocation $\theta \mapsto \min\{\beta(\theta), q\}$ induces total utility

$$U(q) \coloneqq \mathbb{E}\{u(\min\{\beta(\theta), q\}, \theta)\};$$

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$$U'(q) = \int_{[b(q),1]} g'(q) + \theta \, \mathrm{d}F(\theta) \leq g'(q) + \mathbb{E}\{\theta\};$$

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$$V'(q) < V'(q) + \mathrm{Rents}'(q) = U'(q).$$

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Productive inefficiency addendum 2/3

Productive inefficiency arises because:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$

The monopoly that produces quality q implies total surplus

$$V(q) + U(q) - c(q),$$

with $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \overline{q}\} \, \mathrm{d}\theta' \, \mathrm{d}F(\theta)$ (Envelope Theorem).

The marginal surplus is $V'(q) + U'(q)$ and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.

Productive inefficiency addendum 3/3

WTS: $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$.

1. $U'(q) = \int_{[b(q),1]} \theta - b(q) \, \mathrm{d}F(\theta)$,
   because the marginal $u(\boldsymbol{q}(\theta), \theta)$ increases at rate $g'(q) + \theta$ and the marginal transfer at rate $g'(q) + b(q)$, for $\theta > b(q)$ and $\boldsymbol{q}(\cdot) = \min\{\beta(\cdot), q\}$;
2. Using $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$, we have

$$V'(q) + U'(q) = (1 - F(b(q)))g'(q) + \int_{[b(q),1]} \theta \, \mathrm{d}F(\theta).$$

Note that $U'(q) > 0$ for all $q > 0$, because $b(q) \leq \varphi^{-1}(0) < 1$ for all $q \geq 0$.)

The game

Type θ buys quality $D_p(\theta) \in Q := \mathbb{R}_+$ from firm $\iota_p(\theta)$, given the pricing functions in $(p_1, \dots, p_N) = p$. The set of firms is $\mathcal{N} = (1, \dots, N)$.

The revenues of i given the pricing functions in $(p_1, \dots, p_N) = p$ are

$$R_i(p_1, \dots, p_N) := \int_{\{\theta | \iota_p(\theta) = i\}} p_i(D_p(\theta)) \, \mathrm{d}F(\theta).$$

The set of pure strategies for firm i is $S_i := Q \times \boldsymbol{P}_i$, letting $\boldsymbol{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$ be the set of "conditional" pricing functions of firm i.

The *payoff* of firm i from the profile $s := (\dots, (\overline{q}_i^s, P_i^s), \dots) \in \times_{i=1}^N S_i$ is

$$\Pi_i(s) := R_i(P_1^s[\overline{q}^s], \dots, P_N^s[\overline{q}^s]) - c(\overline{q}_i^s).$$

The equilibrium

A strategy for player i is a pair of a cap distribution and conditional pricing function, (H_i, P_i), in which H_i is a distribution function with support contained in Q and $P_i \in \mathbf{P}_i$ for all i.

A strategy profile $((H_1, P_1), \ldots, (H_N, P_N))$ is an *equilibrium* if it is a subgame-perfect Nash equilibrium of the game:

1. The conditional pricing functions maximize conditional profits, that is, for all cap profiles $\overline{q} = (\overline{q}_1, \ldots, \overline{q}_N)$,

$$R_i(P_i[\overline{q}], P_{-i}[\overline{q}]) \geq R_i(p_i', P_{-i}[\overline{q}]) \quad \text{for all } p_i' \in \mathbb{R}^Q,\ i.$$

2. The cap distributions maximize expected profits, that is, for all $\overline{q}_i$ in the support of H_i we have

$$\int_{Q^{N-1}} R_i(P_i[\overline{q}], P_{-i}[\overline{q}]) \,\mathrm{d}H_{-i}(\overline{q}_{-i}) - c(\overline{q}_i) \geq$$
$$\int_{Q^{N-1}} R_i(P_i[\overline{q}_i', \overline{q}_{-i}], P_{-i}[\overline{q}_i', \overline{q}_{-i}]) \,\mathrm{d}H_{-i}(\overline{q}_{-i}) - c(\overline{q}_i')$$

for all $\overline{q}_i' \in Q$, letting H_{-i} denote the joint distribution of the caps of i's opponents under $((H_1, P_1), \ldots, (H_N, P_N))$, and for all i.

Auxiliary monopoly problem

The allocation $\boldsymbol{q}$ is x-y *second best*, for qualities x, y with $x \leq y$, if there exists a transfer function $t(\cdot)$ such that: $(\boldsymbol{q}, t(\cdot))$ solves

$$\sup_{\boldsymbol{q},\ t(\cdot)} \int t(\theta)\,\mathrm{d}F(\theta) \text{ subject to: } y \leq \boldsymbol{q}(\theta) \leq x \text{ for all } \theta \in \Theta.$$

subject to: IC, IR.

Lemma 4

The allocation $\boldsymbol{q}$ is x-y second best iff $\boldsymbol{q}(\theta) = \max\{\min\{\boldsymbol{\gamma}(\theta), x\}, y\}$ for all θ, in which $\boldsymbol{\gamma}$ is a nondecreasing allocation such that $\boldsymbol{\gamma}(\theta) = \boldsymbol{\beta}(\theta)$ almost everywhere.

The pricing subgame

The pricing game given the quality profile $(q_1, \ldots, q_N)$ is the strategic-form game $\Gamma(q_1, \ldots, q_N) = \big(\mathcal{N}, (\mathbb{R}^{[0,q_i]}, R_i(\cdot))_{i\in\mathcal{N}}\big)$.

The profile of pricing functions $(p_1, \ldots, p_N)$ is a $(q_1, \ldots, q_N)$ *equilibrium* if $(p_1, \ldots, p_N)$ is a Nash equilibrium of $\Gamma(q_1, \ldots, q_N)$.

We study the subgame starting at the given production profile $(q_1, \ldots, q_N) \in Q^N$ with $x \coloneqq \max\{q_1, \ldots, q_N\}x$ and $y \coloneqq \max\{q_1, \ldots, q_N\} \setminus \{x\}$.

Lemma 5

For every $(q_1, \ldots, q_N)$ equilibrium $(p_1, \ldots, p_N)$, the following formulas hold:

$$R_i(p_1, \ldots, p_N) = (V(q_i) - V(y))_+,$$

and

$$D_{(p_1,\ldots,p_N)}(\theta) = \max\{\min\{\gamma(\theta), x\}, y\},$$

for all θ, in which γ is a nondecreasing allocation such that $\gamma(\theta) = \beta(\theta)$ almost everywhere.

The production game

Every Nash equilibrium of $\Gamma(q_1, \ldots, q_N)$ induces the same revenues, i.e.,

$$R_i(p_1^*, \ldots, p_N^*) = R_i(p_1^{**}, \ldots, p_N^{**})$$

for all $(q_1, \ldots, q_N)$ equilibria $(p_1^*, \ldots, p_N^*)$, $(p_1^{**}, \ldots, p_N^{**})$. We call $\overline{R}_i(q_1, \ldots, q_N)$ the unique equilibrium revenues of i in the pricing game $\Gamma(q_1, \ldots, q_N)$, that is:

$$\overline{R}_i(q_1, \ldots, q_N) = (V(q_i) - V(y))_+.$$

The *production game* is the strategic-form game $\Gamma = \big(\mathcal{N}, (Q, \overline{R}_i(\cdot) - c(\cdot))_{i\in\mathcal{N}}\big)$. By mixed strategies, we refer to probability distributions over Q identified by ther distribution functions, and we extend payoffs of Γ to mixed-strategy profiles defining

$$\begin{aligned} \Pi_i \colon (\Delta Q)^N &\to \mathbb{R} \\ (H_1, \ldots, H_N) &\mapsto \int_Q \cdots \int_Q \overline{R}_i(q_1, \ldots, q_N) - c(q_i) \,\mathrm{d}H_1(q_1) \cdots \mathrm{d}H_N(q_N). \end{aligned}$$

Auxiliary result for the production game

Necessary conditions for mixed-strategy equilibria:

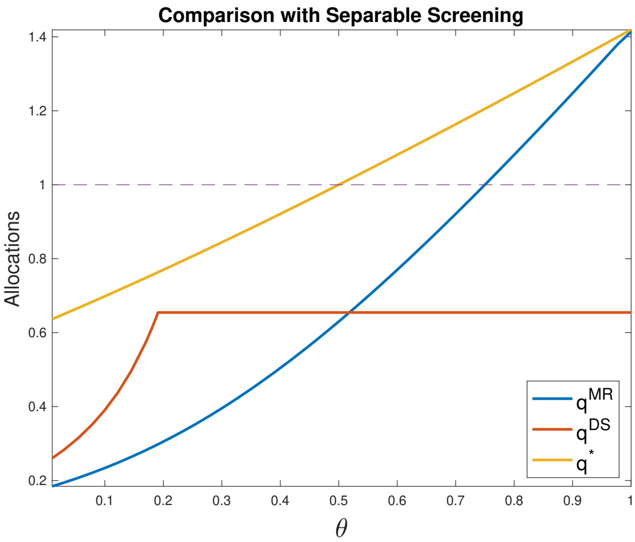
Lemma 6

If $(H_1, \ldots, H_N)$ is a Nash equilibrium in mixed strategies with multiple active players, then for all i with $H_i(q) > 0$ for some $q > 0$ we have:

$$\prod_{j \in \mathcal{N} \setminus \{i\}} H_j(q) = \begin{cases} \frac{c'(q)}{V'(q)}, & \text{if } q \in (0, \overline{q}^M), \\ 1, & \text{if } q \geq \overline{q}^M. \end{cases}$$

Note: There are only two kinds of equilibria in mixed strategies that are symmetric among active players: those with at least 2 active players and the monopolistic equilibria.

Single buyer



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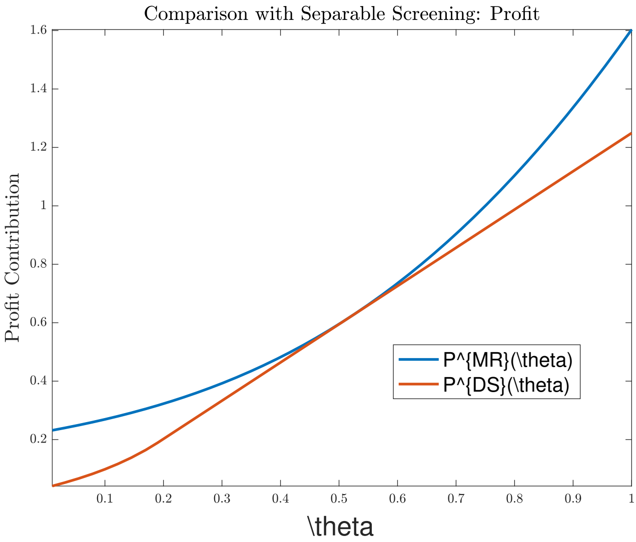
Single buyer



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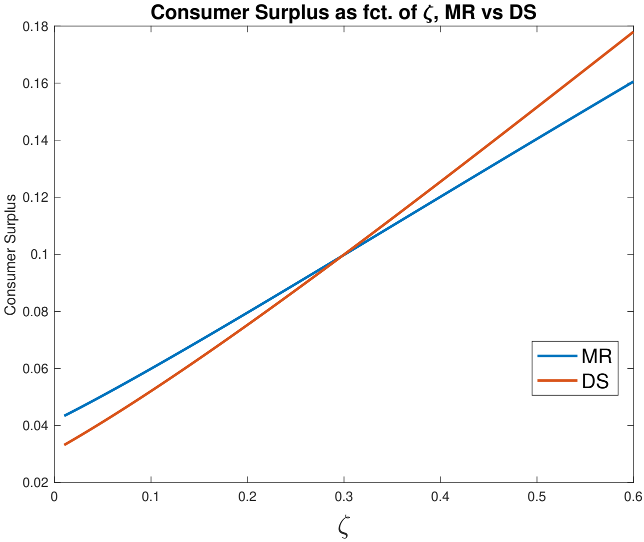
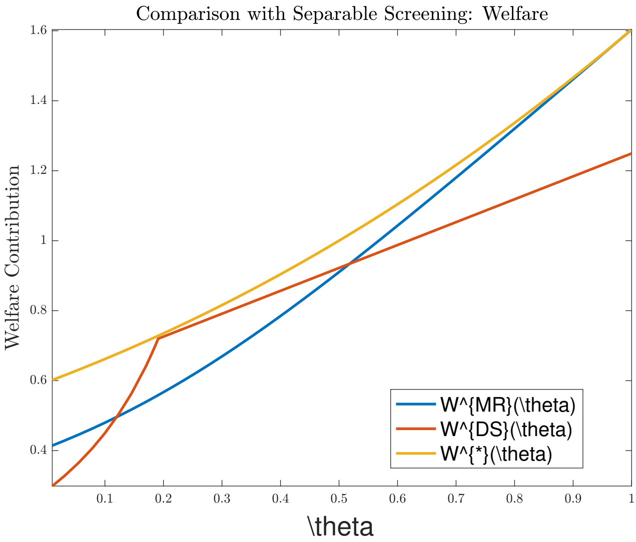
Single buyer



Consumer surplus of $\boldsymbol{q}$ is $\int_{\Theta} \boldsymbol{q}(\theta)(1 - F(\theta))\,\mathrm{d}\theta$, with

$$u(q, \theta) = \zeta g(q) + \theta q.$$

Single buyer



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References

Bergemann, Dirk, Alessandro Bonatti, and Alex Smolin (2018), "The design and price of information." *American Economic Review*, 108(1), 1–48.

Bergemann, Dirk, Yang Cai, Grigoris Velegkas, and Mingfei Zhao (2022), "Is selling complete information (approximately) optimal?" In *Proceedings of the 23rd ACM Conference on Economics and Computation*, EC '22, 608–663, Association for Computing Machinery, New York, NY, USA.

Bergemann, Dirk and Marco Ottaviani (2021), "Chapter 8 - information markets and nonmarkets." volume 4 of *Handbook of Industrial Organization*, 593–672, Elsevier.

Bimpikis, Kostas, Davide Crapis, and Alireza Tahbaz-Salehi (2019), "Information sale and competition." *Management Science*, 65(6), 2646–2664.

Boik, Andre and Hidenori Takahashi (2020), "Fighting bundles: The effects of competition on second-degree price discrimination." *American Economic Journal: Microeconomics*, 12(1), 156–87.

Bonatti, Alessandro, Munther Dahleh, Thibaut Horel, and Amir Nouripour (2024), "Selling information in competitive environments." *Journal of Economic Theory*, 216, 105779.

Borenstein, Severin and Nancy L. Rose (1994), "Competition and price dispersion in the u.s. airline industry." *Journal of Political Economy*, 102(4), 653–683.

Chade, Hector and Jeroen Swinkels (2021), "Screening in vertical oligopolies." *Econometrica*, 89(3), 1265–1311.

Champsaur, Paul and Jean-Charles Rochet (1989), "Multiproduct duopolists." *Econometrica*, 57(3), 533–557.

Corrao, Roberto, Joel P. Flynn, and Karthik A. Sastry (2023), "Nonlinear pricing with underutilization: A theory of multi-part tariffs." *American Economic Review*, 113(3), 836–60.

Crawford, Gregory S., Oleksandr Shcherbakov, and Matthew Shum (2019), "Quality overprovision in cable television markets." *American Economic Review*, 109(3), 956–95.

Deneckere, Raymond J. and R. Preston McAfee (1996), "Damaged goods." *Journal of Economics & Management Strategy*, 5(2), 149–174.

Garrett, Daniel F., Renato Gomes, and Lucas Maestri (2019), "Competitive screening under heterogeneous information." *The Review of Economic Studies*, 86(4), 1590–1630.

Gerardi, Kristopher S. and Adam Hale Shapiro (2009), "Does competition reduce price dispersion? new evidence from the airline industry." *Journal of Political Economy*, 117(1), 1–37.

Grubb, Michael D. (2009), "Selling to overconfident consumers." *American Economic Review*, 99(5), 1770–1807.

Johnson, Justin P. and David P. Myatt (2003), "Multiproduct quality competition: Fighting brands and product line pruning." *American Economic Review*, 93(3), 748–774.

Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk, and Ming Li (2017), "Persuasion of a privately informed receiver." *Econometrica*, 85(6), 1949–1964.

Kreps, David M. and José A. Scheinkman (1983), "Quantity precommitment and bertrand competition yield cournot outcomes." *The Bell Journal of Economics*, 14(2), 326–337.

Lehmann, Etienne, Laurent Simula, and Alain Trannoy (2014), "Tax me if you can! optimal nonlinear income tax between competing governments." *The Quarterly Journal of Economics*, 129(4), 1995–2030.

Maskin, Eric and John Riley (1984), "Monopoly with incomplete information." *The RAND Journal of Economics*, 15(2), 171–196.

Mensch, Jeffrey and Doron Ravid (2025), "Monopoly, product quality, and flexible learning."

Mussa, Michael and Sherwin Rosen (1978), "Monopoly and product quality." *Journal of Economic Theory*, 18(2), 301–317.

Oren, Shmuel, Stephen Smith, and Robert Wilson (1985), "Capacity pricing." *Econometrica*, 53(3), 545–566.

Rochet, Jean-Charles and Lars A. Stole (2002), "Nonlinear pricing with random participation." *The Review of Economic Studies*, 69(1), 277–311.

Rodríguez Olivera, Rosina (2024), "Strategic incentives and the optimal sale of information." *American Economic Journal: Microeconomics*, 16(2), 296–353.

Rothschild, Michael and Joseph Stiglitz (1976), "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information." *The Quarterly Journal of Economics*, 90(4), 629–649.

Shapiro, Carl and Hal R. Varian (1998), "Versioning: The smart way to sell information." *Harvard Business Review*, 76(6), 106–118.

Stole, Lars A. (2007), "Chapter 34 price discrimination and competition." volume 3 of *Handbook of Industrial Organization*, 2221–2299, Elsevier.

Thereze, João (2025), "Screening costly information."

Wilson, Robert (1993), *Nonlinear pricing*. Oxford University Press.

Yang, Kai Hao (2022), "Selling consumer data for profit: Optimal market-segmentation design and its consequences." *American Economic Review*, 112(4), 1364–93.