Screening in digital monopolies

Pietro Dall'Ara Elia Sartori

1st Capri, in theory Workshop, 2025

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Examples of **digital goods**:

- 1. Software goods;
- 2. Digital audio content;
- 3. Data.

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Up to 120,000 variables		-	-	~	~	~
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Up to 2.14 billion		~	~	~	~	~
Up to 20 billion		-	-	~	~	~

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Listen with friends in real time		⊘
Organize listening queue		•

Spotify's HiFi streaming could finally arrive this year



/ A \$6/ month "Music Pro" tier could include lossless audio and perks like discounted concert tickets.

by Quantym Kennemer Feb 17, 2025, 1018 PM GBIT-1

Spatistic Elements: is a writer who helps The Verge's readers save money by surfacing the best tech deals and presenting the latest product recommendations from our experts. He has covered tech and gaming for all of his 15-glabs year casers for publications like Forbes, Business ledder, and more.

Equifax offers three standard Business Credit Reports: BCR Complete View, BCR Risk View and BCR Profile View. The difference between reports is the number of sections that are available in the report:

REPORT SECTION TITLE	COMPLETE VIEW	RISK VIEW	PROFILE VIEW
Business Information & Firmographics	⊘	⊘	⊘
Report Highlights & Alerts	⊘	⊘	⊘
Index Values	⊘	⊘	⊘
Quarterly Index Value & Payment History	⊘	⊘	⊘
Credit Risk Scores	⊘	\bigcirc	
Trade Details (Industry & Financial)	⊘		
Negative Occurrences (Bankruptcy, Collections, Returned Cheques)	⊘		
Legal Data (Suits, Judgments, Liens)	⊘		
Inquiries	⊘		
Other	⊘		

Plan

- 1. Model;
- 2. Monopoly;
- 3. Monopoly inefficiencies;
- 4. No screening case;
- 5. Conclusion.

Model

A continuum of buyer **types**, $\theta \in [0,1] = \Theta$, interact with a seller.

Type θ is privately informed about $\theta \sim F$ for twice diff. F.

- F is regular for these slides and $\mathbb{E}\{\cdot\}$ refers to the r.v. θ .

Type θ has payoff from **quality** $q \in \mathbb{R}_+$ and transfer $t \in \mathbb{R}$:

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q,\theta)} -t,$$

for a concave, nondecreasing, and twice diff. g (Chade-Swinkels '21.) $q: \Theta \to \mathbb{R}_+$ is a quality allocation.

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 $q \colon \Theta \to \mathbb{R}_+$ is a quality allocation.

The cost of a quality allocation q is

$$C(q) = c(\sup q(\Theta)),$$

for a **production cost** c, increasing, strictly convex, twice diff. with c(0) = c'(0) = 0 and $\lim_{q \to \infty} c(q) = \infty$.

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 $q:\Theta\to\mathbb{R}_+$ is a quality allocation.

With separable costs, the cost of quality allocation q is

$$K(q) = \mathbb{E}\{k(q(\theta))\},\$$

for some k (Mussa-Rosen '78.)

The total surplus induced by the allocation $q(\cdot)$ is

$$\mathbb{E}\{u(\boldsymbol{q}(\theta),\theta)\}-c(\sup \boldsymbol{q}(\Theta)).$$

The efficient quality allocation q^* maximizes total surplus.

Proposition 1

The efficient quality allocation satisfies $\mathbf{q}^*(\theta) = q^*$ for all θ , in which q^* is the unique quality q such that

$$\underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\text{Marginal total utility}} = c'(q).$$

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▶ It holds that $\mathbb{E}\{u(\sup q(\Theta), \theta)\} \ge \mathbb{E}\{u(q(\theta), \theta)\}.$

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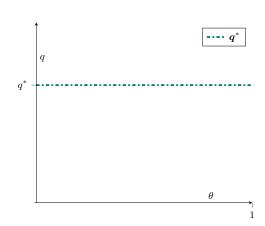
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 \blacktriangleright Versioning is in efficient.



The monopolist problem is:

$$\mathcal{P}^{M} \qquad \max_{\boldsymbol{q}, \ t(\cdot)} \int_{\Theta} t(\theta) \, \mathrm{d}F(\theta) - c(\sup \boldsymbol{q}(\Theta)) \text{ subject to:}$$

$$u(\boldsymbol{q}(\theta), \theta) - t(\theta) \ge u(\boldsymbol{q}(\hat{\theta}), \theta) - t(\hat{\theta}),$$

$$u(\boldsymbol{q}(\theta), \theta) - t(\theta) \ge 0, \text{ for all } (\theta, \hat{\theta}) \in \Theta^{2}.$$

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The monopolist quality allocation q^M solves \mathcal{P}^M .

With separable costs: the monopolist problem can be solved via "pointwise maximization".

The q contingent problem and its value V(q) are:

$$\begin{split} \mathcal{P}(q) \qquad V(q) \coloneqq \max_{\boldsymbol{q},\ t(\cdot)} \int_{\Theta} t(\theta) \, \mathrm{d}F(\theta) - \underline{c(\sup_{\boldsymbol{q}}(\Theta))} \text{ subject to:} \\ & \underline{\boldsymbol{q}}(\theta) \leq q, \\ & u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq u(\boldsymbol{q}(\hat{\theta}),\theta) - t(\hat{\theta}), \\ & u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq 0, \text{ for all } (\theta,\hat{\theta}) \in \Theta^2. \end{split}$$

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Lemma 1 (Invest then distribute)

The quality allocation q solves \mathcal{P}^M if and only if:

- (1) q solves $\mathcal{P}(q^M)$, where
- (2) q^M solves $\max_q V(q) c(q)$.

The q contingent problem and its value, V(q), are:

$$\begin{split} \mathcal{P}(q) \qquad V(q) \coloneqq \max_{\boldsymbol{q},\ t(\cdot)} \int_{[0,1]} \underbrace{g(\boldsymbol{q}(\theta)) + \varphi(\theta) \boldsymbol{q}(\theta)}_{\text{Virtual surplus}} \mathrm{d}F(\theta) \text{ subject to:} \\ \boldsymbol{q}(\theta) \leq q \text{ for all } \theta \in \Theta, \\ \boldsymbol{q} \text{ is nondecreasing;} \end{split}$$

in which
$$\varphi(\theta) := \theta - \frac{1 - F(\theta)}{F'(\theta)}$$
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Lemma 1 (Invest then distribute)

The quality allocation \mathbf{q} solves \mathcal{P}^{M} if and only if:

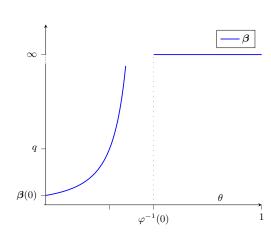
- (1) \boldsymbol{q} solves $\mathcal{P}(q^M)$, in which
- (2) q^M solves $\max_q V(q) c(q)$.

The virtual-surplus maximizer β is

$$\beta(\theta) \in \operatorname*{argmax}_{q} g(q) + \varphi(\theta)q,$$

and is such that:

1.
$$\boldsymbol{\beta}(\theta) = \infty \text{ if } \theta > \varphi^{-1}(0);$$

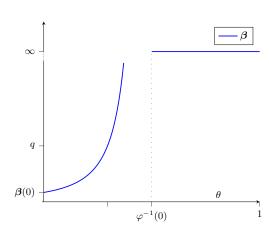


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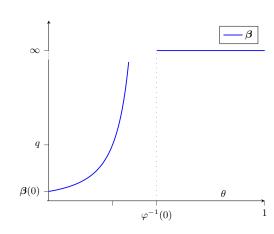


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- 1. $\beta(\theta) = \infty$ if $\theta > \varphi^{-1}(0)$;
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- 3. $\beta(0) > 0$ if g is "Inada".



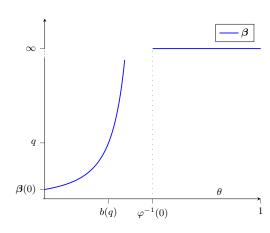
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b is the inverse of β .



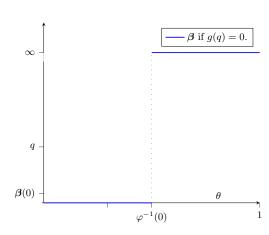
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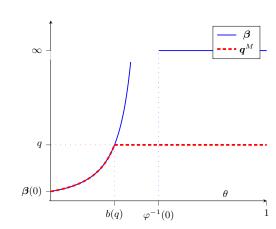
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Lemma 2
The quality allocation q solves $\mathcal{P}(q)$ iff:

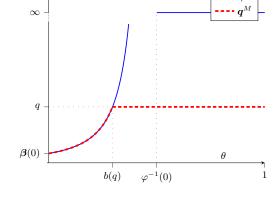
$$\mathbf{q}(\theta) = \min\{\boldsymbol{\beta}(\theta), q\},\$$

for all θ .



Distributive properties of the monopolist allocation for Inada g:

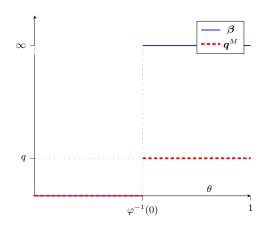
- 1. Bunching at the top; $\beta(\theta) = \infty$ for $\theta > \varphi^{-1}(0)$
- 2. Distributional inefficiency at the bottom or full bunching; β increasing on $[0, \varphi^{-1}(0))$
- 3. No exclusion. $\beta(0) > 0$



Linear preferences

Distributive properties of the monopolist allocation for g(q) = 0:

- 1. Bunching at the top; $\beta(\theta) = \infty$ for $\theta > \varphi^{-1}(0)$
- 2. Exclusion at the bottom; β is 0 on $[0, \varphi^{-1}(0))$
- 3. Single-quality menu.

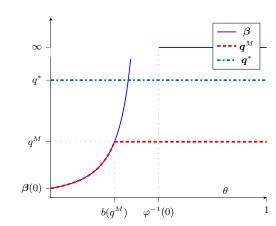


The monopolist allocation

Proposition 2

The monopolist allocation q^M is such that:

- (1) $q^M : \theta \mapsto \min{\{\beta(\theta), q^M\}},$
- (2) for $0 < q^M < q^*$.

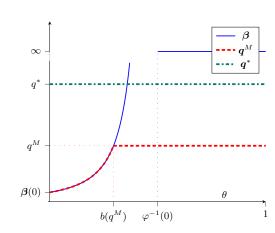


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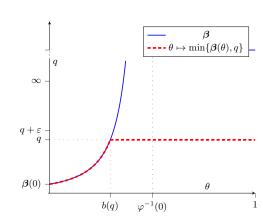
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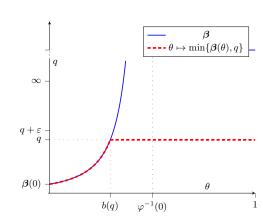
- (1) $q^M : \theta \mapsto \min{\{\beta(\theta), q^M\}},$
- (2) for $0 < q^M < q^*$.
 - ► Productive inefficiency occurs.



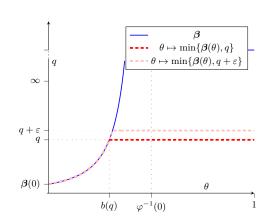
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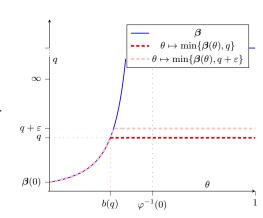


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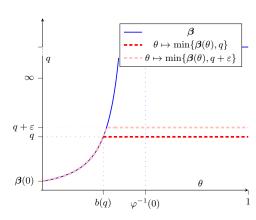
$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{Mass of bunched types}} \underbrace{(g'(q) + b(q))}_{\text{Marginal } u(\cdot, b(q))}.$$



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$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{Mass of bunched types}} \underbrace{(g'(q) + b(q))}_{\text{Marginal bunched types}}.$$

▶ V is not differentiable if F is not regular: a kink occurs if β "bunches" at q.



Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\begin{subarray}{c} {\rm Marginal\ revenues} \\ {\rm given}\ \theta\mapsto \min\{\beta(\theta),q\} \end{subarray}} < \underbrace{g'(q)+\mathbb{E}\{\theta\}}_{\begin{subarray}{c} {\rm Marginal\ total\ utility} \\ {\rm given}\ \theta\mapsto q \end{subarray}}$$

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$$U(q) := \mathbb{E}\{u(\min\{\boldsymbol{\beta}(\theta), q\}, \theta)\};$$

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2. $U'(q) < \text{marginal total utility given } \theta \mapsto q$, because

$$U'(q) = \int_{[b(q),1]} g'(q) + \theta \, dF(\theta) \le g'(q) + \mathbb{E}\{\theta\};$$

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▶ The q contingent no-damaging problem and its value, $V_N(q)$, are:

$$\begin{split} V_N(q) \coloneqq \max_{\boldsymbol{q}(\cdot),\,t(\cdot)} \int_{\Theta} t(\theta) \, \mathrm{d}F(\theta) \text{ subject to:} \\ \boldsymbol{q}(\theta) \le q, \text{IC, IR, } \boldsymbol{q}(\theta) \in \{0, \sup \boldsymbol{q}(\Theta)\} \text{ for all } \theta \in \Theta. \end{split}$$

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▶ The monopolist chooses the **marginally excluded** type n(q) and the revenues are

$$V_N(q) = (1 - F(n(q)))(g(q) + n(q)q)$$
 for $g(q) + \varphi(n(q))q = 0$.

► With damaging:

$$V'(q) = (1 - F(b(q)))(g'(q) + b(q)), \text{ for } g'(q) + \varphi(b(q)) = 0.$$

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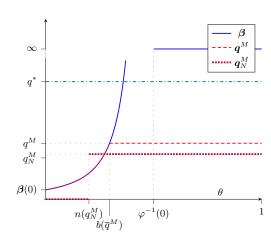
$$V'(q) = (1 - F(b(q)))(g'(q) + b(q)), \text{ for } g'(q) + \varphi(b(q)) = 0.$$

Productive inefficiency is worse: $q_N^M < q^M$, because

$$b(q) \in \underset{\theta}{\operatorname{argmax}} (1 - F(\theta))(g'(q) + \theta).$$

Compared to q^M , the no-damaging quality allocation q_N^M features:

- ► Less production;
- ► Less damaging;
- ► Potential exclusion for "Inada" g.



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- 5. **Competition** is beneficial for damaging inefficiency and harmful for productive efficiency.

▶ More details

Literature

Multi-product monopoly

Mussa and Rosen (1978); Maskin and Riley (1984); Deneckere and McAfee (1996) ... Grubb (2009); Corrao et al. (2023).

Mechanism & information design

Bergemann et al. (2025); Mensch and Ravid (2025); Thereze (2025).

Pricing of information

Bergemann et al. (2018); Bergemann and Ottaviani (2021); Yang (2022); Bergemann et al. (2022) . . .

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Thanks!

Extra slides

Interpretation

The profit expression admits two **interpretations**:

$$\underbrace{\int_{\Theta} t(\theta) \, \mathrm{d}F(\theta) - c(\sup \boldsymbol{q}(\Theta))}_{\text{Magnitude gap}} \quad \text{v.} \quad \underbrace{\int_{\Theta} t(\theta) - c(\sup \boldsymbol{q}(\Theta)) \, \mathrm{d}F(\theta)}_{\text{Costly replication}}.$$

- 1. Maintained interpretation: $c(\sup q(\Theta))$ is paid once and is "incomparable" with the utility of a single buyer.
 - ▶ Population \uparrow by $\alpha \iff \text{Costs} \downarrow \text{by } \alpha$.
 - $ightharpoonup q^M$ and $b(q^M)$ increase as population scales up.
- 2. Alternative interpretation: $c(\sup q(\Theta))$ is paid for every buyer.
 - Free replication does not hold: the cheapest way to produce $q \leq \sup q(\Theta)$ is to damage $\sup q(\Theta)$.
 - ightharpoonup Profits scale up by α as population scales up by α .

▶ Back

Competition

The game among N firms has two stages:

- 1. Every firm i simultaneously chooses a quality q_i .
- 2. Every firm i, observing all stage-1 qualities, simultaneously chooses a pricing function $p_i : \mathbb{R}_+ \to \mathbb{R}$, with $p_i(q) = \infty$ if $q > q_i$.

Then: each type buys a good from a firm i, or does not buy any good for a payoff of 0.

Firms play a subgame-perfect Nash equilibrium.

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Definition 1

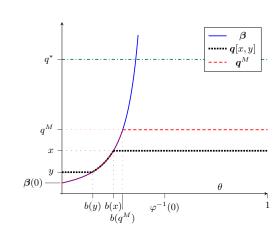
An n equilibrium is an equilibrium in which exactly n firms are active; an n equilibrium is symmetric if active firms play the same strategy.

Competitive allocations

Let's order qualities (q_1, \ldots, q_N) so that:

$$x > y > \cdots$$

Every quality below y comes at zero price.



Competitive equilibria

Lemma 2

In any pure-strategy equilibrium: one firm produces q^M and other firms are idle.

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Proposition 3

- 1. For all $n \leq N$, there exists a symmetric n equilibrium.
- 2. Every symmetric and competitive n equilibrium induces the random allocation $q[\hat{x}, \hat{y}]$, letting \hat{x} and \hat{y} be, resp., the first and second order statistics of the n i.i.d. draws from the CDF

$$H_n(q) = \left(\frac{c'(q)}{V'(q)}\right)^{\frac{1}{n-1}}, \text{ for } q \in [0, q^M].$$

Properties of competitive equilibria

Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

- 1. The lowest quality is positive and free;
- 2. The highest quality is strictly lower than q^M .

In the paper:

- 1. Equilibrium welfare with $n \geq 2$ active firms decreases in n.
- 2. Monopoly dominates duopoly if monopoly fully bunches.
- 3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

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with $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \overline{q}\} d\theta' dF(\theta)$ (Envelope Theorem).

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The marginal surplus is V'(q) + U'(q) and satisfies

$$V'(q) < V'(q) + U'(q) \le g'(q) + \mathbb{E}\{\theta\}.$$

- 1. Monopolist does not internalize buyer surplus;
- 2. Damaging inefficiency.

WTS:
$$V'(q) + U'(q) \le g'(q) + \mathbb{E}\{\theta\}.$$

- 1. $U'(q) = \int_{[b(q),1]} \theta b(q) dF(\theta)$, because the marginal $u(\mathbf{q}(\theta), \theta)$ increases at rate $g'(q) + \theta$ and the marginal transfer at rate g'(q) + b(q), for $\theta > b(q)$ and $\mathbf{q}(\cdot) = \min\{\boldsymbol{\beta}(\cdot), q\}$;
- 2. Using V'(q) = (1 F(b(q)))(g'(q) + b(q)), we have

$$V'(q) + U'(q) = (1 - F(b(q)))g'(q) + \int_{[b(q),1]} \theta \, dF(\theta).$$

(Note that U'(q) > 0 for all q > 0, because $b(q) \le \varphi^{-1}(0) < 1$ for all $q \ge 0$.)

The game

Type θ buys quality $D_{(p_1,\ldots,p_N)}(\theta)$ from firm $\iota_{(p_1,\ldots,p_N)}(\theta)$, given the pricing functions p_1,\ldots,p_N .

The revenues of i given pricing functions p_1, \ldots, p_N are

$$R_i(p_1,\ldots,p_N) := \int_{\left\{\theta: \iota_{(p_1,\ldots,p_N)}(\theta)=i\right\}} p_i(D_{(p_1,\ldots,p_N)}(\theta)) \, \mathrm{d}F(\theta).$$

The set of strategies for firm i is $S_i := Q \times P_i$, letting $P_i \subseteq (\mathbb{R}^Q)^{Q^N}$ be the set of "conditional" pricing functions of firm i.

The payoff of firm i from the profile $s := (\dots, (\overline{q}_i^s, P_i^s), \dots) \in \times_{i=1}^N S_i$ is

$$\Pi_i(s) := R_i(P_1^s[\overline{q}^s], \dots, P_N^s[\overline{q}^s]) - c(\overline{q}_i^s).$$

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