Screening in digital monopolies

Pietro Dall'Ara   Elia Sartori

1st Capri, in theory Workshop, 2025

Free damaging and replication

Several goods exhibit:

1. Free replication;

Free damaging and replication

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2. Free damaging.

Free damaging and replication

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This paper studies monopoly provision of goods exhibiting free replication and free damaging.

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Examples of **digital goods**:

1. Software goods;
2. Digital audio content;
3. Data.

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Up to 32,767 variables	-	✓	✓	✓	✓
Up to 120,000 variables	-	-	✓	✓	✓
Maximum number of observations					
Up to 2.14 billion	✓	✓	✓	✓	✓
Up to 20 billion	-	-	✓	✓	✓

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What you'll get	Spotify's Free plan	Spotify's Premium plans
Ad-free music listening	—	✓
Download to listen offline	—	✓
Play songs in any order	—	✓
High audio quality	—	✓
Listen with friends in real time	—	✓
Organize listening queue	—	✓

NEWS

Spotify's HiFi streaming could finally arrive this year

Image: Cath Virginia / The Verge

/ A $6/ month "Music Pro" tier could include lossless audio and perks like discounted concert tickets.

by Quentyn Kennemer
Feb 17, 2025, 10:18 PM GMT+1

Quentyn Kennemer is a writer who helps The Verge's readers save money by surfacing the best tech deals and presenting the latest product recommendations from our experts. He has covered tech and gaming for all of his 15-plus-year career for publications like Forbes, Business Insider, and more.

Equifax offers three standard Business Credit Reports: BCR Complete View, BCR Risk View and BCR Profile View. The difference between reports is the number of sections that are available in the report:

REPORT SECTION TITLE	COMPLETE VIEW	RISK VIEW	PROFILE VIEW
Business Information & Firmographics	✓	✓	✓
Report Highlights & Alerts	✓	✓	✓
Index Values	✓	✓	✓
Quarterly Index Value & Payment History	✓	✓	✓
Credit Risk Scores	✓	✓	
Trade Details (Industry & Financial)	✓		
Negative Occurrences (Bankruptcy, Collections, Returned Cheques)	✓		
Legal Data (Suits, Judgments, Liens)	✓		
Inquiries	✓		
Other	✓		

Plan

1. Model;
2. Monopoly;
3. Monopoly inefficiencies;
4. No screening case;
5. Conclusion.

Model

A continuum of buyer **types**, $\theta \in [0,1] = \Theta$, interact with a seller.

Type θ is privately informed about $\theta \sim F$ for twice diff. F.

- F is regular for these slides and $\mathbb{E}\{\cdot\}$ refers to the r.v. θ.

Type θ has payoff from **quality** $q \in \mathbb{R}_+$ and transfer $t \in \mathbb{R}$:

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q,\theta)} - t,$$

for a concave, nondecreasing, and twice diff. g (Chade-Swinkels '21.)

$\boldsymbol{q}\colon \Theta \to \mathbb{R}_+$ is a **quality allocation**.

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$\boldsymbol{q}\colon \Theta \to \mathbb{R}_+$ is a **quality allocation**.

The cost of a quality allocation $\boldsymbol{q}$ is

$$C(\boldsymbol{q}) = c(\sup \boldsymbol{q}(\Theta)),$$

for a **production cost** c, increasing, strictly convex, twice diff. with $c(0) = c'(0) = 0$ and $\lim_{q\to\infty} c(q) = \infty$.

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$\boldsymbol{q}\colon \Theta \to \mathbb{R}_+$ is a **quality allocation**.

With *separable* costs, the cost of quality allocation $\boldsymbol{q}$ is

$$K(\boldsymbol{q}) = \mathbb{E}\{k(\boldsymbol{q}(\theta))\},$$

for some k (Mussa-Rosen '78.)

Efficiency

The *total surplus* induced by the allocation $\boldsymbol{q}(\cdot)$ is

$$\mathbb{E}\{u(\boldsymbol{q}(\theta), \theta)\} - c(\sup \boldsymbol{q}(\Theta)).$$

The *efficient* quality allocation $\boldsymbol{q}^*$ maximizes total surplus.

Proposition 1

The efficient quality allocation satisfies $\boldsymbol{q}^*(\theta) = q^*$ for all θ, in which q^* is the unique quality q such that

$$\underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\text{Marginal total utility}} = c'(q).$$

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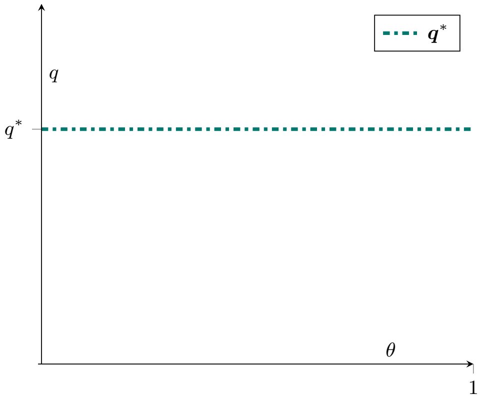
Efficiency

- Versioning is inefficient.



Monopoly

The monopolist problem is:

$$\mathcal{P}^M \qquad \max_{\boldsymbol{q},\, t(\cdot)} \int_\Theta t(\theta)\,\mathrm{d}F(\theta) - c(\sup \boldsymbol{q}(\Theta)) \text{ subject to:}$$

$$u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq u(\boldsymbol{q}(\hat\theta),\theta) - t(\hat\theta),$$
$$u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq 0, \text{ for all } (\theta,\hat\theta) \in \Theta^2.$$

The *monopolist* quality allocation $\boldsymbol{q}^M$ solves $\mathcal{P}^M$.

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The *monopolist* quality allocation $\boldsymbol{q}^M$ solves $\mathcal{P}^M$.

With separable costs: the monopolist problem can be solved via "pointwise maximization".

Monopoly

The q *contingent problem* and its value $V(q)$ are:

$$\mathcal{P}(q) \qquad V(q) := \max_{\boldsymbol{q},\, t(\cdot)} \int_{\Theta} t(\theta)\,\mathrm{d}F(\theta) - \cancel{c(\sup \boldsymbol{q}(\Theta))} \text{ subject to:}$$

$$\begin{aligned}
&\boldsymbol{q}(\theta) \leq q,\\
&u(\boldsymbol{q}(\theta),\theta) - t(\theta) \geq u(\boldsymbol{q}(\hat\theta),\theta) - t(\hat\theta),\\
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Lemma 1 (Invest then distribute)

The quality allocation $\boldsymbol{q}$ *solves* $\mathcal{P}^M$ *if and only if:*

(1) $\boldsymbol{q}$ *solves* $\mathcal{P}(q^M)$*, where*

(2) q^M *solves* $\max_q V(q) - c(q)$.

Monopoly

The q *contingent problem* and its value, $V(q)$, are:

$$\mathcal{P}(q) \qquad V(q) := \max_{\boldsymbol{q},\, t(\cdot)} \int_{[0,1]} \underbrace{g(\boldsymbol{q}(\theta)) + \varphi(\theta)\boldsymbol{q}(\theta)}_{\text{Virtual surplus}} \mathrm{d}F(\theta) \text{ subject to:}$$

$$\boldsymbol{q}(\theta) \leq q \text{ for all } \theta \in \Theta,$$

$$\boldsymbol{q} \text{ is nondecreasing;}$$

in which $\varphi(\theta) := \theta - \frac{1-F(\theta)}{F'(\theta)}$.

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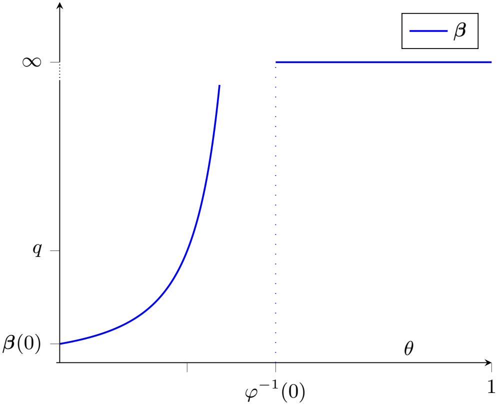
Virtual surplus maximization

The *virtual-surplus maximizer* $\boldsymbol{\beta}$ is

$$\boldsymbol{\beta}(\theta) \in \operatorname*{argmax}_{q} g(q) + \varphi(\theta) q,$$

and is such that:

1. $\boldsymbol{\beta}(\theta) = \infty$ if $\theta > \varphi^{-1}(0)$;



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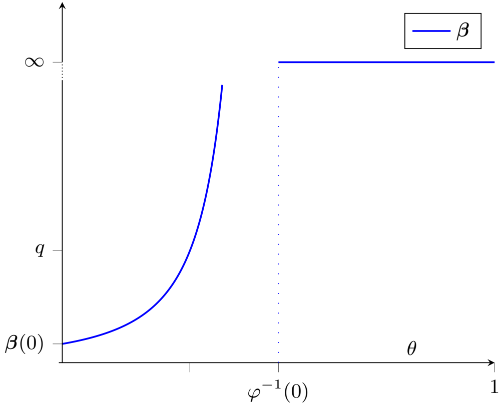

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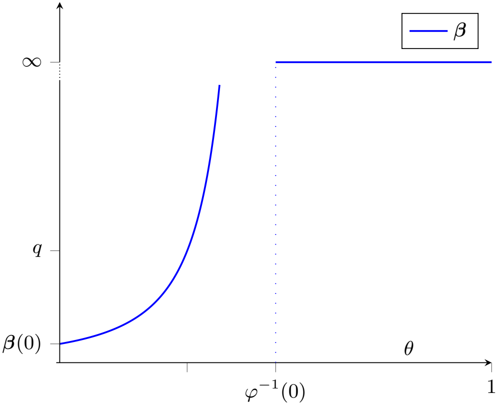

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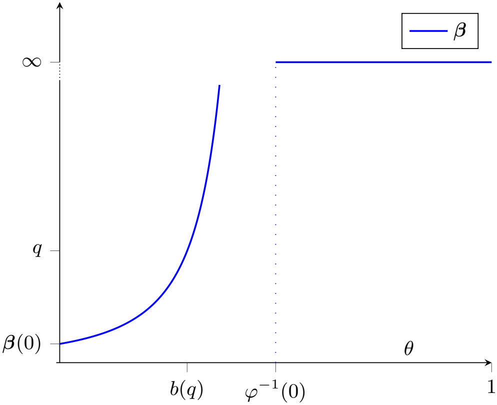
b is the inverse of $\boldsymbol{\beta}$.



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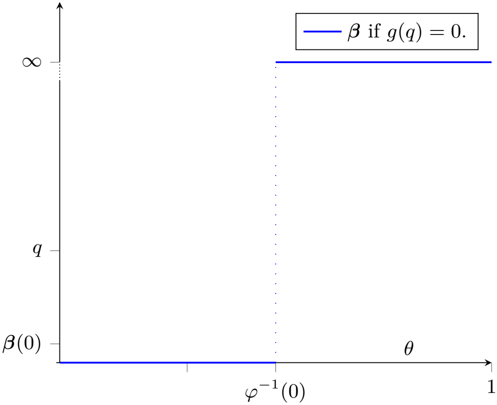


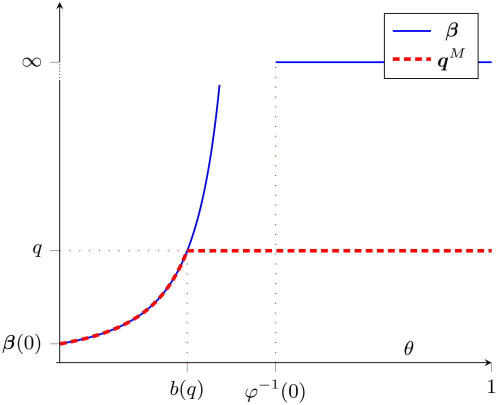
Virtual surplus maximization

Lemma 2

The quality allocation $\boldsymbol{q}$ solves $\mathcal{P}(q)$ iff:

$$\boldsymbol{q}(\theta) = \min\{\boldsymbol{\beta}(\theta), q\},$$

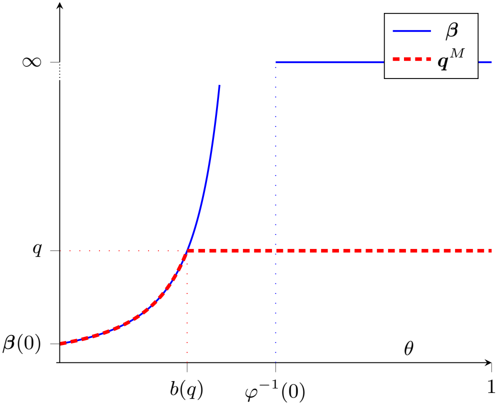
for all θ.



Virtual surplus maximization

Distributive properties of the monopolist allocation for Inada g:

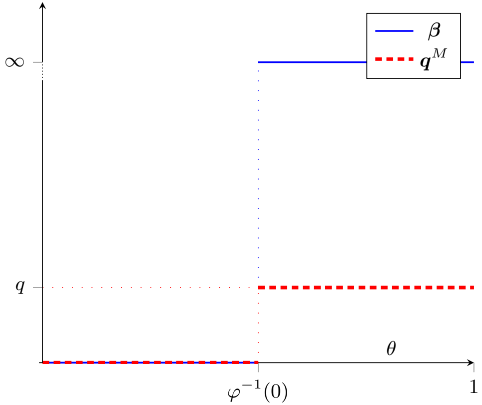
1. Bunching at the top;
   $\boldsymbol{\beta}(\theta) = \infty$ for $\theta > \varphi^{-1}(0)$
2. Distributional inefficiency at the bottom or full bunching;
   $\boldsymbol{\beta}$ increasing on $[0, \varphi^{-1}(0))$
3. No exclusion.
   $\boldsymbol{\beta}(0) > 0$



Linear preferences

Distributive properties of the monopolist allocation for $g(q) = 0$:

1. Bunching at the top; $\boldsymbol{\beta}(\theta) = \infty$ for $\theta > \varphi^{-1}(0)$
2. Exclusion at the bottom; $\boldsymbol{\beta}$ is 0 on $[0, \varphi^{-1}(0))$
3. **Single-quality menu**.



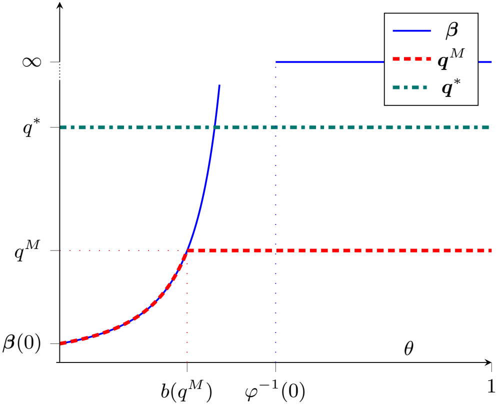
The monopolist allocation

Proposition 2

The monopolist allocation $\boldsymbol{q}^M$ *is such that:*

(1) $\boldsymbol{q}^M : \theta \mapsto \min\{\boldsymbol{\beta}(\theta), q^M\}$,

(2) *for* $0 < q^M < q^*$.



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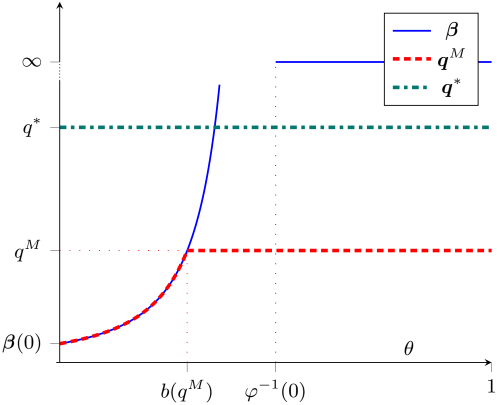
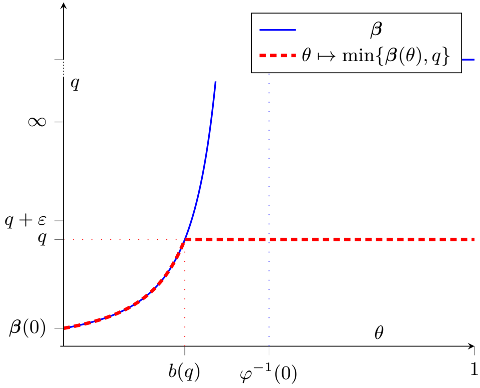
- Productive inefficiency occurs.



Marginal revenues

$V'(q)$ is the marginal revenue given the q constrained allocation $\theta \mapsto \min\{\boldsymbol{\beta}(\theta), q\}$,



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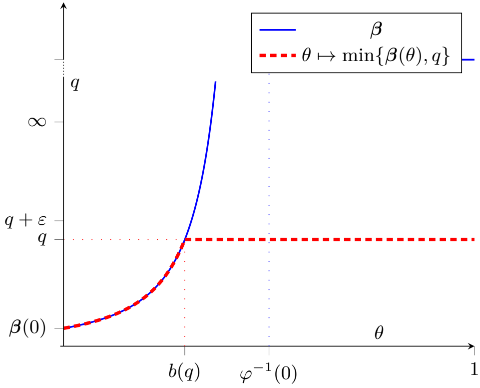

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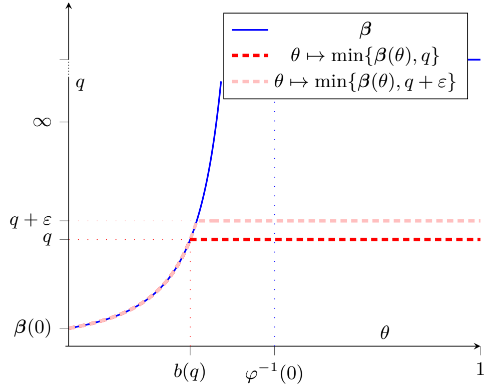
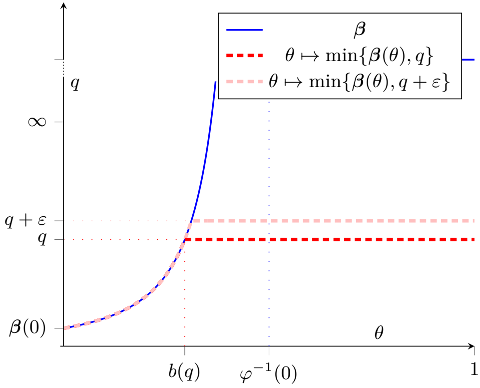


Marginal revenues

$V'(q)$ is the marginal revenue given the q constrained allocation $\theta \mapsto \min\{\boldsymbol{\beta}(\theta), q\}$, and

$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{Mass of bunched types}} \underbrace{(g'(q) + b(q))}_{\text{Marginal } u(\cdot, b(q))}.$$



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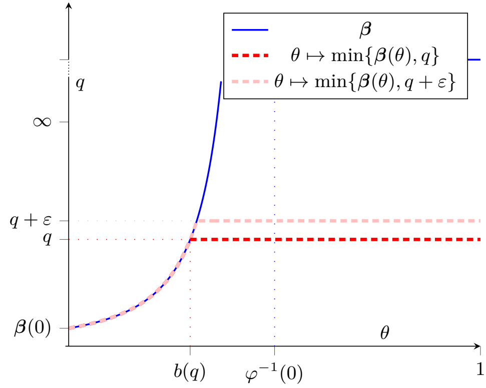
- V is not differentiable if F is not regular: a kink occurs if $\boldsymbol{\beta}$ "bunches" at q.



Productive inefficiency

Productive inefficiency arises if:

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2. $U'(q) <$ marginal total utility given $\theta \mapsto q$, because

$$U'(q) = \int_{[b(q),1]} g'(q) + \theta \, \mathrm{d}F(\theta) \leq g'(q) + \mathbb{E}\{\theta\};$$

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No damaging

By technological constraints, damaging may be infeasible.

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- The q *contingent no-damaging problem* and its value, $V_N(q)$, are:

$$V_N(q) \coloneqq \max_{\boldsymbol{q}(\cdot),\, t(\cdot)} \int_\Theta t(\theta)\,\mathrm{d}F(\theta) \text{ subject to:}$$
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- The monopolist chooses the **marginally excluded** type $n(q)$ and the revenues are

$$V_N(q) = (1 - F(n(q)))(g(q) + n(q)q) \quad \text{for} \quad g(q) + \varphi(n(q))q = 0.$$

- With damaging:

$$V'(q) = (1 - F(b(q)))(g'(q) + b(q)), \quad \text{for} \quad g'(q) + \varphi(b(q)) = 0.$$

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- With damaging:

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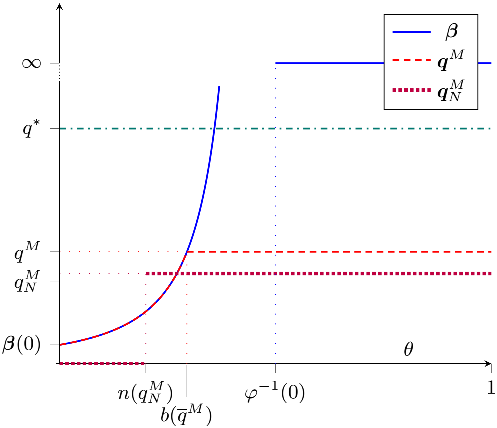
Productive inefficiency is worse: $q_N^M < q^M$, because

$$b(q) \in \operatorname*{argmax}_{\theta} (1 - F(\theta))(g'(q) + \theta).$$

No damaging

Compared to $\boldsymbol{q}^M$, the no-damaging quality allocation $\boldsymbol{q}_N^M$ features:

- Less production;
- Less damaging;
- Potential exclusion for "Inada" g.



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5. **Competition** is beneficial for damaging inefficiency and harmful for productive efficiency.

More details

Literature

Multi-product monopoly
??? . . . **??**.

Mechanism & information design
???.

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Thanks!

Extra slides

Interpretation

The profit expression admits two **interpretations**:

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1. Maintained interpretation: $c(\sup \boldsymbol{q}(\Theta))$ is paid once and is "incomparable" with the utility of a single buyer.
   - Population $\uparrow$ by $\alpha \iff$ Costs $\downarrow$ by α.
   - q^M and $b(q^M)$ increase as population scales up.
2. Alternative interpretation: $c(\sup \boldsymbol{q}(\Theta))$ is paid for every buyer.
   - Free replication does not hold: the cheapest way to produce $q \leq \sup \boldsymbol{q}(\Theta)$ is to damage $\sup \boldsymbol{q}(\Theta)$.
   - Profits scale up by α as population scales up by α.

Back

Competition

The game among N firms has two stages:

1. Every firm i simultaneously chooses a quality q_i.
2. Every firm i, observing all stage-1 qualities, simultaneously chooses a pricing function $p_i \colon \mathbb{R}_+ \to \mathbb{R}$, with $p_i(q) = \infty$ if $q > q_i$.

Then: each type buys a good from a firm i, or does not buy any good for a payoff of 0.

Firms play a subgame-perfect Nash equilibrium.

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Definition 1

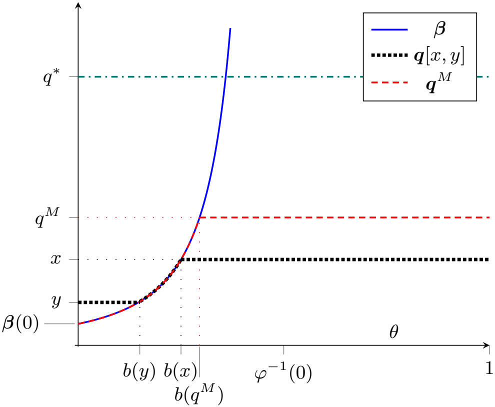
An n equilibrium is an equilibrium in which exactly n firms are active; an n equilibrium is symmetric if active firms play the same strategy.

Competitive allocations

Let's order qualities $(q_1, \ldots, q_N)$ so that:

$$x > y > \cdots$$

Every quality below y comes at zero price.



Competitive equilibria

Lemma 2
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Proposition 3

1. *For all $n \leq N$, there exists a symmetric n equilibrium.*
2. *Every symmetric and competitive n equilibrium induces the random allocation $\boldsymbol{q}[\hat{x}, \hat{y}]$, letting $\hat{x}$ and $\hat{y}$ be, resp., the first and second order statistics of the n i.i.d. draws from the CDF*

$$H_n(q) = \left(\frac{c'(q)}{V'(q)}\right)^{\frac{1}{n-1}}, \quad \textit{for } q \in [0, q^M].$$

Properties of competitive equilibria

Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. *The lowest quality is positive and free;*
2. *The highest quality is strictly lower than* q^M*.*

In the paper:

1. Equilibrium welfare with $n \geq 2$ active firms decreases in n.
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.

Productive inefficiency from damaging 1

Productive inefficiency arises because:

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$$V(q) + U(q) - c(q),$$

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The marginal surplus is $V'(q) + U'(q)$ and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.

Productive inefficiency from damaging 2

WTS: $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$.

1. $U'(q) = \int_{[b(q),1]} \theta - b(q)\, \mathrm{d}F(\theta)$,
   because the marginal $u(\boldsymbol{q}(\theta), \theta)$ increases at rate $g'(q) + \theta$ and the marginal transfer at rate $g'(q) + b(q)$, for $\theta > b(q)$ and $\boldsymbol{q}(\cdot) = \min\{\boldsymbol{\beta}(\cdot), q\}$;
2. Using $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$, we have

$$V'(q) + U'(q) = (1 - F(b(q)))g'(q) + \int_{[b(q),1]} \theta\, \mathrm{d}F(\theta).$$

(Note that $U'(q) > 0$ for all $q > 0$, because $b(q) \leq \varphi^{-1}(0) < 1$ for all $q \geq 0$.)

The game

Type θ buys quality $D_{(p_1,\ldots,p_N)}(\theta)$ from firm $\iota_{(p_1,\ldots,p_N)}(\theta)$, given the pricing functions $p_1,\ldots,p_N$.

The revenues of i given pricing functions $p_1,\ldots,p_N$ are

$$R_i(p_1,\ldots,p_N) \coloneqq \int_{\left\{\theta:\iota_{(p_1,\ldots,p_N)}(\theta)=i\right\}} p_i(D_{(p_1,\ldots,p_N)}(\theta))\,\mathrm{d}F(\theta).$$

The set of strategies for firm i is $S_i \coloneqq Q \times \boldsymbol{P}_i$, letting $\boldsymbol{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$ be the set of "conditional" pricing functions of firm i.

The *payoff* of firm i from the profile $s \coloneqq (\ldots,(\overline{q}_i^s, P_i^s),\ldots) \in \times_{i=1}^N S_i$ is

$$\Pi_i(s) \coloneqq R_i(P_1^s[\overline{q}^s],\ldots,P_N^s[\overline{q}^s]) - c(\overline{q}_i^s).$$

References