

# Screening in digital monopolies

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# Free damaging and replication

Several goods exhibit:

1. Free replication;
2. Free damaging.

└ taste heterogeneity



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This paper studies monopoly provision of goods whose production features free replication and free damaging.



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Examples of **digital goods**:

1. Software goods;
2. Digital content;
3. Data.



6-month	Annual	Perpetual	
<b>Stata/BE</b> For mid-sized datasets.  <b>\$225 USD</b> perpetual <a href="#">Buy</a>	<b>Stata/SE</b> For larger datasets.  <b>\$425 USD</b> perpetual <a href="#">Buy</a>	<b>Stata/MP 2-core</b> ⓘ Faster & for the largest datasets.  <b>\$595 USD</b> perpetual <a href="#">Buy</a>	<b>Stata/MP 4-core</b> Even faster.  <b>\$795 USD</b> perpetual <a href="#">Buy</a>

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			2-core	4-core	6+
Maximum number of variables ⓘ					
Up to 2,048 variables	✓	✓	✓	✓	✓
Up to 32,767 variables	-	✓	✓	✓	✓
Up to 120,000 variables	-	-	✓	✓	✓
Maximum number of observations ⓘ					
Up to 2.14 billion	✓	✓	✓	✓	✓
Up to 20 billion	-	-	✓	✓	✓



Equifax offers three standard Business Credit Reports: BCR Complete View, BCR Risk View and BCR Profile View. The difference between reports is the number of sections that are available in the report:

REPORT SECTION TITLE	COMPLETE VIEW	RISK VIEW	PROFILE VIEW
Business Information & Firmographics	✓	✓	✓
Report Highlights & Alerts	✓	✓	✓
Index Values	✓	✓	✓
Quarterly Index Value & Payment History	✓	✓	✓
Credit Risk Scores	✓	✓	
Trade Details (Industry & Financial)	✓		
Negative Occurrences (Bankruptcy, Collections, Returned Cheques)	✓		
Legal Data (Suits, Judgments, Liens)	✓		
Inquiries	✓		
Other	✓		



# Preview

This paper studies screening when production features free replication and free damaging.

The **monopoly** provision features two interdependent inefficiencies:

1. **Productive inefficiency**: the best quality is lower than its efficient level;
2. **Distributional inefficiency**: some buyers purchase a damaged good.



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Extensions:

- ▶ A no-damaging constraint makes productive inefficiency stronger;
- ▶ **Competition** is harmful for productive efficiency and beneficial for damaging inefficiency.



# Plan

1. Model;
2. Efficiency benchmark;
3. Monopoly allocation and inefficiencies;
4. No-damaging constraint;
5. Single buyer;
6. Competition.



**Model**



# Model

A unit mass of buyers, each drawing a **type**  $\theta \in [0, 1] = \Theta$ , interacts with a seller.

Type  $\theta$  is privately informed about  $\theta \sim F$ , for twice diff.  $F$ ;

$\hookrightarrow F$  is regular except in dedicated slide,  $\mathbb{E}\{\cdot\}$  refers to the r.v.  $\theta$ .

Type  $\theta$  has payoff from **quality**  $q \in \mathbb{R}_+$  and transfer  $t \in \mathbb{R}$ :

$$\underbrace{g(q) + \theta q}_{\text{utility } u(q, \theta)} - t,$$

for a strictly concave, increasing, and twice diff.  $g$  with  $g(0) = 0$ .

An **allocation** is a measurable  $\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ .



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An **allocation** is a measurable  $\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ .

The cost of allocation  $\mathbf{q}$  is

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)),$$

for a **production cost**  $c$ , increasing, strictly convex, twice diff., with  $c'(0) = 0$  and  $\lim_{q \rightarrow \infty} c'(q) = \infty$ .



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An **allocation** is a measurable  $\mathbf{q}: \Theta \rightarrow \mathbb{R}_+$ .

With *separable* costs, the cost of  $\mathbf{q}$  is

$$C(\mathbf{q}) = \mathbb{E}\{k(\mathbf{q}(\theta))\},$$

for some  $k$  (Mussa-Rosen '78.)



# Model

1. The cost of allocation  $\mathbf{q}$  is

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) \\ \neq \int_{\Theta} c(\text{what } \theta \text{ gets}) dF(\theta),$$

for a **production cost**  $c$ , increasing, strictly convex, twice diff., with  $c'(0) = 0$  and  $\lim_{q \rightarrow \infty} c'(q) = \infty$ .

2. The main results hold with increasing-differences  $u$ ,
  - 2.1 Basic task (text editing):  $g(q)$ ;
  - 2.2 Advanced task (computing):  $\theta q$ .



# Model

1. In general,

$$C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \int_{\Theta} k(\mathbf{q}(\theta)) \, dF(\theta),$$

for some  $k$ .

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**Efficiency**



# Efficiency

The *surplus* induced by allocation  $\mathbf{q}$  is

$$\mathbb{E}\{u(\mathbf{q}(\theta), \theta)\} - c(\sup \mathbf{q}(\Theta)).$$

The *efficient* allocation  $\mathbf{q}^*$  maximizes surplus.

## Proposition 1

The efficient allocation is given by  $\mathbf{q}^*(\theta) = q^*$  for all  $\theta$ , in which  $q^*$  is the unique quality  $q$  such that

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# Efficiency

The *surplus* induced by allocation  $\theta \mapsto q$  is

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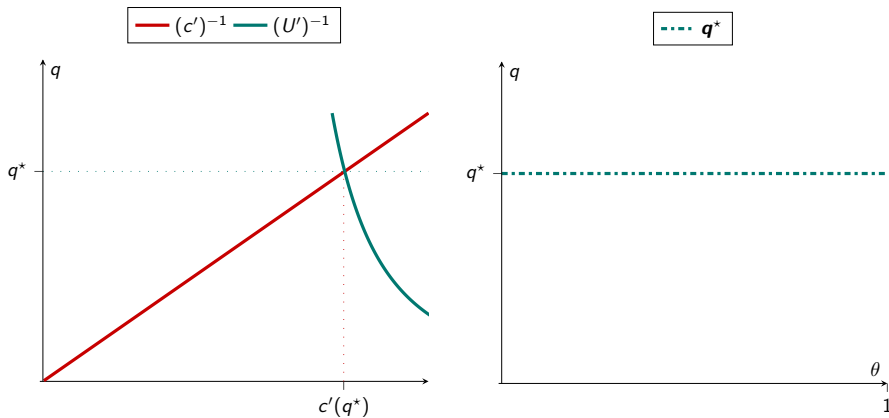
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# Efficiency



The utility induced by allocation  $\theta \mapsto q$  is  $U(q) = g(q) + \mathbb{E}\{\theta\}q$ .



# Monopoly



# Monopoly

The monopolist problem is:

$$(\mathcal{P}^M) \quad \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) \, dF(\theta) - c(\sup \mathbf{q}(\Theta)) \text{ subject to:}$$

$$\begin{aligned} u(\mathbf{q}(\theta), \theta) - t(\theta) &\geq u(\mathbf{q}(\hat{\theta}), \theta) - t(\hat{\theta}), \text{ for all } (\theta, \hat{\theta}), \\ u(\mathbf{q}(\theta), \theta) - t(\theta) &\geq 0, \text{ for all } \theta. \end{aligned}$$

- The *monopolist* allocation  $\mathbf{q}^M$  solves  $\mathcal{P}^M$  for some  $t(\cdot)$ .



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- ▶ The *monopolist* allocation  $\mathbf{q}^M$  solves  $\mathcal{P}^M$  for some  $t(\cdot)$ .
- ▶ Without separable costs: the monopolist problem cannot be solved via “pointwise maximization”.



# Monopoly

The  $q$ -constrained problem is:

$$(\mathcal{P}(q)) \quad V(q) := \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) - \cancel{c(\sup_{\theta} \mathbf{q}(\theta))} \text{ subject to:}$$

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## Lemma 1 (Invest then distribute)

The allocation  $\mathbf{q}$  solves  $\mathcal{P}^M$  if and only if:

1.  $\mathbf{q}$  solves  $\mathcal{P}(q^M)$ ,
2.  $q^M$  solves  $\max_q V(q) - c(q)$ .



# Monopoly

The  $q$ -constrained problem is:

$$(\mathcal{P}(q)) \quad V(q) := \max_{\mathbf{q}} \int_{\Theta} \underbrace{g(\mathbf{q}(\theta)) + \varphi(\theta)\mathbf{q}(\theta)}_{\text{Virtual surplus}} dF(\theta) \text{ subject to:}$$
$$\mathbf{q}(\theta) \leq q, \text{ for all } \theta,$$
$$\mathbf{q} \text{ is nondecreasing;}$$

$$\text{in which } \varphi(\theta) := \theta - \frac{1-F(\theta)}{F'(\theta)}.$$

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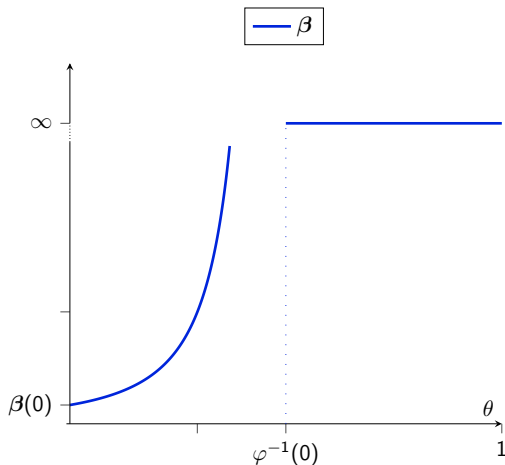
# Virtual surplus maximization

The *virtual-surplus maximizer*

$$\beta(\theta) \in \operatorname{Argmax}_q g(q) + \varphi(\theta)q$$

satisfies:

1.  $\beta(\theta) = \infty$  if  $\theta \geq \varphi^{-1}(0)$ ;
2.  $\beta$  is increasing;
3.  $\beta(0) > 0$  ("lnada"  $g$ ).





# Virtual surplus maximization

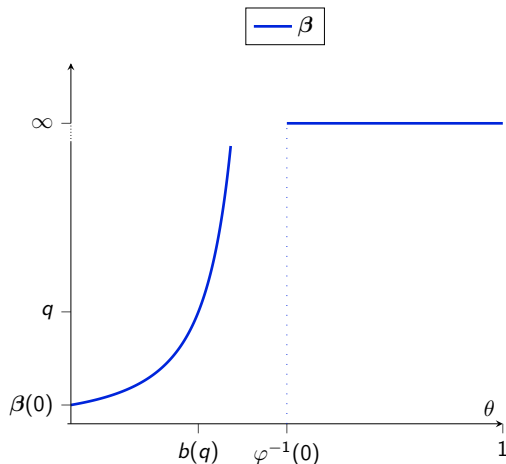
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$b$  is the inverse of  $\beta$ .



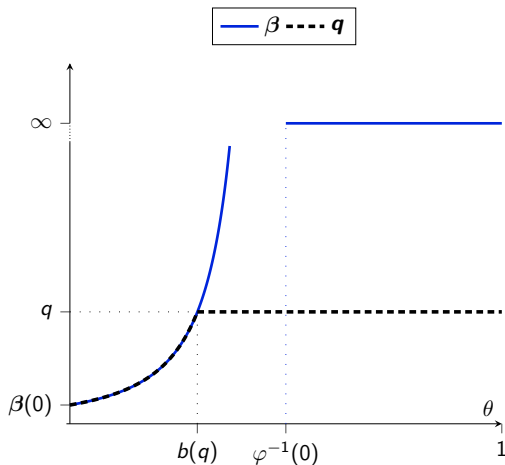


# Virtual surplus maximization

## Lemma 2

Allocation  $\mathbf{q}$  solves  $\mathcal{P}(\mathbf{q})$  iff:

$$\mathbf{q}(\theta) = \min\{\beta(\theta), q\}, \text{ for all } \theta.$$





# Virtual surplus maximization

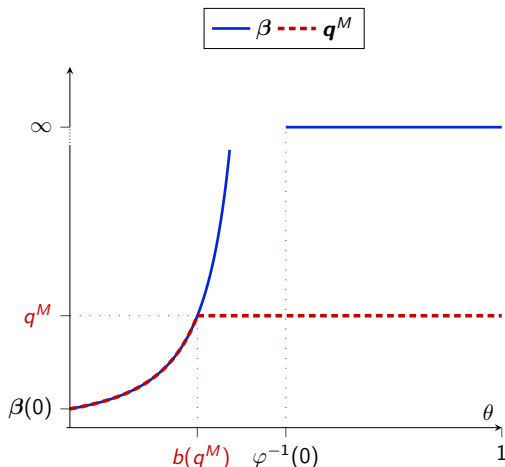
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Distributive properties of  $\mathbf{q}^M$ :

1. Bunching at the top;
2. Distributional inefficiency at the bottom or full bunching;
3. No exclusion (if  $q^M > 0$ .)





# Virtual surplus maximization

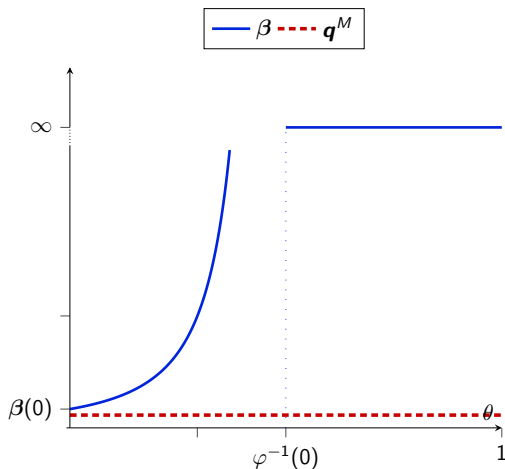
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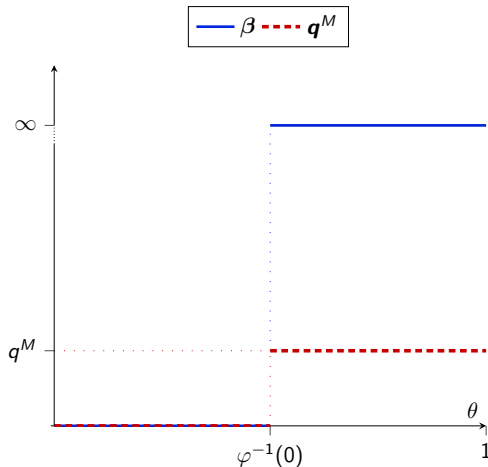
# Linear preferences

Distributive properties if  $g(q) = 0$ :

1. Bunching at the top;  
 $\beta(\theta) = \infty$  for  $\theta \geq \varphi^{-1}(0)$
2. Exclusion at the bottom;  
 $\beta(\theta) = 0$  for  $\theta < \varphi^{-1}(0)$

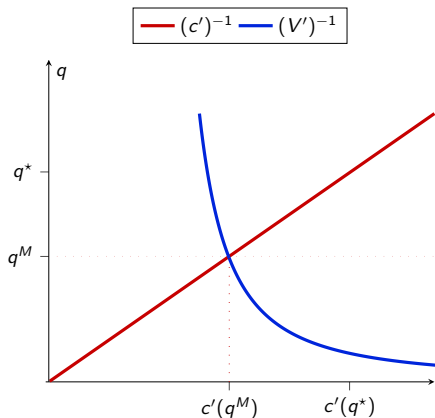
$\Rightarrow$  **single version.**

Richness in digital markets is due solely to preferences.

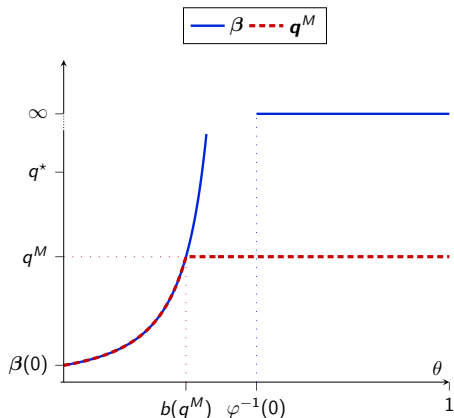




# The monopolist allocation



**Acquisition**

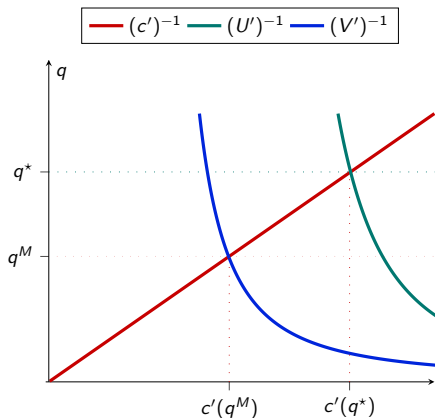


**Distribution:**

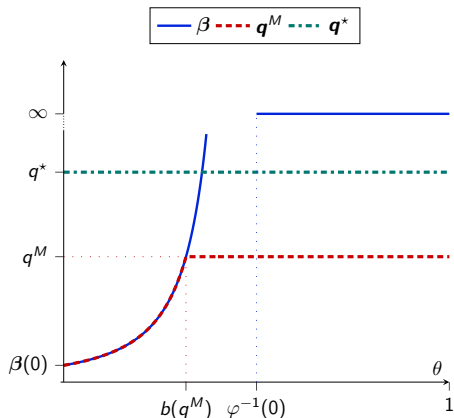
$$q^M(\theta) = \min\{\beta(\theta), q^M\}.$$



# The monopolist allocation



**Acquisition**



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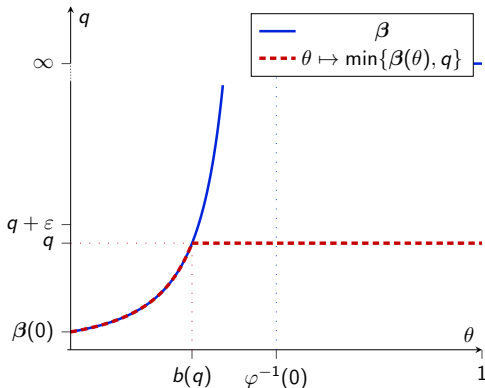
# Marginal revenues

The return from increasing the cap of the  $q$ -constrained allocation is:

$$V'(q) = \underbrace{(1 - F(b(q)))}_{\text{bunched types}} \underbrace{(g'(q) + b(q))}_{u_q \text{ of } b(q)}.$$

The change from  $q$  to  $q + \varepsilon$  leads to:

1. same revenues from  $q' < q$ :  
 $q'$  sold to the same  $\theta$ , and  $\theta$  gets the same **rent**;
2. higher quality for bunched types;
3. higher price by  $u_q(q, b(q))$ .





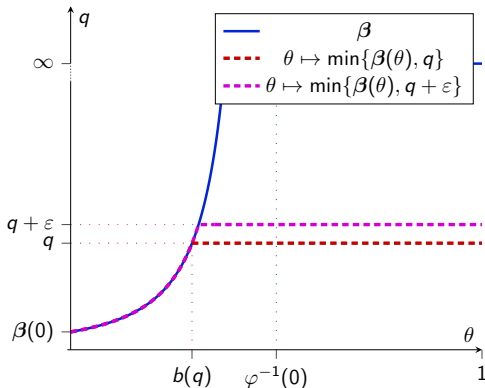
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1. By the distributive properties

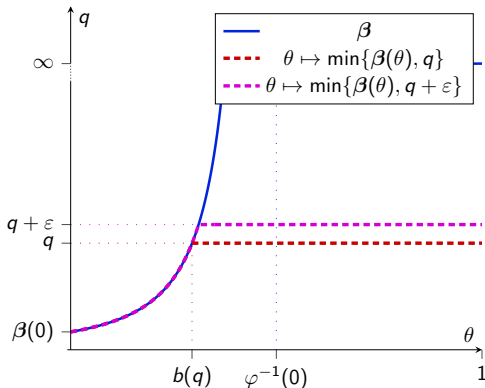
$$1 - F(b(q)) < 1;$$

2. By Markov's inequality:

$$(1 - F(b(q)))b(q) \leq \mathbb{E}\{\theta\};$$

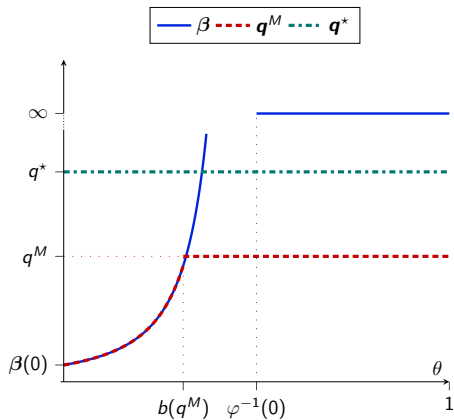
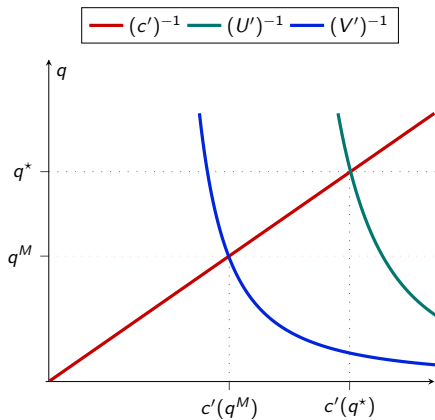
So:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$





# Productive inefficiency





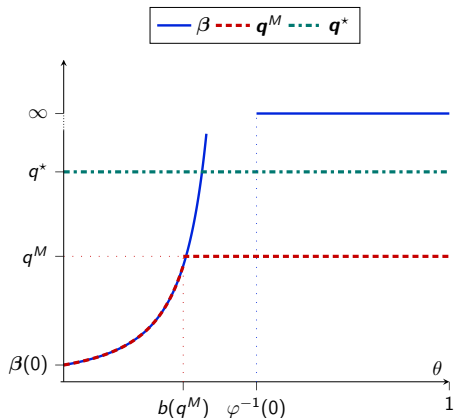
# Productive inefficiency

## Proposition 2

The monopolist allocation is given by  $q^M(\theta) = \min\{\beta(\theta), q\}$  for all  $\theta$ , in which  $q^M$  is the unique  $q$  solving

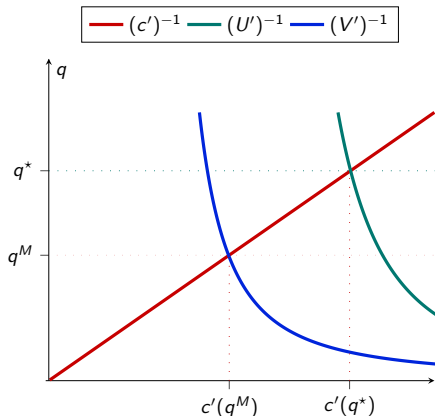
$$V'(q) = c'(q).$$

Moreover, it holds that:  $q^M < q^*$ .

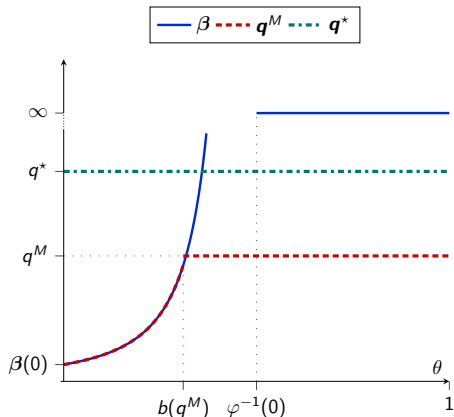




# Productive inefficiency



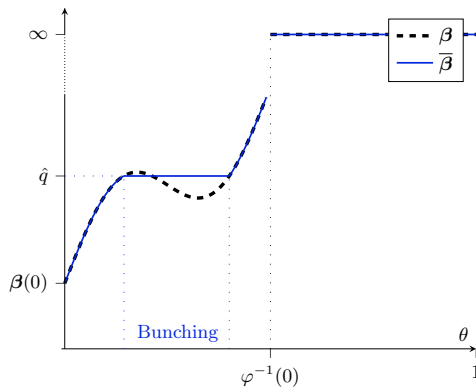
**Acquisition:**  $q^M$  is the quality  $q$  solving:  
 $(1 - F(b(q)))u_q(q, b(q)) = c'(q).$



**Distribution:**  
 $q^M(\theta) = \min\{\beta(\theta), q^M\}.$



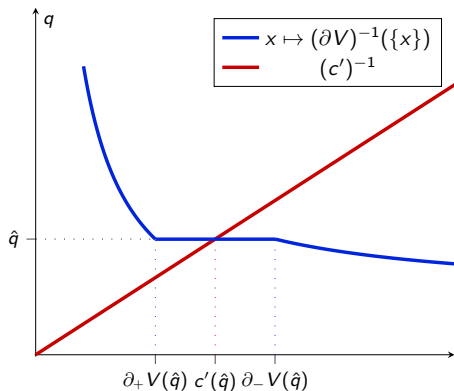
# Non-regular distribution



- $\beta$  is ironed to obtain  $\bar{\beta}$ ;
- By Lemma 1,  
 $\theta \mapsto \min\{\bar{\beta}(\theta), q\}$  solves  $\mathcal{P}(q)$ ;



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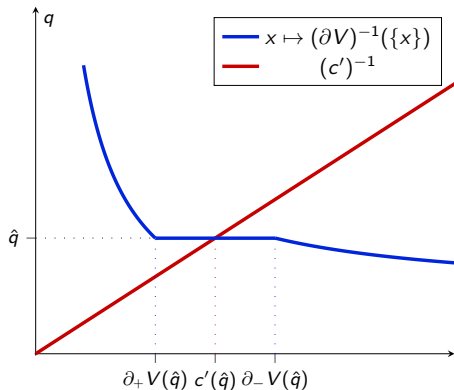
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- ▶ If types in  $(\theta', \theta'')$  are bunched  
 “at”  $\hat{q} \in (0, q)$ ,

$$\partial_- V(\hat{q}) > \partial_+ V(\hat{q}),$$

the extra revenues from  $\hat{q} + \varepsilon$   
 come from types higher than  $\theta''$ .



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## Proposition 3

The monopolist allocation is  $\mathbf{q}^M(\theta) = \min\{\bar{\beta}(\theta), q^M\}$ , in which  $q^M$  is the unique  $q$  with  $c'(q) \in \partial V(q)$ . Moreover, it holds that  $q^M < q^*$ .



**No damaging constraint**



# The highest quality increment

- ▶ **Taxation principle:** The problem  $\mathcal{P}(q)$  can be stated as the choice of a **tariff**  $T: \mathbb{R}_+ \rightarrow \mathbb{R}$ .
- ▶ The **price increment**  $T'(q)$  of the optimal tariff  $T$  satisfies

$$T'(q) \text{ solves } \max_{p \in \mathbb{R}_+} p \Pr(\{\theta \mid g'(q) + \theta \geq p\})$$

(Wilson, 1993).



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- ▶ The price increment at  $q$  is  $g'(q) + b(q)$ .

$$\implies g'(q) + b(q) \text{ solves } \max_{p \in \mathbb{R}_+} p \Pr(\{\theta \mid g'(q) + \theta \geq p\}).$$

## Fact 1

The type  $b(q)$  solves  $\max_{\theta \in \Theta} (g'(q) + \theta)(1 - F(\theta))$ .



# No damaging

Without damaging, the  $q$ -constrained problem is:

$$V_N(q) := \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) dF(\theta) \text{ subject to:}$$

IC, IR,  $\mathbf{q}(\theta) \in \{0, q\}$ , for all  $\theta$ .

The constraint is irrelevant under:

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2. Linear preferences.



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$$V_N(q) = (1 - F(n(q)))(g(q) + n(q)q), \quad \text{for } g(q) + \varphi(n(q))q = 0.$$

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- ▶ Intuitively: damaging ban  $\implies n(q) \leq b(q)$ , strictly if  $b(q) > 0$ ,
- ▶ so productive inefficiency is worse:

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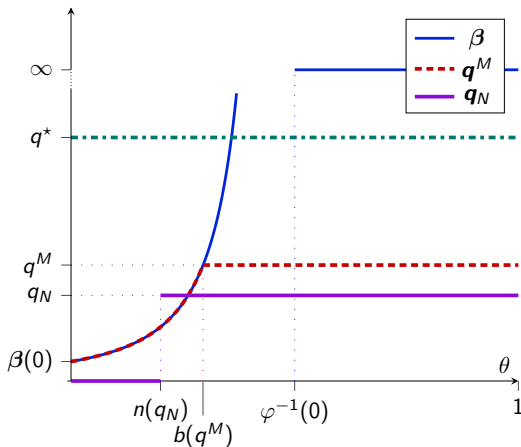


# No damaging

The no-damaging allocation  $q_N$  features:

- ▶ Less production;
- ▶ No damaging;
- ▶ (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.



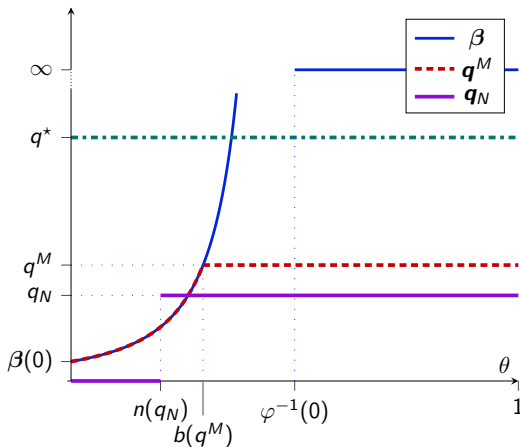


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The no-damaging allocation  $q_N$  features:

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- ▶ No damaging;
- ▶ (Possibility of) exclusion.

The welfare comparison is type specific and ambiguous.



## Proposition 4

Without damaging, the monopolist allocation is  $q_N(\theta) = \mathbf{1}_{[b_N(q_N), 1]}(\theta)q_N$ , in which  $q_N$  is the unique  $q$  solving  $V'_N(q) = c'(q)$ . Moreover, we have  $q_N \leq q^M$ , strictly if  $b(q^M) > 0$ .



**Separable costs**



# Cost interpretation

For digital goods:  $\Pi(\mathbf{q}) := \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{per-agent revenues}} - \underbrace{c(\sup \mathbf{q}(\Theta))}_{\text{per-agent unit cost}},$

1. Payment  $t(\theta)$  and production cost  $c(q^M)$  have different size; (Shapiro and Varian, 1998).



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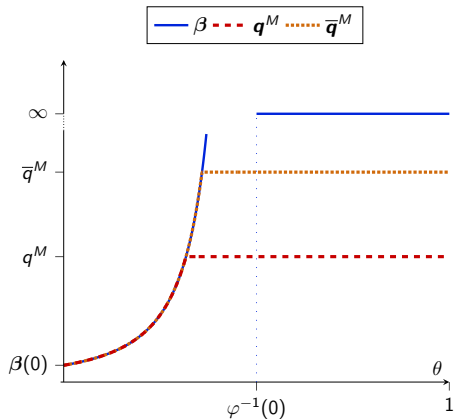
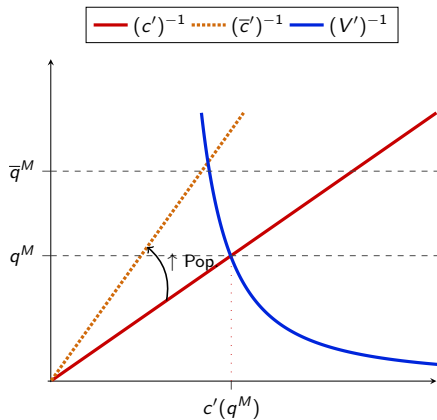
For separable costs:  $\Pi^{\text{M-R}}(\mathbf{q}) := \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{per-agent revenues}} - \underbrace{\int_{\Theta} k(\mathbf{q}(\theta)) dF(\theta)}_{\text{per-agent costs}},$

1. Payment  $t(\theta)$  and production cost  $k(\mathbf{q}(\theta))$  are comparable;
2. Population size only scales profits;

In general:  $C(\mathbf{q}) = \int_{\Theta} k(\mathbf{q}(\theta)) dF(\theta) + c(\sup \mathbf{q}(\Theta)).$



# Population size





# Single buyer

$$\Pi^{\text{M-R}}(\mathbf{q}) = \underbrace{\int_{\Theta} t(\theta) dF(\theta)}_{\text{expected revenues}} - \underbrace{\int_{\Theta} c(\mathbf{q}(\theta)) dF(\theta)}_{\text{expected costs}},$$

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3. Free damaging and replication are irrelevant.

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3. Free damaging matters, replication is irrelevant.



# Single buyer

$$\text{M-R: } \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) - c(\mathbf{q}(\theta)) dF(\theta) \quad \text{D-S: } \max_{\mathbf{q}, t(\cdot)} \int_{\Theta} t(\theta) - c(\sup \mathbf{q}(\Theta)) dF(\theta)$$

subject to: IC, IR.

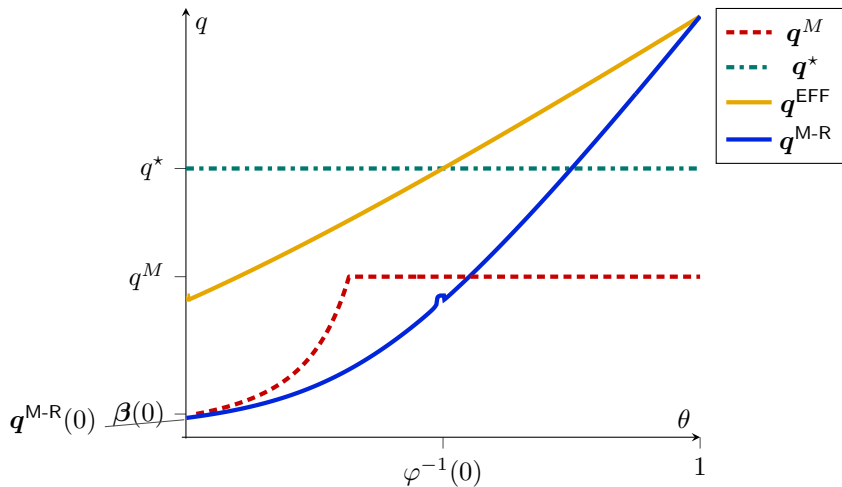
elicit  $\rightarrow$  produce ( $\rightarrow$  damage)

produce  $\rightarrow$  elicit  $\rightarrow$  damage

- ▶ “Invest-then-distribute” Lemma becomes a timing assumption:
  - ▶ seller produces in advance and, after learning the type, can only damage.
- ▶ Free damaging is irrelevant in M-R.
- ▶ No-damaging seller: produce  $\rightarrow$  elicit  ~~$\rightarrow$  damage~~.
- ▶ Payment  $t(\theta)$  and production cost  $c(\sup \mathbf{q}(\Theta))$  are comparable;
- ▶ Efficiency benchmark:  $u_q(q^{\text{EFF}}(\theta), \theta) = c'(q^{\text{EFF}}(\theta)), \uparrow$  in  $\theta$ .

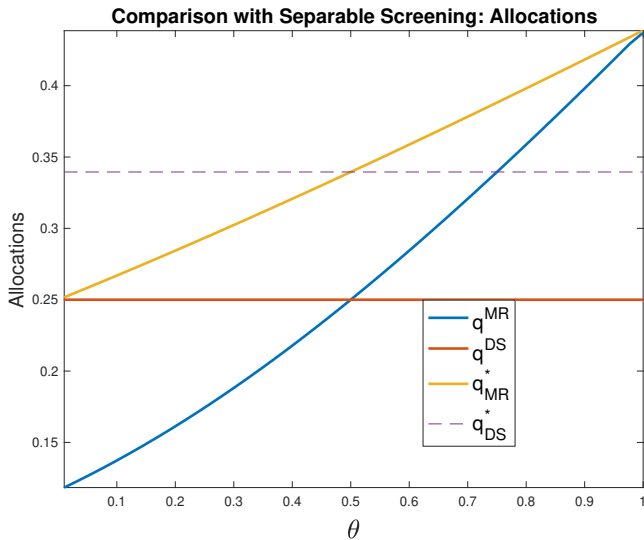


# Single buyer





# Single buyer





# Literature

## **Monopolistic screening**

Mussa and Rosen (1978); Maskin and Riley (1984); Wilson (1993) ...  
Costs are separable.

## **Damaged goods**

Oren, Smith, and Wilson (1985); Deneckere and McAfee (1996); Grubb (2009); Corrao, Flynn, and Sastry (2023).  
Costs are separable, and consumers can damage the good.

## **Pricing of information with buyer's private information**

Bergemann, Bonatti, and Smolin (2018); Bimpikis, Crapis, and Tahbaz-Salehi (2019); Bergemann and Ottaviani (2021); Yang (2022); Bergemann, Cai, Velezgas, and Zhao (2022); Rodríguez Olivera (2024); Bonatti, Dahleh, Horel, and Nouripour (2024) ...

Information is sold to strategic buyers, without production.

Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) ...

Information is allocated without production.

## **Mechanism & information design**

Mensch and Ravid (2025); Thereze (2025).



# Competition



# Competition

$N$  firms play the two-stage game of perfect information with timing:

1. Every firm  $i$  simultaneously chooses a distribution  $H_i$  of her quality, **paying**  $c(q_i)$  for the realization  $q_i$ ;
2. Every firm  $i$  simultaneously chooses a pricing function  $p_i: \mathbb{R}_+ \rightarrow \mathbb{R}$ , with  $p_i(q) = \infty$  if  $q > q_i$ , **observing** realized qualities  $q_1, \dots, q_n$ .

Then, every type  $\theta$  buys one good from a firm, or does not buy any good for a payoff of 0. (tie breaker favors lowest index.)



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The “market” price of  $q$  is  $\min_i p_i(q)$ .

Firms play a subgame-perfect Nash equilibrium.

## Definition 1

An  $n$  *equilibrium* is an equilibrium in which exactly  $n$  firms are active; an  $n$  equilibrium is *symmetric* if active firms play the same strategy.



# Literature

Forms of “market power” that guarantee equilibrium existence (Rothschild and Stiglitz (1976), . . . , Stole (2007)):

1. Incumbent-entrant timing (Borenstein and Rose, 1994; Johnson and Myatt, 2003; Gerardi and Shapiro, 2009; Crawford et al., 2019; Boik and Takahashi, 2020);
2. Rich buyer heterogeneity (Rochet and Stole, 2002; Lehmann et al., 2014; Garrett et al., 2019);
3. “Relevance” assumption (Chade and Swinkels, 2021): every firm has a cost advantage over a quality range.

Related models:

1. Kreps and Scheinkman (1983);  
Quantity-and-price competition, no private information, and Cournot outcome;
2. Champsaur and Rochet (1989);  
Quality range-and-price competition, costless quality ranges, which are disjoint in equilibrium.



## Second stage

Let's order qualities  $(q_1, \dots, q_N)$  so that:  $x \geq y \geq \dots$

**Bertrand force:** Every quality below  $y$  comes at zero price.



## Second stage

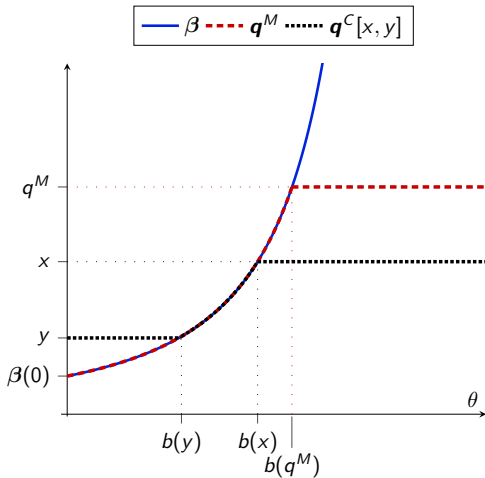
Let's order qualities ( $q_1, \dots, q_N$ ) so that:  $x \geq y \geq \dots$

**Bertrand force:** Every quality below  $y$  comes at zero price.

The allocation in the subgame is  $q^C[x, y]$ , given by:

$$\theta \mapsto \begin{cases} y, & \text{if } \theta \leq b(y), \\ \beta(\theta), & \text{if } \theta \in (b(y), b(x)), \\ x, & \text{if } \theta \geq b(x). \end{cases}$$

**Competitive constraint:** if  $q$  is feasible for some competitor, then  $q$  can be purchased for free. The “monopolist” earns  $V(x) - V(y) - c(x)$





# Competitive equilibria

## Lemma 3

In every pure-strategy equilibrium: one firm produces  $q^M$  and other firms are idle.

Intuition:

1.  $q^M$  is the best response to  $(0, \dots, 0)$ , and  $q \neq q^M$  is **not** a best response to  $(q^M, 0, \dots, 0)$ .
2. In any  $(x, y, \dots)$ ,  $y > 0$  is not a best response, and  $(0, \dots, 0)$  is not an equilibrium.

$\implies$  Every symmetric  $n$  equilibrium is mixed if  $n \geq 2$  (*competitive*.)

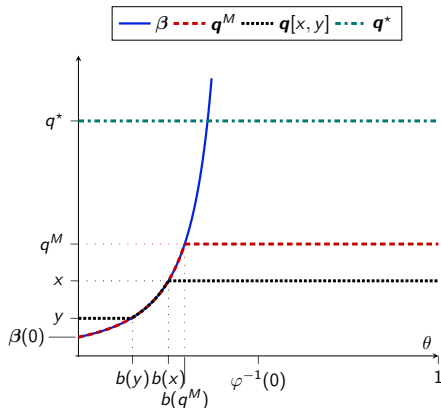


# Competitive equilibria

## Proposition 5

1. For all  $n \leq N$ , there exists a symmetric  $n$  equilibrium.
2. Every symmetric and competitive  $n$  equilibrium induces the random allocation  $\mathbf{q}^C[\hat{x}, \hat{y}]$ , where  $\hat{x}$  and  $\hat{y}$  are the first and second order statistics of the  $n$  i.i.d. draws from the atomless CDF on  $[0, q^M]$  given by

$$H_n(q) = \left( \frac{c'(q)}{V'(q)} \right)^{\frac{1}{n-1}}.$$



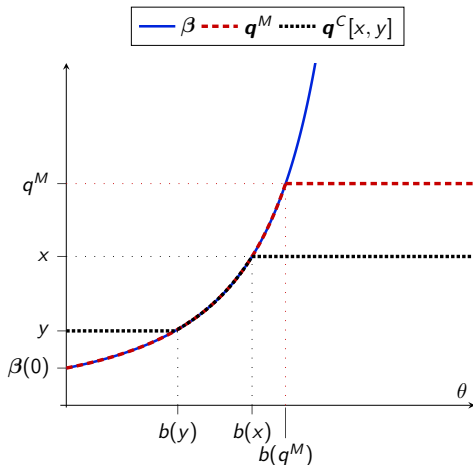


# Competitive equilibria

## Corollary 1

Every symmetric competitive equilibrium leads to an allocation such that, with probability one:

1. The lowest quality is free and  $\geq \beta(0)$ ;
2. The highest quality is strictly lower than  $q^M$ ;
3. The marginally bunched type is lower than  $b(q^M)$ .





# Properties of competitive equilibria

- ▶ Productive inefficiency is stronger;
- ▶ Distributional inefficiency is more prevalent.



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- ▶ Distributional inefficiency is more prevalent.

In the paper:

1. Equilibrium welfare with  $n \geq 2$  active firms decreases in  $n$ .
2. Monopoly dominates duopoly if monopoly fully bunches.
3. Duopoly dominates monopoly if: full bunching does not occur and costs are approximately fixed.



# Conclusion

1. With digital goods, two interdependent inefficiencies arise:  
**productive and damaging.**
2. The *efficiency at the top* insight is revisited:  
**'distributional efficiency for high types.'**



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  - 3.1 Free-replication-and-damaging technology for mass of buyers;
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► More



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Thanks!



**Extra slides**



# Hybrid costs

With more general costs:  $C(\mathbf{q}) = \int_{\Theta} \hat{c}(\mathbf{q}(\theta), \sup \mathbf{q}(\Theta)) dF(\theta)$ ,  
the seller pays:

1. Development / production costs:  $\sup \mathbf{q}(\Theta)$ ;
2. Distribution / replication / damaging costs:  $\mathbf{q}(\theta)$ .

Lemma 1 holds, but the characterization of  $\mathbf{q}^M$  has two complications:

1. Distribution: the solution to  $\mathcal{P}(q)$  does not depend on  $q$  solely through capping;
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If  $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \kappa \log\left(\frac{\sup \mathbf{q}(\Theta)}{\mathbf{q}(\theta)}\right)$ , then **1.** is turned off.



# Damaging costs

If  $C(\mathbf{q}) = c(\sup \mathbf{q}(\Theta)) + \kappa \log\left(\frac{\sup \mathbf{q}(\Theta)}{\mathbf{q}(\theta)}\right)$ , then:

- Production costs + pure-damaging replication / distribution costs;



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- ▶ The efficient allocation is flat: damaging decreases utility and increases costs;
- ▶ The solution to  $\mathcal{P}(q)$  is  $\theta \mapsto \min\{\beta_\kappa(\theta), q\}$ .

1.  $\kappa > 0$  acts as a preference shift ( $\uparrow g$ ) at the distribution stage:

- ▶  $\uparrow \beta_\kappa$  and  $\downarrow b_\kappa(q)$ ;



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2.  $\kappa > 0$  impacts production directly:

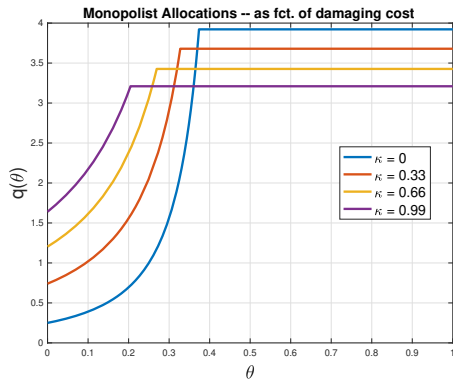
- ▶  $V'(q) = (1 - F(b_\kappa(q)))(g'(q) + b_\kappa(q)) - \kappa \frac{b_\kappa(q)}{q}$ .



# Damaging costs

$\kappa > 0$  implies

1. Less damaging;
2. Lower production.

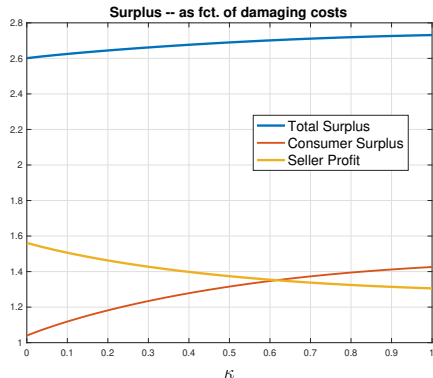




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# Efficiency with general $u$ and $k$

## Proposition 6

The allocation  $\mathbf{q}^*$  is efficient iff  $\mathbf{q}^*(\theta) = \min\{\gamma(\theta), q^*\}$  for all  $\theta$ , in which:  $q^*$  is the unique  $q$  such that  $\int_{[a(q), 1]} u_1(q, \theta) - k'(q) dF(\theta) = c'(q)$ , and  $\gamma$  is an allocation such that  $\gamma(\theta) = \alpha(\theta)$  almost everywhere.

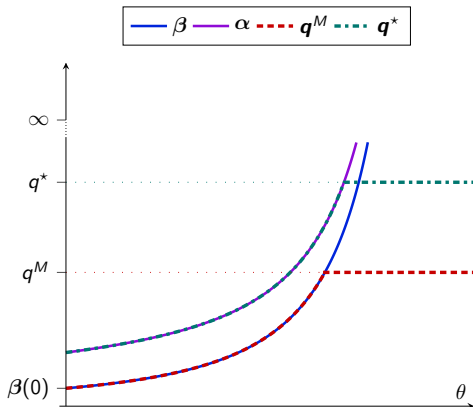
In general,  $q \in [0, \bar{q}]$ , and:

$$J(q, \theta) := u(q, \theta) - \frac{1}{h(\theta)} u_2(q, \theta) - k(q),$$

$\beta(\theta)$  is the largest element of  
 $\operatorname{Argmax}_q J(q, \theta),$

$\alpha(\theta)$  is the largest element of  
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$u$  and  $J$  satisfy incr. differences, and are: twice diff., concave in  $q$  for all  $\theta$ , str. quasiconcave in  $q$  a.e. on  $\Theta$ ;  $k$  is lnada.





# Monopoly with general $u$ and $k$

## Proposition 7

The allocation  $\mathbf{q}^M$  is monopolist iff  $\mathbf{q}^M(\theta) = \min\{\gamma(\theta), q^M\}$  for all  $\theta$ , in which:  $q^M$  is the unique  $q$  such that  $\int_{[b(q), 1]} J_1(q, \theta) dF(\theta) = c'(q)$ , and  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere. Moreover,  $0 < q^M < q^*$ .

In general,  $q \in [0, \bar{q}]$ , and:

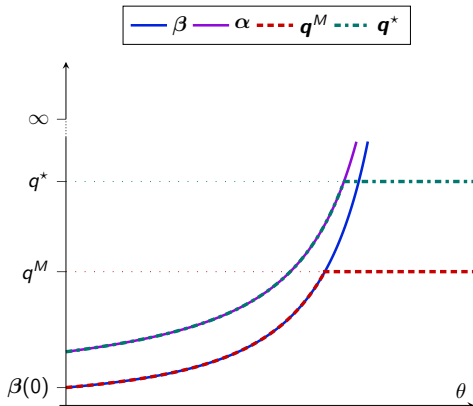
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► Back





# No-damaging monopoly with general $u$ and $k$

Assumption:  $J(0, \theta) = 0$  for all  $\theta$  and  $J(q, \cdot)$  is increasing for all  $q > 0$ .

## Proposition 8

The allocation  $\mathbf{q}_N$  is no screening iff  $\mathbf{q}_N(\theta) = [\theta \geq b_N(q_N)]q_N$  for all  $\theta \neq b_N(q_N)$  and  $\mathbf{q}_N(b_N(q_N)) \in \{0, q_N\}$ , in which  $q_N$  is the unique  $q$  such that:  $\int_{[b_N(q), 1]} J_1(q, \theta) dF(\theta) = c'(q)$ . Moreover, it holds that:

1.  $0 < q_N \leq q^M$ ;
2.  $q_N < q^M$  if  $b(q^M) > b_N(q^M)$ .

We use Iverson brackets:  $[P] = 1$  if  $P$  is true, and  $[P] = 0$  otherwise.



# Productive inefficiency addendum 1/3

Productive inefficiency arises if:

$$\underbrace{V'(q)}_{\substack{\text{Marginal revenues} \\ \text{given } \theta \mapsto \min\{\beta(\theta), q\}}} < \underbrace{g'(q) + \mathbb{E}\{\theta\}}_{\substack{\text{Marginal total utility} \\ \text{given } \theta \mapsto q}} .$$

1. The  $q$  constrained allocation  $\theta \mapsto \min\{\beta(\theta), q\}$  induces total utility

$$U(q) := \mathbb{E}\{u(\min\{\beta(\theta), q\}, \theta)\};$$



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$$U(q) := \mathbb{E}\{u(\min\{\beta(\theta), q\}, \theta)\};$$

2.  $U'(q) < \text{marginal total utility given } \theta \mapsto q$ , because

$$U'(q) = \int_{[b(q), 1]} g'(q) + \theta \, dF(\theta) \leq g'(q) + \mathbb{E}\{\theta\};$$

3. It holds that

$$V'(q) < V'(q) + \text{Rents}'(q) = U'(q).$$



# Productive inefficiency addendum 1/3

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3. It holds that

$$V'(q) < V'(q) + \text{Rents}'(q) = U'(q).$$



## Productive inefficiency addendum 2/3

Productive inefficiency arises because:

$$V'(q) < g'(q) + \mathbb{E}\{\theta\}.$$

The monopoly that produces quality  $q$  implies total surplus

$$V(q) + U(q) - c(q),$$

with  $U(q) = \int_{[0,1]} \int_{[0,\theta]} \min\{\beta(\theta'), \bar{q}\} d\theta' dF(\theta)$  (Envelope Theorem).

The marginal surplus is  $V'(q) + U'(q)$  and satisfies

$$V'(q) < V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}.$$

1. Monopolist does not internalize buyer surplus;
2. Damaging inefficiency.



## Productive inefficiency addendum 3/3

WTS:  $V'(q) + U'(q) \leq g'(q) + \mathbb{E}\{\theta\}$ .

1.  $U'(q) = \int_{[b(q), 1]} \theta - b(q) \, dF(\theta)$ ,  
because the marginal  $u(\mathbf{q}(\theta), \theta)$  increases at rate  $g'(q) + \theta$  and the marginal transfer at rate  $g'(q) + b(q)$ , for  $\theta > b(q)$  and  $\mathbf{q}(\cdot) = \min\{\beta(\cdot), q\}$ ;
2. Using  $V'(q) = (1 - F(b(q)))(g'(q) + b(q))$ , we have

$$V'(q) + U'(q) = (1 - F(b(q)))(g'(q) + b(q)) + \int_{[b(q), 1]} \theta \, dF(\theta).$$

Note that  $U'(q) > 0$  for all  $q > 0$ , because  $b(q) \leq \varphi^{-1}(0) < 1$  for all  $q \geq 0$ .)



# The game

Type  $\theta$  buys quality  $D_p(\theta) \in Q := \mathbb{R}_+$  from firm  $\iota_p(\theta)$ , given the pricing functions in  $(p_1, \dots, p_N) = p$ . The set of firms is  $\mathcal{N} = (1, \dots, N)$ .

The revenues of  $i$  given the pricing functions in  $(p_1, \dots, p_N) = p$  are

$$R_i(p_1, \dots, p_N) := \int_{\{\theta | \iota_p(\theta) = i\}} p_i(D_p(\theta)) \, dF(\theta).$$

The set of pure strategies for firm  $i$  is  $S_i := Q \times \mathbf{P}_i$ , letting  $\mathbf{P}_i \subseteq (\mathbb{R}^Q)^{Q^N}$  be the set of “conditional” pricing functions of firm  $i$ .

The *payoff* of firm  $i$  from the profile  $s := (\dots, (\bar{q}_i^s, P_i^s), \dots) \in \times_{i=1}^N S_i$  is

$$\Pi_i(s) := R_i(P_1^s[\bar{q}^s], \dots, P_N^s[\bar{q}^s]) - c(\bar{q}_i^s).$$



# The equilibrium

A strategy for player  $i$  is a pair of a cap distribution and conditional pricing function,  $(H_i, P_i)$ , in which  $H_i$  is a distribution function with support contained in  $Q$  and  $P_i \in \mathbf{P}_i$  for all  $i$ .

A strategy profile  $((H_1, P_1), \dots, (H_N, P_N))$  is an *equilibrium* if it is a subgame-perfect Nash equilibrium of the game:

1. The conditional pricing functions maximize conditional profits, that is, for all cap profiles  $\bar{q} = (\bar{q}_1, \dots, \bar{q}_N)$ ,

$$R_i(P_i[\bar{q}], P_{-i}[\bar{q}]) \geq R_i(p'_i, P_{-i}[\bar{q}]) \quad \text{for all } p'_i \in \mathbb{R}^Q, i.$$

2. The cap distributions maximize expected profits, that is, for all  $\bar{q}_i$  in the support of  $H_i$  we have

$$\int_{Q^{N-1}} R_i(P_i[\bar{q}], P_{-i}[\bar{q}]) dH_{-i}(\bar{q}_{-i}) - c(\bar{q}_i) \geq \int_{Q^{N-1}} R_i(P_i[\bar{q}'_i, \bar{q}_{-i}], P_{-i}[\bar{q}'_i, \bar{q}_{-i}]) dH_{-i}(\bar{q}_{-i}) - c(\bar{q}'_i)$$

for all  $\bar{q}'_i \in Q$ , letting  $H_{-i}$  denote the joint distribution of the caps of  $i$ 's opponents under  $((H_1, P_1), \dots, (H_N, P_N))$ , and for all  $i$ .



# Auxiliary monopoly problem

The allocation  $\mathbf{q}$  is  $x$ - $y$  *second best*, for qualities  $x, y$  with  $x \leq y$ , if there exists a transfer function  $t(\cdot)$  such that:  $(\mathbf{q}, t(\cdot))$  solves

$$\sup_{\mathbf{q}, t(\cdot)} \int t(\theta) dF(\theta) \text{ subject to: } y \leq \mathbf{q}(\theta) \leq x \text{ for all } \theta \in \Theta.$$

subject to: IC, IR.

## Lemma 4

The allocation  $\mathbf{q}$  is  $x$ - $y$  second best iff  $\mathbf{q}(\theta) = \max\{\min\{\gamma(\theta), x\}, y\}$  for all  $\theta$ , in which  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere.



# The pricing subgame

The pricing game given the quality profile  $(q_1, \dots, q_N)$  is the strategic-form game  $\Gamma(q_1, \dots, q_N) = (\mathcal{N}, (\mathbb{R}^{[0, q_i]}, R_i(\cdot)))_{i \in \mathcal{N}}$ .

The profile of pricing functions  $(p_1, \dots, p_N)$  is a  $(q_1, \dots, q_N)$  *equilibrium* if  $(p_1, \dots, p_N)$  is a Nash equilibrium of  $\Gamma(q_1, \dots, q_N)$ .

We study the subgame starting at the given production profile  $(q_1, \dots, q_N) \in Q^N$  with  $x := \max\{q_1, \dots, q_N\}$  and  $y := \max\{q_1, \dots, q_N\} \setminus \{x\}$ .

## Lemma 5

For every  $(q_1, \dots, q_N)$  equilibrium  $(p_1, \dots, p_N)$ , the following formulas hold:

$$R_i(p_1, \dots, p_N) = (V(q_i) - V(y))_+,$$

and

$$D_{(p_1, \dots, p_N)}(\theta) = \max\{\min\{\gamma(\theta), x\}, y\},$$

for all  $\theta$ , in which  $\gamma$  is a nondecreasing allocation such that  $\gamma(\theta) = \beta(\theta)$  almost everywhere.



# The production game

Every Nash equilibrium of  $\Gamma(q_1, \dots, q_N)$  induces the same revenues, i.e.,

$$R_i(p_1^*, \dots, p_N^*) = R_i(p_1^{**}, \dots, p_N^{**})$$

for all  $(q_1, \dots, q_N)$  equilibria  $(p_1^*, \dots, p_N^*)$ ,  $(p_1^{**}, \dots, p_N^{**})$ . We call  $\bar{R}_i(q_1, \dots, q_N)$  the unique equilibrium revenues of  $i$  in the pricing game  $\Gamma(q_1, \dots, q_N)$ , that is:

$$\bar{R}_i(q_1, \dots, q_N) = (V(q_i) - V(y))_+.$$

The *production game* is the strategic-form game

$\Gamma = (\mathcal{N}, (Q, \bar{R}_i(\cdot) - c(\cdot))_{i \in \mathcal{N}})$ . By mixed strategies, we refer to probability distributions over  $Q$  identified by their distribution functions, and we extend payoffs of  $\Gamma$  to mixed-strategy profiles defining

$$\Pi_i: (\Delta Q)^N \rightarrow \mathbb{R}$$

$$(H_1, \dots, H_N) \mapsto \int_Q \cdots \int_Q \bar{R}_i(q_1, \dots, q_N) - c(q_i) dH_1(q_1) \cdots dH_N(q_N).$$



# Auxiliary result for the production game

Necessary conditions for mixed-strategy equilibria:

## Lemma 6

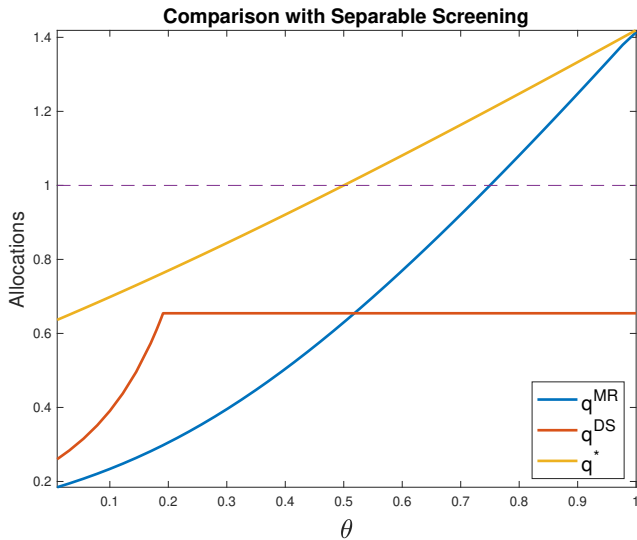
If  $(H_1, \dots, H_N)$  is a Nash equilibrium in mixed strategies with multiple active players, then for all  $i$  with  $H_i(q) > 0$  for some  $q > 0$  we have:

$$\prod_{j \in \mathcal{N} \setminus \{i\}} H_j(q) = \begin{cases} \frac{c'(q)}{V'(q)}, & \text{if } q \in (0, \bar{q}^M), \\ 1, & \text{if } q \geq \bar{q}^M. \end{cases}$$

Note: There are only two kinds of equilibria in mixed strategies that are symmetric among active players: those with at least 2 active players and the monopolistic equilibria.

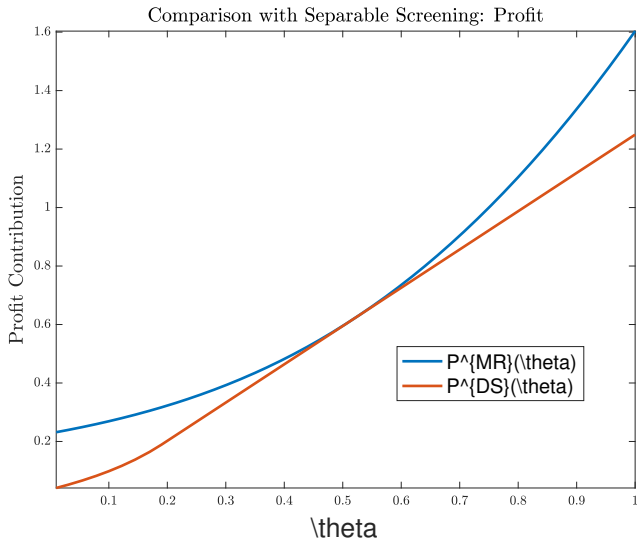


# Single buyer



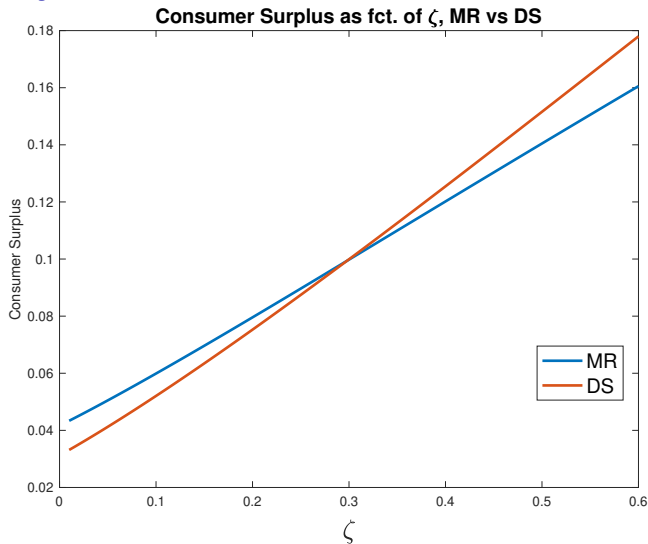


# Single buyer





# Single buyer

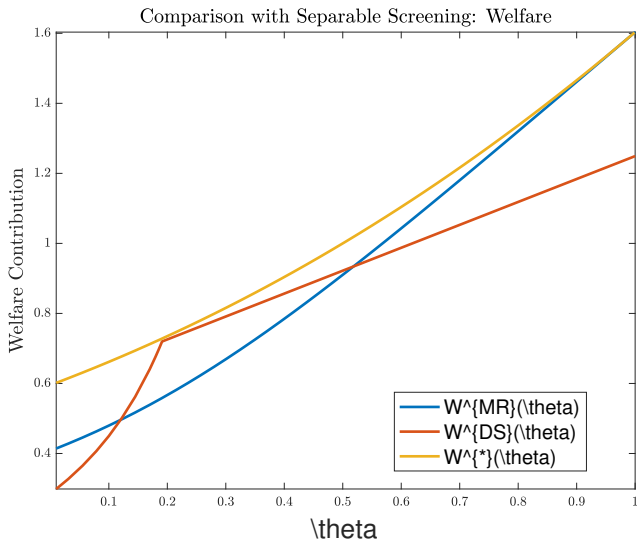


Consumer surplus of  $\mathbf{q}$  is  $\int_{\Theta} \mathbf{q}(\theta)(1 - F(\theta)) d\theta$ , with

$$u(q, \theta) = \zeta g(q) + \theta q.$$



# Single buyer





## References

- Bergemann, Dirk, Alessandro Bonatti, and Alex Smolin (2018), "The design and price of information." *American Economic Review*, 108(1), 1–48.
- Bergemann, Dirk, Yang Cai, Grigoris Velezgas, and Mingfei Zhao (2022), "Is selling complete information (approximately) optimal?" In *Proceedings of the 23rd ACM Conference on Economics and Computation*, EC '22, 608–663, Association for Computing Machinery, New York, NY, USA.
- Bergemann, Dirk and Marco Ottaviani (2021), "Chapter 8 - information markets and nonmarkets." volume 4 of *Handbook of Industrial Organization*, 593–672, Elsevier.
- Bimpikis, Kostas, Davide Cripps, and Alireza Tahbaz-Salehi (2019), "Information sale and competition." *Management Science*, 65(6), 2646–2664.
- Boik, Andre and Hidenori Takahashi (2020), "Fighting bundles: The effects of competition on second-degree price discrimination." *American Economic Journal: Microeconomics*, 12(1), 156–87.
- Bonatti, Alessandro, Munther Dahleh, Thibaut Horel, and Amir Nouripour (2024), "Selling information in competitive environments." *Journal of Economic Theory*, 216, 105779.
- Borenstein, Severin and Nancy L. Rose (1994), "Competition and price dispersion in the u.s. airline industry." *Journal of Political Economy*, 102(4), 653–683.



- Chade, Hector and Jeroen Swinkels (2021), "Screening in vertical oligopolies." *Econometrica*, 89(3), 1265–1311.
- Champsaur, Paul and Jean-Charles Rochet (1989), "Multiproduct duopolists." *Econometrica*, 57(3), 533–557.
- Corrao, Roberto, Joel P. Flynn, and Karthik A. Sastry (2023), "Nonlinear pricing with underutilization: A theory of multi-part tariffs." *American Economic Review*, 113(3), 836–60.
- Crawford, Gregory S., Oleksandr Shcherbakov, and Matthew Shum (2019), "Quality overprovision in cable television markets." *American Economic Review*, 109(3), 956–95.
- Deneckere, Raymond J. and R. Preston McAfee (1996), "Damaged goods." *Journal of Economics & Management Strategy*, 5(2), 149–174.
- Garrett, Daniel F., Renato Gomes, and Lucas Maestri (2019), "Competitive screening under heterogeneous information." *The Review of Economic Studies*, 86(4), 1590–1630.
- Gerardi, Kristopher S. and Adam Hale Shapiro (2009), "Does competition reduce price dispersion? new evidence from the airline industry." *Journal of Political Economy*, 117(1), 1–37.
- Grubb, Michael D. (2009), "Selling to overconfident consumers." *American Economic Review*, 99(5), 1770–1807.



- Johnson, Justin P. and David P. Myatt (2003), "Multiproduct quality competition: Fighting brands and product line pruning." *American Economic Review*, 93(3), 748–774.
- Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk, and Ming Li (2017), "Persuasion of a privately informed receiver." *Econometrica*, 85(6), 1949–1964.
- Kreps, David M. and José A. Scheinkman (1983), "Quantity precommitment and bertrand competition yield cournot outcomes." *The Bell Journal of Economics*, 14(2), 326–337.
- Lehmann, Etienne, Laurent Simula, and Alain Trannoy (2014), "Tax me if you can! optimal nonlinear income tax between competing governments." *The Quarterly Journal of Economics*, 129(4), 1995–2030.
- Maskin, Eric and John Riley (1984), "Monopoly with incomplete information." *The RAND Journal of Economics*, 15(2), 171–196.
- Mensch, Jeffrey and Doron Ravid (2025), "Monopoly, product quality, and flexible learning."
- Mussa, Michael and Sherwin Rosen (1978), "Monopoly and product quality." *Journal of Economic Theory*, 18(2), 301–317.
- Oren, Shmuel, Stephen Smith, and Robert Wilson (1985), "Capacity pricing." *Econometrica*, 53(3), 545–566.



- Rochet, Jean-Charles and Lars A. Stole (2002), "Nonlinear pricing with random participation." *The Review of Economic Studies*, 69(1), 277–311.
- Rodríguez Olivera, Rosina (2024), "Strategic incentives and the optimal sale of information." *American Economic Journal: Microeconomics*, 16(2), 296–353.
- Rothschild, Michael and Joseph Stiglitz (1976), "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information." *The Quarterly Journal of Economics*, 90(4), 629–649.
- Shapiro, Carl and Hal R. Varian (1998), "Versioning: The smart way to sell information." *Harvard Business Review*, 76(6), 106–118.
- Stole, Lars A. (2007), "Chapter 34 price discrimination and competition." volume 3 of *Handbook of Industrial Organization*, 2221–2299, Elsevier.
- Thereze, João (2025), "Screening costly information."
- Wilson, Robert (1993), *Nonlinear pricing*. Oxford University Press.
- Yang, Kai Hao (2022), "Selling consumer data for profit: Optimal market-segmentation design and its consequences." *American Economic Review*, 112(4), 1364–93.