Persuading an inattentive and privately informed receiver

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Abstract

This paper studies the persuasion of a receiver who accesses information only if she exerts costly attention effort. A sender designs an experiment to persuade the receiver to take a specific action. The experiment affects the receiver's attention effort, that is, the probability that she updates her beliefs. As a result, persuasion has two margins: extensive (effort) and intensive (action). The receiver's utility exhibits a supermodularity property in information and effort. By leveraging this property, we prove a general equivalence between experiments and persuasion mechanisms à la Kolotilin et al. (2017). In applications, the sender's optimal strategy involves censoring favorable states.

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1 Introduction

In the "information age," consumers evaluate whether information sources are worth their attention because learning takes effort and time (Simon, 1996; Floridi, 2014). The persuasion literature studies how a sender, such as an advertiser or media outlet, provides information to persuade a receiver to take a specific action (Kamenica, 2019). When attention is costly, the sender faces a dual problem: the receiver can be persuaded only if she pays attention. This paper studies a persuasion model in which the sender's information affects the attention effort of a receiver who privately knows the costs and benefits of information.

The *intensive* margin of persuasion refers to the intensity of the sender's influence on the receiver's action, given that she is attentive, whereas the *extensive* margin refers to whether the receiver pays attention to the information. The study of the extensive margin is important to understand how consumers allocate attention to product advertisements and news content. This allocation of attention ultimately determines the success of marketing campaigns and the spread of information across heterogeneous audiences.

To study the extensive and intensive margins of persuasion, we introduce the receiver's attention decision into a persuasion game between two players: Sender (he) and Receiver (she). In the first stage of the game, Sender designs a signal, a random variable that is jointly distributed with an unknown state. Receiver chooses her attention *effort* knowing the signal's distribution but not its realization. Increasing effort is costly and raises the probability of observing the signal's realization. In the last stage of the game, Receiver takes a binary action: 1 or 0. The players' interests conflict because Receiver chooses action 1 only if she expects the state to exceed her outside option, whereas Sender wants her to choose 1 regardless of the state. The Receiver's outside option and effort cost constitute her private *type*. The outside option reflects the benefits of information, because it is unlikely that a piece of information is useful if the available outside option is extremely beneficial. Similar games are applied to study the persuasion of voters, electoral manipulation, and credit-rating agencies (Alonso and Câmara, 2016; Gehlbach and Simpser, 2015;

Bizzotto and Vigier, 2021).

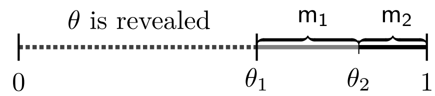
Sender considers that increasing the correlation between the state and the signal affects both the Receiver's attention effort e (the extensive margin) and her action upon observing the signal (the intensive margin). Specifically, Receiver updates her beliefs with probability e and does not update with the remaining probability. Effort represents the acquisition of information and is associated with costs that can be monetary, such as subscription fees, or cognitive, such as mental exertion. This attention model is less general than those with flexible information acquisition (Caplin et al., 2022; Denti, 2022; Pomatto et al., 2023), as Receiver only chooses the probability with which she uniformly observes every signal realization. This parsimonious model accommodates both asymmetric information and a general functional form of effort cost.[1]

In the model, the Receiver's utility is supermodular in information and effort (Corollary 1). In particular, the return from effort increases in a type-specific informativeness order, which is a completion of Blackwell's order. This property is a complementarity between information and attention effort. Complementarity is a feature of information acquisition that is likely to arise from sources like news outlets and advertisements. For instance, when voters' willingness to subscribe to a newspaper increases as the newspaper dedicates more space to election coverage, and when TV audiences pay more attention to informative advertisements. (There is empirical evidence that product awareness increases in the informative content of ads, e.g., Honka et al. (2017); Tsai and Honka (2021).) This paper analyzes the extent of persuasion in such settings.

We establish the equivalence between persuasion mechanisms and signals (Theorem 1). A persuasion mechanism is a menu of signals, one for every Receiver's report of her type. Under a persuasion mechanism, Receiver makes a report and chooses her effort. Specifically, Receiver chooses the probability with which she observes the signal that corresponds to her report. For every persuasion mechanism, there is a signal that induces the same action and effort choices of all Receiver's types. The key step in the proof is to construct a signal that "allocates" to each type the same type-specific informativeness as the mechanism. This step establishes the equivalence with respect

[1] Typical applications of flexible information acquisition rely on functional-form assumptions and define cost functions over belief distributions, which this model avoids — the Receiver's information cost is "experimental" (Denti et al., 2022) because she chooses mixtures of full information and null information about the Sender's signal.

(a) An upper censorship is a signal that reveals the state θ if θ is below a threshold $\overline{\theta}$ and sends a single message, m, if the state is above $\overline{\theta}$.



(b) A bi-upper censorship is a signal that reveals the state θ if θ is below a lower threshold θ_1, sends a message m_1 if the state is between θ_1 and an upper threshold θ_2, and sends a different message m_2 if the state is above θ_2.

Figure 1: An upper censorship (a) and a bi-upper censorship (b), for a state θ with support $[0, 1]$.

to effort choices. The constructed signal also replicates Receiver's optimal action, by simple convex analysis given the representation of signals as convex functions (Gentzkow and Kamenica, 2016). So, the equivalence in Kolotilin et al. (2017) arises as the particular case of costless effort. An a result, an information provider need not offer a fine collection of targeted experiments and the analysis of the extensive margin can be done with single signals.

We characterize the optimal signal in applications and demonstrate that it censors high states. An upper censorship is a signal that reveals low states and pools high states, as shown in Figure 1a. Upper censorships are optimal if the Receiver's outside option follows a single-peaked distribution (Theorem 3). In the costless-attention case, the result follows directly from the shape of the noise in the Receiver's action given her posterior belief. The noise — perceived by Sender — is exogenous and due to asymmetric information. Our result accounts for the endogenous randomness due to the Receiver's choice of effort. Moreover, any equilibrium upper censorship provides less information if effort is costless than if effort comes at a small cost (Proposition 1). We also consider an extension inspired by models of media capture à la Gehlbach and Sonin (2014). In this model, Sender values Receiver's effort directly — i.e., not only because effort ultimately affects the Receiver's action."Bi-upper censorships" are optimal signals (Proposition 2, Figure 1b). In the proof, the additional censorship region allows Sender to separately control the extensive and intensive margins. Overall, these results suggest that attention constraints can act as a push for additional information by interested information providers.

Related literature Existing work considers persuasion without Receiver's information acquisition.[2] The optimality properties of upper censorships are known, and the equivalence between persuasion mechanisms and signals is shown by Kolotilin et al. (2017). We generalize these results to the case of Receiver's costly effort and privately known effort cost. This paper's model is not nested in the fruitful "mean-measurable" paradigm because, in equilibrium, effort is a function of the entire posterior-mean distribution, not of a single posterior mean, as implied by Lemma 2 and the Sender's maximand in Lemma B.4. So, the techniques of Kolotilin (2018) and Dworczak and Martini (2019) do not apply.

The persuasion of an inattentive Receiver is studied without private information. In Wei (2021), Receiver's attention cost is posterior separable. As a result of costly attention and symmetric information, the optimal signal is binary, and, in equilibrium, Receiver pays full attention. In the main model of Bloedel and Segal (2021), Receiver's attention cost is proportional to the entropy reduction in her belief and upper censorships are optimal signals. In a separate model, the authors study the same effort-cost structure as in this paper. The connection with these approches is discussed in Section 6. Certain dynamic models of persuasion include costly Receiver's attention (Liao, 2021; Jain and Whitmeyer, 2022; Au and Whitmeyer, 2023; Che et al., 2023), although the focus of these binary-state models is on the intertemporal flow of information.[3]

Other work studies Receiver's information acquisition with different Sender's incentives or Receiver's sources than in this paper. The "attention-management" literature considers Receiver's attention given a benevolent Sender, who maximizes Receiver's material payoff ignoring attention cost, (Lipnowski et al., 2020, 2022). The literature on persuasion with acquisition of "outside-information" studies Receiver's costs of acquiring extra information beyond what Sender provides (Brocas and Carrillo, 2007; Felgenhauer, 2019; Bizzotto et al., 2020; Dworczak and Pavan, 2022; Matysková and Montes, 2023). The focus is on how payoffs and information change as outside information becomes cheaper. The belief of a "psychological" Receiver arises from an

[2] *Inter alia*: Rayo and Segal (2010); Kamenica and Gentzkow (2011); Kolotilin (2018); Dworczak and Martini (2019). For upper censorships, see also: Gentzkow and Kamenica (2016); Kleiner et al. (2021); Kolotilin et al. (2022); Arieli et al. (2023); Feng et al. (2024); for persuasion mechanisms, see also Guo and Shmaya (2019).

[3] Related research includes the dynamic models in Knoepfle (2020) and Hébert and Zhong (2024), in which Sender only values attention, and the search models in Branco et al. (2016) and Board and Lu (2018).

optimization problem, which typically occurs after the signal realization (Lipnowski and Mathevet, 2018; Galperti, 2019; Beauchêne et al., 2019; de Clippel and Zhang, 2022; Augias and Barreto, 2024) — and not before, as in this paper.

Outline Section 2 describes the model and Section 3 analyzes the Receiver's equilibrium attention and action. Section 4 describes the equivalence between persuasion mechanisms and signals, and Section 5 considers upper censorships and applications. Section 6 discusses alternative approaches of incorporating inattention in information design. Omitted proofs are in Appendix B.

2 Model

2.1 Players, actions, and payoffs

Two players, Sender (he) and Receiver (she), play the following persuasion game. Receiver chooses action $a \in \{0, 1\}$ and effort $e \in [0, 1]$, knowing her type $(c, \lambda) \in [0, 1]^2$. The material payoff of action a, given state $\theta \in [0, 1]$, is $a(\theta - c)$, and the cost of effort e is $\lambda k(e)$, for a continuous function $k\colon [0, 1] \longrightarrow \mathbb{R}$ and given the Receiver's type (c, λ). The *cutoff type* c represents the opportunity cost of taking the risky action, 1, and the *attention type* λ scales the effort cost. The Receiver's utility U_R is her material payoff net of effort cost and is given by

$$U_R(\theta, a, e, c, \lambda) := a(\theta - c) - \lambda k(e).$$

Sender chooses a signal about the state, a measurable $\pi\colon [0, 1] \longrightarrow \Delta M$, in which ΔM is the set of Borel probability distributions over the rich message space M.[4] The Sender's utility is given by $U_S(a) := a$. The results in Section 4 do not depend on the Sender's utility, and in Section 5 we consider a linear function of a and e as Sender's utility.

[4]For this game, letting $M = [0, 1]$ is sufficient (Appendix A.2); the representation of signals as convex functions used in the rest of the paper is in Section 2.3.

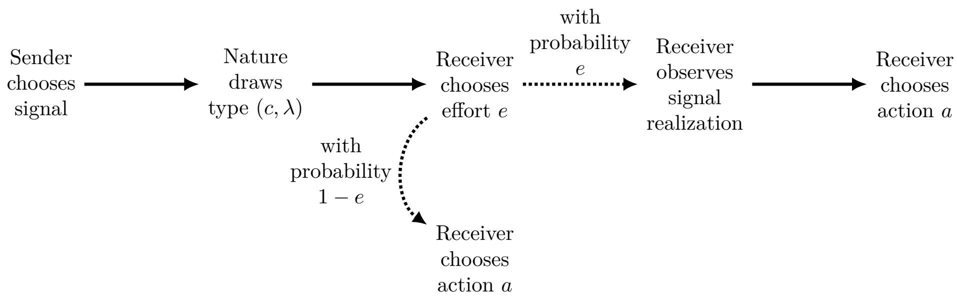


Figure 2: The timeline of the game.

2.2 Information and timing

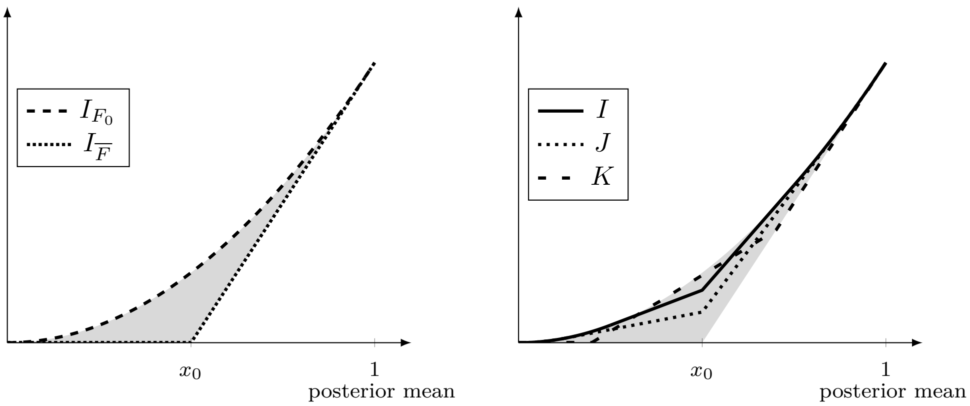
Information The state θ is distributed according to an atomless distribution $F_0 \in \mathcal{D}$, the *prior belief*, with mean x_0, letting $\mathcal{D}$ be the set of distributions over $[0, 1]$ identified by their distribution functions. The Receiver's type is independent of θ and admits a marginal distribution of the attention type λ, $G \in \mathcal{D}$, and a conditional distribution of the cutoff c given λ, $G(\cdot|\lambda) \in \mathcal{D}$.

Timing First, Sender chooses a signal, without knowing either the state or the Receiver's type (c, λ). Second, Receiver chooses effort e, knowing her type (c, λ) and the signal. Third, Nature draws the state θ according to F_0, and the signal realization from $\pi(\theta)$. Afterwards, with probability e, Receiver observes the signal realization, she updates her belief about the state using Bayes' rule and chooses an action given her posterior belief; with probability $1 - e$, Receiver does not observe the signal realization and chooses an action given the prior belief. The equilibrium notion is Perfect Bayesian Equilibrium (Appendix A.2).

2.3 Information policies

Without loss, signals can be represented by the distributions of the posterior belief's mean induced on a Bayesian player who observes the signal realization.[5] Given the presence of Receiver's effort, it pays off to represent signals by the integrals of such distributions, called "information policies".

[5]Signals can be represented by the their posterior-mean distributions in "mean-measurable" models — as this model — with costless Receiver's attention — unlike this model. Appendix C.1 shows that the equivalence holds for this model.



(a) The set $\mathcal{I}$ is the set of convex functions that lie between I_{F_0}, corresponding to a fully informative signal, and $I_{\overline{F}}$, corresponding to an uninformative signal, so that I takes values in the shaded region.

(b) Information policy I is more informative than information policy J in the Blackwell sense iff: $I(x) \geq J(x)$ for all $x \in \mathbb{R}_+$. Information policies K and I are not comparable.

Figure 3: Panel (a) illustrates the set of information policies, panel (b) illustrates the Blackwell's order of information policies; the prior F_0 is a uniform distribution for these figures and the following ones.

Let's define the *information policy of* $F \in \mathcal{D}$ as the function $I_F \colon \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ such that

$$I_F \colon x \longmapsto \int_0^x F(y)\,\mathrm{d}y,$$

the set of feasible distributions $\mathcal{F} := \{F \in \mathcal{D} \mid I_F(1) = I_{F_0}(1),\ I_F(x) \leq I_{F_0}(x) \text{ for all } x \in \mathbb{R}_+\}$, and the set of information policies $\mathcal{I} := \{I \colon \mathbb{R}_+ \longrightarrow \mathbb{R}_+ \mid I \text{ is convex, } I_{\overline{F}}(x) \leq I(x) \leq I_{F_0}(x) \text{ for all } x \in \mathbb{R}_+\}$, in which $\overline{F}$ is the distribution putting full mass at the prior mean. Figure 3 illustrates the set $\mathcal{I}$ and Blackwell's order on $\mathcal{I}$. We identify signals with information policies by the results of Gentzkow and Kamenica (2016) and Kolotilin (2018) stated in Lemma A.1.

Hence, Sender chooses $I \in \mathcal{I}$ in the first stage of the game and the Receiver's posterior mean is drawn from the distribution I' with probability corresponding to her effort, and is equal to x_0 with the remaining probability (Figure 2).

Definition 1. An *equilibrium* is a tuple $\langle I^*, e(\cdot), \alpha \rangle$, in which $I^* \in \mathcal{I}$ is the Sender's information policy, $e(c, \lambda, I) \in [0, 1]$ is the Receiver's effort given her type (c, λ) and information policy I, and $\alpha(c, \lambda, x) \in [0, 1]$ is the probability that Receiver chooses action 1 given type (c, λ) and posterior mean x, in a Perfect Bayesian Equilibrium (Appendix A.2).

Notation We let $I'(x)$ and $\partial I(x)$ denote the right derivative and subdifferential of $I \in \mathcal{I}$ at $x \in \mathbb{R}_+$, respectively. The function $g\colon \mathbb{R}^2 \longrightarrow \mathbb{R}$ exhibits *strictly increasing differences* if $t \longmapsto g(s', t) - g(s, t)$ is increasing for all $s',\ s \in \mathbb{R}$ with $s < s'$.

3 Persuasion

3.1 Receiver's action and effort

This section studies Receiver's equilibrium choices for a given type (c, λ).

Given the posterior mean x, Receiver chooses action 1 if $x > c$ and action 0 if $x < c$. Because $\theta \longmapsto U_R(\theta, a, e; c, \lambda)$ is affine, the Receiver's expected utility from choosing the action optimally given posterior mean x is

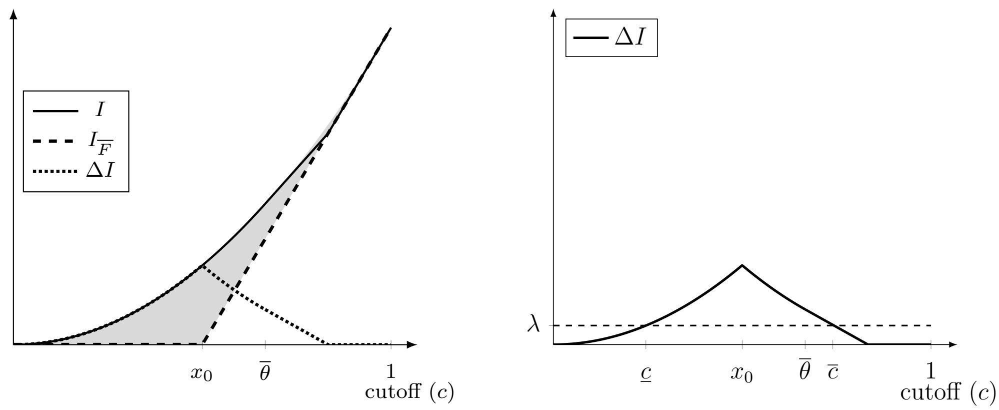
$$U_R(x, e, c, \lambda) := \max_{a \in \{0,1\}} U_R(x, a, e, c, \lambda).$$

To characterize the equilibrium effort, let's define the *marginal benefit of effort given information policy I* as the difference in expected utility with and without the information contained in I: $\int_{[0,1]} U_R(x, e, c, \lambda) - U_R(x_0, e, c, \lambda)\, \mathrm{d}I'(x)$. The marginal benefit of effort given I is also referred to as the value of information in the literature. The *net informativeness of information policy I* is the difference between I and the uninformative-signal information policy, $I_{\overline{F}}$ (Figure 4a). The following result shows that the marginal benefit of effort is given by the net informativeness evaluated at c, using the operator $\Delta\colon I \longmapsto I - I_{\overline{F}}$ to express the net informativeness succintly.

Lemma 1 (Net informativeness)**.** *Given information policy I and Receiver's effort e, the following holds:*

$$\int_{[0,1]} U_R(x, e, c, \lambda)\, \mathrm{d}I'(x) - U_R(x_0, e, c, \lambda) = \Delta I(c).$$

The net informativeness ΔI is single peaked, with a peak at the prior mean x_0, by construction, as in Figure 3b. Intuitively, extreme-cutoff types benefit the least



(a) The net informativeness of I at cutoff c, $\Delta I(c)$, is obtained by subtracting the value of the uninformative-signal information policy at c, $I_{\overline{F}}(c)$, to $I(c)$. The function $c \longmapsto \Delta I(c)$ is single peaked, with peak at the prior mean x_0, by construction.

(b) The marginal benefit of effort equals the marginal cost for cutoff types $\underline{c}$ and $\overline{c}$, given attention type λ (Lemma 1). Receiver chooses effort 1 if $c \in (\underline{c}, \overline{c})$, and does not exert effort if $c \in [0,1] \setminus [\underline{c}, \overline{c}]$.

Figure 4: Panel (a) illustrates the construction of the net informativeness of information policy I, panel (b) illustrates the subset of cutoff types that exert positive effort, given I and linear k. The information policy I is an "upper censorship" in both panels, defined in Section 5.

from observing the signal realization because they are the most certain about the optimal action when left at the prior belief.

The following result characterizes Receiver's equilibrium choices.

Lemma 2 (Receiver's rationality)**.** *If $\langle I^*, e(\cdot), \alpha \rangle$ is an equilibrium, then, for every information policy I:*

1. $1 - \int_{[0,1]} \alpha(c, \lambda, x) \, \mathrm{d}I'(x) \in \partial I(c)$;
2. $e(c, \lambda, I) \in \operatorname{Arg\,max}_{e \in [0,1]} e\Delta I(c) - \lambda k(e)$.

Proof. Part 1. follows from the definition of information policies and the equilibrium properties of α, part 2. follows from Lemma 1 and the equilibrium properties of e. **QED**

The takeaway of Lemma 2 is part 2., which identifies the net informativeness of I at the Receiver's cutoff as a sufficient statistic for her effort decision. As an implication, the two dimensions of Receiver's type, c and λ, represent her private information about, respectively, her benefit and cost of attention. Part 1. restates the equilibrium conditions that the Receiver's action satisfies.

3.2 Interval structure of the extensive margin

This section studies the Receiver's choice of effort.

The Receiver's *value of information policy* I, given type (c, λ) and effort e, is $V_\lambda(e, \Delta I(c)) := e\Delta I(c) - \lambda k(e)$.[6] By Lemma 2, part 2., the Receiver's equilibrium effort maximizes the value of the Sender's information policy, given type (c, λ). The value of I exhibits strictly increasing differences in net informativeness and effort by Lemma 2.

Corollary 1 (Supermodularity)**.** *The Receiver's value of information policy I, $V_\lambda(e, \Delta I(c))$, exhibits strictly increasing differences in e and $\Delta I(c)$.*

As an implication, a more informative Sender's information policy, in the Blackwell sense, makes Receiver better off. In particular, we note that I is Blackwell more informative than J iff: $J \leq I$. So, if I is more informative than J, I allocates more net informativeness to every type than J. Finally, by the increasing-differences property and the envelope theorem (Lemma C.11), Receiver is better off facing I than J.[7] The following result characterizes the set of types that exert positive effort.

Lemma 3 (Interval structure)**.** *Let $\langle I^*, e(\cdot), \alpha\rangle$ be an equilibrium and define the function $e_\lambda \colon c \longmapsto e(c, \lambda, I)$ for information policy I and attention type λ. The set $e_\lambda^{-1}((0, 1])$ is an interval if type (x_0, λ) chooses positive effort, i.e., $e_\lambda(\Delta I(x_0)) > 0$, and is empty otherwise.*

Proof. Let $\langle I^*, e(\cdot), \alpha\rangle$ be an equilibrium, and let's fix $\lambda \in [0, 1]$ and $I \in \mathcal{I}$. We start with three preliminary observations. First, $e(c, \lambda, I)$ equals $e^* \circ \Delta I(c)$ for some selection e^* from $\Delta J(c) \longmapsto \operatorname{Arg\,max}_{e\in[0,1]} V_\lambda(e, \Delta J(c))$, via Lemma 2. Second, every selection from $\Delta J(c) \longmapsto \operatorname{Arg\,max}_{e\in[0,1]} V_\lambda(e, \Delta J(c))$ is nondecreasing, because V_λ satisfies strictly increasing differences, via Corollary 1 and known results (Topkis, 1978, Theorem 6.3). From these observations, it follows that $e^* \circ \Delta I$ is nondecreasing

[6] Receiver's expected utility equals $V_\lambda(e, \Delta I(c))$ plus a constant, because we have $V_\lambda(e, \Delta I(c)) = \int_{[0,1]} U_R(x, e, c, \lambda)\, \mathrm{d}I'(x) + x_0 - c + I_{\overline{F}}(c)$.

[7] This observation is also an implication of Blackwell's theorem; Corollary 1 is a stronger result which we need for the results in Section 4.

on $[0, x_0]$ and nonincreasing on $[x_0, 1]$ because ΔI is nondecreasing on $[0, x_0]$ and ΔI is nonincreasing on $[x_0, 1]$.

If $e^*(\Delta I(x_0)) = 0$, then every cutoff c has $e^*(\Delta I(c)) = 0$, by the above observations. Let's suppose that $e^*(\Delta I(x_0)) > 0$. We define $\underline{c}_\lambda(\Delta I) = \sup\{c \in [0, x_0] : e^* \circ \Delta I(c) = 0\}$, if $\{c \in [0, x_0] : e^* \circ \Delta I(c) = 0\} \neq \emptyset$, and $\underline{c}_\lambda(\Delta I) = 0$ otherwise. We define $\overline{c}_\lambda(\Delta I) = \inf\{c \in [x_0, 1] : e^* \circ \Delta I(c) = 0\}$, if $\{c \in [x_0, 1] : e^* \circ \Delta I(c) = 0\} \neq \emptyset$, and $\overline{c}_\lambda(\Delta I) = 1$ otherwise. First, we note that $e^* \circ \Delta I(c) > 0$ only if: $c \in [\underline{c}, \overline{c}]$; second, $c \in (\underline{c}, \overline{c})$ only if $e^* \circ \Delta I(c) > 0$. Thus, either no type (c, λ) chooses positive effort or $e_\lambda^{-1}((0, 1])$ is an interval. The lemma follows from the fact that $\Delta I(x_0) \geq \Delta I(c)$ for all $c \in [0, 1]$. **QED**

For intuition, let's assume linear effort cost, i.e., $k(e) = e$, capturing a market price or fixed cost of information. Receiver compares the marginal cost and marginal benefit of effort. As shown in Figure 4b, in equilibrium, exerting effort 1 is optimal only if $\Delta I(c) \geq \lambda$, and no effort is optimal only if $\Delta I(c) \leq \lambda$. Moreover, the net informativeness of I at a cutoff is single peaked as a function of the cutoff (Figure 3). As an implication, the set of cutoff types that exert positive effort is an interval. (The effort of indifferent type is not relevant in equilibrium for atomless cutoff distributions, by Lemma B.4.) The proof of Lemma 3 generalizes the first part of the argument. Specifically, the Receiver's effort is nondecreasing in net informativeness at her cutoff type, by supermodularity of V_λ through comparative statics à la Topkis (1978).

4 Persuasion mechanisms

This section studies the equivalence between information policies and persuasion mechanisms.

Definition 2. A *persuasion mechanism* $I_\bullet$ is a list of information policies: $I_\bullet = (I_r)_{r \in R}$, with R equal to the support of the Receiver's type. A persuasion mechanism $I_\bullet$ is *incentive compatible* (IC) if

$$\max_{e \in [0,1]} V_\lambda(e, \Delta I_{(c,\lambda)}(c)) \geq \max_{e \in [0,1]} V_\lambda(e, \Delta I_r(c)),$$

for every type (c, λ) and report r.

Our focus on IC mechanisms references to an auxiliary game of screening. First, Sender publicly commits to a mechanism that selects an information policy for every *type report*. Second, Receiver reports a type $r \in R$, knowing her true type (c, λ). The rest of the game proceeds as in Section 2.2: Receiver chooses effort e, then she observes the realization of a signal corresponding to information policy I_r with probability e, and lastly chooses an action. We are interested in equilibria in which Receiver truthfully reports the type, which is without loss by a revelation-principle argument.

We consider a persuasion mechanism $I_\bullet$ to be implementable by information policy J if all types chooses the same action and effort under truthful reporting given $I_\bullet$ as in some equilibrium of the subgame that starts with the Sender's choice of information policy J (Section 2.2).

Definition 3. An IC persuasion mechanism $I_\bullet$ is *equivalent to information policy J* if, for every type (c, λ):

1. $\operatorname*{Arg\,max}_{e \in [0,1]} V_\lambda(e, \Delta I_{(c,\lambda)}(c)) \subseteq \operatorname*{Arg\,max}_{e \in [0,1]} V_\lambda(e, \Delta J(c))$,
2. $\partial I_{(c,\lambda)}(c) \subseteq \partial J(c)$ if $(0,1] \cap \operatorname*{Arg\,max}_{e \in [0,1]} V_\lambda(e, \Delta I_{(c,\lambda)}(c)) \neq \emptyset$.

If effort is costless, Definition 3 is the same as in Kolotilin et al. (2017, p. 1954). The novelty is item 1., which requires type (c, λ) to choose the same effort under $I_\bullet$ as under the signal that implements $I_\bullet$. Item 2. in Definition 3 does not deal with types who exert effort 0 under truthful reporting given $I_\bullet$. The reason is that the equilibrium action given the prior belief does not depend on Sender's information.[8]

Every IC persuasion mechanism is equivalent to a signal.

Theorem 1. *Every IC persuasion mechanism is equivalent to an information policy.*

This result guarantees that the characterization of the extensive margin of persuasion in Section 3 holds in more general environments, including applications in which multiple information structures are available to decision-makers.

[8]Formally, the reason is that the equivalence of the action decision holds as a consequence of item 1. "for this type." Specifically, $\operatorname{Arg\,max}_{e\in[0,1]} V_\lambda(e, \Delta I_{(c,\lambda)}(c)) = \{0\}$ implies that $0 \in \operatorname{Arg\,max}_{e\in[0,1]} V_\lambda(e, \Delta J(c))$ by item 1., and the optimal action at the prior belief given $I_\bullet$ is the same as given J, possibly via equilibrium selection.

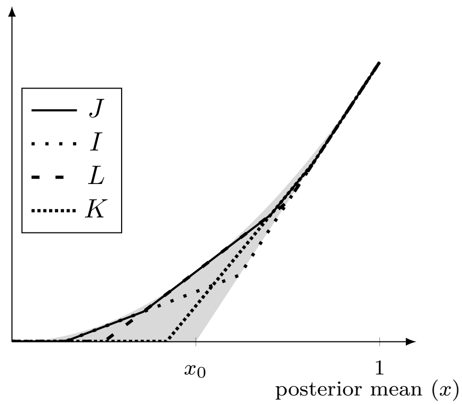


Figure 5: The upper envelope J of the information policies in the persuasion mechanism $I_\bullet = (I, L, K)$.

We sketch the intuition and proof of Theorem 1, which leverage Corollary 1. The proof verifies that supermodularity is key by establishing the result for more general Receiver's payoff functions (Appendix B.2). Let's claim that the IC mechanism $I_\bullet$ is equivalent to its upper envelope J (Figure 5), defined as $J\colon x \longmapsto \sup_{r\in R} I_r(x)$. Let's also fix Receiver's type (c, λ) exerting positive effort. A report r is *active* at x if $I_r(x) \geq I_{r'}(x)$ for all $r' \in R$. First, we observe that an active report at c maximizes Receiver's expected utility. By Lemma 2, report r affects Receiver's utility only through the local net informativeness $\Delta I_r(c)$. By increasing differences, an active report at c makes type (c, λ) weakly better off than any other report (Corollary 1, via the envelope theorem for supermodular programming, Lemma C.11.) Hence, an active report at c maximizes Receiver's expected utility at the reporting stage.

Towards the equivalence with respect to effort, we strengthen the observation: Receiver is *strictly* worse off with an inactive report than with an active report. This conclusion uses both the fact that Corollary 1 establishes *strictly* increasing differences and type (c, λ)'s positive effort (Lemma C.11). To build on this conclusion, let's order information policies according to the type-specific relation $\leq_c$, defined by $\hat{I} \leq_c \hat{J}$ iff $\Delta\hat{I}(c) \leq \Delta\hat{J}(c)$. The linear order $\leq_c$ is a completion of Blackwell's order and ranks the menu's items according to Receiver's expected utility. By the IC property of the mechanism $I_\bullet$, the policy I_r maximizes $\leq_c$ on $I_\bullet$ only if $\Delta I_r(c) = \Delta I_{(c,\lambda)}$.[9] Hence, $J(c) = I_{(c,\lambda)}(c) \geq I_r(c)$, for every report r. An application of Lemma 2

[9] Blackwell's theorem does not suffice for this conclusion, which uses (i) Corollary 1, (ii) the envelope theorem (Appendix, Lemma C.11), and (iii) completeness of $\leq_c$.

completes the argument for the equivalence with respect to effort. In particular, the net informativeness, $\Delta J(c)$, is the only component of the information policy $I_{(c,\lambda)}$ that affects the effort decision in the IC mechanism $I_\bullet$.

The equivalence with respect to action decisions follows from simple convex analysis, due to our results. In particular, we have that $\partial I_r(x) \subseteq \partial J(x)$ if report r is active at x. The proof uses a continuity argument to cover the case of zero effort.

5 Optimality properties of upper censorships

This section discusses the properties of the following class of information policies.

Definition 4. The $\overline{\theta}$ *upper censorship*, for state $\overline{\theta} \in [0, 1]$, is the unique information policy $I_{\overline{\theta}} \in \mathcal{I}$ such that

$$I_{\overline{\theta}}(x) = \begin{cases} I_{F_0}(x), & x \in [0, \overline{\theta}] \\ \max\{I_{F_0}(\overline{\theta}) + F_0(\overline{\theta})(x - \overline{\theta}), I_{\overline{F}}(x)\}, & x \in (\overline{\theta}, \infty). \end{cases}$$

The case of a single-peaked marginal distribution of the cutoff type is relevant for applications (Romanyuk and Smolin, 2019; Kolotilin et al., 2022; Gitmez and Molavi, 2023; Shishkin, 2024; Augias and Barreto, 2024; Sun et al., 2024).

Assumption 1. The conditional distribution of the cutoff type given attention type λ admits a density $g(\cdot|\lambda)$ such that: (i) $g(\cdot|\lambda)$ is absolutely continuous, and (ii) there exists $p \in [0, 1]$ such that: for all λ, $g(\cdot|\lambda)$ is nondecreasing on $[0, p]$ and nonincreasing on $[p, 1]$.

The class of single-peaked distributions includes the standard uniform and the $[0, 1]$-truncated normal. The assumption rules out the symmetric-information benchmark, which is treated in Appendix. We say that *strict single-peakedness* holds if: Assumption 1 holds and $g(\cdot|\lambda)$ is increasing on $[0, p]$ and decreasing on $[p, 1]$.

We first establish that an equilibrium exists and that the Sender's equilibrium expected utility is unique.

Theorem 2. *Under Assumption 1, there exists an equilibrium and the Sender's expected utility is the same in every equilibrium.*

In the Appendix (Lemma B.4), we establish that continuity of the cutoff distribution ensures that Sender is indifferent among all Receiver's best responses.[10]

The following result shows that an optimal signal that is an upper censorship exists.

Theorem 3. *Under Assumption 1, there exists an equilibrium in which the Sender's information policy is an upper censorship.*

Given Theorem 1, Theorem 3 shows that the extensive margin of a complicated optimal persuasion mechanism can be studied via an upper censorship. Moreover, Theorem 3 reduces the Sender's optimization to a uni-dimensional problem.

In the case of costless attention and Sender-optimal equilibria, the argument for Theorem 3 rests on the shape of the exogenous noise in Receiver's action given a posterior belief, from the Sender's viewpoint. The Sender's expected utility at posterior mean x is $H(x)$, letting H denote the distribution of the cutoff type. By single-peakedness, H is "S shaped." So, Sender is risk lover conditionally on low posterior means, i.e., $x < p$, and he is risk averse around high posterior means. In particular, a mean-preserving spread around a low posterior mean increases his expected utility. Second-order dominance is related to the informativeness of Sender's signal because: $F \in \mathcal{F}$ is a mean-preserving spread of $\hat{F} \in \mathcal{F}$ iff I_F is more Blackwell informative than $I_{\hat{F}}$. Moreover, the upper censorship $I_{\overline{\theta}}$ induces either full information conditionally on the state being lower than the threshold $\overline{\theta}$, or no information except that $\theta > \overline{\theta}$. Hence, intuitively, upper censorships induce posterior-mean distributions that align with the Sender's interests.

Let's adjust the intuition for the case of endogenous effort, i.e., in which the relevant information policy is $x \longmapsto eI(x) + (1-e)I_{\overline{F}}(x)$ if the Receiver's effort is e. We claim that effort is affected by the signal's informativeness in a way that aligns with the Sender's interests. Let's suppose that Sender increases the net informativeness of posterior mean x: $\Delta I(x)$. This change induces cutoff type x to pay extra attention, via the envelope theorem for supermodular optimization (lemmata 2 and C.11.) If cutoff type x increases her effort, she gathers more information, because $x \longmapsto eI(x) + (1-e)I_{\overline{F}}(x)$ increases in the Blackwell's order as e increases. Thus, by

[10] Lipnowski et al. (2024) show that uniqueness obtains in a general model, which does not nest ours. Their Corollary 1 is similar to our observation, even if our proof leverages the convexity of (i) information policies and (ii) Receiver's interim utility $a \longmapsto \max_{e\in[0,1]} V_\lambda(e,a)$. The latter result obtains from the envelope theorem for supermodular optimization, Lemma C.11.

increasing the net informativeness, Sender spreads out the Receiver's posterior-mean distribution around x. This argument, however, is "local." Specifically, the net informativeness $\Delta I(x)$ increases only via switching to an information policy satisfying the convexity constraint in $\mathcal{I}$. The proof addresses this point by constructing an upper censorship that improves upon I, for arbitrary I.

The following result shows that Sender provides more information as Receiver's attention cost increases, for small attention costs. We say that $I \in \mathcal{I}$ is *optimal* if there exists an equilibrium in which Sender chooses I.

Proposition 1. *Let strict single-peakedness hold, F_0 admit a density, k be linear, and the attention type put full mass at λ. Let I_{θ_ε} be an optimal upper censorship if $\lambda = \varepsilon$, and I_η be an optimal upper censorship if $\lambda = 0$, with $\eta \in (0,1)$. We have: $\theta_\varepsilon > \eta$ for all sufficiently small $\varepsilon > 0$.*

The same qualitative result holds in Wei (2021, Proposition 7). Let's describe the intuition in the symmetric-information benchmark, for $c > x_0$. Sender solves the maximization of the Receiver's action subject to the constraint that she exerts effort 1. Let's claim that the "participation constraint" binds (Lemma B.5). Let's suppose this were not the case. Sender increases the probability of a posterior mean x with $x \geq c$ as much as possible. Specifically, he induces the mean $x = c$ with the highest probability that satisfies Bayes' rule (Kamenica and Gentzkow, 2011). Hence, Receiver faces two contingencies: either she is indifferent between the actions or she finds it optimal to go for the the riskless action. So, information brings no value, which is a contradiction: the constraint binds. Thus, Sender provides "better" information if $\lambda > 0$ than if $\lambda = 0$. Proposition 1 shows that the insight generalizes, for small λ. In general, a change in the censorship state θ_ε affects the extensive margin because of private information. However, only the extensive margin's upper bound ($\bar{c}$ in Figure 4b) is affected by small changes in θ_ε around η, because a nontrivial upper censorship is optimal if $\lambda = 0$.[11] This argument leads to Proposition 1.

In applications to media capture, Sender cares directly about Receiver's attention (Gehlbach and Sonin, 2014). In this case, Sender is a dictator and owns a state's media, so he collects advertisement revenues. The next result shows that an extension of the class of upper censorships contains an optimal information policy in these

[11] The net informativeness of $I_{\overline{\theta}}$ is 0 at a cutoff type weakly greater than the conditional expectation of θ given $\theta \geq \overline{\theta}$ (Figure 4).

applications.[12] A *bi-upper censorship* is an information policy I such that

$$I(x) = \begin{cases} I_{F_0}(x), & x \in [0, \theta_1], \\ I_{F_0}(\theta_1) + F_0(\theta_1)(x - \theta_1), & x \in (\theta_1, x_1], \\ I_{\overline{F}}(x_2) - m(x_2 - x), & x \in (x_1, x_2], \end{cases}$$

for $m = \frac{I_{\overline{F}}(x_2) - [I_{F_0}(\theta_1) + F_0(\theta_1)(x_1 - \theta_1)]}{x_2 - x_1}$ and $0 \leq \theta_1 \leq x_1 \leq x_2 \leq 1$ (Figure 1).

Proposition 2. *Let Assumption 1 hold with $p \geq x_0$, k be linear, the attention type put full mass at λ, and the Sender's utility be given by $U_G(\theta, a, e, c, \lambda) := a + \gamma e$ for $\gamma \geq 0$. For every equilibrium $\langle I, e(\cdot), \alpha \rangle$, there exists a bi-upper censorship with a weakly greater Sender's expected utility than I, given $e(\cdot)$ and α.*

The intuition clarifies that the additional threshold state is constructed to increase the marginal benefit of effort of certain types in case $I_{\overline{\theta}}$ induces fewer cutoff types than I to exert effort. The proof constructs a bi-upper censorship that improves upon an arbitrary information policy in terms of expected Receiver's action, utility and extensive margin. First, we construct an upper censorship $I_{\overline{\theta}}$ that improves upon a given I for $\gamma = 0$, thanks to the same intuition as for Theorem 3. Second, we take into account the endogeneity of the extensive margin: we modify $I_{\overline{\theta}}$ in a way that replicates the extensive margin of I by censoring extreme states on either sides of the state space. At this stage, we have a candidate "improved" information policy that is not a bi-upper censorship. As a last step, we leverage single-peakedness to note that increasing the lower bound of the extensive margin is beneficial for Sender, as in the discussion following Proposition 1. The lower bound is maximized by choosing to fully reveal low states, so this argument returns a bi-upper censorship.

The Sender's preferences are introduced by Gehlbach and Sonin (2014), who assume binary state and Sender's signal. The case of $\gamma = 0$ is studied by Kolotilin et al. (2022), who show that upper censorships are optimal signals for costless attention. The requirement that the cutoff's peak satisfies $p \geq x_0$ represents sufficient ex-ante disagreement between Sender and Receiver, as in Shishkin (2024) and for symmetric cutoff densities.

[12] Equilibrium existence is not established for this extension. The difficulty lies in establishing continuity of the extensive margin — i.e., continuity of $F \longmapsto \underline{c}_\lambda(\Delta I_F)$ and $F \longmapsto \overline{c}_\lambda(\Delta I_F)$, defined in the proof of Lemma 3 — when $\mathcal{F}$ is endowed with the L^1-norm topology.

6 Discussion and interpretation

The term $\lambda k(e)$ in the Receiver's utility represents her attention cost. In particular, let's consider e as representing an attention effort and look at the effort-choice stage for nondecreasing k. An increase in attention effort results in a more informed Receiver in the Blackwell's sense (Figure 3) and higher costs. The general functional form of effort cost allows the model to capture a range of attention- and non-attention-related phenomena. Examples of costly attention include cognitive difficulties and memory limits. In contrast, the opportunity cost of being attentive is relevant when evaluating media subscription or exposure.

Costless attention Kolotilin et al. (2017) study the special case of the model in which the distribution of λ puts full mass at 0. There exists an optimal signal that is an upper censorship for a single-peaked distribution of the cutoff type and signals are equivalent to persuasion mechanisms.

Symmetric information Receiver does not have private information if the type distribution is degenerate. Bloedel and Segal (2021) and Wei (2021) propose alternatives to this symmetric-information model. In Bloedel and Segal (2021), the cost of attention is proportional to an expected entropy reduction in Receiver's belief (described in the following paragraph), and the Receiver's strategy space contains the present one: Receiver chooses any signal about the Sender's signal realization. The optimal Sender's signal is an upper censorship, although for a different reason than in this model. In particular, Sender perceives Receiver's action as random, given a signal realization, because of her attention strategy; in this model, instead, the randomness arises due to both the Receiver's effort and asymmetric information (as discussed in Section 5.) Bloedel and Segal also consider our symmetric-information benchmark, as an alternative to their model. In Wei (2021), the state is binary, the Receiver's attention cost is posterior separable (described in the following paragraph) and her strategy space is fully general. Wei shows that the same message as Proposition 1 holds: Sender provides more information as attention becomes costlier.

Alternative models of information design with costly attention The literature offers several models of information design with costly Receiver's attention and motivations that overlap with ours: Lipnowski et al. (2020); Bloedel and Segal (2021); Dworczak and Pavan (2022); Matysková and Montes (2023).

In Lipnowski et al. (2020) (LMW), the attention cost is proportional to the reduction in the uncertainty about the state. Receiver incurs a cost for what she learns about the state. LMW is a model of *"delegated"* learning (Bloedel and Segal, 2021), which fits applications with: a separate entity from Receiver researching about θ and Receiver learning through that research. As an illustration, LMW captures the problem of a firm (Receiver) that processes data provided by an information intermediary (Sender). Wei (2021) applies this paradigm to study state-independent Sender's preferences.

In the main model of Bloedel and Segal (2021) (BS), the attention cost is proportional to the entropy reduction of the Receiver's belief about the Sender's message.[13] Receiver incurs a cost for what she learns about the Sender's talk. BS is a model of learning *from communication*, fitting applications in which communication is costly to process. As an illustration, BS captures the problem of a social-media user (Receiver) who learns from the advertisement of an influencer (Sender) at a cost that involves deciphering words and situations portrayed in the ad.

In this model, the attention cost is independent of the information provided by Sender, and fully flexible in this class. Receiver incurs a cost for exposure to the Sender's communication. We model learning *via exposure*, fitting applications in which the Receiver's strategy has a cost irrespectively of Sender's information provision. Paying full attention to a communication that turns out uninformative is allowed to have any cost here, whereas this strategy is costless in BS and LMW. As an illustration, this model captures the problem of a platform user (Receiver) who devotes a share of her mental energy to learning about current affairs from her news feed engineered by the platform (Sender). If the feed contains only friends' updates and product ads, searching for news is both costly and fruitless. The independence restriction is well suited for abstract applications, in which cognitive costs are addressed less granularly than in BS-LMW and possibly aggregated with costs of different nature.

Our model builds upon BS and LMW by constraining the Receiver's strategies to mixtures of full and null information about the Sender's message. In the rational-inattention tradition (Sims, 2003), the Receiver of BS-LMW flexibly allocates her

[13] We describe the main model of BS with state-independent Sender's preferences and entropy-based cost, even though the paper includes other preferences and costs.

cognitive resources — she can learn in any conceivable way about the Sender's message —, but such flexibility comes with tracking multiple signal structures and using extensions of entropy-based costs. By focusing on effort as a single attention variable, our framework abstracts from these complexities while preserving the fundamental tradeoff of rational inattention. By extending the "constrained BS-LMW", we analyze additional questions related to screening and the shape of the extensive margin, which complement the current literature. Moreover, departures from flexibility reflect real-life psychological and technological constraints. For instance, a consumer may only choose the time and mental energy to spend in front of the TV and a voter may choose how many articles to sample randomly and learn fully from in a newspaper. Lastly, rational inattention is not the only explanation for costly effort, which could refer to the opportunity cost of learning or a transfer paid to "infomediaries" — including Sender, as in Proposition 2 — in applications.

In Matysková and Montes (2023), Receiver pays a cost to access additional information beyond what the Sender provides. In Dworczak and Pavan (2022), Receiver may have access to extra information sources than just the Sender's one. These models target a fundamentally different strategic context than ours and fit applications in which the Sender's communication is costless to understand. The Sender's tradeoff involves (i) inducing favorable actions and (ii) preventing the access to external information that may hinder (i). In the model with binary state and action of Matysková and Montes, Sender provides more information as the Receiver's cost increases, similarly as in Proposition 2. However, the channel is different: in Matysková and Montes (2023), Sender provides more information so as to disincentivize (extra) attention, whereas here Sender does it to incentivize attention.

7 Conclusion

This paper proposes a model of inattention within the canonical persuasion framework, which underscores the complementarity between information and attention effort. This complementarity leads to the equivalence of persuasion mechanisms and experiments. The sender's optimization problem is solved by censoring favorable states, a strategy relevant in contexts in which attention is directly valued, such as media capture.

In general, complementarity may hold only "locally", across audiences and information structures, for instance due to information overload and psychological constraints. A study of the extensive margin of persuasion that incorporates these distinctions offers an open avenue for future research.

Appendix

A Equilibrium

A.1 The equivalence between signals and information policies

Lemma A.1. *The following hold:*

1. *If $F \in \mathcal{F}$, then $I_F \in \mathcal{I}$;*

2. *If $I \in \mathcal{I}$, then $I' \in \mathcal{F}$, extending I to take value 0 at every $x < 0$.*

Proof. See Gentzkow and Kamenica (2016) and Kolotilin (2018). **QED**

A.2 Equilibrium definition

We define a Perfect Bayesian Equilibrium in which Sender directly chooses an experiment $F \in \mathcal{F}$. From Section C.1, this approach is without loss. From Lemma A.1, the equilibrium notion is essentially the same as in the text (Section 2.2). Let T denote the support of Receiver's type. Given $F \in \mathcal{F}$ and effort $\varepsilon \in [0,1]$, we define $\varepsilon \odot F = \varepsilon F + (1-\varepsilon)\overline{F}$, and note that $\varepsilon \odot F \in \mathcal{F}$. An equilibrium is a tuple $\langle F, e, \alpha \rangle$, in which $F \in \mathcal{F}$, $e(\cdot, \hat{F}) \colon T \longrightarrow [0,1]$ is measurable for all $\hat{F} \in \mathcal{F}$, $\alpha(\cdot, x) \colon T \longrightarrow [0,1]$ is measurable for all $x \in [0,1]$, and $\alpha(c, \lambda, \cdot) \colon [0,1] \longrightarrow [0,1]$ is measurable for all $(c, \lambda) \in T$, such that:

1. α satisfies a Opt:

$$\alpha(c, \lambda, x) > 0 \text{ only if } 1 \in \operatorname*{Arg\,max}_{a \in \{0,1\}} a(\theta - c)$$

   for all $x \in [0,1]$, $(c, \lambda) \in T$;

2. e satisfies e Opt:

$$e(c, \lambda, \hat{F}) \in \operatorname*{Arg\,max}_{e \in [0,1]} \int_{[0,1]} \max_{a \in \{0,1\}} U_R(x, a, e, c, \lambda) \, \mathrm{d}(e(c, \lambda, \hat{F}) \odot \hat{F})(x)$$

   for all $(c, \lambda) \in T$, $\hat{F} \in \mathcal{F}$;

3. F is rational for Sender given (α, e), that is: F maximizes

$$\hat{W}(\cdot, e, \alpha) \colon \hat{F} \longmapsto \int_{[0,1]} \int_{[0,1]} \int_{[0,1]} \alpha(x, c, \lambda) \, \mathrm{d}(e(c, \lambda, \hat{F}) \odot \hat{F})(x) \, \mathrm{d}G(c|\lambda) \, \mathrm{d}G(\lambda)$$

on $\mathcal{F}$.

The set of maximizers in e Opt is nonempty because the function $e \longmapsto U_R(x, a, e, c, \lambda)$ is continuous for all x, a, c, λ. Lemmata B.2 and B.4 establish that the maximization in (3.) is well-defined, given (α, e) satisfying items (1.) and (2.).

B Proofs

We endow $\mathcal{F}$ with the L^1 norm, which metrizes weak convergence (Machina, 1982, Lemma 1). We endow $\mathcal{I}$ with the pointwise order, denoted by $\leq$. We define the functions

$$W_\lambda \colon F \longmapsto \int_{[0,1]} V_\lambda(\Delta I_F(c)) \frac{\partial g}{\partial c}(c|\lambda) \, \mathrm{d}c$$

and $W \colon F \longmapsto \int_{[0,1]} W_\lambda(F) \, \mathrm{d}G(\lambda)$. The function $g \colon \mathbb{R}^2 \longrightarrow \mathbb{R}$ exhibits *increasing differences* if $t \longmapsto g(s', t) - g(s, t)$ is nondecreasing for all $s',\ s \in \mathbb{R}$ with $s < s'$.

Proofs that are mainly technical or follow from known arguments are relegated to Appendix C.

Definition 5. The experiment F is W *maximal* if F maximizes W on $\mathcal{F}$. The experiment $\hat{F} \in \mathcal{F}$ is an *equilibrium experiment* if there exists an equilibrium $\langle F, e, \alpha \rangle$ with $\hat{F}(x) = F(x)$ for all $x \in \mathbb{R}$. The Receiver's *value of* $F \in \mathcal{F}$ is $V_\lambda(\Delta I_F(c)) := \max_{e \in [0,1]} V_\lambda(e, \Delta I_F(c))$. There are *multiple Sender's payoffs* if there exist equilibria $\langle F, e, \alpha \rangle$ and $\langle \tilde{F}, \tilde{e}, \tilde{\alpha} \rangle$ such that $\hat{W}(F, e, \alpha) \neq \hat{W}(\tilde{F}, \tilde{e}, \tilde{\alpha})$.

Remark B.1. Let's fix an equilibrium $\langle F, e(\cdot), \alpha \rangle$. We have $e(c, \lambda, I) = e^*_\lambda \circ \Delta I(c)$ for some selection e^*_λ from $\Delta J(c) \longmapsto \operatorname{Arg\,max}_{e \in [0,1]} V_\lambda(e, \Delta J(c))$ by e Opt. We define $\underline{c}_\lambda(\Delta I) = \sup\{c \in [0, x_0] : e^*_\lambda \circ \Delta I(c) = 0\}$, if $\{c \in [0, x_0] : e^*_\lambda \circ \Delta I(c) = 0\} \neq \emptyset$, and $\underline{c}_\lambda(\Delta I) = 0$ otherwise. We define $\bar{c}_\lambda(\Delta I) = \inf\{c \in [x_0, 1] : e^*_\lambda \circ \Delta I(c) = 0\}$, if $\{c \in [x_0, 1] : e^*_\lambda \circ \Delta I(c) = 0\} \neq \emptyset$, and $\bar{c}_\lambda(\Delta I) = 1$ otherwise.

B.1 Proof of Lemma 1

Proof. Let's fix Receiver's type (c, λ) and $I \in \mathcal{I}$. By definition of U_R, letting $\alpha(c, x)$ be any probability measure over $\{0, 1\}$ such that $\alpha(c, x)\Big(\operatorname{Arg\,max}_{a \in \{0,1\}} a(x - c)\Big) = 1$

for all $x \in [0,1]$, we have

$$\begin{aligned}\int_{[0,1]} U_R(x,e,c,\lambda)\,\mathrm{d}I'(x) + \lambda k(e) &= \int_{[c,1]} x - c\,\mathrm{d}I'(x) \\ &\qquad - \Big(1-\alpha(c,c)(\{1\})\Big)\Big(I'(c)-I'(c^-)\Big)(c-c), \\ &= \int_{[c,1]} x - c\,\mathrm{d}I'(x).\end{aligned}$$

Moreover,

$$\begin{aligned}\int_{[0,1]} U_R(x,e,c,\lambda)\,\mathrm{d}I'(x) + \lambda k(e) &= (1-c) - \int_{[c,1]} I'(x)\,\mathrm{d}x, \\ &= x_0 - c + I(c).\end{aligned}$$

in which the first equality is due to Riemann–Stieltjes integration by parts (Machina, 1982, Lemma 2) and the second to absolute continuity of I. It follows that

$$\int_{[0,1]} U_R(x,e,c,\lambda)\,\mathrm{d}I'(x) - \int_{[0,1]} U_R(x,e,c,\lambda)\,\mathrm{d}\overline{F}(x) = \Delta I(c).$$

QED

B.2 Proof of Theorem 1

Theorem 1 is implied by the result proved in this section as Proposition B.1. For this section, we fix a function $f\colon [0,1]\times[0,1] \longrightarrow \mathbb{R}^2$ that satisfies strictly increasing differences, and such that: $f(\cdot,a)$ is continuous for all $a \in [0,1]$, $f(e,\cdot)$ is nondecreasing for all $e \in [0,1]$, the derivative with respect to the variable a, $\frac{\partial f}{\partial a}(e,\cdot)$, exists, is nonnegative and bounded for all $e \in [0,1]$, and $f(e,\cdot)$ is increasing for all $e \in (0,1]$. We maintain the definitions of the main text except that the following definitions replace the corresponding ones in the main text: The *value of* $I \in \mathcal{I}$, given type (c,λ) and effort e, is $V_\lambda(e,\Delta I(c)) := f(e,\Delta I(c)) - K(e,\lambda)$, and the cost of effort $e \in [0,1]$ is $K(e,\lambda)$ for a continuous function $K(\cdot,\lambda)$. We use the shorthand $t = (c_t,\lambda_t)$ and we define the set of optimal efforts

$$E_{\lambda_t}(\Delta I(c_t)) := \operatorname*{Arg\,max}_{e\in[0,1]} V_{\lambda_t}(e,\Delta I(c_t)),$$

and $V_{\lambda_t}(\Delta I(c_t)) := \max_{e\in[0,1]} V_{\lambda_t}(e, \Delta I(c_t))$, for $I \in \mathcal{I}$. A persuasion mechanism $I_\bullet$ is *incentive compatible* (IC) if:

$$t \in \operatorname*{Arg\,max}_{r\in R} V_{\lambda_t}(\Delta I_r(c_t)), \text{for all types } t \in T.$$

Definition 6. An IC persuasion mechanism $I_\bullet$ is *equivalent to an experiment* if there exists information policy I such that, for all $t \in T$:

1. $E_{\lambda_t}(\Delta I_t(c_t)) \subseteq E_{\lambda_t}(\Delta I(c_t))$,
2. $\partial I_t(c_t) \subseteq \partial I(c_t)$ if $(0,1] \cap E_{\lambda_t}(\Delta I_t(c_t)) \neq \emptyset$.

Proposition B.1. *Every IC persuasion mechanism is equivalent to an experiment.*

Proof. Let's fix an IC persuasion mechanism $I_\bullet$. The proof has three steps: (1) we define an information policy J, (2) we show that J induces the same effort and (3) action as $I_\bullet$.

(1) Definition of information policy J Let's define the function $I\colon [0,1] \longrightarrow [0,1]$ as

$$I(c) := \sup_{r\in R} I_r(c),\ c \in [0,1].$$

$I(c)$ is well defined because $0 \leq I_r(c) \leq I_{F_0}(c) \leq 1 - x_0$, $c \in [0,1]$. I is the pointwise supremum of a family of convex functions, so I is convex. We have $I_{\overline{F}}(c) \leq I(c) \leq I_{F_0}(c)$, $c \in [0,1]$, because $I_r \in \mathcal{I}, r \in R$. We extend I on $(1,\infty)$, so that the resulting extended function $J\colon \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is an information policy, by defining $J(c) = I_{F_0}(c)$, $c \in (1,\infty)$, and $J(c) = I(c)$, $c \in [0,1]$. Thus, $J \in \mathcal{I}$.

(2) Effort distribution There are two cases.

1. $E_{\lambda_t}(\Delta I_t(c_t)) \cap (0,1] \neq \emptyset$.
2. $E_{\lambda_t}(\Delta I_t(c_t)) = \{0\}$.

First, we consider case (1.). By the envelope theorem (Lemma C.11), we have:

$$V_{\lambda_t}(a) - V_{\lambda_t}(\Delta I_t(c_t)) = \int_{\Delta I_t(c_t)}^{a} \frac{\partial f}{\partial e}(\tilde{a}, e(\tilde{a}))\, \mathrm{d}\tilde{a},$$

for a selection e of E_{λ_t}. Because f exhibits strictly increasing differences, $e(\tilde{a}) \geq e(\Delta I_t(c_t))$ if $\tilde{a} \geq \Delta I_t(c_t)$. By the assumption that $\frac{\partial f}{\partial e}(\tilde{a}, \cdot) > 0$ on $(0, 1]$ for all $\tilde{a}$

$$V_{\lambda_t}(a) - V_{\lambda_t}(\Delta I_t(c_t)) > 0, \text{ for all } a > \Delta I_t(c_t).$$

Thus, in case (1.), IC implies that

$$\sup_{r \in R} \Delta I_r(c_t) = \Delta I_t(c_t).$$

Let's consider case (2.), and, towards a contradiction, let's suppose $0 \notin E_{\lambda_t}(\Delta J(c_t))$. By Berge's Maximum Theorem (Aliprantis and Border, 2006, Theorem 17.31), E_{λ_t} is upper hemi-continuous and has compact values. Hence, by the sequential characterization of upper hemi-continuity of compact-valued correspondences (Aliprantis and Border, 2006, Theorem 17.16), there exists $\overline{a} \in (\Delta I_t(c_t), \Delta J(c_t))$ and $f > 0$ such that $f \in E_{\lambda_t}(\overline{a})$ (else, define $a_n := \frac{1}{n}\Delta I_t(c_t) + \left(1 - \frac{1}{n}\right)\Delta J(c_t),\ n \in \mathbb{N}$, to get: $a_n \longrightarrow \Delta J(c_t)$ as $n \longrightarrow \infty$, $E_{\lambda_t}(a_n) = \{0\},\ n \in \mathbb{N}$, and $0 \notin E_{\lambda_t}(\Delta J(c_t))$, which contradicts upper hemi-continuity of E_{λ_t}.) By the assumption that $\frac{\partial f}{\partial e}(\tilde{a}, \cdot) > 0$ on $(0, 1]$ for all $\tilde{a}$

$$V_{\lambda_t}(\Delta J(c_t)) - V_{\lambda_t}(\overline{a}) > 0.$$

The above inequality and the envelope theorem imply that

$$V_{\lambda_t}(\Delta J(c_t)) - V_{\lambda_t}(\Delta I_t(c_t)) > 0.$$

Hence, IC does not hold, which is a contradiction. Thus, $0 \in E_{\lambda_t}(\Delta J(c_t))$.

(3) Action distribution Let's suppose that $d \in \partial I_s(c_s)$ and $d \notin \partial J(c_s)$ for some type $s \in T$. Because I_s and J are information policies, they have the same extension on $(-\infty, 0)$ and, so, $c_s > 0$. We have that d is a subgradient of I_s at c_s, and d is not subgradient of J at c_s; from the fact that $J(c_s) = I_s(c_s)$ — established above —, there exists $x \in \mathbb{R}$ such that

$$I_s(x) \geq I_s(c_s) + d(x - c_s) > J(x),$$

which implies $I_s(x) > J(x)$. The last inequality contradicts the definition of J. **QED**

B.3 Proof of Theorem 2

In this section, we maintain the assumption that: the conditional density of the cutoff type given the attention type λ, $g(\cdot|\lambda)$, is absolutely continuous for all λ.

Lemma B.2. *The function W is continuous on $\mathcal{F}$.*

Lemma B.3. *There exists a measurable selection from $(c, \lambda, x) \longmapsto \operatorname{Arg\,max}_{a \in \{0,1\}} a(\theta - c)$, for all $e \in [0, 1]$, and there exists a measurable selection from $(c, \lambda) \longmapsto \operatorname{Arg\,max}_{e \in [0,1]} e\Delta I_F(c) - \lambda k(e)$, for all $F \in \mathcal{F}$.*

Proof. The nontrivial part is the second one. The maximand is a real-valued function that is continuous in c, λ, and e. So, the Measurable Maximum Theorem holds (Aliprantis and Border, 2006, Theorem 18.19). **QED**

The next result establishes that the Sender's expected utility given $I \in \mathcal{I}$ the same in every equilibrium adopting a slightly stronger uniqueness condition than in Definition 5. The comparison holds for two reasons. First, Definition 5 compares Sender's expected utility given the *equilibrium information policy* across equilibria, whereas the proof compares Sender's expected utility given an arbitrary and fixed information policy across equilibria. Second, the proof looks at the conditional expected utility given λ.

Lemma B.4. *The experiment F is an equilibrium experiment if, and only if: F is W maximal. Moreover, there are not multiple Sender's payoffs.*

Proof. We first show that: F is W maximal if, and only if: F is rational for Sender given (α, e), α satisfies a Opt, and e satisfies e Opt. It suffices to show that the function

$$D_\lambda(\cdot, \alpha, e) \colon F \longmapsto \int_{[0,1]} \int_{[0,1]} \alpha(x, c, \lambda) \, \mathrm{d}(e(c, \lambda, F) \odot F)(x) \, \mathrm{d}G(c|\lambda) - W_\lambda(F)$$

is constant for all λ. First, let's express the Sender's equilibrium–conditional-expected utility given λ as

$$\hat{W}_\lambda(F) := \int_{[0,1]} \int_{[0,1]} e^*_\lambda(\Delta I_F(c))(\alpha(x, c, \lambda) - \alpha(x_0, c, \lambda)) \, \mathrm{d}F(x) \, \mathrm{d}G(c|\lambda)$$
$$+ \int_{[0,1]} \alpha(x_0, c, \lambda) \, \mathrm{d}G(c|\lambda),$$

for a selection e^*_λ from $a \longmapsto \operatorname{Arg\,max}_{e\in[0,1]} V_\lambda(e,a)$, via Remark B.1. By Lemma 2, there exists a selection d^1_I from the subdifferential of ΔI_F on $[0,x_0]$ and a selection d^2_I from the subdifferential of ΔI_F on $(x_0,1]$ such that:

$$-(\hat{W}_\lambda(F) - \hat{W}_\lambda(\overline{F})) = \int_{[0,x_0]} e^*_\lambda(\Delta I_F(c)) d^1_I(c)\,\mathrm{d}G(c|\lambda) + \int_{(x_0,1]} e^*_\lambda(\Delta I_F(c)) d^2_I(c)\,\mathrm{d}G(c|\lambda)$$

By the envelope theorem (Lemma C.11), e^*_λ is a selection from the subdifferential of the convex and nondecreasing function V_λ. By $\Delta I_F \in \mathcal{A}$, ΔI_F is: (i) convex on $[0,x_0]$, and (ii) convex on $(x_0,1]$. Hence: by the rules of subdifferential calculus (Fact C.1), there exists a selection d from the subdifferential of $V_\lambda \circ \Delta I_F$ such that: $d(c) = e^*_\lambda(\Delta I_F(c)) d^1_I(c)$, for all $c \in [0,x_0]$, and $d(c) = e^*_\lambda(\Delta I_F(c)) d^2_I(c)$, for all $c \in (x_0,1]$. Hence:

$$\begin{aligned}
-(\hat{W}_\lambda(F) - \hat{W}_\lambda(\overline{F})) &= \int_{[0,x_0]} d(c)\,\mathrm{d}G(c|\lambda) + \int_{(x_0,1]} d(c)\,\mathrm{d}G(c|\lambda) \\
&= \int_{[0,x_0]} d(c)\,\mathrm{d}G(c|\lambda) + \int_{[x_0,1]} d(c)\,\mathrm{d}G(c|\lambda),
\end{aligned}$$

in which the second equality uses absolute continuity of $G(\cdot|\lambda)$. By Fact C.1, the composition $V_\lambda \circ \Delta I_F$ is a convex function on $[0,x_0]$, so $V_\lambda \circ \Delta I_F$ is the integral of any selection from the its subdifferential on $[0,x_0]$ (Rockafellar, 1970, Corollary 24.2.1.) Similarly, $V_\lambda \circ \Delta I_F$ is a convex function on $[x_0,1]$. By absolute continuity of $g(\cdot|\lambda)$, we integrate by parts to obtain

$$\begin{aligned}
-(\hat{W}_\lambda(F) - \hat{W}_\lambda(\overline{F})) &= V_\lambda \circ \Delta I_F(1) g(1|\lambda) - V_\lambda \circ \Delta I_F(0) g(0|\lambda) \\
&\qquad - \int_{[0,1]} V_\lambda \circ \Delta I_F(c) \frac{\partial g}{\partial c}(c|\lambda)\,\mathrm{d}c.
\end{aligned}$$

The fact that $\Delta I_F(1) = \Delta I_F(0) = 0$ implies

$$-(\hat{W}_\lambda(F) - \hat{W}_\lambda(\overline{F})) = (g(1|\lambda) - g(0|\lambda)) V_\lambda(0) - \int_{[0,1]} V_\lambda \circ \Delta I_F(c) \frac{\partial g}{\partial c}(c|\lambda)\,\mathrm{d}c.$$

Hence,

$$\hat{W}_\lambda(F) = W(F) + \hat{W}_\lambda(\overline{F}) - (g(1|\lambda) - g(0|\lambda)) V_\lambda(0).$$

So,

$$D_\lambda(F, \alpha, e) = \int_{[0,1]} \alpha(x_0, c, \lambda)\, \mathrm{d}G(c|\lambda) - (g(1|\lambda) - g(0|\lambda))V_\lambda(0).$$

As a result, $D_\lambda(\cdot, \alpha, e)$ is constant on $\mathcal{F}$. Hence, F is W maximal if, and only if: F is rational for Sender, given (α, e), α satisfies a Opt, and e satisfies e Opt.

From the above equivalence, it follows that: if $\langle \hat{F}, e, \alpha \rangle$ is an equilibrium, then $\hat{F}$ is W maximal. For the other direction, let F be W maximal. By Lemma B.3, there exist e and α that satisfy the equilibrium measurability conditions, a Opt, and e Opt, given F. Because F is W maximal, F is rational for Sender, given (α, e), by the above equivalence. Thus, $\langle F, e, \alpha \rangle$ is an equilibrium.

As an implication, there are not multiple Sender's payoffs. **QED**

Proposition B.2. *There exists an equilibrium.*

Proof. The set $\mathcal{F}$ is compact in the topology induced by the L^1 norm (Kleiner et al., 2021, Proposition 1.) The result follows from Lemma B.4 via upper semi continuity of the Sender's maximand in the definition of W maximality (Lemma B.2). **QED**

Proof of Theorem 2

Proof. Theorem 2 is implied by Lemma B.4 and Proposition B.2, given that Assumption 1 contains the continuity requirements assumed in this section. **QED**

B.4 Proof of Theorem 3

Theorem 3 is a consequence of Lemma B.4 and the following property of upper censorship. A version of the property is in the working paper Lipnowski et al., 2021, Appendix A.5; Kolotilin et al. (2017, Theorem 2) and Romanyuk and Smolin (2019, Theorem 2) establish similar results.

Lemma B.5. *Let $I \in \mathcal{I}$ and $\zeta \in [0, 1]$. There exists $\theta \in [0, \zeta]$ such that:*

(1.) $I_\theta(\zeta) = I(\zeta)$*;*

(2.) $I'_\theta(\zeta^-) \leq I'(\zeta^-)$*, and*

$$I_\theta(x) - I(x) \geq 0, \text{ for all } x \in [0, \zeta],$$
$$I_\theta(x) - I(x) \leq 0, \text{ for all } x \in [\zeta, \infty).$$

.

Proof of Theorem 3

Proof. By Lemma B.4, if $F^* \in \mathcal{F}$ maximizes

$$W\colon F \longmapsto \int_{[0,1]} \int_{[0,p]} V_\lambda(\Delta I_{\hat{F}}(c)) \frac{\partial g}{\partial c}(c|\lambda)\, \mathrm{d}c + \int_{[p,1]} V_\lambda(\Delta I_{\hat{F}}(c)) \frac{\partial g}{\partial c}(c|\lambda)\, \mathrm{d}c\, \mathrm{d}G(\lambda),$$

then there exists an equilibrium in which F^* is the Sender's experiment. Suppose two experiments $F, H \in \mathcal{F}$ such that $I_F(x) \geq I_H(x)$ for all $x \in [0,p]$ and $I_F(x) \leq I_H(x)$ for all $x \in [p,1]$. Because (i) V_λ is nondecreasing, (ii) $\frac{\partial g}{\partial c}(\cdot|\lambda)$ is nonnegative on $[0,p]$ and nonnpositive on $[p,1]$, it follows that $W(F) \geq W(H)$. Hence, the result follows from Lemma B.5. In particular, by Proposition B.2, there exists an equilibrium experiment $\hat{F}$, and by Lemma B.5 there exists F^* such that F^* weakly improves upon $\hat{F}$ in terms of W and I_{F^*} is an upper censorship. **QED**

B.5 Proof of Proposition 1

The proof has four steps. First, we establish a single-crossing property of the derivative of the Sender's payoff given I_θ with respect to θ, in three claims. Second, we establish a monotonicity property of the Sender's payoff given I_θ with respect to θ given certain conditions, in two claims. The third step verifies that the optimality properties and the hypotheses in the statement of the Proposition imply the aforementioned conditions. The final step completes the argument.

Proof. Let's fix an equilibrium $\langle F, e, \alpha \rangle$.

(1.) Let strict single-peakedness hold. We claim that the function $(\theta, \zeta) \longmapsto \int_{[\theta,\zeta]} (c-\theta) \frac{\partial}{\partial c} g(c|\lambda)\, \mathrm{d}c$ crosses zero at most once and from above, that is:

$$\int_{[\theta,\zeta]} (c-\theta) \frac{\partial}{\partial c} g(c|\lambda)\, \mathrm{d}c \leq 0 \implies \int_{[\theta',\zeta']} (c-\theta') \frac{\partial}{\partial c} g(c|\lambda)\, \mathrm{d}c < 0,$$

for all $\theta \leq \theta'$ and $\zeta \leq \zeta'$, with $\theta' < \zeta'$, $\theta < \zeta$. If $p \leq \theta'$, the result holds. If $\int_{[\theta,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c \leq 0$, then $p < \zeta$. We have

$$\int_{[\theta,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c = \int_{[\theta,\theta')}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c + \int_{[\theta',p)}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c + \int_{[p,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c.$$

Let $\int_{[\theta,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c \leq 0$. Then:

$$\int_{[\theta,\theta')}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c + \int_{[\theta',p)}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c \leq -\int_{[p,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c,$$

which implies, by $\theta' < p$:

$$\int_{[\theta',p)}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c < -\int_{[p,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c.$$

From the above inequality and $p < \zeta$, we have:

$$\int_{[\theta',p)}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c + \int_{[p,\zeta]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c + \int_{(\zeta,\zeta']}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c < 0,$$

so the claim follows.

(2.) Let strict single-peakedness hold. $\int_{[\theta,\overline{c}]}(c-\theta)\frac{\partial}{\partial c}g(c|\lambda)\,\mathrm{d}c$ is increasing in λ if $p \leq \overline{c}$, for $\overline{c} := \overline{c}_\lambda(\Delta I_\theta)$ and $\overline{c} \in (x_0, 1)$. The claim holds because $\lambda \longmapsto \overline{c}_\lambda(\Delta I_\theta)$ is decreasing under our hypotheses.

(3.) Let Assumption 1 hold. We claim that $\overline{c}_\lambda(\Delta I_\theta) > \theta$, $\underline{p} \leq \overline{c}_\lambda(\Delta I_\theta)$, and $\overline{c}_\lambda(\Delta I_\theta) \in (x_0, 1)$, if: I'_θ maximizes W on $\mathcal{F}$ and F_0, $\overline{F}$ do not maximize W on $\mathcal{F}$. If $\overline{c}_\lambda(\Delta I_\theta) \leq \theta$, then F_0 maximizes W on $\mathcal{F}$. If $\overline{c}_\lambda(\Delta I_\theta) < \underline{p}$, then F_0 maximizes W on $\mathcal{F}$. The rest of the claim follows from similar arguments.

(4.) Let $x_{\overline{\theta}} := \frac{\int_{\overline{\theta}}^1 \theta\,\mathrm{d}F_0(\theta)}{1-F_0(\overline{\theta})}$, for threshold state $\overline{\theta} \in [0,1]$. By Lemma B.4, we compute the derivative of the Sender's expected utility, given information policy $I_{\overline{\theta}}$, with respect to $\overline{\theta}$, which is:

$$\frac{\partial}{\partial \overline{\theta}} W(I'_{\overline{\theta}}) = \begin{cases} \frac{\partial F_0}{\partial \theta}(\overline{\theta}) \int_{[\max\{\overline{\theta}, \underline{c}_\lambda(\Delta I_{\overline{\theta}})\}, \overline{c}_\lambda(\Delta I_{\overline{\theta}})]} (x - \overline{\theta}) \frac{\partial g}{\partial c}(x|\lambda) \, \mathrm{d}x, & \text{if } \theta < \overline{c}_\lambda(\Delta I_\theta) \\ 0, & \text{if } \theta > \overline{c}_\lambda(\Delta I_\theta). \end{cases}$$

As claimed above, under our hypotheses, $\theta_\varepsilon < \overline{c}_\lambda(\Delta I_{\theta_\varepsilon})$. Moreover, by strict single-peakedness, there exists a unique optimal upper censorship I_η if $\lambda = 0$, with $\eta \in (0, 1)$ (Kolotilin et al., 2022, Lemma 7.) Let's complete the proof.

First, claim 1. implies that $\overline{\theta} \longmapsto W(I'_{\overline{\theta}})$ crosses zero only once and from above: at θ_ε. By claims 2. and 3., $\theta_\varepsilon > \eta$, for $\varepsilon > 0$. **QED**

B.6 Proof of Proposition 2

The proof of Proposition 2 has two steps. The first and main step has the same structure as that of Theorem 3. In particular, Lemma B.6 generalizes the construction of Lemma B.5 to construct: an information policy $I^\star$ that preserves the extensive margin and improves upon an arbitrary information policy I, for large p. $I^\star$ induces two censorship regions, separated by a full-revelation region. The second step of the proof: (1) adds a second censorship region at the top to include the general case of $p > x_0$, and (2) verifies that eliminating the bottom censorship region improves upon Sender's payoff. For the rest of this section, we omit reference to λ and we fix an equilibrium $\langle F, e(\cdot), \alpha \rangle$.

Lemma B.6. *Let $I \in \mathcal{I}$ and define $c^* := \overline{c}(\Delta I)$. There exists an information policy $I^\star$ that satisfies the following properties:*

1. *(FEAS) $I^\star$ is feasible, i.e., $I^\star \in \mathcal{I}$;*
2. *(EM) $I^\star$ produces the same extensive margin as I, i.e., $\overline{c}(\Delta I^\star) = c^*$ and $\underline{c}(\Delta I^\star) = \underline{c}(\Delta I)$;*
3. *(IMPR) $\Delta I^\star(x) \geq 0$, for all $x \in [\underline{c}(\Delta I), c^*]$;*

4. (CENS) There exist $x_\ell, \theta_\ell, \theta_m, x_m$ such that $0 \leq x_\ell \leq \theta_\ell \leq \theta_m \leq x_m \leq 1$ and

$$I^\star(x) = \begin{cases} I_{\overline{F}}(x) & , x \in [0, x_\ell] \\ I_{F_0}(\theta_\ell) + F_0(\theta_\ell)(x - \theta_\ell) & , x \in (x_\ell, \theta_\ell] \\ I_{F_0}(x) & , x \in (\theta_\ell, \theta_m] \\ I_{F_0}(\theta_m) + F_0(\theta_m)(x - \theta_m) & , x \in (\theta_m, x_m] \\ I_{\overline{F}}(x) & , x \in (x_m, \infty). \end{cases}$$

Proof. We use the notation: $\overline{c}(\Delta I) =: \overline{c}$, $\underline{c}(\Delta I) =: \underline{c}$. In the first step, we prove the result for the case in which there is a feasible information policy that is a straight line between the points $\underline{p} := (\underline{c}, I(\underline{c}))$ and $\overline{p} := (\overline{c}, I(\overline{c}))$. In the second step we analyze the other case.

First Step. Let's define the line i such that $x \longmapsto I(\underline{c}) + \lambda^*(x - \underline{c})$, with slope $\lambda^* := \frac{I(\overline{c}) - I(\underline{c})}{\overline{c} - \underline{c}}$. We claim that $i^\star(x) := \max\{i(x), I_{\overline{F}}(x)\}$ satisfies all properties. $i^\star$ is FEAS by hypothesis. $i^\star$ is EXT because $i(\underline{c}) = I(\underline{c})$ and $i(\overline{c}) = I(\overline{c})$. $i^\star$ is IMPR because I is convex and $i^\star$ is EXT. $i^\star$ is CENS with $\theta_\ell = \theta_m = x_m$, because: (i) EXT of $i^\star$ and convexity of I imply that $i^\star$ is affine on $[\underline{c}, \overline{c}]$, (ii) $\lambda^* \in [0, 1]$ and EXT imply, with $I \in \mathcal{I}$, that there are intersection points $\widetilde{x}_1, \widetilde{x}_2$, with $\widetilde{x}_1 \leq \underline{c} \leq \overline{c} \leq \widetilde{x}_2$, such that: $i^\star(x) = I(x)$ if $x \in [0, \widetilde{x}_1] \cup [\widetilde{x}_2, 1]$.

Second Step. In this case, $i^\star$ is not FEAS. Because $i^\star$ satisfies FEAS at x if $x \leq \underline{c}$ and if $x \geq \overline{c}$, there exists a point $x^* \in (\underline{c}, \overline{c})$ such that: $i(x^*) > I_{F_0}(x^*)$. Let's define:

$$L := \{\lambda \in [I'(\underline{c}), 1] : I(\underline{c}) + \lambda(x - \underline{c}) \leq I_{F_0}(x) \text{ for all } x \in [\underline{c}, \infty)\},$$
$$M := \{\lambda \in [0, I'(\overline{c})] : I(\overline{c}) + \lambda(x - \overline{c}) \leq I_{F_0}(x) \text{ for all } x \in [0, \overline{c}]\},$$

$\ell := \max L$, $m := \min M$, and the lines

$$y_\ell : x \longmapsto I(\underline{c}) + \ell(x - \underline{c}),$$
$$y_m : x \longmapsto I(\overline{c}) + m(x - \overline{c}).$$

As part of the rest of the proof, we establish some lemmata.

Lemma B.7. *It holds that ℓ and m are well-defined.*

Proof. L is nonempty because $I'(\underline{c}) \in L$, which follows from: (i) $I_{F_0}(x) \geq I(x)$ for all x and (ii) $I'(\underline{c}) \in \partial I(\underline{c})$. M is nonempty because $I'(\overline{c}) \in M$, which follows from:

(i) $I_{F_0}(x) \geq I(x)$ for all x and (ii) $I'(\bar{c}) \in \partial I(\bar{c})$. L, M are closed because I_{F_0} is continuous. L, M are bounded. **QED**

Lemma B.8. *There exists a unique pair of numbers* $(\theta_\ell, \theta_m) \in [\underline{c}, 1] \times [0, \bar{c}]$ *such that:* $y_\ell(\theta_\ell) = I_{F_0}(\theta_\ell)$ *and* $y_m(\theta_m) = I_{F_0}(\theta_m)$.

Proof. Suppose there does not exists such θ_ℓ. There exists a sufficiently small $\varepsilon > 0$ such that: (i) $\ell + \varepsilon \in L$ and (ii) $I(\underline{c}) + (\ell + \varepsilon)(x - \underline{c}) < I_{F_0}(x)$ for all $x \in [\underline{c}, \infty)$; we note that $\theta_\ell = 1$ contradicts $\ell \in L$ because $I'_{F_0}(x) < 1$ if $x < 1$. Uniquenss of θ_ℓ follows from convexity of I_{F_0}.

Suppose there does not exists such θ_m. There exists a sufficiently small $\varepsilon > 0$ such that: (i) $\ell - \varepsilon \in M$ and (ii) $I(\bar{c}) + (m - \varepsilon)(x - \bar{c}) < I_{F_0}(x)$ for all $x \in [0, \bar{c})$; we note that $\theta_m = 0$ contradicts $I \neq I_{\overline{F}}$. Uniquenss of θ_m follows from convexity of I_{F_0}. **QED**

Lemma B.9. *It holds that* $\theta_\ell \leq \theta_m$.

Proof. Let's first prove that: it suffices to show that $\ell \leq m$. Suppose $\ell \leq m$, then, from $\ell \in \partial I_{F_0}(\theta_\ell)$, $m \in \partial I_{F_0}(\theta_m)$, and I_{F_0} being strictly convex, we have: $\theta_\ell \leq \theta_m$.

Next, we show that $\ell \leq \lambda^*$. Suppose that: $\ell > \lambda^*$. Then: $I(x) + \ell(x - \underline{c}) > I(\underline{c}) + \lambda^*(x - \underline{c})$ for all $x > \underline{c}$. Therefore, because $\ell > 0$, we get:

$$I_{F_0}(x^*) \geq I(\underline{c}) + \lambda^*(x^* - \underline{c}).$$

We reach a contradiction with the definition of x^*, so: $\ell \leq \lambda^*$.

Let's prove that $m \geq \lambda^*$. Suppose $m < \lambda^*$. Then: $I(x) + m(x - \bar{c}) > I(\bar{c}) + \lambda^*(x - \bar{c})$ for all $x < \bar{c}$. Therefore, because $m > 0$, we get:

$$I_{F_0}(x^*) \geq I(\underline{c}) + \lambda^*(x^* - \underline{c}).$$

We reach a contradiction with the definition of x^*, so: $m \geq \lambda^*$. Therefore, we have $m \geq \lambda^* \geq \ell$, which implies $\theta_m \geq \theta_\ell$. **QED**

We define a candidate $I^\star$ and verify that $I^\star$ has the desired properties.

$$I^\star(x) := \begin{cases} \max\{I_{\overline{F}}(x), I(\underline{c}) + \ell(x - \underline{c})\} & , x \in [0, \theta_\ell] \\ I_{F_0}(x) & , x \in [\theta_\ell, \theta_m] \\ \max\{I_{\overline{F}}(x), I(\bar{c}) + m(x - \bar{c})\} & , x \in [\theta_m, \infty) \end{cases}$$

Let's first verify that $I^\star$ is well-defined. We know that $\ell \in \partial I_{F_0}(\theta_\ell)$ and $m \in \partial I_{F_0}(\theta_m)$. Because $I(\underline{c})+\ell(0-\underline{c}) < I_{F_0}(0)$ and $I(\underline{c}) \geq I_{F_0}(\underline{c})$, $\max\{I_{F_0}(x), I(\underline{c})+\ell(x-\underline{c})\} = I_{F_0}(x)$ if $x < x_0$; and $\max\{I_{F_0}(x), I(\underline{c})+\ell(x-\underline{c})\} = I(\underline{c})+\ell(x-\underline{c})$ if $x > x_0$; for some $x_0 \in [0, \theta_\ell]$. In a similar way, we can show that there exists a $x_2 \in [\theta_m, 1]$ such that: $\max\{I_{F_0}(x), I(\overline{c})+m(x-\overline{c})\} = I_{F_0}(x)$ if $x > x_2$, and $\max\{I_{F_0}(x), I(\overline{c})+m(x-\overline{c})\} = I(\overline{c})+m(x-\overline{c})$ if $x < x_2$.

1. CENS follows from the definition of $I^\star$ and the conclusion of the above paragraph.

2. IMPR on $[\underline{c}, \theta_\ell]$ and $[\theta_m, \overline{c}]$ follows from convexity of I, and on $[\theta_\ell, \theta_m]$ follows from FEAS of I in that region.

3. EM follows by construction of $I^\star$.

4. FEAS is established as in the last step of the proof of Lemma B.5.

QED

Proof of Proposition 2

Proof. Let's define information policy J by: letting J equal $I^\star$, constructed as in Lemma B.6 by replacing c^* with p, for $x \in [0, x_m^\circ]$, defining the point x_m° in which $I^\star$ intercepts the line $j\colon x \longmapsto I(\overline{c}) + I'(\overline{c})(x-\overline{c})$; and letting J equal $x \longmapsto \max\{I_{\overline{F}}(x), j(x)\}$ on $[x_m^\circ, \infty)$.

It suffices to show that: if the resulting information policy J induces a censorship region at the bottom, then there is an improvement over J that is a bi-upper censorship. Suppose that $I^\star$ is affine on $[x_\ell, \theta_\ell]$ and $I^\star$ equals $I_{\overline{F}}$ on $[0, x_\ell]$, for $0 < x_\ell < \theta_\ell$ (for notation, see Lemma B.6.) By construction, $I^\star(\theta_\ell) = I_{F_0}(\theta_\ell)$. Let's define information policy K by

$$K(x) = \begin{cases} I_{F_0}(x) & , 0 \leq x \leq \theta_\ell, \\ J(x) & , x \geq \theta_\ell. \end{cases}$$

We have $K \geq J$, so K induces a weakly lower $\underline{c}_\lambda$, than J. Hence, by $\gamma \geq 0$, it suffices to verify that the expected Receiver's action is weakly higher under K than under J. Because $p \geq x_0$, the argument of Theorem 3 suffices. Specifically, by Lemma B.4, we have

$$
\begin{aligned}
W(K') - W(J') &= \int_{[0,\theta_\ell]} \Big(V_\lambda(\Delta K(c)) - V_\lambda(\Delta J(c))\Big) \frac{\partial g}{\partial c}(c|\lambda)\,\mathrm{d}c \\
&\geq 0,
\end{aligned}
$$

in which the inequality follows from the definition of $I^\star$, which includes $p \geq \theta_\ell$. Hence K is a bi-upper censorship that improves upon I, for arbitrary I, in terms of U_G. **QED**

C Supplementary material

C.1 Preliminaries

We claim that the Sender's signal affects the decisions and payoffs of both Sender and Receiver only through the distribution of the posterior mean that it induces on a Bayesian agent who always observes the signal realization.

Type-t Receiver's optimal action, given posterior belief $\mu \in \mathcal{D}$ and $t = (c, \lambda)$, depends on the belief μ only through its mean $\overline{x}_\mu := \int_{[0,1]} \theta \,\mathrm{d}\mu(\theta)$. The Receiver's expected material payoff given belief μ and is given by

$$
v_t(\mu) := \begin{cases} \int_{[0,1]} (\theta - c)\,\mathrm{d}\mu(\theta), & \text{if } \overline{x}_\mu \geq c, \\ 0, & \text{if } \overline{x}_\mu < c. \end{cases}
$$

We note that $v_t(\mu)$ depends on the belief μ only through x_μ. If the Sender's signal induces the Bayes-plausible distribution over posterior beliefs p (Kamenica and Gentzkow, 2011), type-t Receiver chooses $e \in [0, 1]$ to maximize her expected utility

$$
e \int_{\mathcal{D}} v_t(\mu)\,\mathrm{d}p(\mu) + (1 - e)v_t(F_0) - \lambda k(e).
$$

Thus, Receiver's action, effort, and her payoff depend on the Sender's signal only via the distribution of the posterior mean (i.e., the distribution of x_μ implied by p.) The claim follows from the Sender's payoff function, which depends on the signal only via the Receiver's choice of action. The same conclusion holds under the hypothesis of Proposition 2.

C.2 Symmetric-information benchmark

For this section, the type distribution puts full mass at (ζ, κ), k is linear, and F_0 admits a density. The Sender's *problem* is:

$$\max_{I \in \mathcal{I}} \Big(1 - I'(\zeta^-)\Big) \mathbf{1}_{\{I \in \mathcal{I} | \Delta I(\zeta) \geq \kappa\}}(I),$$

because an experiment F is an equilibrium experiment iff I_F solves the above maximization, due to a generalization of the argument of Gentzkow and Kamenica (2016). If $\zeta > 1$, any information policy is optimal. If $\zeta < x_0$, $I_{\overline{F}}$ is optimal. Let $1 \geq \zeta \geq x_0$.

Lemma C.10. *There exists $\theta \in [0, \zeta]$ such that: I_θ solves the Sender's problem and $\Delta I_\theta \leq \kappa$, with equality if $\theta > 0$.*

Proof. Let $\mathcal{I}^u := \{I \in \mathcal{I} \mid I = I_\theta,\ \theta \in [0, \zeta]\}$. Without loss of optimality by Lemma B.5, we consider solutions in $\mathcal{I}^u$. Suppose there exists a solution $I \in \mathcal{I}^u$, such that $I = I_{\theta^\star}$, for some $\theta^\star \in (0, 1)$. We distinguish three cases.

(1) If $\Delta I(\zeta) < \kappa$, then Sender is indifferent between I and $I_{\overline{F}}$, so the lemma holds. (2) If $\Delta I(\zeta) = \kappa$, the lemma holds. (3) If $\Delta I(\zeta) > \kappa$, then, by definition of I at $y = I(\zeta)$,

$$I_{F_0}(\theta^\star) + F_0(\theta^\star)(\zeta - \theta^\star) - y = 0.$$

By the implicit function theorem, there exists a differentiable function $t \colon (0, 1) \longrightarrow (0, 1)$ such that $t \colon y \longmapsto \theta^\star$ and

$$t'(y) = \begin{cases} \frac{1}{(\zeta - t(y)) \frac{\partial F_0}{\partial \theta}(t(y))}, & 0 < \zeta < t(y), \\ \frac{1}{\frac{\partial F_0}{\partial \theta}(t(y))} & , 1 > \zeta \geq t(y). \end{cases}$$

Let's define the value of the Sender's maximand at I_θ as $v \colon (0, 1) \longrightarrow [0, 1]$ such that $v \colon \theta \longmapsto 1 - I'_\theta(\zeta^-)$. Because $I'_{\theta^\star}(\zeta^-) = F_0(\theta^\star)$, v is differentiable in θ at $\theta^\star$. The derivative of v with respect to $I(\zeta)$ is:

$$-\frac{\partial F_0}{\partial \theta}(t(I(\zeta))) \frac{1}{(\zeta - t(I(\zeta))) \frac{\partial F_0}{\partial \theta}(t(I(\zeta)))},$$

if $\zeta > t(I(\zeta))$, and -1 otherwise. It follows that we can consider without loss solutions $I \in \mathcal{I}^u$ that satisfy: $\Delta I_\theta(\zeta) = \kappa$ and $I = I_\theta$, or $\Delta I(\zeta) < \kappa$. **QED**

C.3 Auxiliary results

Fact C.1 (Subdifferential of convex functions)**.** *Let $S \subseteq \mathbb{R}$, $f\colon S \longrightarrow \mathbb{R}$ be convex and $\varphi\colon \mathbb{R} \longrightarrow \mathbb{R}$ be a nondecreasing convex function on the range of f. The following hold:*

1. *The function $\varphi \circ f$ is convex on S;*

2. *For all $y \in S$, letting $t = f(y)$, we have:*

$$\{\alpha u : (\alpha, u) \in \partial\varphi(t) \times \partial f(y)\} = \partial\varphi \circ f(y).$$

Proof. See Bauschke and Combettes (2011, Proposition 8.21 and Corollary 16.72.) **QED**

Lemma C.11 (Envelope theorem)**.** *Let $f\colon [0,1]^2 \longrightarrow \mathbb{R}$ exhibit increasing differences and be such that: $f(\cdot, a)$ is continuous for all $a \in [0,1]$, $f(e, \cdot)$ is nondecreasing for all $e \in [0,1]$, the derivative with respect to the variable a, $\frac{\partial f}{\partial a}(e, \cdot)$, exists and is bounded for all $e \in [0,1]$. The following hold.*

1. *We have $\operatorname{Arg\,max}_{e\in[0,1]} f(e,a) \neq \emptyset$ for all $a \in [0,1]$.*

2. *The function $a \longmapsto \max_{e\in[0,1]} f(e,a)$ is nondecreasing and absolutely continuous.*

3. *If $a \longmapsto \frac{\partial f}{\partial a}(e,a)$ is nondecreasing for all $e \in [0,1]$, then $a \longmapsto \max_{e\in[0,1]} f(e,a)$ is convex.*

4. *If f exhibits strictly increasing differences, $a \longmapsto \frac{\partial f}{\partial a}(e,a)$ is nondecreasing, $f(e,\cdot)$ is increasing for all $e \in (0,1]$, $\operatorname{Arg\,max}_{e\in[0,1]} f(e,a) \cap (0,1] \neq \emptyset$, and $1 \geq a' > a \geq 0$, then*

$$\max_{e\in[0,1]} f(e,a') > \max_{e\in[0,1]} f(e,a).$$

Proof. By upper semi-continuity of f, $\operatorname{Arg\,max}_{e\in[0,1]} f(e,a) \neq \emptyset$, so 1. holds. Then, by the increasing-differences property of f, there exists a nondecreasing selection $e^\star\colon a \longmapsto \operatorname{Arg\,max}_{e\in[0,1]} f(e,a)$ on $[0,1]$ (Topkis, 1978). By our hypotheses, we apply the envelope theorem (Milgrom and Segal, 2002), letting $V(a) := \max_{e\in[0,1]} f(e,a)$, to establish that V is absolutely continuous and

$$V(a) = V(0) + \int_{[0,a]} \frac{\partial f}{\partial a}(e^{\star}(\tilde{a}), \tilde{a})\, \mathrm{d}\tilde{a}.$$

V is nondecreasing because $\frac{\partial f}{\partial a} \geq 0$. Hence, 2. holds.

Let's establish that V is convex if $a \longmapsto \frac{\partial f}{\partial a}(e, a)$ is nondecreasing. By the increasing-differences property of f: (i) $e \longmapsto \frac{\partial f}{\partial a}(e, a)$ is nondecreasing, and (ii) there exists a nondecreasing $e^{\star}\colon a \longmapsto \operatorname{Arg\,max}_{e \in [0,1]} f(e, a)$. As a result, $a \longmapsto \frac{\partial f}{\partial a}(e^{\star}(a), a)$ is nondecreasing. Thus, V is convex (Rockafellar, 1970, Theorem 24.8.) Hence, 3. holds.

Let $a' > a$, for $a', a \in [0, 1]$, and $e' \in \operatorname{Arg\,max}_{e \in [0,1]} f(e, a) \cap (0, 1]$. Then: $V(a') - V(a) = \int_{[a,a']} \frac{\partial f}{\partial a}(e^{*}(\tilde{a}), \tilde{a})\, \mathrm{d}\tilde{a}$ for every selection e^{*} of $\operatorname{Arg\,max}_{e \in [0,1]} f(e, a) \cap (0, 1]$. We have the following chain of inequalities under the additional hypotheses stated in part 4.:

$$\begin{aligned} V(a') - V(a) &\geq \int_{[a,a']} \frac{\partial f}{\partial a}(e', \tilde{a})\, \mathrm{d}\tilde{a} \\ &\geq \int_{[a,a']} \frac{\partial f}{\partial a}(e', a)\, \mathrm{d}\tilde{a}, \end{aligned}$$

in which the first inequality follows from the strict increasing-differences property of f and the definition of e', the second inequality holds because $a \longmapsto \frac{\partial f}{\partial a}(e, a)$ is nondecreasing (for the first inequality, in particular, we note that: (i) every selection e^{*} of $\operatorname{Arg\,max}_{e \in [0,1]} f(e, a) \cap (0, 1]$ is nondecreasing, (ii) there exists a selection e^{*} of $\operatorname{Arg\,max}_{e \in [0,1]} f(e, a) \cap (0, 1]$ such that $e^{*}(a) = e'$.) Item 4. holds because $\int_{[a,a']} \frac{\partial f}{\partial a}(e', a)\, \mathrm{d}\tilde{a} = (a' - a)\frac{\partial f}{\partial a}(e', a)$. **QED**

C.4 Proof of Lemma B.2

Proof. Let's fix λ, $F \in \mathcal{F}$, and $\varepsilon > 0$, and define $p_{\lambda} := \int_{[0,1]} \left| \frac{\partial g}{\partial c}(c|\lambda) \right| \mathrm{d}c$. Let $\delta := \frac{\varepsilon}{p_{\lambda}}$ if $p_{\lambda} > 0$, and let δ be an arbitrary positive number otherwise. Let $H \in \mathcal{F}$ be such that $\int_{[0,1]} |H(x) - F(x)|\, \mathrm{d}x < \delta$.

We first establish the claim that: $|V_{\lambda}(\Delta I_H(c)) - V_{\lambda}(\Delta I_F(c))| < \delta$. By definition of V_{λ} and the envelope theorem (Lemma C.11), there exists a selection e from

$c \longmapsto \operatorname{Arg\,max}_{e\in[0,1]} e\Delta I_F(c) - \lambda k(e)$ such that:

$$|V_\lambda(\Delta I_H(c)) - V_\lambda(\Delta I_F(c))| = \int_{[\min\{\Delta I_H(c),\Delta I_F(c)\},\max\{\Delta I_H(c),\Delta I_F(c)\}]} e(a)\,\mathrm{d}a.$$

The codomain of e is $[0,1]$, so, by the above equality:

$$|V_\lambda(\Delta I_H(c)) - V_\lambda(\Delta I_F(c))| \leq |\Delta I_H(c) - \Delta I_F(c)|.$$

We have the following chain of inequalities,

$$\begin{aligned} |V_\lambda(\Delta I_H(c)) - V_\lambda(\Delta I_F(c))| &\leq \left|\int_{[0,c]} H(x) - F(x)\,\mathrm{d}x\right| \\ &\leq \int_{[0,c]} |H(x) - F(x)|\,\mathrm{d}x \\ &\leq \delta, \end{aligned}$$

which establishes the claim.

We establish the continuity of the function W_λ on $\mathcal{F}$. We have the following chain of inequalities,

$$\begin{aligned} |W_\lambda(H) - W_\lambda(F)| &\leq \int_{[0,1]} |V_\lambda(\Delta I_H(c)) - V_\lambda(\Delta I_F(c))| \left|\frac{\partial g}{\partial c}(c|\lambda)\right| \mathrm{d}c \\ &\leq \delta p_\lambda \\ &\leq \varepsilon. \end{aligned}$$

Thus, W_λ is continuous on $\mathcal{F}$. The result follows from the following chain of inequalities,

$$\begin{aligned} |W(H) - W(F)| &\leq \int_{[0,1]} |W_\lambda(H) - W_\lambda(F)|\,\mathrm{d}G(\lambda) \\ &\leq \varepsilon. \end{aligned}$$

QED

C.5 Proof of Lemma B.5

Proof. Let $\zeta \in [0,1]$. Let $M := \{m \in [0, I'(\zeta^-)] : I(\zeta) + m(x-\zeta) \leq I_{F_0}(x) \text{ for all } x \in [0,\zeta]\}$, and $m := \min M$. We construct an information policy starting from the line

$x \longmapsto I(\zeta) + m(x-\zeta)$, via the next three claims.

(1) m is well-defined. (i) M is nonempty, because $0 \leq I'(\zeta^-) \leq 1$ (which follows from $I \in \mathcal{I}$), $I'(\zeta^-) \in \partial I(\zeta^-)$ and $I(x) \leq I_{F_0}(x)$ for all x; (ii) M is closed, because the mapping $m \longmapsto I(\zeta) + m(x-\zeta)$ is a continuous function on $[0, I'(\zeta^-)]$; (iii) M is bounded because $I'(\zeta^-) \leq 1$, from $I \in \mathcal{I}$.

(2) There exists $\theta \in [0,\zeta]$ such that $I_{F_0}(\theta) = I(\zeta) + m(\theta - \zeta)$. If $m = 0$, then $0 = I_{F_0}(0) \geq I(\zeta) \geq 0$. Hence, taking $\theta = 0$ verifies our claim. Let $m > 0$, and suppose there does not exist $\theta \in [0,\zeta]$ such that $I_{F_0}(\theta) = I(\zeta) + m(\theta - \zeta)$. There exists $\overline{\varepsilon} > 0$ such that: $I(\zeta) + (m-\varepsilon)(x-\zeta) < I_{F_0}(x)$ for all $x \in [0,\zeta]$ and $0 < \varepsilon \leq \overline{\varepsilon}$. Moreover, for a sufficiently small $\varepsilon > 0$, we have $m - \varepsilon \in M$. Thus, we have a contradiction with the definition of m.

(3) $m \in \partial I_{F_0}(\theta)$ and $I(\zeta) + m(x-\zeta) = I_{F_0}(\theta) + (x-\theta)F_0(\theta)$ for all x. First, we argue that $m \in \partial I_{F_0}(\theta)$. By convexity of I_{F_0} and definition of θ, $x \longmapsto I(\zeta)+m(x-\zeta)$ is tangent to I_{F_0} at θ. Thus, m is a subgradient of I_{F_0} at θ. Now, we argue that $I(\zeta) + m(x-\zeta) = I_{F_0}(\theta) + (x-\theta)F_0(\theta)$ for all x. $m = F_0(\theta)$ because I_{F_0} is differentiable (by the fact that $F_0(x^-) = F_0(x), x \in \mathbb{R}$.) The equality follows because $x \longmapsto I(\zeta) + m(x-\zeta)$ is equal to I_{F_0} at $x = \theta$.

We define the following function.

$$I^u \colon x \longmapsto \begin{cases} I_{F_0}(x) & , x \in [0,\theta] \\ I(\zeta) + m(x-\zeta) & , x \in (\theta, \zeta] \\ \max\{I(\zeta) + m(x-\zeta), I_{\overline{F}}(x)\} & , x \in (\zeta, \infty). \end{cases}$$

Now, we claim that $I^u = I_\theta$. It suffices to show that: (i) for some $x_u \in [0,1]$

$$I^u(x) = \begin{cases} I_{F_0}(x) & , x \in [0,\theta] \\ I_{F_0}(\theta) + (x-\theta)F_0(\theta) & , x \in (\theta, x_u] \\ I_{\overline{F}}(x) & , x \in (x_u, \infty), \end{cases}$$

and (ii) $I^u \in \mathcal{I}$. We claim that (i) holds by means of the next three claims.

There exists $x_u \in [\zeta, 1]$ such that:

$$\begin{aligned} I(\zeta) + m(x-\zeta) &\geq I_{\overline{F}}(x) & , x \in [0, x_u] \\ I(\zeta) + m(x-\zeta) &\leq I_{\overline{F}}(x) & , x \in (x_u, 1]. \end{aligned}$$

Let's note that: (a) $I(\zeta) \geq I_{\overline{F}}(\zeta)$; (b) by $m \in \partial I_{F_0}(\theta)$ and $I_{F_0}(1) = I_{\overline{F}}(1)$, we have that $I_{\overline{F}}(1) \geq I(\zeta) + m(1-\zeta)$, and (c) the two functions, $x \longmapsto I(\zeta) + m(x-\zeta)$ and $I_{\overline{F}}$, are affine with slopes, respectively, m and 1, such that: $m \leq 1$.

We proceed to verify that (ii) holds, i.e. $I^u \in \mathcal{I}$, via the next two claims.

(1) $I_{\overline{F}}(x) \leq I^u(x) \leq I_{F_0}(x)$ for all $x \in \mathbb{R}_+$ and I^u locally convex at all $x \notin \{\theta, x_u\}$. If $x \in [0,\theta)$, I^u is locally convex and $I_{\overline{F}}(x) \leq I^u(x) \leq I_{F_0}(x)$. If $x \in (\theta, \zeta)$, I^u is affine, $I_{\overline{F}}(x) \leq I(x) \leq I^u(x)$ by construction of I^u and definition of I, and $I^u(x) \leq I_{F_0}(x)$ by $m \in \partial I_{F_0}(x)$. If $x \in [\zeta, \infty)$, I is locally convex (because it is the maximum of affine functions), $I_{\overline{F}}(x) \leq I^u(x)$ by construction of I^u, $I^u(x) \leq I_{F_0}(x)$ because: (i) $m \in \partial I_{F_0}(\zeta)$ and (ii) $I_{\overline{F}}(x) \leq I_{F_0}(x)$. To verify global convexity, it suffices to verify the next claim.

(2) I^u is subdifferentiable at $x \in \{\theta, x_u\}$. First, we argue that m is a subgradient of I^u at θ. This follows from the fact that the slope of I^u at θ is a subgradient of I_{F_0} at θ, and $I^u(\theta) = I_{F_0}(\theta)$. On $[0,\theta]$, $I^u = I_{F_0}$, and on $[\theta_u, \infty)$ I^u is above the line $x \longmapsto I(\zeta) + m(x-\zeta)$. Thus, $m \in \partial I^u(\theta)$. Second, the fact that m is a subgradient of I^u at x_u follows from the definition of x_u.

We established that $I^u(x) = I_\theta(x)$ for all $x \in [0,1]$. (1.) and (2.) hold by construction. **QED**

References

Aliprantis, Charalambos D. and Kim C. Border (2006), *Infinite Dimensional Analysis: A Hitchhiker's Guide*, third edition. Springer Berlin, Heidelberg.

Alonso, Ricardo and Odilon Câmara (2016), "Persuading voters." *American Economic Review*, 106(11), 3590–3605.

Arieli, Itai, Yakov Babichenko, Rann Smorodinsky, and Takuro Yamashita (2023), "Optimal persuasion via bi-pooling." *Theoretical Economics*, 18(1), 15–36.

Au, Pak Hung and Mark Whitmeyer (2023), "Attraction versus persuasion: Information provision in search markets." *Journal of Political Economy*, 131(1), 202–245.

Augias, Victor and Daniel M. A. Barreto (2024), "Persuading a wishful thinker." Working Paper.

Bauschke, Heinz H. and Patrick L. Combettes (2011), *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, second edition. Springer Cham.

Beauchêne, Dorian, Jian Li, and Ming Li (2019), "Ambiguous persuasion." *Journal of Economic Theory*, 179, 312–365.

Bizzotto, Jacopo, Jesper Rüdiger, and Adrien Vigier (2020), “Testing, disclosure and approval.” *Journal of Economic Theory*, 187, 105002.

Bizzotto, Jacopo and Adrien Vigier (2021), “Fees, reputation, and information production in the credit rating industry.” *American Economic Journal: Microeconomics*, 13(2), 1–34.

Bloedel, Alexander W. and Ilya Segal (2021), “Persuading a Rationally Inattentive Agent.” Working Paper.

Board, Simon and Jay Lu (2018), “Competitive information disclosure in search markets.” *Journal of Political Economy*, 126(5), 1965–2010.

Branco, Fernando, Monic Sun, and J. Miguel Villas-Boas (2016), “Too much information? information provision and search costs.” *Marketing Science*, 35(4), 605–618.

Brocas, Isabelle and Juan D. Carrillo (2007), “Influence through ignorance.” *The RAND Journal of Economics*, 38(4), 931–947.

Caplin, Andrew, Mark Dean, and John Leahy (2022), “Rationally inattentive behavior: Characterizing and generalizing shannon entropy.” *Journal of Political Economy*, 130(6), 1676–1715.

Che, Yeon-Koo, Kyungmin Kim, and Konrad Mierendorff (2023), “Keeping the listener engaged: A dynamic model of bayesian persuasion.” *Journal of Political Economy*, 131(7), 1797–1844.

de Clippel, Geoffroy and Xu Zhang (2022), “Non-bayesian persuasion.” *Journal of Political Economy*, 130(10), 2594–2642.

Denti, Tommaso (2022), “Posterior separable cost of information.” *American Economic Review*, 112(10), 3215–59.

Denti, Tommaso, Massimo Marinacci, and Aldo Rustichini (2022), “Experimental cost of information.” *American Economic Review*, 112(9), 3106–23.

Dworczak, Piotr and Giorgio Martini (2019), “The Simple Economics of Optimal Persuasion.” *Journal of Political Economy*, 127(5), 1993–2048.

Dworczak, Piotr and Alessandro Pavan (2022), “Preparing for the worst but hoping for the best: Robust (bayesian) persuasion.” *Econometrica*, 90(5), 2017–2051.

Felgenhauer, Mike (2019), “Endogenous persuasion with costly verification.” *The Scandinavian Journal of Economics*, 121(3), 1054–1087.

Feng, Yiding, Chien-Ju Ho, and Wei Tang (2024), “Rationality-robust information design: Bayesian persuasion under quantal response.” In *Proceedings of the 2024 ACM-SIAM Symposium on Discrete Algorithms, SODA 2024, Alexandria, VA, USA, January 7-10, 2024* (David P. Woodruff, ed.), 501–546, SIAM.

Floridi, Luciano (2014), *The Fourth Revolution: How the Infosphere is Reshaping Human Reality*. Oxford University Press, Oxford.

Galperti, Simone (2019), “Persuasion: The art of changing worldviews.” *American Economic Review*, 109(3), 996–1031.

Gehlbach, Scott and Alberto Simpser (2015), “Electoral manipulation as bureaucratic control.” *American Journal of Political Science*, 59(1), 212–224.

Gehlbach, Scott and Konstantin Sonin (2014), “Government control of the media.” *Journal of Public Economics*, 118, 163–171.

Gentzkow, Matthew and Emir Kamenica (2016), “A Rothschild-Stiglitz Approach to Bayesian Persuasion.” *American Economic Review*, 106(5), 597–601.

Gitmez, A. Arda and Pooya Molavi (2023), “Informational autocrats, diverse societies.”

Guo, Yingni and Eran Shmaya (2019), “The interval structure of optimal disclosure.” *Econometrica*, 87(2), 653–675.

Honka, Elisabeth, Ali Hortaçsu, and Maria Ana Vitorino (2017), “Advertising, consumer awareness, and choice: evidence from the u.s. banking industry.” *The RAND Journal of Economics*, 48(3), 611–646.

Hébert, Benjamin and Weijie Zhong (2024), “Engagement maximization.” URL https://arxiv.org/abs/2207.00685.

Jain, Vasudha and Mark Whitmeyer (2022), “Competitive disclosure of information to a rationally inattentive agent.” Working Paper.

Kamenica, Emir (2019), “Bayesian Persuasion and Information Design.” *Annual Review of Economics*, 11, 249–272.

Kamenica, Emir and Matthew Gentzkow (2011), “Bayesian Persuasion.” *American Economic Review*, 101(6), 2590–2615.

Kleiner, Andreas, Benny Moldovanu, and Philipp Strack (2021), “Extreme Points and Majorization: Economic Applications.” *Econometrica*, 89(4), 1557–1593.

Knoepfle, Jan (2020), “Dynamic competition for attention.” Working Paper.

Kolotilin, Anton (2018), “Optimal information disclosure: a linear programming approach.” *Theoretical Economics*, 13(2), 607–635.

Kolotilin, Anton, Timofiy Mylovanov, and Andriy Zapechelnyuk (2022), “Censorship as optimal persuasion.” *Theoretical Economics*, 17(2), 561–585.

Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk, and Ming Li (2017), “Persuasion of a Privately Informed Receiver.” *Econometrica*, 85(6), 1949–1964.

Liao, Xiaoye (2021), “Bayesian persuasion with optimal learning.” *Journal of Mathematical Economics*, 97, 102534.

Lipnowski, Elliot and Laurent Mathevet (2018), "Disclosure to a psychological audience." *American Economic Journal: Microeconomics*, 10(4), 67–93.

Lipnowski, Elliot, Laurent Mathevet, and Dong Wei (2020), "Attention management." *American Economic Review: Insights*, 2(1), 17–32.

Lipnowski, Elliot, Laurent Mathevet, and Dong Wei (2022), "Optimal attention management: A tractable framework." *Games and Economic Behavior*, 133, 170–180.

Lipnowski, Elliot, Doron Ravid, and Denis Shishkin (2021), "Persuasion via weak institutions." Working Paper. Electronic copy available at: https://ssrn.com/abstract=3168103, July 20, 2021.

Lipnowski, Elliot, Doron Ravid, and Denis Shishkin (2024), "Perfect bayesian persuasion."

Machina, Mark J. (1982), "Expected utility analysis without the independence axiom." *Econometrica*, 50(2), 277–324.

Matysková, Ludmila and Alfonso Montes (2023), "Bayesian Persuasion With Costly Information Acquisition." *Journal of Economic Theory*, 211, 105678.

Milgrom, Paul and Ilya Segal (2002), "Envelope theorems for arbitrary choice sets." *Econometrica*, 70(2), 583–601.

Pomatto, Luciano, Philipp Strack, and Omer Tamuz (2023), "The cost of information: The case of constant marginal costs." *American Economic Review*, 113(5), 1360–93.

Rayo, Luis and Ilya Segal (2010), "Optimal Information Disclosure." *Journal of Political Economy*, 118(5), 949–987.

Rockafellar, R. Tyrell (1970), *Convex Analysis*. Princeton University Press, Princeton.

Romanyuk, Gleb and Alex Smolin (2019), "Cream skimming and information design in matching markets." *American Economic Journal: Microeconomics*, 11(2), 250–76.

Shishkin, Denis (2024), "Evidence Acquisition and Voluntary Disclosure." Working Paper.

Simon, Herbert A. (1996), "Working paper no. 96–2 — Knowledge and the time to attend to it." In *Carnegie Bosch Institute for Applied Studies in International Management*.

Sims, Christopher A. (2003), "Implications of rational inattention." *Journal of Monetary Economics*, 50(3), 665–690.

Sun, Junze, Arthur Schram, and Randolph Sloof (2024), "Publicly persuading voters: The single-crossing property and the optimality of interval revelation." Working Paper.

Topkis, Donald M. (1978), "Minimizing a submodular function on a lattice." *Operations Research*, 26(2), 305–321.

Tsai, Yi-Lin and Elisabeth Honka (2021), "Informational and noninformational advertising content." *Marketing Science*, 40(6), 1030–1058.

Wei, Dong (2021), “Persuasion under costly learning.” *Journal of Mathematical Economics*, 94, 102451.