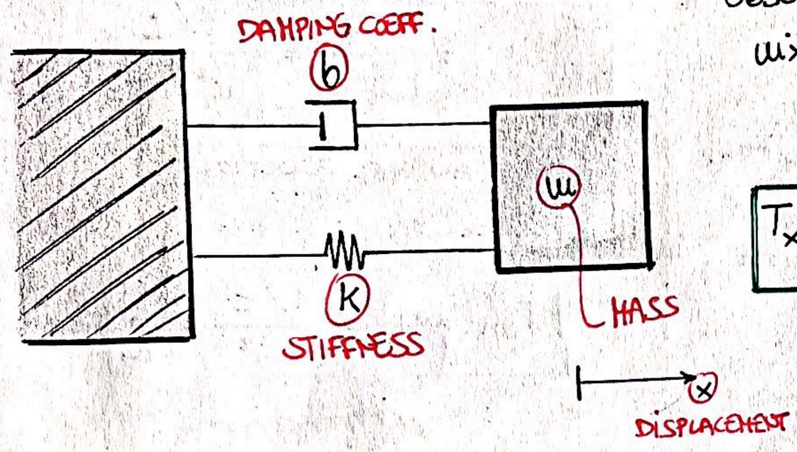
FORMULARIO DI MEMS & MICROSENSORS

the SPRING-MASS-DAMPER SYSTEM



described by:

$$m\ddot{x} + b\dot{x} + kx = F_{ext}$$

$\downarrow \mathcal{L}$ (laplace)

$$T_{xF}(s) = \frac{x(s)}{F_{ext}(s)} = \frac{1/m}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)} = \frac{1/m}{\left(s^2 + \frac{b}{m}s + \frac{k}{m}\right)}$$

TRANSFER FUNCTION DISPLACEMENT FORCE

- for $\omega \ll \omega_0$:

$$\boxed{|T_{xF}(j\omega)| = \frac{1/m}{\omega_0^2} = \frac{1}{k}}$$

- for $\omega = \omega_0$:

$$\boxed{|T_{xF}(j\omega)| = \frac{Q/m}{\omega_0^2} = \frac{Q}{k}}$$

- for $\omega \gg \omega_0$:

$$\boxed{|T_{xF}(j\omega)| = \frac{1/m}{\omega^2}}$$

$$\boxed{\omega_0 = \sqrt{k/m}}$$ RESONANCE FREQUENCY

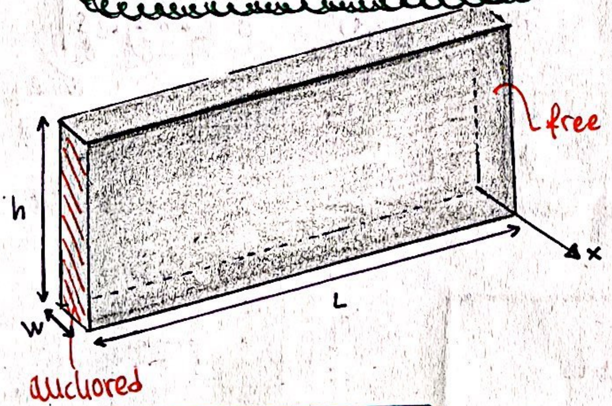
$$\boxed{Q = \omega_0 \frac{m}{b}}$$ QUALITY FACTOR

$$\boxed{\tau = Q/(\pi f_0)}$$ TIME-CONSTANT

since $b \propto \sqrt{T}$

$\hookrightarrow Q \propto 1/\sqrt{T}$

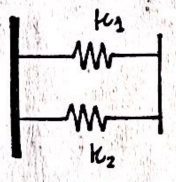
MECHANICAL STIFFNESS



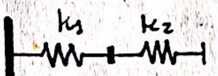
$$\boxed{k_x = E \cdot \frac{h}{4}\left(\frac{w}{L}\right)^3}$$

FREE END + ANCHORED END IP MOTION STIFFNESS

YOUNG'S MODULUS



$$\boxed{k_{TOT_{PAR}} = \sum_i k_i}$$ SPRINGS in PARALLEL



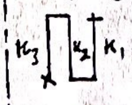
$$\boxed{\frac{1}{k_{TOT_{SER}}} = \sum_i \frac{1}{k_i}}$$ SPRINGS in SERIES

- SPRINGS USED ARE OFTEN GUIDED-END:

as two free spring in series

$$\boxed{k_{spring} = E h \left(\frac{W}{L}\right)^3}$$ GUIDED-END MECHANICAL STIFFNESS SPRING

- IF A PARAMETER IS NOT ALLOWED WE CAN FOLD SPRINGS: (to get lower ω).

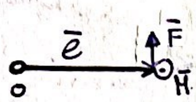


$$\boxed{k_{folded} = \frac{1}{N_{FOLD}} k_{spring}}$$ FOLDED SPRING STIFFNESS

TORSIONAL MECHANICAL STIFFNESS and SPRING-MASS-DAMPER SYSTEM

we define:

- **SHEAR MODULUS G**: ratio between shear stress and shear strain
- **TORQUE** $\vec{M} = \vec{\ell} \times \vec{F}$



- **TORSIONAL STIFFNESS k_θ**:

$$\boxed{k_\theta = G \frac{h}{3} \cdot \frac{W^3}{L}} \quad [N \cdot m]$$

- **MOMENT OF INERTIA I**: it's a moment of mass with respect to distance from an axis

$$I = \int_m r^2 \, dm$$

the spring-mass-damper system become:

$$\boxed{I\ddot{\theta} + b\dot{\theta} + k_\theta \theta = M_{ext}}$$

TORSIONAL SMD SYSTEM

the transfer function becomes:

$$\boxed{T_{\theta M}(j\omega) = \frac{\theta(j\omega)}{M_{ext}(j\omega)} = \frac{1/I}{\left(\omega_0^2 - \omega^2 + j\frac{\omega_0 \omega}{Q}\right)}}$$

TRANSFER FUNCTION

for $\omega \ll \omega_0$:

$$\boxed{|T_{\theta M}(j\omega)| = \frac{1/I}{\omega_0^2} = \frac{1}{k_\theta}}$$

DAMPING COEFFICIENT b

$$\boxed{\begin{aligned} b &\propto v_{mol} \cdot n_{mol} = \frac{p}{k_B T}\sqrt{k_B T} \propto \sqrt{T} \\ b &= b_{area} \cdot 2 \cdot L_{pp} \cdot H \cdot N_{pp} = (const \cdot P_0) 2 L_{pp} H N_{pp} \end{aligned}}$$

DAMPING COEFF.

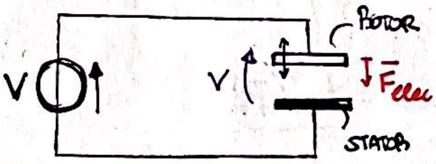
H: HEIGHT OF PROCESS

L_{pp}: LENGTH OF P.P.

MEMS MASS

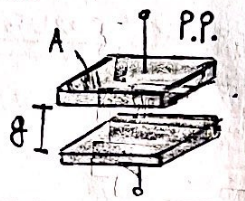
simply exploit the MATERIAL DENSITY ρ and volume dimensions

ELECTROSTATIC FORCES and EQUIVALENT ELECTROSTATIC STIFFNESS



$$\boxed{F_{elec} = \frac{V^2}{2} \cdot \frac{dC}{dx}}$$

ELECTROSTATIC FORCE (always directed to inside of capacitor)



- **GAP VARYING CAPACITOR** (parallel plate)

$$\boxed{C_{pp} = \varepsilon_0 \frac{AN}{g \pm x}} \qquad \frac{\partial C}{\partial x} = \varepsilon_0 \frac{AN}{(g-x)^2} = \frac{C_0}{g}\left[\frac{1}{1+\left(\frac{x}{g}\right)^2 - 2\frac{x}{g}}\right]$$

usually $x \ll g$

- **AREA VARYING CAPACITOR** (comb-finger)

$$\boxed{C_{CF} = 2 \varepsilon_0 N \frac{H(x_0 \pm x)}{g}} \qquad \frac{\partial C}{\partial x} = 2\varepsilon_0 \frac{HN}{g}$$

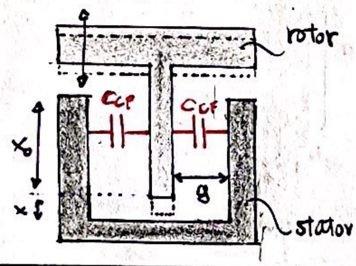
2: geometry

N: number

obtained from $F_{elec} + F_{elas}$

$$\boxed{k_{elec} = -2V_{DD}^2 \frac{C_0}{g^2}}$$

EQUIVALENT ELECTROSTATIC STIFFNESS (only PP)



MODULATION and DEMODULATION

IT'S USEFUL TO MOVE THE SIGNAL AWAY FROM FLICKER NOISE and FROM DC OFFSET

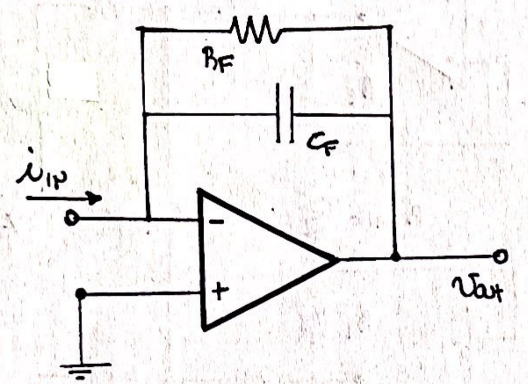
↳ the CAPACITANCE VARIATION IS MULTIPLIED BY A SINUSOIDAL WAVE AT f_{MOD}

→ WE ARE MODULATING THE ROTOR VOLTAGE, THAT MODULATES THE CURRENT BUT DOES NOT CHANGE THE TRANSFER FUNCTION OF THE ELECTRONIC NOISE TO THE OUTPUT

after DEMODULATION and LPF, the SIGNAL is moved back to BASE-BAND and LOW FREQ. NOISE IS SHIFTED AT HIGH FREQ. and then CUT-OFF.

COMMON ELECTRONICS BLOCKS

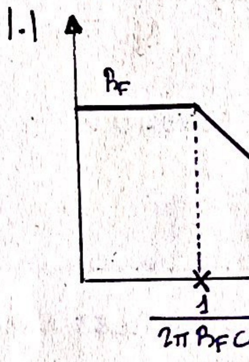
CHARGE AMPLIFIER and TRANSIMPEDANCE AMPLIFIER



IT'S THE GENERIC TOPOLOGY OF THE FRONT-END STAGE THAT READS OUT THE MOTIONAL CURRENT FLOWING THROUGH THE SENSE ELECTRODE OF A MEMS STRUCTURE

↓

ITS ROLE IS TO READOUT THE MOTIONAL CURRENT and TRANSLATE IT INTO A VOLTAGE



$$T(s) = \frac{R_F}{1 + s R_F C_F}$$

- **TIA**: in a TRANSIMPEDANCE AMPLIFIER, the feedback is DOMINATED by the RESISTANCE. Indeed the pole MUST BE at least a DECADE AFTER the operating system frequency

$$\omega_F = \frac{1}{R_F C_F} \gg \omega_0$$

$$T_{TIA}(s)\Big|_{\omega \simeq \omega_0} \simeq R_F$$

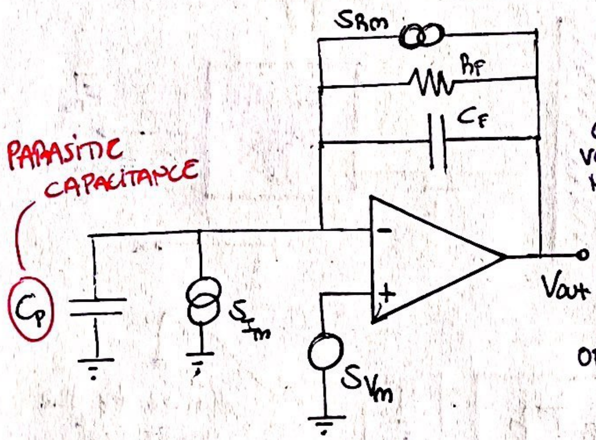
- **CA**: in a CHARGE AMPLIFIER, the feedback is DOMINATED by the CAPACITANCE. Indeed the pole should be at least a decade before the working freq.

$$\omega_F = \frac{1}{R_F C_F} \ll \omega_0$$

$$T_{CA}(s)\Big|_{\omega \simeq \omega_0} = \frac{1}{s C_F}$$

- CA can be preferred because the SNR is dependent on $\sqrt{R_F}$ but not on capacitance. Indeed we prefer the highest R_F which happen with low pole. We can make feedback resistance noise negligible
- in addition with TIA we cannot independently choose SNR and the GAIN as they both depend on R_F

TIA and CA - NOISE



for NOISE CONTRIBUTIONS:

$$\sqrt{S_{m,tot}} = \sqrt{\sum_i S_{m,i}} \quad \left[\frac{\dots}{\sqrt{Hz}}\right]$$

$$\boxed{S^{OUT}_{V_m}} = S_{V_m}\left(1 + \frac{R_F \,||\, \frac{1}{sC_F}}{\frac{1}{sC_P}}\right)^2 \underset{\omega R_F C_F \gg 1}{\simeq} \boxed{S_{V_m}\left(1 + \frac{C_P}{C_F}\right)^2} \quad [V^2/Hz]$$

OP-AMP VOLTAGE NOISE

(by doing $\sqrt{S^{out}_{V_m}}$ / SENSITIVITY we find the noise in the quantity to be measured)

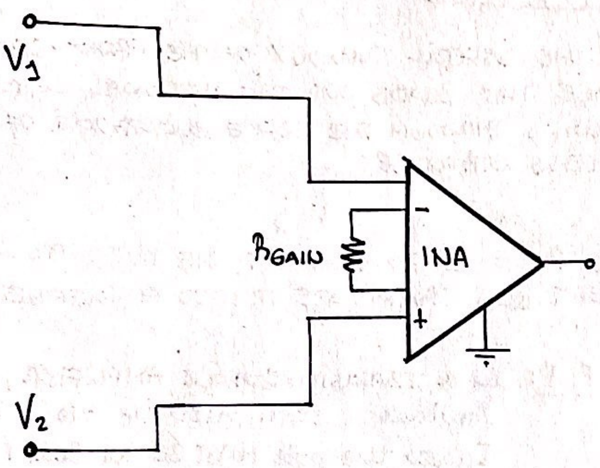
$$\boxed{S^{OUT}_{I_m}} = S_{I_m}\left(\frac{R_F}{1 + sC_F R_F}\right)^2 \underset{\omega R_F C_F \gg 1}{\simeq} \boxed{S_{I_m}\left(\frac{1}{sC_F}\right)^2 \left[\frac{V^2}{Hz}\right]}$$

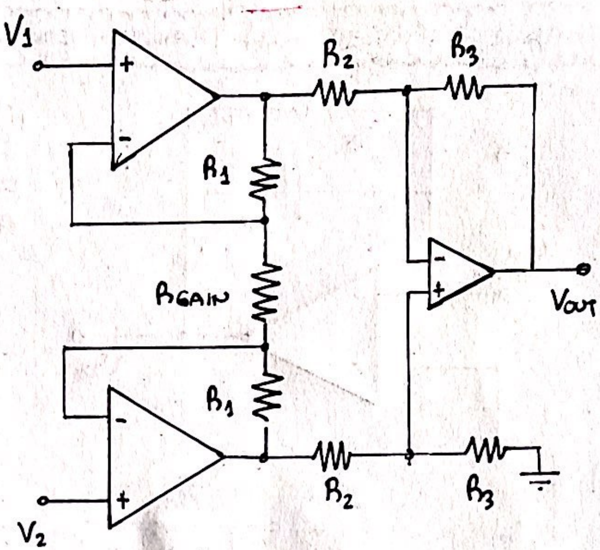
OP-AMP CURRENT NOISE

$$\boxed{S^{OUT}_{R_{F_m}}} = \frac{4k_BT}{R_F}\left(\frac{R_F}{1 + sC_F R_F}\right)^2 \simeq \boxed{\frac{4k_BT}{R_F}\left(\frac{1}{sC_F}\right)^2 \left[\frac{V^2}{Hz}\right]}$$

FEEDBACK RESISTANCE THERMAL NOISE

INA - INSTRUMENTATION AMPLIFIER





IT HAS LOW OFFSET AND NOISE, HIGH INPUT IMPEDANCE (THANKS TO BUFFERS).
IT AMPLIFY A MEASURED ΔV_{out} AND REFER IT TO GROUND.

INA NOISE

INPUT REFERRED INA NOISE IS DUE TO THE TWO OP-AMPS VOLTAGE NOISE SOURCES and the THERMAL NOISE OF GAIN RESISTANCE

$$S_{n_V, INA} = 2 S_{n_V, OP\text{-}AMP} + S_{n_V, R_{INA}}$$

dove:

$$\boxed{S_{n_V, R_{INA}} = 4 k_B T R_{GAIN}}$$

GAIN RESISTANCE NOISE

and

OP-AMP VOLTAGE NOISE

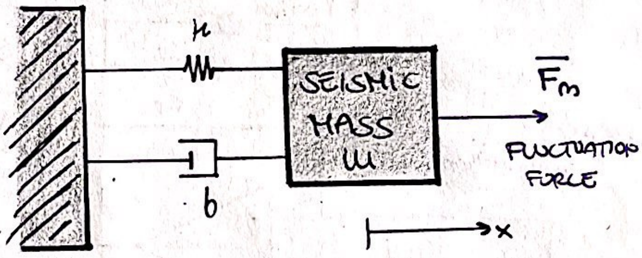
$$\boxed{S_{n_V, OP\text{-}AMP} = 2 \cdot \frac{4 k_B T \gamma}{g_m}}$$

two transistors

γ = 2/3 MOSFET in SATURATION

THERMOMECHANICAL NOISE

ANY DISSIPATIVE MECHANISM THAT RESULTS IN A MECHANICAL DAMPING MUST BE BALANCED BY A FLUCTUATION FORCE TO OBTAIN MACROSCOPIC ENERGY BALANCE. IT'S THE FORCE THAT PASS BETWEEN THE MEMS and the ENVIRONMENT.



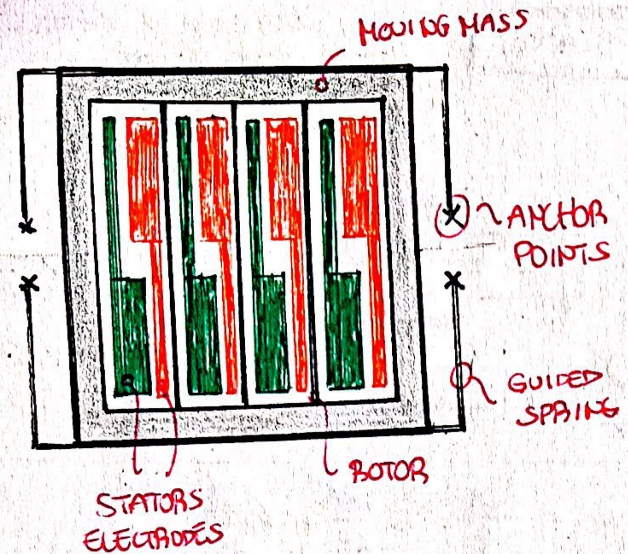
described by a NOISE SPECTRUM

$$\boxed{S_{F_n} = 4 k_B T b} \quad \left[\frac{N^2}{Hz}\right]$$

MEMS NOISE POWER SPECTRAL DENSITY

divide by the sensitivity² to obtain noise in the quantity to be measured

ACCELEROMETER



Parallel Plates Accelerometers

IT'S A SPRING-MASS-DAMPER SYSTEM described by:

$$m\ddot{x} + b\dot{x} + K_{TOT}\,x = m a_{ext}$$

where

$$\boxed{K_{TOT} = K_{mech} + \underbrace{K_{elec}}_{(<0)}}$$

an ACCELEROMETER can be
- PARALLEL PLATE / COMB-FINGER
- IN-PLANE MOTION / OUT-of-PLANE MOTION

it usually works at LOW FREQUENCES ($\omega \ll \omega_0$), indeed

$$|T_{xF}(j\omega)| \simeq 1/K_{TOT}$$

THE MECHANICAL SENSITIVITY IS:

$$\boxed{S_{mech} = \frac{dC}{da} = \underbrace{2}_{\text{DIFFERENTIAL GAIN}} \cdot \underbrace{\frac{dx}{da}}_{\text{ACCELERATION TO DISPLACEMENT GAIN}} \cdot \underbrace{\frac{dC}{dx}}_{\text{DISPLACEMENT TO CAPACITANCE GAIN}}}$$

MECHANICAL SENSITIVITY

DISPLACEMENT v ACCELERATION

$\boxed{\frac{dx}{da}}$ term is given by the spring-mass-damper system description. Usually the ACCELEROMETER working REGION is $\omega \ll \omega_0$ then

$$\left|\frac{X(s)}{F(s)}\right| = \left|\frac{X(s)}{mA(s)}\right|_{\omega \ll \omega_0} \simeq \frac{1}{K_{TOT}}$$

$$\Downarrow$$

$$\boxed{\frac{dx}{da} = \frac{m}{K_{TOT}} = \frac{1}{\omega_0^2}}$$

$$\boxed{S_{mech_{PP}} = 2\,\frac{C_0}{g}\cdot\frac{1}{\omega_0^2}}$$

PARALLEL-PLATE

$$\boxed{S_{mech_{CF}} = 2\,\frac{C_0}{x_0}\cdot\frac{1}{\omega_0^2}}$$

COMB-FINGER

CAPACITANCE VARIATION

$\boxed{\frac{dC}{dx}}$ term depends on the type of ACCELEROMETER

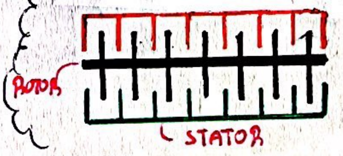
- PARALLEL PLATE:

$$= \frac{C_0}{g}$$

- COMB-FINGER:

$$= \frac{C_0}{x_0}$$

NOTE THAT COMB-FINGER:
- IS good BECAUSE THE ELECTROSTATIC FORCE IS NOT FUNCTION OF DISPLACEMENT (NO PULL-IN RISK) and NO-NON LINEARITY in CAPACITIVE READOUT NO TRADE-OFF SENSITIVITY v FSR
- IS bad BECAUSE WITH SAME PARAMETERS THE NUMBER OF FINGERS WHICH CAN FIT IS LOWER



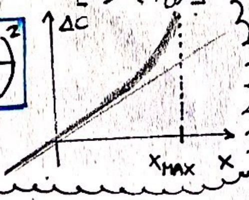
PP-CONFIG. NON-LINEARITY

the real capacitance variation is not $2\frac{C_0}{g}x$ but $C_0\left[\frac{2x/g}{1-(x/g)^2}\right]$

the non-linearity is:

$$\varepsilon_{\%}(x) = \frac{\Delta C_{REAL} - \Delta C_{LIN}}{\Delta C_{REAL}} \rightarrow \boxed{\varepsilon_{\%}^{MAX} = 100\left(\frac{x_{MAX}}{g}\right)^2}$$

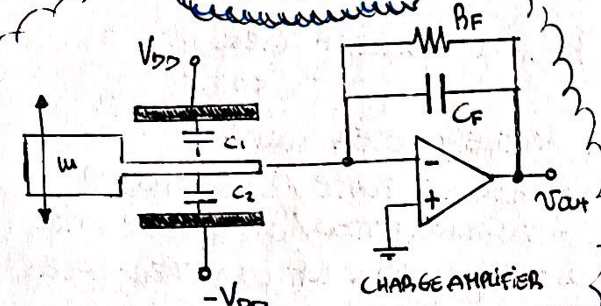
PP LINEARITY ERROR



READOUT CONFIGS

6 THERE ARE TWO CONFIGS

DC READOUT



$$i_{C_i} = \frac{dQ_i}{dt} = \cancel{C_i \frac{dV_i}{dt}} + V_i \frac{dC_i}{dt} = V_i \frac{dC_i}{dt}$$

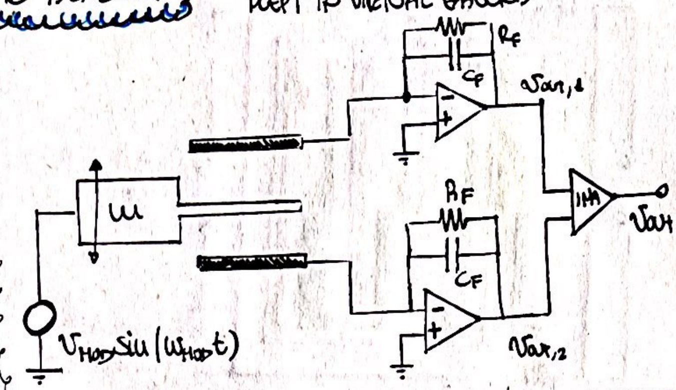
$$v_{out} = -i\,\frac{1}{sC_F} = -\left[V_{DD}\, s\, C(s)\right]\frac{1}{sC_F}$$

$$\boxed{\frac{dV_{out}}{dC} = \frac{V_{DD}}{C_F}}$$ CAPACITANCE VARIATION TO VOLTAGE GAIN (DC)

☹ CANNOT MEASURE DC ACCELERATIONS (WORKING REGION, AFTER $R_F C_F$ POLE; WE NEED C.A.

AC READOUT

IT CONSIST IN A HIGH-FREQUENCY MODULATION OF THE SUSPENDED MASS, WITH EACH OF STATORS KEPT IN VIRTUAL GROUND



$$i_{C_i} = \frac{dQ_i}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

by assuming $C = C_0 + C_a[\cos(\omega_a t)]$ (ω_a: acceleration freq.)

$$\frac{C\frac{dV}{dt}}{V\frac{dC}{dt}} \approx \frac{\omega_{MOD}}{\omega_a}$$

if $\omega_{MOD} \gg \omega_a$, $V\frac{dC}{dt}$ is negl.

$$V_{out,i} = -\frac{1}{C_F}\int C\,\frac{dV}{dt}\,dt = -\frac{C_0 \pm \Delta C}{C_F} V_{MOD}\sin(\omega_{MOD}t)$$

$$\boxed{\frac{dV_{out}}{dC} = \frac{V_{MOD}}{C_F}\sin(\omega_{MOD}t)}$$ CAPACITANCE TO VOLTAGE GAIN (AC)

☺ - CAN MEASURE DC ACCELERATIONS
- MOVE AWAY FROM FLICKER NOISE AND DC OFFSET
- NEED TO DEMODULATE AND LPF TO CUT-OFF NOISE SHIFTED AT HIGH FREQ.
- $V_{DD} = V_{MOD}$

$$S_{TOT} = \frac{dV_{OUT}}{da} = \frac{dV_{out}}{dC}\,S_{mech}$$

ONLY IF MODULATION APPROACH

ONLY IF WE HAVE DIFFERENTIAL READOUT

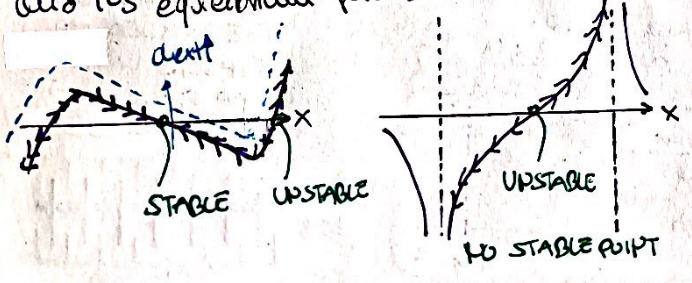
$$\boxed{\begin{aligned} S_{TOT}^{PP} &= 2\,\frac{1}{2}\frac{V_{DD}}{C_F}\,2\,\frac{C_0}{g}\cdot\frac{1}{\omega_0^2} \\ S_{TOT}^{CF} &= 2\,\frac{1}{2}\frac{V_{DD}}{C_F}\,2\,\frac{C_0}{x_0}\cdot\frac{1}{\omega_0^2} \end{aligned}}$$

OVERALL SENSITIVITY

CAPACITANCE VARIATION

PULL-IN and STABILITY

we can analyze the system total FORCE and its equilibrium points



IT COULD HAPPEN THAT THERE ARE NO STABLE POINT AND THE ROTOR PLATES SNAP ONTO THE STATOR ONES (PULL-IN CONDITION).

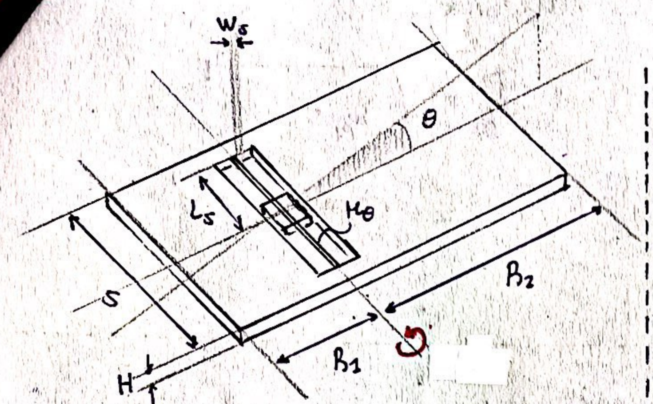
WE NEED:

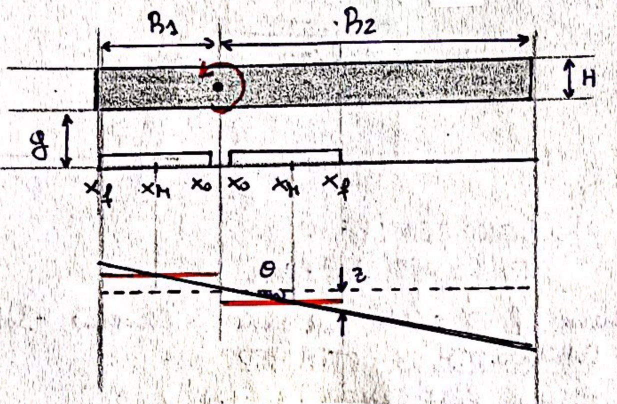
$$\boxed{V_{DD} < V_{PULL\text{-}IN} = \sqrt{\frac{g^3 k}{2\varepsilon_0 A N}}}$$

NO PULL-IN CONDITION

$$\boxed{|k_{mech}| > |k_{elec}|}$$

T-OF-PLANE ACCELEROMETER





we take the AVERAGE DISPLACEMENT z for CAPACITANCE VARIATIONS | gap in x_M

$$C_i = \varepsilon_0 \frac{(x_f - x_0)\cdot e_{PP}}{g \pm z}$$

$$z = x_M \tan(\theta) \simeq x_M \theta$$

$$\boxed{S_{mech} = \underbrace{2\alpha \frac{C_0}{g}}_{\frac{\partial C}{\partial z}} \cdot \underbrace{x_M}_{\frac{\partial z}{\partial \theta}} \cdot \underbrace{\frac{1}{\omega_0^2}\frac{e_M}{I}}_{\frac{\partial \theta}{\partial a}}}$$

MECHANICAL SENSITIVITY

comes from

$$\theta = \frac{1}{\omega_0^2}\frac{M}{I} = \frac{1}{\omega_0^2}\frac{\bar{e}\times(m\bar{a})}{I}$$

THE SYSTEM IS DESCRIBED BY:

$$I\ddot{\theta} + b\dot{\theta} + k_{TOT}\theta = M_{ext} \quad \text{and} \quad |T_{\theta M}| \simeq \frac{1}{k_\theta}\Big|_{\omega \ll \omega_0}$$

$$\Delta C_{REAL} = \varepsilon_0 \frac{A_{PP}}{g-z} - \varepsilon_0 \frac{A_{PP}}{g+z} = C_0 \frac{2\,z/g}{1-(z/g)^2}\alpha$$

$$\Delta C_{lin} = \Delta C_{REAL}\Big|_{z \ll g} \simeq 2 C_0 \frac{z}{g}\alpha$$

$$\boxed{\varepsilon_{lin\%} = 100\left(\frac{z_{FSR}}{g}\right)^2 = 100\left(\frac{x_M\cdot\theta_{FSR}}{g}\right)^2}$$

LINEARITY ERROR

$$M_{FSR} = M_1 + M_2 = \bar{B}_1 \times \bar{F}_1 + \bar{B}_2 \times \bar{F}_2 =$$
$$= -\frac{B_1}{2} m_1 FSR\,\hat{g} + \frac{B_2}{2} m_2 FSR\,\hat{g}$$

force applied at half

$$\boxed{\theta_{FSR} = \frac{M_{FSR}}{k} = \frac{1}{\omega_0^2}\cdot\frac{M_{FSR}}{I}} \qquad \boxed{\omega_0 = \sqrt{\frac{k}{I}}}$$

$$\Delta C_{FSR} = 2\alpha \frac{C_0}{g} z_{FSR}$$

HOLES TRANSDUCTION COEFF.

$$M_{elec} = 2F_{elec}x_M = 2\alpha \frac{C_0}{g^2} V_{DD}^2 z x_M = 2\alpha\frac{C_0}{g^2} V_{DD}^2 \theta x_M^2$$

$$\boxed{\begin{aligned} k_{elec} &= -2\alpha \frac{C_0}{g^3} V_{DD}^2 x_M^2 \\ k_{mech} &= 2G\frac{H}{3}\cdot\frac{w_s^3}{l_s} \end{aligned}}$$

STIFFNESS k_{TOT}

2x (two springs)

the MOMENT OF INERTIA is

$$I = I_1 + I_2 = \int_{m_1} r^2 dm + \int_{m_2} r^2 dm$$

$$dm = sH\rho\, dr$$

$$\boxed{I = \frac{B_1^2 m_1 + B_2^2 m_2}{3}}$$

MOMENT OF INERTIA

MECHANICAL OFFSET

DUE TO PROCESS NON-UNIFORMITIES IN FABRICATION PROCESS
- GAPS OF TWO DIFFERENTIAL PP
- RESIDUAL MECHANICAL STRESS IN WAFER BENDING

IT'S CALLED ZGO - ZERO G OUTPUT

$$ZGO(T) = \frac{x_{os}(T)}{\frac{dx}{da_{ext}}}$$

IT ADDS NON-LINEARITY

$$\boxed{\Delta C_{diff} = C_0 \frac{x}{g}\left[\frac{-1}{1+\frac{x}{g-x_{os}}} + \frac{1}{1-\frac{x}{(g+x_{os})}}\right]}$$

to correct it we can
- apply WAFER-LEVEL CALIBRATION
- apply IN-MOTION CALIBRATION

8 NEAD

it's the NOISE EQUIVALENT ACCELERATION DENSITY

$$\boxed{NEAD = \sqrt{S_{A_m}} = \frac{\sqrt{S_{F_m}}}{m}} \quad \left[\frac{m/s^2}{\sqrt{Hz}}\right]$$

$$= \sqrt{\frac{4 k_B T b}{m}} = \sqrt{\frac{4 k_B T \omega_0}{m Q}}$$

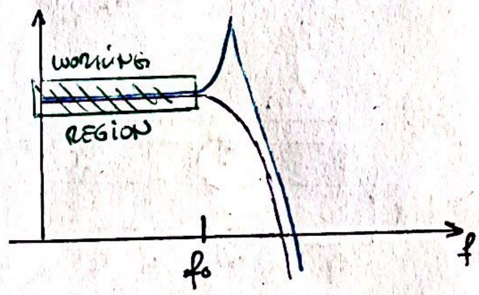
for overall contribution we need bandwidth BW

$$\bar{a}_m = \sqrt{S_{A_m} \cdot BW} = NEAD \cdot \sqrt{BW}$$

CHOICHE OF QUALITY FACTOR Q

the QUALITY FACTOR Q is a key factor, it appears in:

- NEAD (HIGH Q, LOW NOISE)
- TIME RESPONSE
- SPECTRUM TRANSFER FUNCTION



IF MEMS NOISE DOMINATES ⟶ HIGH Q ⟶ REQUIRED LPF TO LOWER THE PEAK

IF ELECTRONIC NOISE DOMINATES ⟶ $Q \simeq 0.5$

⟶ because if $Q < 0.5$ the gain is lowered −3dB and BANDWIDTH is OPTIMIZED (if HIGH Q flat region is less!)

TRADE-OFFS

- about SENSITIVITY:

$$S_{tot} = 2 \frac{V_{DD}}{C_F} \cdot \frac{C_0}{g} \cdot \frac{m}{\left(k - 2V_{DD}^2 \frac{C_0}{g^2}\right)}$$

- $g \downarrow$: $S_{TOT} \uparrow$ PULL-IN RISK ↑
- $m \uparrow$: $S_{TOT} \uparrow$ AREA ↑ BANDWIDTH ↓
- $k_{TOT} \downarrow$: $S_{TOT} \uparrow$ PULL-IN RISK ↑ BANDWIDTH ↓
- $V_{DD} \uparrow$: $S_{TOT} \uparrow$ PULL-IN RISK ↑ POWER CONSUMPTION ↑
- H : $S_{TOT} \uparrow$ AREA, m, k ↑

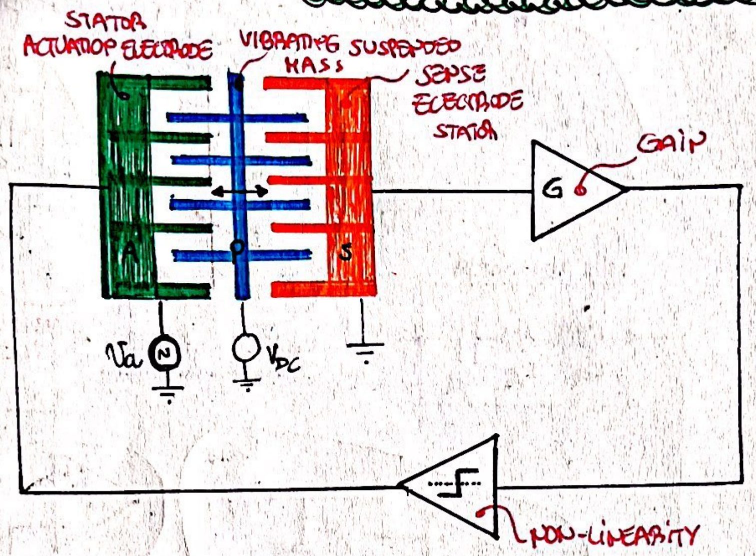
- about BANDWIDTH:

NO PULL-IN $k_{TOT} \uparrow$ $f_0 \uparrow$ SENSITIVITY ↓

$a_{MAX} \uparrow$ $\omega_0^2 \uparrow$ SENSITIVITY ↓ (because $a_{MAX} = x_{MAX} \omega_0^2$)

BANDWIDTH − NOISE (about Q)

RESONATORS and OSCILLATORS



PARAMETERS

- N_{CF} : number of ROTATOR FINGERS
- g : gap
- L_{ov} : finger overlap at rest
- m : mass
- k : stiffness
- b : damping coeff.
- H : process height
- A : $H \cdot L_{ov}$ area

ELECTROSTATIC FORCES

BY CHOOSING

$V_P = V_{DC}$

$V_A = v_a \sin(\omega_0 t)$

$V_S = 0V$

$$|F_{elec}| = \frac{V^2}{2} \cdot \frac{\partial C}{\partial x} \; ; \quad \text{for COMB-FINGER } \frac{\partial C}{\partial x} = \varepsilon_0 \frac{N_{CF} H}{g} = \frac{C_0}{x_0}$$

$$|F_{elec,TOT}| = |F_{elec,1} - F_{elec,2}|$$

$$= \left| \frac{1}{2} \cdot \frac{\varepsilon_0 N_{CF} H}{g} \left[(V_{DC} - v_a \sin \omega_0 t)^2 - V_{DC}^2 \right] \right|$$

$$= \frac{1}{2} \varepsilon_0 \frac{N_{CF} H}{g} \left[2 V_{DC} v_a \sin(\omega_0 t) + \cancel{v_a^2 \sin^2(\omega_0 t)} \right]$$

$$\simeq \frac{\varepsilon_0 N_{CF} H}{g} V_{DC} v_a \sin(\omega_0 t) \qquad v_a \ll V_{DC}$$

ACTUATION TRANSDUCTION FACTOR

$$\boxed{\eta_A = \frac{F_{elec}(s)}{v_a(s)} = V_{DC} \cdot \frac{dC_A}{dx}}$$

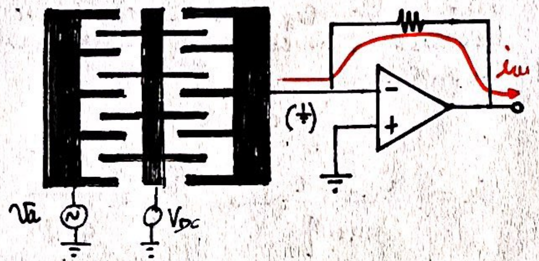
SENSE PORT READOUT

IN ORDER TO READOUT ROTOR DISPLACEMENT AND CALIBRATE THE LOOP GAIN WE NEED TO SENSE THE CURRENT INDUCED BY CAPACITANCE VARIATION

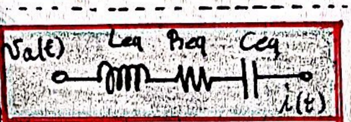
$$i_m = V_{DC} \cdot \frac{dC_s}{dt} = V_{DC} \frac{dC_s}{dx} \cdot \frac{dx}{dt} = V_{DC} \frac{dC_s}{dx} \dot{x}$$

$$\boxed{\eta_S = \frac{i_m(s)}{s X(s)} = V_{DC} \frac{dC_s}{dx}}$$

ELECTROMECHANICAL TRANSDUCTION FACTOR



$$\frac{i_m(s)}{V_a(s)} = \frac{1}{\left(L_{eq} s + R_{eq} + \frac{1}{s C_{eq}}\right)}$$



$L_{eq} = m/\eta^2 \quad R_{eq} = b/\eta^2 \quad C_{eq} = \eta^2/k$

EQUIVALENT CIRCUIT OF RESONATOR

ELECTRICAL MODEL

CONSIDERING $\eta_A = \eta_S$ WE FIND:

$$\frac{X(s)}{F_{elec}(s)} = \frac{1}{m s^2 + b s + k}$$

(ADMITTANCE)

$$\boxed{\frac{i_m(s)}{V_a(s)} = \eta^2 s \frac{1/m}{\left(s^2 + \frac{b}{m} s + \frac{k}{m}\right)} = \eta^2 s \frac{1/m}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)}}$$

$\omega \ll \omega_0$: $T_{Ia} = \frac{\eta^2}{k} s$

$\omega = \omega_0$: $T_{Ia} = \eta^2 / b$

$\omega \gg \omega_0$: $T_{Ia} = \frac{\eta^2}{m} \cdot \frac{1}{s}$

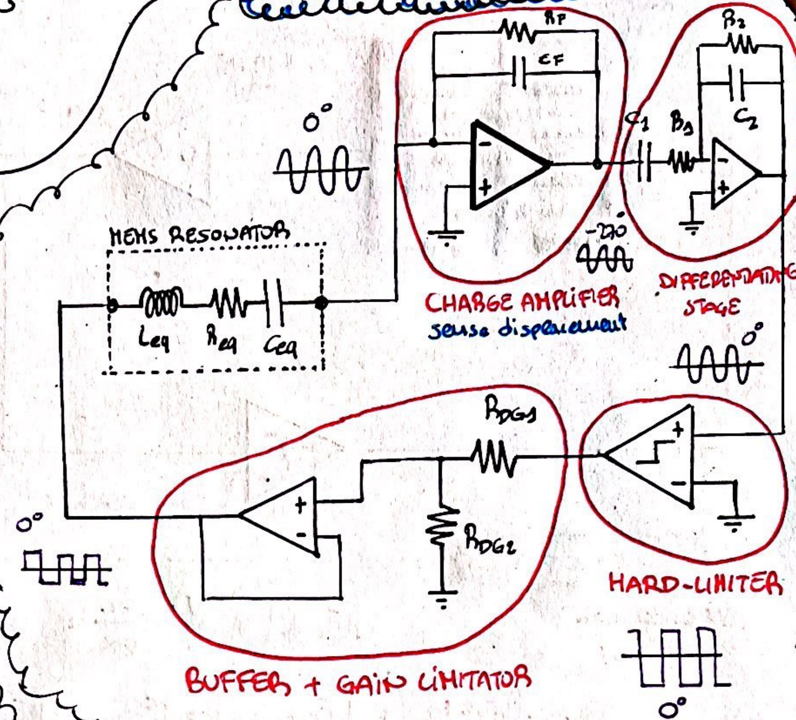
BARKHAUSEN CRITERIA and SUBSTAINING CIRCUIT

IN ORDER TO HAVE A STABLE OSCILLATION THE BARKHAUSEN CRITERIA SHOULD BE MET:

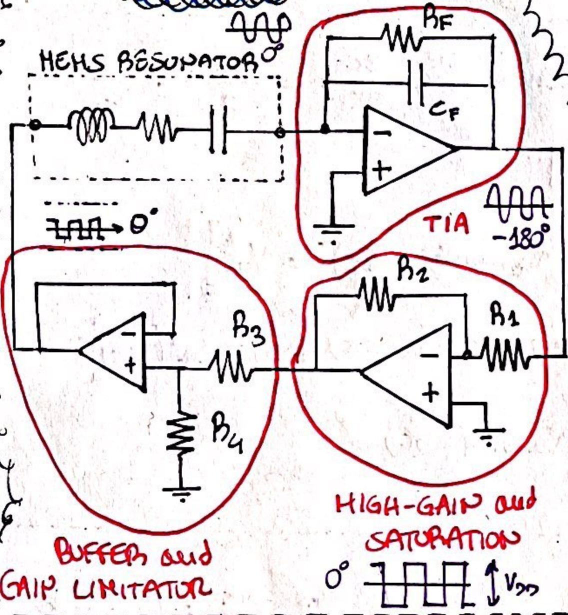
$$\left| G_{loop}(j\omega_0) \right| = 1$$
$$\measuredangle \left[G_{loop}(j\omega_0) \right] = 360°$$

BARKHAUSEN CRITERIA

CHARGE AMPLIFIER CONFIG.



TRANSIMPEDANCE AMPLIFIER CONFIG.



- **TIA**: $G_{TIA}(j\omega)\Big|_{\omega \simeq \omega_0} \simeq R_F \quad \measuredangle -180°$

  R_F dominates over C_F

  $$\omega_F = \frac{1}{R_F C_F} \gg \omega_0$$

- **HIGH GAIN AND SATURATION**:

  $$G_{HG} = -\frac{R_2}{R_1}$$

  OUTPUT SATURATES at $\pm V_{DD}$

- **BUFFER and GAIN LIMITATOR**:

  ''

CA is to be preferred than TIA because of $SNR \propto \sqrt{R_F}$

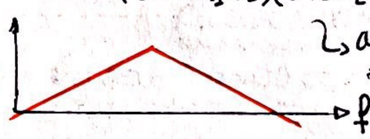
- **CHARGE AMPLIFIER**: sense the displacement

  $$G_{CA}(j\omega)\Big|_{\omega \simeq \omega_0} = \frac{1}{j\omega C_F} \qquad \measuredangle: -180° -90° = -270°$$

  ADD $\measuredangle = 180 - \arctan(f_0/f_{pCA})$

- **DIFFERENTIATING STAGE**: adjust the phase by adding $-90°$ (total reach $-360°$)

  $$G_{DIF}(s) = \frac{-sC_1R_2}{(1+sC_1R_1)(1+sC_2R_2)}$$



  ↳ at high freq

- **HARD-LIMITER**: high voltage, saturates at $\pm V_{DD}$. It's output cannot be used to directly drive the resonator because it would violate small signal hypothesis

  $$\frac{V_a}{4} \ll V_{DC}$$

  $$V_a = \frac{x_{a\,MAX} k}{Q_{MAX} \eta} \longrightarrow V_{a\,sq} = \frac{V_a}{4/\pi}$$

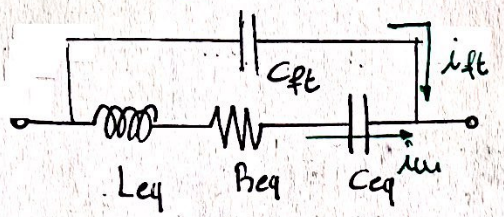
- **BUFFER and GAIN LIMITATOR**:

  $$G_{DG} = \frac{V_{a,sq}}{V_{DD}} = \frac{R_{DG2}}{R_{DG1}+R_{DG2}}$$

  ↓ >100 LOOP GAIN at start-up

$$\boxed{G_{loop}(j\omega_0) = \frac{1}{R_{eq}} G_{CA} G_{DIFF} G_{HL} G_{DG}}$$

EFFECTS OF FEEDTHROUGH CAPACITANCE

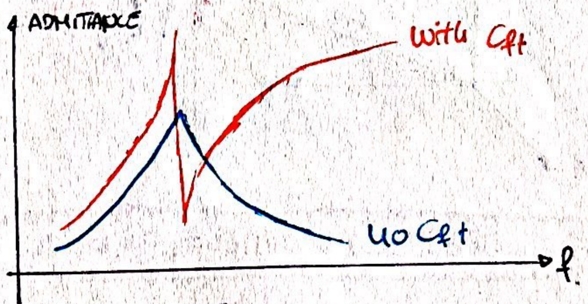


TRANSFER FUNCTION WITH FEEDTHROUGH CAPACITANCE

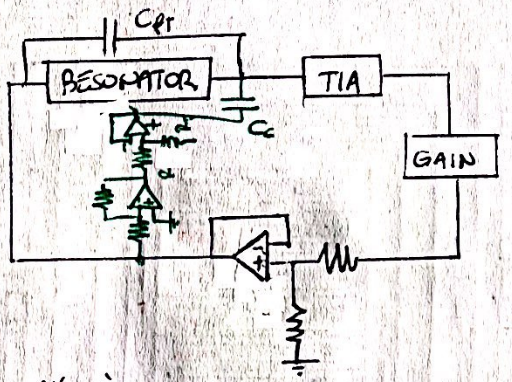
$$\boxed{\frac{i(s)}{v_a(s)} = \frac{1}{sL_{eq} + R_{eq} + \frac{1}{sC_{eq}}} + sC_{ft}}$$

↳ we want to lower the Gap right after the resonance peak

- we can solve it with a 2ND POLE near after the peak we can avoid other freq. to be sustained



- introduce an OPPOSITE CURRENT to the feedthrough contribution using a suitable capacitive gain excited by a current in antiphase to feedthrough signal



$$i_c = C_c \frac{dv_a}{dt}\left(-\frac{R_T}{R_T + R}\right)$$

$$C_{ft} = C_c\left(\frac{A_T}{R_T + R}\right)$$

try to annul with i_{cc} i_{ft}

TUNING FREQUENCY

WE TRY TO EXPLOIT ELECTROSTATIC SOFTENING TO DOWNTUNE WITH PARALLEL PLATES

N_{tun} : number of pp

C_{pp} : overall capacitance

g_{pp} : gap

V_{tun} : drive tuning

$$\Delta\omega = \sqrt{\frac{k_{TOT}}{m}} - \sqrt{\frac{k - 2(V_{DC} - V_{tun})^2 \frac{C_{PP}}{g_{PP}^2}}{m}}$$

$$= \sqrt{\frac{|k_{mech}| - |k_{el}|}{m}} - \sqrt{\frac{k_{mech} - |k_{el}| + |k_{tun}|}{m}}$$

$$k_{tun} = 2V_{tun}^2 \frac{C_0 N_{tun}}{g_{tun}^2}$$

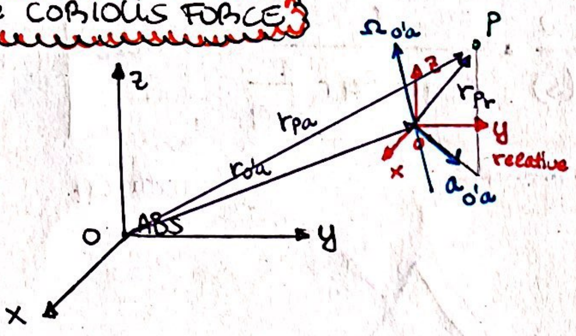
FREQ. SHIFT

TRADE-OFFS

- an HIGH Q could be desiderable as electronic gain could be lower
- an HIGH BIAS VOLTAGE on ROTOR would increase η but also increases POWER CONSUMPTION
- we need low displacements to avoid non-linearities

GYROSCOPES

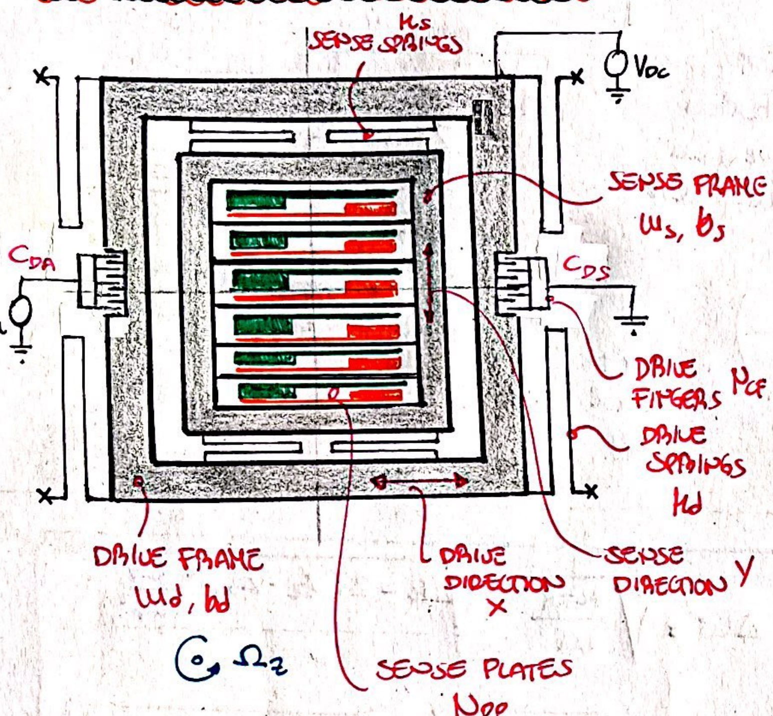
the CORIOLIS FORCE



$$\vec{a}_{Pa} = \vec{a}_{Pr} + \vec{a}_{o'a} + \cancel{(\dot{\vec{\Omega}}_{o'a} \times \vec{r}_{Pr})}^{<<} + \vec{\Omega}_{o'a} \times \cancel{(\vec{\Omega}_{o'a} + \vec{r}_{Pr})}^{<<} + 2(\vec{\Omega}_{o'a} \times \vec{v}_{Pr})$$

) CORIOLIS ACCELERATION

ARCHITECTURE and WORKING PRINCIPLE



there are two motion directions of interest

- DRIVE MODE: along which the device oscillates creating a desired v_x
- SENSE MODE: direction along which device moves only under angular rates

it's a combination of

COMB DRIVEN RESONATOR + PARALLEL PLATE ACCELEROMETER

MODE MATCHED GYROSCOPE

$$\left.\begin{aligned} C_{DA} &= \frac{2\varepsilon_0 H (L_{ov} - x) N_{CF}}{g} \\ C_{DD} &= \frac{2\varepsilon_0 H (L_{ov} + x) N_{CF}}{g} \end{aligned}\right\} \longrightarrow |F_{elec}| \cong \frac{\varepsilon_0 H N_{CF}}{g}\left[2 V_{DC} V_a \sin(\omega_0 t)\right]$$

$$\boxed{Q = \omega_0 \frac{m}{b} \qquad \Delta\omega_{BW} = \frac{\omega_0}{2Q}}$$

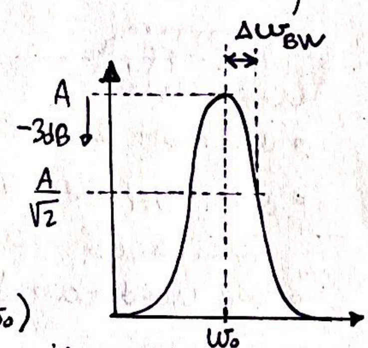
$$x_{D,0} = F_{elec,0}\,\frac{Q_D}{k_D} = \frac{Q_D}{k_D}\cdot\frac{\varepsilon_0 H N_{CF}}{g}\,2V_{DC}V_a$$

$$v_D = \dot{x}_D \longrightarrow v_{D,0} = \omega_0 x_{D,0}$$

$$F_{COR_0} = m_s \cdot a_{COR_0} = -2 m_s v_{D_0} \Omega \quad \text{(modulated at } \omega_0\text{)}$$

$$y_{S,0} = F_{COR,0}\cdot\frac{Q_s}{k_s} = 2 m_s x_{D,0}\,\omega_0\,\frac{Q_s}{k_s}\,\Omega \implies \boxed{\frac{y_{S,0}}{\Omega} = \frac{x_{D,0}}{b_s/2m_s} = \frac{x_{D,0}}{\Delta\omega_{BW}}}$$

if $\omega_D = \omega_S$



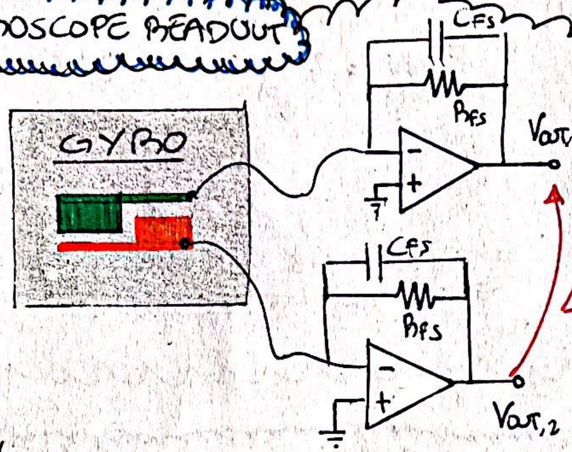
We need to find $\partial C_s/\partial y$:

WITH PARALLEL PLATES SENSING: $C_s = \frac{\varepsilon_0 A_{PP} N_{PP}}{g \pm y} \rightarrow \frac{\partial C_s}{\partial y} \simeq \frac{C_s}{g}$ $(y \ll g_s)$

$$\boxed{S_{mech} = 2\frac{dC}{dy}\cdot\frac{dy}{d\Omega} = 2\cdot\frac{C_s}{g}\cdot\frac{x_{D,0}}{\Delta\omega_{BW}}}$$

MECHANICAL SENSITIVITY

GYROSCOPE READOUT



$$\boxed{S = \frac{\Delta V_{out,0}}{\Omega} = \frac{V_{DC}}{C_{FS}}\cdot S_{mech}}$$

OVERALL SENSITIVITY

$$\boxed{G^{AGC}_{loop} = \frac{2}{\pi} G_{INA}\cdot\frac{4}{\pi}\cdot\frac{1}{R_{eq}}\cdot\left(1/2\pi f_{0D} C_F\right)}$$

$V^{IP}_{FWR} = V_a \sin(\omega_0 t)$

$V^{OUT}_{FWR} = V_a|\sin\omega_0 t|$

$V^{OUT}_{LPF} = \frac{2}{\pi} V_a$ & V^{OUT}_{INA}

$$\boxed{\frac{\Delta x_0}{x_0} = \frac{\Delta Q}{Q}\cdot\frac{1}{1-G_{loop}}}$$

DRIVE MODE SYSTEM OVERVIEW

IT MAKES THE DRIVE FRAME OSCILLATE AND SETS $x_{D,0}$ WHICH APPEARS IN S_{mech}

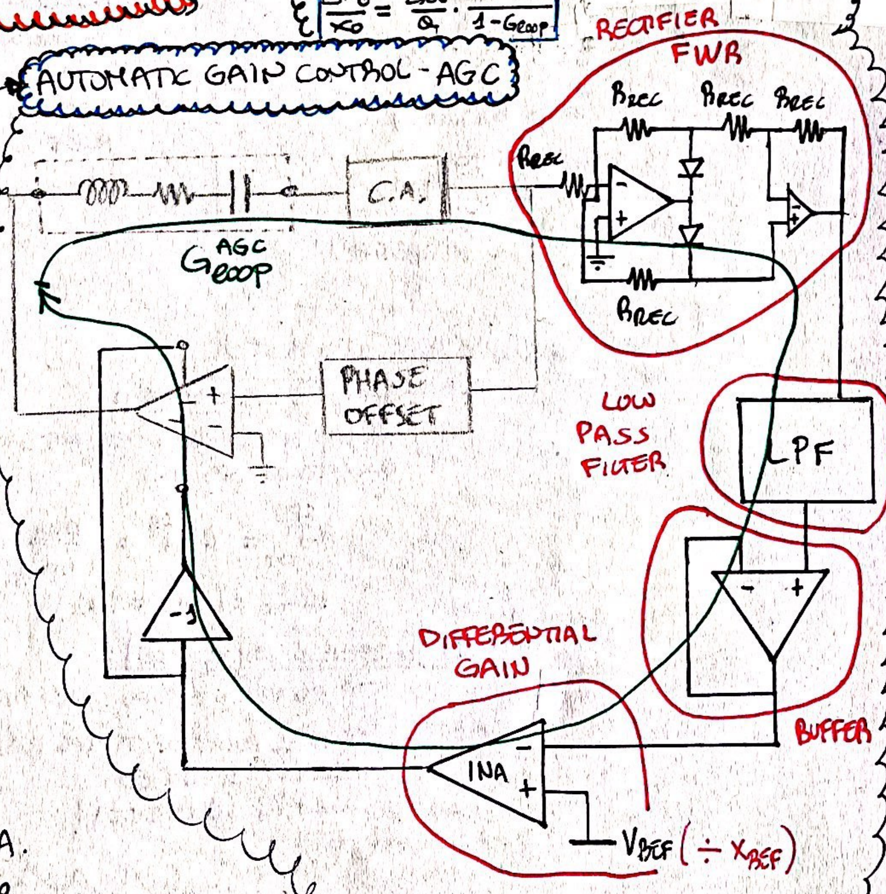
$x_{D,0}$ SHOULD BE STABLE

HOW ABOUT TEMPERATURE CHANGES?

$Q \propto 1/\sqrt{T}$

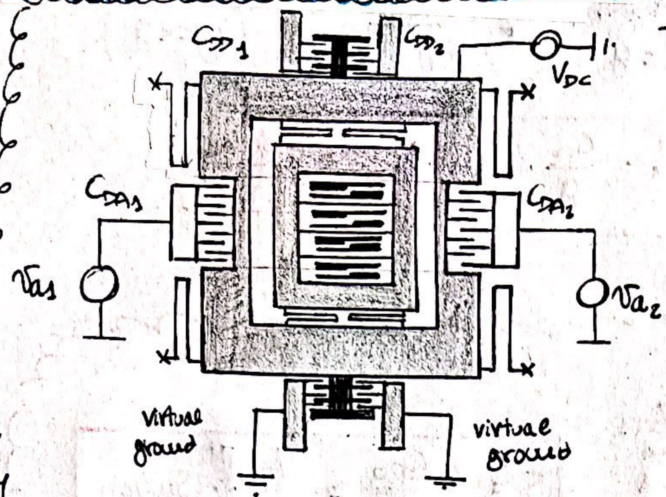
$\hookrightarrow \frac{\Delta Q}{Q} = -\frac{1}{2}\frac{\Delta T}{T}$

we need a circuit to drive correct $x_{D,0}$ to a reference value



if $x_0 > x_{0,REF}$ the C.A. output voltage will be higher. It will be compared with V_{REF} ($>V_{REF}$) and the voltage biasing the comparator will be lowered, reducing x_0 until... $\rightarrow$ ... $\boxed{x_0 \simeq x_{0,REF}}$

PUSH-PULL ACTUATION for DRIVE MODE



$v_{a1} = v_a \sin(\omega_d t)$
$v_{a2} = -v_a \sin(\omega_d t)$

THERE ARE TWO DRIVE SIGNALS AND TWO PAIR OF ELECTRODES FOR ACTUATION (C_{DA}) and DETECTION (C_{DD}) SUPPOSED TO BE EQUAL IN PAIRS

BY CALCULATING THE TOTAL F_{elec} DRIVE DETECTION TERMS GET CANCELED AND WE HAVE:

$$F_{elec,\,DRIVE} = 2\eta_{DA} v_a \sin(\omega_d t)$$

TRASDUCTION COEFF. OF ONE DRIVE-ACTUATION ELECTRODE

IN THIS WAY THE $2\omega_d$ TERM IS DELETED AND NOT NEGLECTED!

EFFECTS OF EXTERNAL ACCELERATIONS and IN-PHASE/ANTIPHASE DRIVE MODE

$$T_{YF}\Big|_{\omega \ll \omega_0} = \frac{1/m}{\omega_0^2}$$

$$\hookrightarrow y_s = \frac{a}{\omega_0^2} \gg y_0$$

(due to ROTATION)

IT'S IMPORTANT TO NOT HAVE FREQs OF GYRO FALLING INTO DISTURBANCES RANGE (SOUND or VIBRATION)

TO SOLVE THIS WE CAN (COUPLING WITH TUNING FORK)

- USE TWO GYROS WITH OPPOSITE DRIVE DIRECTION TO HAVE
  - CORIOLIS FORCES IN OPPOSITE DIRECTION
  - ACCELERATIONS IN THE SAME

IN ORDER TO CANCEL AS COMMON MODE SIGNAL THE COMMON ACCELERATIONS (ALSO DOUBLED SENSITIVITY but DOUBLED AREA)

→ THERE ARE TWO MODES

I) $\boxed{\text{IN-PHASE DRIVE MODE}}$:

TUNING FORK SPRING DEOS NOT BEND AND DON'T CONTRIBUTE ONLY DRIVE-SPRINGS

$$k_{d,TOT}^{IP} = 2k_d^{IP} = \frac{n_{PAR}}{n_{FOLD}} EH\left(\frac{w_d}{l_d}\right)^3 \qquad f_d^{IP} = \frac{1}{2\pi}\sqrt{\frac{k_{TOT}^{IP}}{m_{TOT}}}$$

$$m_{TOT} = (m_d + m_s)2$$

II) $\boxed{\text{ANTIPHASE DRIVE MODE}}$:

TUNING FORK SPRING TAKES PART

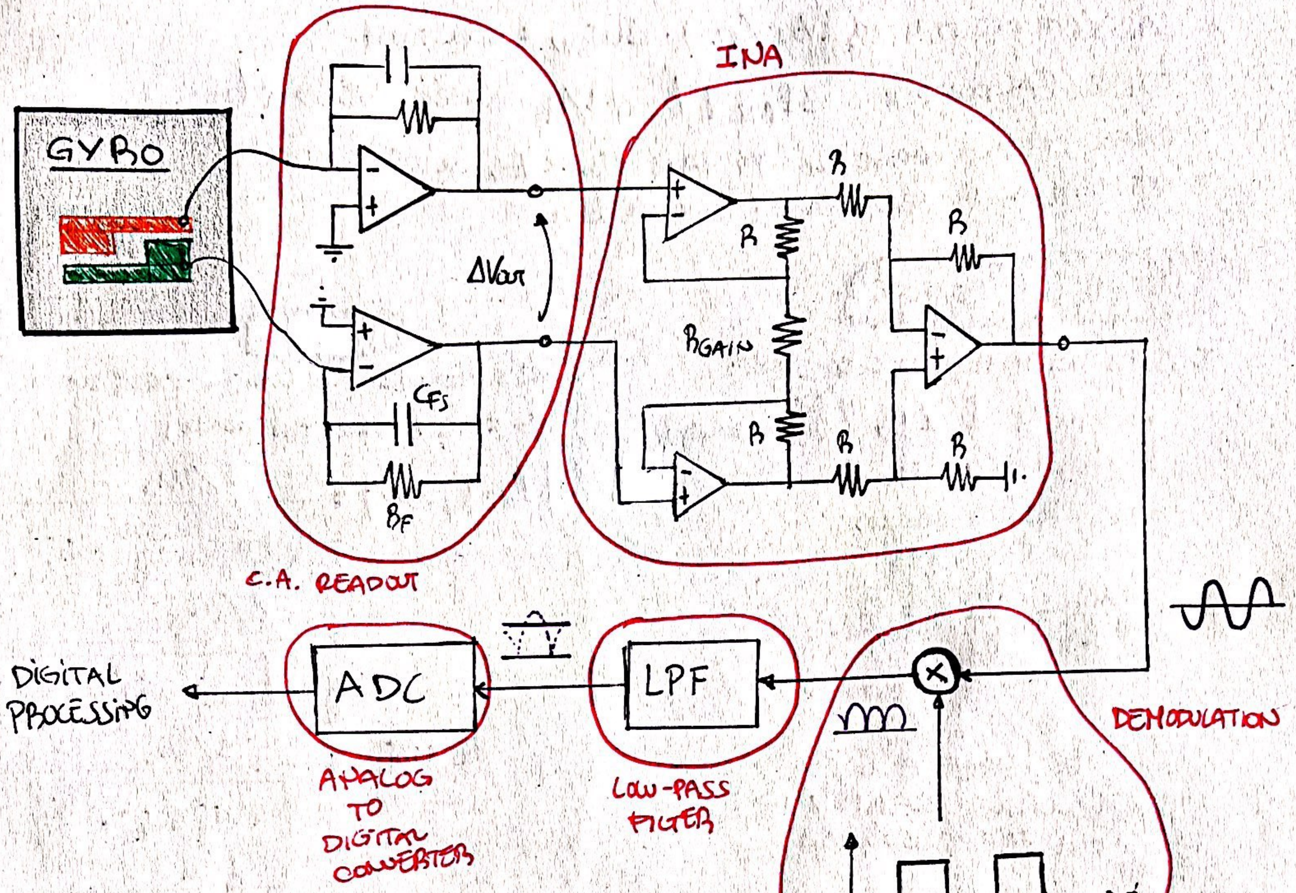
CENTRAL POINT OF TUNING FORK VIRTUALLY ANCHORED

Used in sensitivity!

$$k_{tf} = \frac{n_{PAR,f}}{n_{FOLD,f}} EH\left(\frac{w_{tf}}{l_{tf}}\right)^3$$

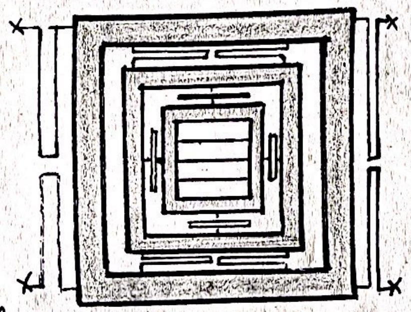
$$k_{TOT} = k_{ft} + k_{IP}$$

SENSE MODE SYSTEM OVERVIEW



DOUBLE DECOUPLING

SENSE MODE CAP SEE SMALL CAPACITANCE CHANGES BECAUSE OF FRINGE EFFECT → FURTHER DECOUPLING



MODE-MATCHED NOISE

- DRIVE MODE NOISE NEGLIGIBLE
- SENSE MODE

$$S_{y_m} = S_{F_m} \cdot \left(\frac{Q_S}{k_S}\right)^2 \left[\frac{m^2}{Hz}\right]$$

$$S_{\Omega_m} = S_{y_m} \cdot \left(\frac{\Delta\omega_{BW}}{x_{D0}}\right)^2 \left[\frac{(rad/s)^2}{Hz}\right]$$

$$\boxed{NERD = \sqrt{S_{\Omega,m}}}$$

NOISE EQUIVALENT RATE DENSITY

in dps by multiplying × 180/π

- FEEDBACK RESISTANCE : $\sqrt{S_{\Omega m,RF}} = \left(\sqrt{2 \cdot \frac{4k_BT}{R_F} \cdot \left(\frac{1}{\omega_S C_{FS}}\right)^2}\right) \Big/ \frac{\Delta V_{out}}{\Omega} \cdot \left(\frac{180}{\pi}\right)$
- OP-AMP : $\sqrt{S_{\Omega m,CA}} = \left(\sqrt{2 S_{n,OP}} \left(1 + \frac{C_P}{C_{FS}}\right)\right) \Big/ \frac{\Delta V_{out}}{\Omega} \left(\frac{180}{\pi}\right)$

MODE-MATCHED GYRO TRADE-OFFS

- GAIN/BANDWIDTH $(\Delta V_{out}/\Omega)\cdot\Delta\omega_{BW} = \text{constant}$
- NOISE/BANDWIDTH $\Delta f_{BW} \propto b_s \propto 1/Q$ while $NERD \propto b_s$
- USEFUL TO HAVE LARGE Q_D TO HAVE $V_a \ll V_{DC}$ in DRIVE MODE
- NOT HIGH Q_s for SENSE MODE (trade-off above)

MODE-SPLIT GYROSCOPE

ISSUES WITH MODEMATCHED GYROs:

- usually $f_s \neq f_D$ (for non-uniformity)
- above tradeoffs
- sensitivity changes for T changes

$$\frac{dQ_s}{Q_s} = \frac{db_s}{b_s} = \frac{1}{2}\frac{dT}{T}$$

WE WANT A GYRO THAT WORKS UNIFORMLY IN DIFFERENT CONDITIONS

MODE SPLIT

let's modulate the coriolis-force at $\omega_D < \omega_s$ (before the peak)

$$|T_{YF}(j\omega_D)| = \frac{1/m_s}{\left(-\omega_D^2 + j\frac{\omega_D\omega_s}{Q_s} + \omega_s^2\right)} = \frac{1/m_s}{\sqrt{(\omega_s^2-\omega_D^2)^2 + \left(\frac{\omega_D\omega_s}{Q_s}\right)^2}}$$

$$\overset{\omega_{MS} \ll \omega_s}{\simeq} \frac{1/m_s}{\sqrt{4\omega_s^2\Delta\omega_{MS}^2 + \frac{\omega_s^4}{Q_s^2}}} = \frac{1/m_s}{2\omega_s\sqrt{\Delta\omega_s^2 + \Delta\omega_{BW}^2}}$$

$$\overset{\Delta\omega_{MS} \gg \Delta\omega_{BW}}{\simeq} \frac{1}{k_s}\cdot\frac{\omega_s}{2\Delta\omega_s} = \frac{1}{k_s}Q_{eff}$$

$$\boxed{Q_{eff} = \frac{\omega_s}{2\Delta\omega_{MS}}}$$ EFFECTIVE QUALITY FACTOR

SACRIFICE SENSITIVITY FOR BANDWIDTH

$$\boxed{\frac{y_{s,0}}{\Omega} = \frac{x_{D,0}}{\Delta\omega_{BW}}}$$

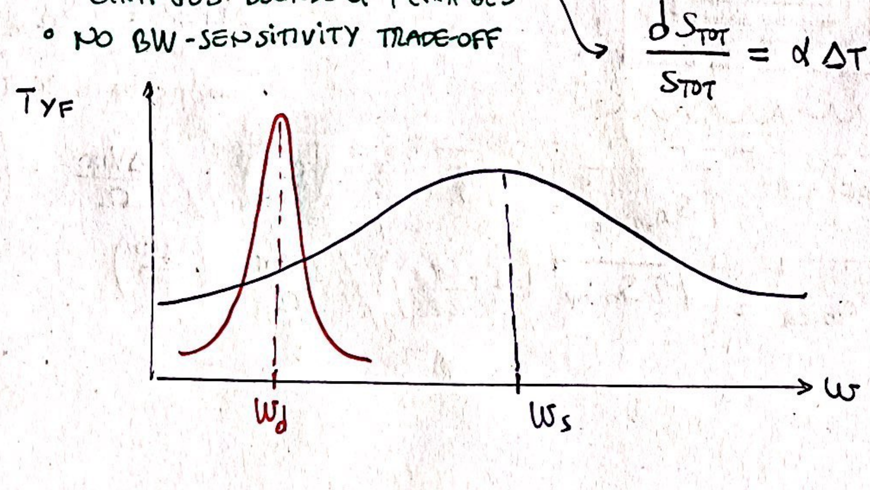
MODE-SPLIT SENSITIVITY

☺
- DOES NOT DEPEND ANYMORE ON b and Q, then WE CAN LOWER b TO MINIMIZE NERD
- MUCH MORE TOLERANT TO f CHANGES BECAUSE OF T CHANGES

$$\frac{dS_{TOT}}{S_{TOT}} = \alpha\,\Delta T \quad \text{with } \alpha = -30\,\frac{ppm}{K}$$

- NO BW-SENSITIVITY TRADE-OFF

☹
- GAIN REDUCTION
- NEED HIGH PERFORMANCE ELECTRONIC



MAGNETOMETERS

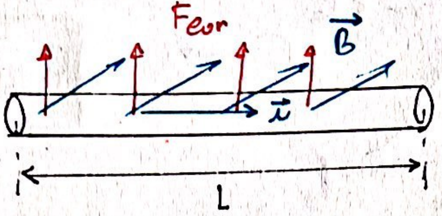
the LORENTZ FORCE

$$\boxed{\vec{F}_{lor} = q(\vec{v} \times \vec{B})}$$

LORENTZ FORCE

$$\boxed{\vec{F}_{lor} = i\,L \times \vec{B}}$$

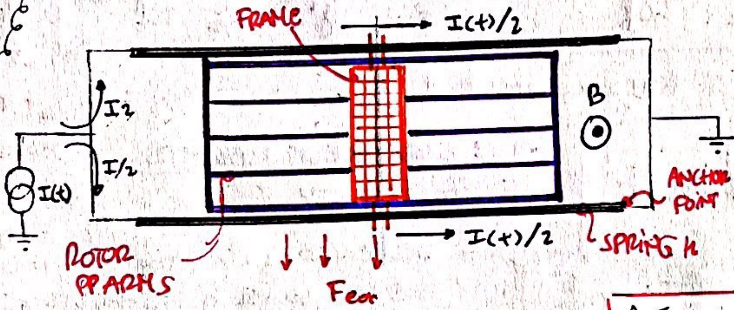
i: CURRENT, L: WIRE LENGTH



$F_{lor} \simeq 1\,mN$
with
$L = 1000\,\mu m$
$i = 0.2\,mA$

ARCHITECTURE

Z-AXIS MAGNETOMETER - SIMPLE ARCH



RESONANT OPERATION

force applied uniformly not only center

$I(t) = i_0 \sin(\omega_0 t)$

$F_{lor}(t) = BL\,i_0 \sin(\omega_0 t)$

$x = \frac{Q}{2k}\, iBL$ at RESONANCE

to be readout with a capacitance variation

if $\Delta C_{PP} = 2\frac{C_0}{g}x \rightarrow \boxed{\frac{\Delta C}{B} = \frac{C_0}{g}\cdot\frac{Q}{k}\,iL = S_{mech}}$ MECHANICAL SENSITIVITY

$\hookrightarrow S_{TOT} = \frac{V_{DD}}{C_F}\cdot S_{mech}$ (with charge amplifier and INA readout)

AGAIN, LIKE FOR GYROS THE INFORMATION ABOUT B IS IN THE LORENTZ FORCE MODULATE AT ω_0 OF CURRENT GEN.

MAXIMUM BANDWIDTH

$$\boxed{\Delta\omega_{BW} = \frac{\omega_0}{2Q} = b/2m}$$

(−3dB off resonance peak)

RESONANT MODE NOISE

$$\sqrt{S_{F_m}} = \sqrt{4k_BTb}$$

$$NEMD = \sqrt{S_{B_m}} = \left[\left(\sqrt{S_{F_m}}\cdot\frac{Q}{k}\right)\Big/\left(\frac{x}{B}\right)\right] = \frac{4}{iL}\sqrt{4k_BTb}$$

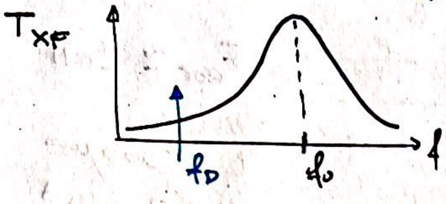
RESONANT OPERATION MODE:

TRADE-OFFs

- again, BW vs NEMD

OFF-RESONANCE OPERATION MODE - MULTI-LOOP

TO SOLVE THE NOISE-BANDWIDTH TRADE-OFF LET'S CHOOSE A DRIVE FREQ. f_D OF $F_{lor} < f_0$



WE SHOULD FIX SENSITIVITY AND FORMULAS WITH

$$\boxed{Q_{eff} = \frac{f_0}{2\Delta f} = \frac{\omega_0}{2\Delta\omega}}$$

EFFECTIVE QUALITY FACTOR

FREQ OF MAGNETOMETER SENSING MODE

TO FURTHER IMPROVE THE LOST GAIN DUE TO $Q_{eff} < Q$ WE CAN REUSE THE CURRENT IN LOOPS and BOOST F_{lor} by N_{loop}

$$S_{mech} = \frac{\Delta C}{B} = 2\frac{C_0}{g} \cdot \frac{Q_{eff}}{2k_{1/2}} i L N_{loop} = \frac{C_0}{g\, k_{1/2}} iL \frac{f_0}{2\Delta f} N_{loop}$$

$$\boxed{S = \frac{\Delta V_{out}}{B} = \frac{V_{DD}}{C_F} \cdot \frac{C_0}{g} \cdot \frac{iL}{k_{1/2}} \cdot \frac{f_0}{2\Delta f} N_{loop} = \frac{V_{DD}}{C_F} \cdot \frac{\varepsilon_0 A N_{pp}}{g^2} \cdot \frac{iL}{k_{1/2}} \cdot \frac{f_0}{2\Delta f} N_{loop}}$$

OVERALL SENSITIVITY

↳ $S \cdot BW = \text{constant}$!

$k_{1/2} = k_{beam} + k_{1/2\,TF}$

TUNING FORK STRUCTURE (HALF DEVICE)

$$\boxed{NEMD = \frac{4}{N_{loop}\, iL} \sqrt{k_B T b}}$$

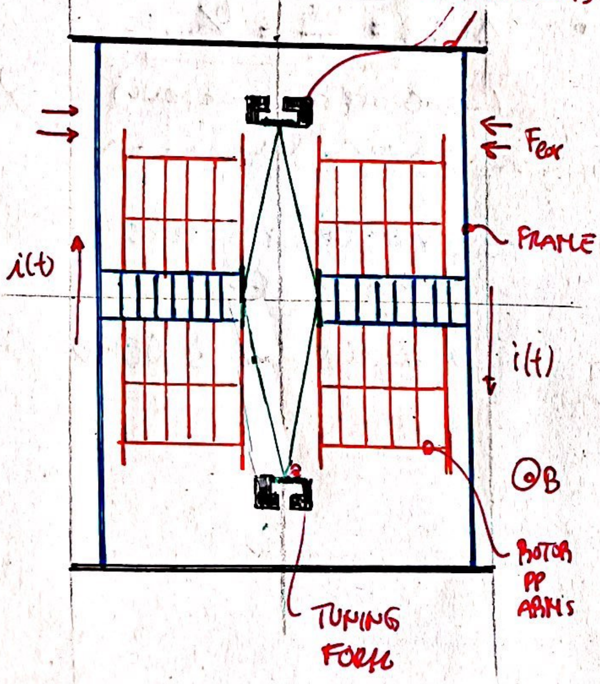
$\left[T/\sqrt{Hz} \right]$

NOISE EQUIVALENT MAGNETIC FIELD DENSITY

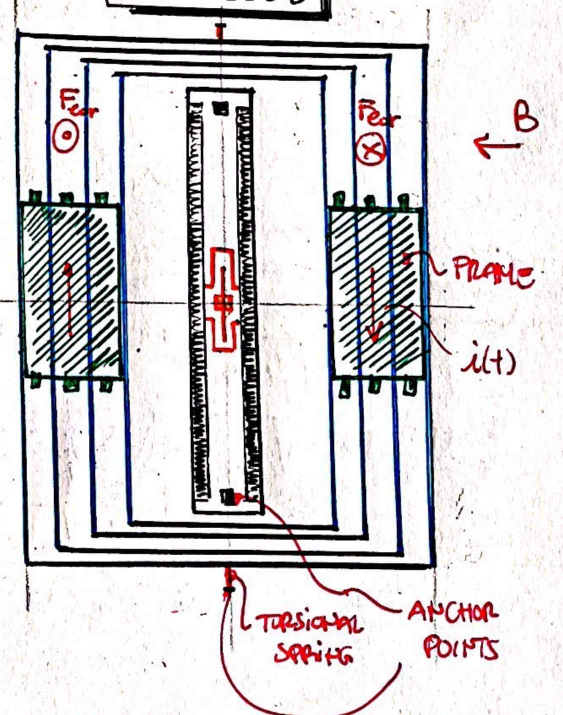
☺
- DECREASED $\propto 1/N_{loop}$ (also ELECTRONIC NOISE)
- WE CAN LOWER b WITHOUT LIMITATIONS
- AREA DECREASED

DIFFERENTIAL ARCHITECTURES

OOP FIELDS



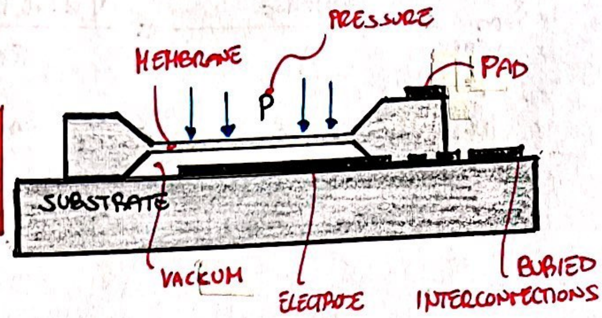
IP FIELDS



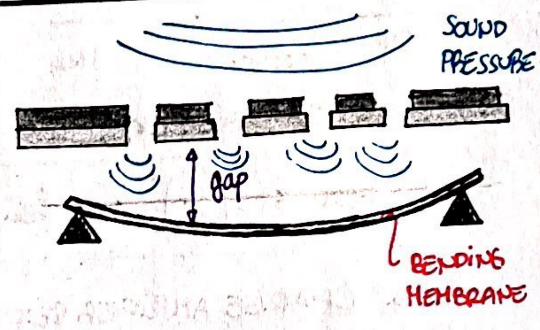
PRESSURE SENSOR

SENSITIVITY

$$\boxed{S = \frac{\Delta V_{OUT}}{P} = \frac{V_{DD}}{C_F} \cdot \frac{C_0}{g} \cdot \frac{dy}{dP} = \frac{V_{DD}}{C_F} \cdot \frac{C_0}{g} \cdot \frac{A}{2k}}$$



MICROPHONE



same parameters:

$$P_{SPL} = 20 \log_{10} \frac{P}{P_{REF}}$$

$P_{REF} = 20\ \mu Pa$) THRESHOLD OF HUMAN HEARING

$$\boxed{\begin{aligned} k_m &= 16 E \cdot \frac{\pi h^3}{r^2} & &\text{(STIFFNESS)} \\ m &= \pi r^2 h \rho & &\text{(MASS)} \\ b &= b_{area}\, \pi r^2 & &\text{(DAMPING)} \end{aligned}}$$

THE VIBRATING MEMBRANE CAN BE APPROXIMATED AS A 1D MASS-SPRING-DAMPER SYSTEM, GIVEN A UNIFORMLY DISTRIBUTED FORCE APPLIED ONTO THE MEMBRANE $\rightsquigarrow$ $\omega_R = \sqrt{\frac{k_m}{m}}$ $\quad y = F / k_m$

THE INPUT SIGNAL OF A MICROPHONE IS SOUND PRESSURE P AS AN AC SOUND WAVE $\longrightarrow$ $\boxed{p = p_a \sin(2\pi f_a t)}$

ACOUSTIC FREQUENCY RANGE 20Hz - 20kHz

THE REQUIREMENTS ARE

$$P_{a_{MIN}} = P_{REF} \cdot 10^{\left(\frac{P_{a_{MIN},SPL}}{20}\right)} = 893\ \mu Pa$$

$$P_{a_{MAX}} = P_{REF} \cdot 10^{\left(\frac{P_{a_{MAX},SPL}}{20}\right)} = 28.3\ Pa$$

THE MINIMUM DETECTABLE SIGNAL BY THE BANDWIDTH OF OUR SENSOR

$$\boxed{S_{mP} = \frac{P_{a,MIN}}{\sqrt{f_{MAX} - f_{MIN}}} \simeq \frac{P_{a,MIN}}{\sqrt{f_{MAX}}}}$$

the FORCE APPLIED is $\boxed{F = pA = p\pi r^2}$

NOTE THAT BOTH THE CHAMBERS FACED BY THE MEMBRANE ARE KEPT AT THE ATMOSPHERIC PRESSURE, SO THAT THE DC DISPLACEMENTS INDUCED BY THE ATMOSPHERIC PRESSURE ARE CANCELED

THE PRESSURE DIFFERENCE APPLIED TO THE MEMBRANE IS

$$P_m = (P_{atm} + P_{sound}) - P_{atm} = P_{sound}$$

NOISE

$$S_{m_{MEMS_P}} = \frac{S_{m_{MEMS_F}}}{A} = \frac{\sqrt{4k_BTb}}{\pi r^2} = \sqrt{\frac{4k_BT\, b_{area}}{\pi r^2}}$$

also should be $S_{m_{MEMS_P}} = \frac{S_{m_{TOT_P}}}{\sqrt{2}}$ for a good balance

WE FIND:

$$\boxed{\left.\frac{\partial y}{\partial P}\right|_{\omega \ll \omega_r} = \frac{Area}{k_m}}$$ DISPLACEMENT v PRESSURE

and some linearity error

$$\boxed{\varepsilon_{lin_{diff}} = \left(\frac{y}{g}\right)^2 = \left(\varepsilon_{lin}\right)^2}$$ LINEARITY ERROR

with a CHARGE AMPLIFIER READOUT:

$$\boxed{T_{TOT}(j\omega) = -\pi r^2 \frac{1}{k + j\omega b - \omega^2 m} \cdot \frac{\partial C}{\partial y} j\omega V_{DC} \frac{R_F}{1 + sC_FR_F}}$$

OVERALL SENSITIVITY WITH TCA

$$T_{TOT}(j\omega) = \frac{V_{OUT}(j\omega)}{P(j\omega)}$$

CMOS IMAGING SENSORS

BASICS OF OPTICS

RAYS are not COLORED! they are made of PHOTONS with a certain ENERGY which corresponds a WAVELENGTH (and a certain color perception from the eye)

$$\boxed{E_{ph} = \frac{hc}{\lambda}}$$ ENERGY of PHOTONS

LIGHT SOURCE $s(\lambda)$ → OBJECT REFLECTION $r(\lambda)$ → IMAGING OPTICS → IMAGING SENSORS → ADC PROCESSING DISPLAY

$$B = \int_{400}^{700} \underbrace{s(\lambda)}_{\text{ILLUMINANT}} \cdot r(\lambda) \cdot \underbrace{t(\lambda)}_{\text{SCENE TRASMITTANCE}} \cdot \underbrace{b(\lambda)}_{\text{BLUE PIXEL RESPONSE}} \, d\lambda$$

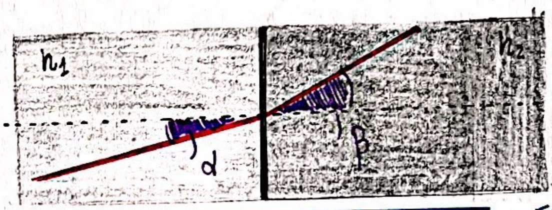
$\simeq 1$

ILLUMINANT: we can approximate as almost perfect BLACK BODIES RADIATORS

$$\boxed{\lambda_{peak} \cdot T = 2{,}9 \cdot 10^{-3} \, m \cdot K}$$ WIEN'S LAW

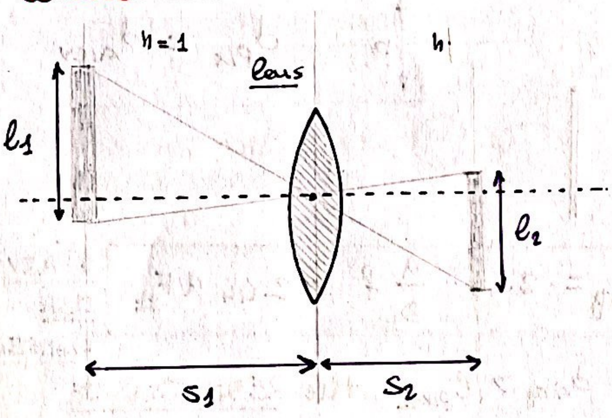
some approximations...

- light is described by RAYS which travels in STRAIGHT LINES while they are deflected by an OPTICAL ELEMENT
- the DEFLECTION is described by the SNELL LAW



$$\boxed{n_1 \sin(\alpha) = n_2 \sin(\beta)}$$ SNELL'S LAW

LENS SYSTEMS and PARAMETERS



$$\boxed{\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f} \quad \text{with} \quad \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

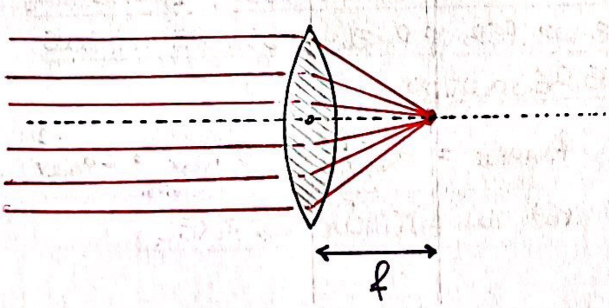
LENS MAKER FORMULA

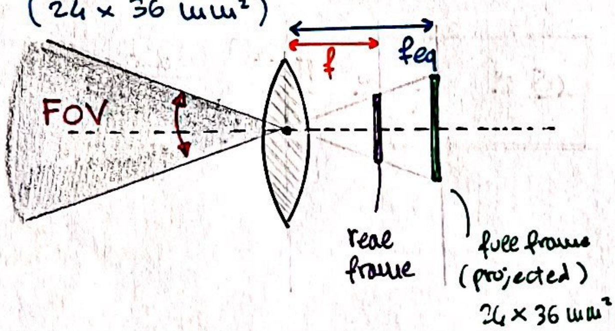
- since $s_1 \gg s_2$ → $\boxed{f \simeq s_2}$ FOCAL LENGTH

$f\downarrow$ with POSITIVE CONVEX LENS

$f\uparrow$ with NEGATIVE CONCAVE LENS

- we can calculate the EQUIVALENT FOCAL LENGTH by referring to FULL-FRAME FORMAT ($24 \times 36 \, mm^2$)





we also define :

- MAGNIFICATION FACTOR

$$\boxed{m = \frac{\ell_2}{\ell_1} = \left(\frac{S_2}{S_1} \simeq \frac{f}{S_1}\right) = \frac{Area^{MIN}_{PIX}}{Area^{MAX}_{SCENE}}}$$ MAGNIFICATION FACTOR

- F# NUMBER

$$\boxed{F_{\#} = f / D}$$ F# NUMBER

D: DIAMETER OF THE LENS

useful to find the

$$\boxed{\theta = \operatorname{atan}\left(\frac{D}{2f}\right) = \operatorname{atan}\left(\frac{1}{2F_{\#}}\right)}$$ MAXIMUM APERTURE OF FOLDED RAYS

ABERRATIONS

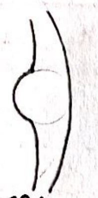
CAUSES IMPERFECTIONS in REPLICAS OF OBJECTS

↓

WORSEN THE RESOLUTION

⇓

WE DON'T HAVE A FOCAL POINT
instead we have a DISK OF CONFUSION
can be reduced using



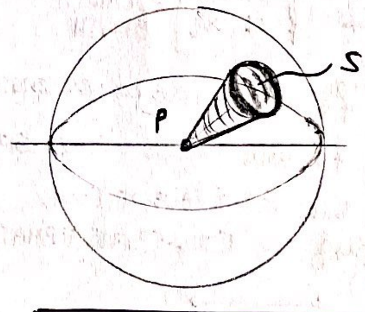
ASPHERIC LENS

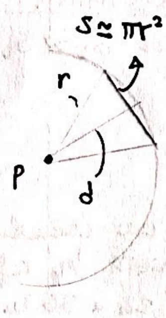
HAPPENS DUE TO :

- errors during 1st order approximations
  (3rd order spherical aberrations)
- the radiation is not 'monochromatic' and the refractive index is function of λ
  (chromatic aberration)

  can be reduced with :
  - LOW DISPERSION GLASSES
  - ACHROMATIC DOUBLETS

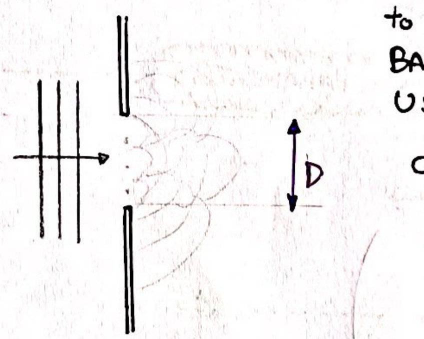
SOLID ANGLES





$$\boxed{\Omega = \frac{S}{d^2} \simeq \frac{\pi r^2}{d^2}}$$ SOLID ANGLE [sr]

DIFFRACTION



to have a BALANCED SYSTEM usually choose

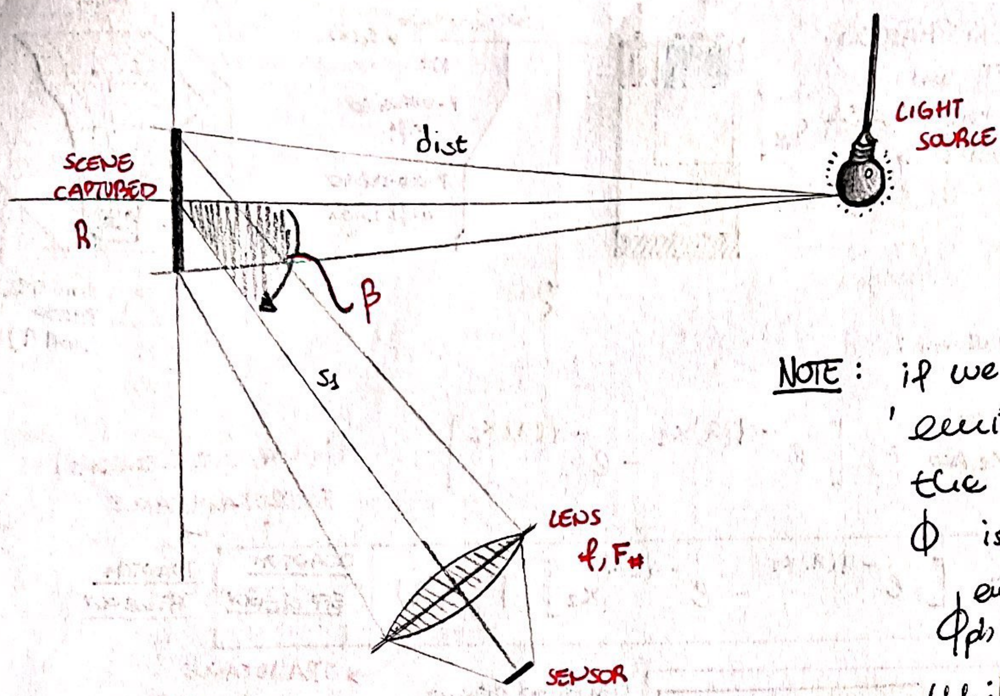
$$d_{PIX} = d_{AIRY}$$

$$\boxed{d_{Airy} = 2.44 \cdot \frac{\lambda}{D} f = 2.44\, \lambda F_{\#}}$$ 1st AIRY DISK DIAMETER

- if $d_{Airy} > \ell_{pixel}$ the RESOLUTION OF THE SENSOR IS LIMITED. INCREASING THE NUMBER OF PIXELS DOES NOT IMPROVE THE RESOLUTION.
- if $f_{SAMPLE} = \ell_{PIX}^{-1} < 2 f_{MAX\,SIGNAL} = 2 d_{AIRY}^{-1}$

  we need an ANTIALIASING FILTER

OPTICAL POWER and PHOTON FLUX



<u>NOTE</u>: if we have an 'emitted' ϕ_{ph} the absorbed ϕ is given by

$$\phi_{ph}^{emit} \cdot A_{PD} \cdot \frac{1}{m^2}$$

m: magnification factor

1 CALCULATE THE AREA OF THE SCENE WHICH CORRESPONDS TO ONE PIXEL

$$A_{\substack{SCENE\\PIXEL}} = \frac{A_{PIXEL}}{(m)^2}$$

m: MAGNIFICATION FACTOR

only photons coming from that area are captured by corresponding pixel

2 STARTING FROM OPTICAL POWER OF THE SOURCE WE EVALUATE THE IMPINGING POWER ON $A_{\substack{SCENE\\PIX}}$

- for an ISOTROPIC LIGHT SOURCE

$$I_{SOURCE} = \frac{P_{SOURCE}}{4\pi\ sr} \quad \left[\frac{W}{sr}\right]$$

- we calculate the solid angle seen from source to $A_{\substack{SCENE\\PIX}}$:

$$\Omega_{\substack{SCENE\\PIX\\SOURCE}} = \frac{A_{\substack{SCENE\\PIX}}}{(dist)^2} \quad [sr]$$

- the OPTICAL POWER IMPINGING ON SCENE AREA WHICH CORRESPONDS TO ONE PIXEL

$$P^{(I)}_{\substack{SCENE\\PIX}} = I_{SOURCE} \cdot \Omega_{\substack{SCENE\\PIX\\SOURCE}} \quad [W]$$

3 EVALUATE THE OPTICAL POWER REFLECTED

$$P^{(R)}_{\substack{SCENE\\PIX}} = P^{(I)}_{\substack{SCENE\\PIX}} \cdot R_{SCENE}$$

R_{SCENE}: SCENE REFLECTANCE

AND THEN THE INTENSITY DEPENDING ON TYPE OF REFLECTING OBJECT:

- <u>ISOTROPIC REFLECTION</u>:

$$I_{R_{ISO}} = P^{(R)} / 2\pi\ sr$$

- <u>LAMBERTIAN REFLECTION</u>:

$$I_{R_{LAMB}} = \frac{P^{(R)}}{\pi} \cos(\beta)$$

4 WE EVALUATE THE POWER IMPINGING ON A PIXEL

$$A_{lens} = \pi\left(\frac{D}{2}\right)^2$$

$$\Omega_{\substack{LENS\\SCENE\,PIX}} = \frac{A_{lens}}{s_1^2}$$

$$P_{PIX} = I_R \cdot \Omega_{\substack{LENS\\SCENE\,PIX}}$$

5 DIVIDE BY ENERGY OF A PHOTON

$$E_{ph} = \frac{hc}{\lambda}$$

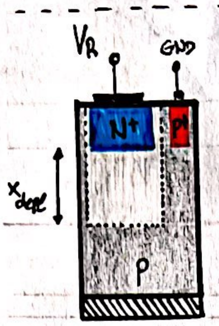
$$N_{ph} = \frac{P_{PIX}}{E_{ph}} \quad \left[\frac{ph}{s}\right]$$

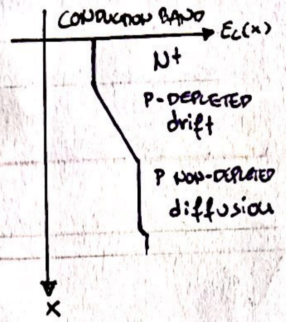
$$\boxed{\phi_{ph} = \frac{N_{ph}}{A_{pix}} \left[\frac{ph}{m^2 \cdot s}\right]}$$

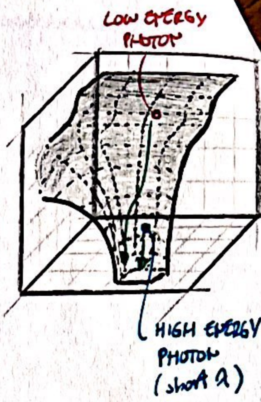
PHOTON FLUX

the PHOTODIODE

CHARGE COLLECTION HAPPEN WITH BOTH DRIFT and DIFFUSION into the N^+ WELL WHERE PHOTOCURRENT is GENERATED







let's define some values:

PHOTON ABSORBED BETWEEN x_1 AND x_2 $= \left[e^{-\alpha(\lambda)x_1} - e^{-\alpha(\lambda)x_2} \right]$

(in CFA, COLOR FILTER ARRAYS)

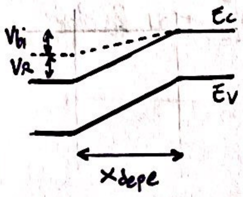
FILTER TRASMITTANCE

$\left[\frac{\text{collected } e^-}{\text{incident ph}}\right]$

$$\boxed{\eta(\lambda) = \left[e^{-\alpha(\lambda)x_1} - e^{-\alpha(\lambda)x_2} \right] T_{Si}(\lambda)\, T_{FIL}(\lambda)}$$

QUANTUM EFFICIENCY

TRASMITTANCE Si-Air (<1)



$$\boxed{x_{depe} = \sqrt{\frac{2\varepsilon_0\varepsilon_{Si}(V_R + V_{bi})}{q N_A}}}$$

DEPLETION REGION LENGTH

$$\boxed{R(\lambda) = \underbrace{\frac{i_{ph}}{P_{IN}}}_{\text{INPUT OPTICAL POWER}} = \frac{q \cdot (\text{collected electrons per second})}{E_{ph} \cdot (\text{incident photons per second})} = \frac{q}{E_{ph}}\eta(\lambda)}$$

RESPONSIVITY

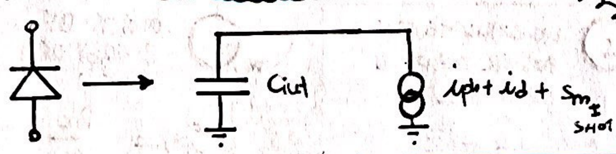
given the POWER IMPINGING ON A PIXEL P_{PIX} or the PHOTON FLUX Φ_{ph}

$$\boxed{i_{ph} = q \cdot \eta(\bar{\lambda}) \cdot \Phi_{ph} \cdot A_{pixel} = R(\bar{\lambda})\, P_{PIX}}$$

GENERATED PHOTOCURRENT

$$\boxed{i_{ph} = q\,\eta_{PIX}\, T_{FIL}\, \frac{\lambda}{hc} \cdot \frac{P_{source}\, A_{PIX}\, R_{scene}}{16\pi (dist)^2 F_\#^2}}$$

EQUIVALENT CIRCUIT



$C_{int} = C_{par} + C_{PD}$

$$\boxed{C_{PD} = \varepsilon_0\varepsilon_{Si}\frac{A_{active}}{x_{dep}}}$$

DARK CURRENT

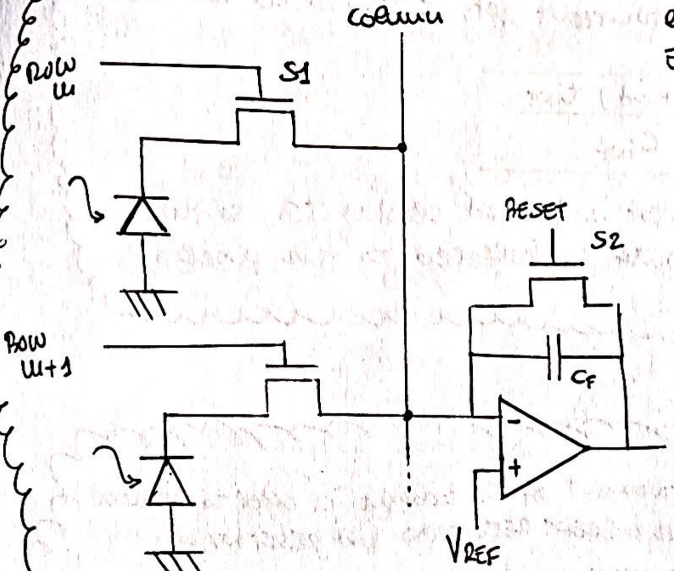
due to thermal generation in depletion region at Si/SiO_2 interface

$$\boxed{i_d = J_d \cdot A_{active} = \frac{q n_i}{2\tau_m} x_{depl} A_{active}}$$

DARK CURRENT doubles every 7°C

READOUT TOPOLOGIES

PASSIVE PIXELS SENSOR - PPS



each pixel is formed just by PHOTOSENSITIVE ELEMENT and a SELECTION TRANSISTOR

- SMALL PIXEL SIZE
- DOES NOT INTRODUCE PIXEL TO PIXEL VARIATIONS

- LIMITED READOUT SPEED
- SPECIALIZED TECHNOLOGY
- HIGH VOLTAGE REQUIRED FOR CHARGE TRANSFER

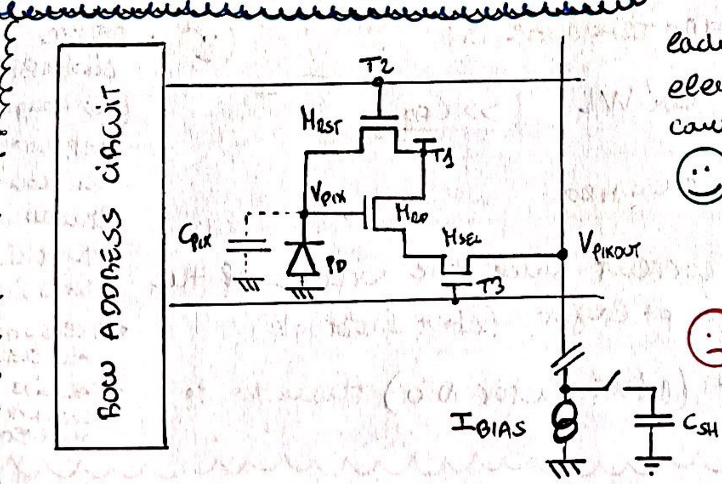
1. OUTPUT INITIALLY RESET TO V_{REF} BY CLOSING BOTH S_1 AND S_2. PHOTOCURRENT OF THE DIODE DOES NOT DETERMINE ANY CHANGE IN THE OUTPUT VOLTAGE (as it flows through the short circuit due to S_2 from the low-impedance output of the operational amplifier)
2. DURING INTEGRATION S_1 IS KEPT OPEN TOGETHER WITH THE CAMERA SHUTTER UNTIL READOUT HAPPEN.
   PHOTOCURRENT AND DARK CURRENTS ARE COLLECTED ON PHOTODIODE CAPACITANCE, WHOSE VOLTAGE DECREASES DURING INTEGRATION TIME t_{int}

$$\boxed{V_{PD}(t) = V_{REF} - \Delta V_{PD}(t) \quad \Delta V_{PD}(t) = \frac{(i_{ph}+i_d)\,t}{C_{int}}}$$

3. IN THE READOUT PHASE, S_1 IS CLOSED AGAIN SOON AFTER S_2 IS OPEN. THE VOLTAGE CHANGE ACROSS THE DIODE CAPACITANCE DETERMINES A CURRENT PULSE FLOWING IN THE FEEDBACK LOOP OF THE AMPLIFIER

$$\boxed{V_{OUT} = V_{REF} + \Delta V_{PD} = V_{REF} + \frac{(i_{ph}+i_d)\,t_{int}}{C_F}}$$

ACTIVE PIXEL SENSOR - APS: 3T TOPOLOGY



each pixel is formed by the photosensitive element and a small charge to voltage conversion

- HIGH SPEED READOUT
- STANDARD FABRICATION PROCESS
- LOWER VOLTAGE
- ONLY SELECTED PIXELS CONSUME POWER

- INTRODUCE GAIN/OFFSET NONUNIFORMITIES
- SAME NODE FOR PHOTOSENSING ELEMENT AND CONVERSION (GAIN - FF TRADE OFF)
- NONLINEARITY DUE TO C_{dep} VARIABILITY

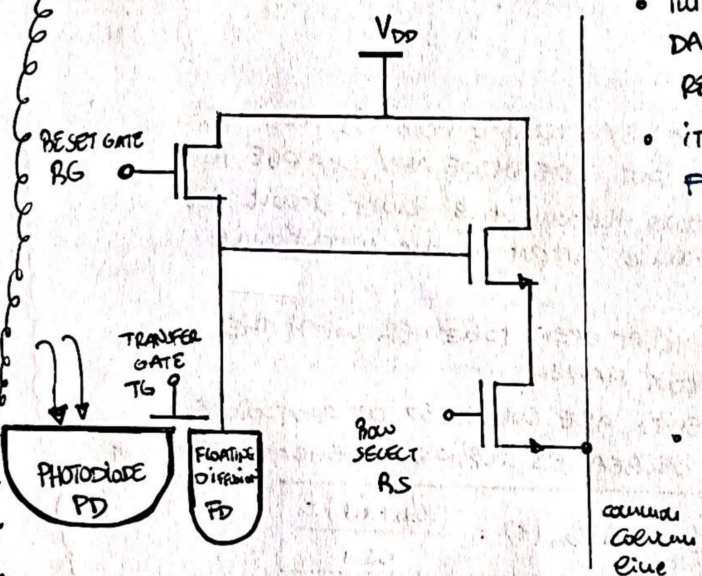
(24) in 3T topology:

1. the INTEGRATION NODE is first RESET to a value equal to V_{DD} by closing the RESET SWITCH.
2. then T2 is open and the photocurrent gets INTEGRATED on the integration capacitance.

$$\boxed{V_{pix} = V_{DD} - \frac{(i_{ph} + i_d)\, t_{INT}}{C_{INT}}}$$

3. finally, the follower transistor is biased closing T3, so that the voltage at the follower gate is buffered to the pixel output

APS - 4T TOPOLOGY



- improvement on 3T topology in order to obtain DARK CURRENT REDUCTION and RESET NOISE REDUCTION
- IT consist in a PINNED PHOTODIODE and FOUR TRANSISTORS
  - MTX TRANSFER GATE
  - MRS RESET TRANSISTOR
  - MRD AMPLIFIER TRANSISTOR
  - MSEL SELECT TRANSISTOR
- WE HAVE ALSO A DOUBLE SAMPLING
  WE SUBTRACT TWO MEASUREMENT:
  - one with photocurrent and offset and noise
  - one only with reset and noise (no signal)

  overall noise is thus reduced!

- DIFFUSION CAPACITANCE can be made very small since it's no longer responsible of photons collection

$$C_{FD} = \varepsilon_0 \varepsilon_R \frac{WL}{X_{dep}}$$

while INPUT CAPACITANCE OF MRD TRANSISTOR is

$$C_{in,MRD} = C_{ox}' WL \quad (\gg C_{FD})$$

$$C_{in} = C_{FD} + C_{in,MRD}$$

- NO MORE GAIN-FF TRADE OFF
- BETTER LINEARITY (no longer dependent on photodiode capacitance that changes with signal)
- PIXELS SHARE SAME ELECTRONICS
- BACKSIDE ILLUMINATION ALLOWED

- also we have lower dark current since the surface of the pinned photodiode is shielded by a p+ layer (about factor ~10)
- also kTC rms value is reduced (about factor ~20) thanks to double sampling

SENSITIVITY, SIGNAL, NON-LINEARITIES

with a constant photocurrent, we can evaluate the collected electrons:

$$Q_{ph} = i_{ph}\, t_{int} \rightarrow N_{ph} = Q_{ph}/q$$

and the gain:

$$G = \frac{\Delta V_{PD}}{N_{ph}} = \frac{Q_{ph}\, q}{C_{PD}\, Q_{ph}} = \frac{q}{C_{PD}} \rightarrow \Delta V_{PD} = G \cdot N_{ph}$$

and finally the sensitivity:

$$\boxed{S = \frac{\Delta V_{PD}}{\phi_{ph}} \quad \left[V \Big/ \frac{ph}{m^2 s} \right]}$$

SENSITIVITY

However in 3T topology we should take into account that photodiode capacitance varies with signal:

$$\Delta V_{PD} = \frac{(i_{ph} + i_d)\, t_{int}}{C_{PD}} \qquad V_{PD} = V_{DD} - \Delta V_{PD}$$

REAL PHOTODIODE CAPACITANCE

$$x_{dep} = \sqrt{\frac{2 \varepsilon_0 \varepsilon_R (V_{PD} + V_{bi})}{q N_A}} \longrightarrow \boxed{C_{PD} = \varepsilon_{Si} \frac{A}{x_{DEP}} = \frac{C_{PD}^0}{\sqrt{1 + \frac{V_{PD}}{V_{bi}}}} \qquad C_{PD}^0 = C_{PD}\Big|_{V_{PD}=0}}$$

we can also find:

$$\boxed{Q_{ph,max}^{REAL} = 2 C_0 V_{bi} \left[1 - \sqrt{1 - \frac{V_{DD}}{V_{bi}}} \right]}$$

MAXIMUM COLLECTABLE CHARGE

and

$$\boxed{\varepsilon_{lin} = \frac{Q_{ph\,MAX}^{REAL} - Q_{ph\,MAX}^{LIN}}{Q_{ph\,MAX}^{REAL}}} \quad \text{with} \quad Q_{ph\,MAX}^{LIN} = V_{DD} \cdot C_{PD}\Big|_{V_{PD}=V_{DD}}$$

LINEARITY ERROR

NOISE CONTRIBUTIONS

		DARK	ILLUMINATED	
FIXED PATTERN NOISE FPN	SPATIAL NOISE	ELECTRONIC OFFSET NON-UNIFORMITY DARK CHARGE NON-UNIFORMITY GAIN NON-UNIFORMITY	PHOTORESPONSE NON UNIFORMITY	at sensor level
		HOT PIXELS	DARK PIXELS	
TEMPORAL NOISE		DARK CURRENT SHOT NOISE	PHOTON CURRENT SHOT NOISE	at pixel level
		READ NOISE (AMPLIFIER), RESET NOISE	QUANTIZATION NOISE	

TEMPORAL NOISE

we are referring to 3T-topology!

CURRENT SHOT NOISE

A READOUT PERFORMED AS THE APPROACH RESET + INTEGRATION CORRESPONDS TO A GATED-INTEGRATOR SCHEME FOR FINITE TIME t_{int} WITH A BANDWIDTH $1/2t_{int}$

$$S^2_{i,shot} = 2q\,(i_{ph} + i_d)\ \left[\frac{A^2}{Hz}\right]$$

we can convert in terms of charge:

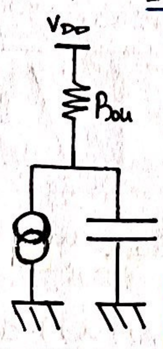
$$\sigma^2_{q,shot} = \sigma^2_{i,shot} \cdot t^2_{int} = \frac{S^2_{i,shot}}{2t_{int}}\, t^2_{int}$$

$$\boxed{\sigma^2_{q_{shot}} = q\,(i_{ph} + i_d)\, t_{int}\ \left[C^2\right]}$$

CURRENT SHOT NOISE

KTC NOISE (RESET NOISE)

WHEN RESET SWITCH OPENS, ONE RANDOM VOLTAGE VALUE WITHIN THE NOISE POWER SPECTRAL DENSITY IS 'SAMPLED' AND FROZEN AS STARTING VOLTAGE FOR THE INTEGRATION. THE BANDWITH IS REPRESENTED BY THE $1/(4RC_{int})$ BANDWIDTH



$$S_{V_{Ron}} = 4kTR_{on}\ \left[\frac{V^2}{Hz}\right]$$

$$\sigma^2_{V,kTC} = \frac{S_{V_{Ron}}}{4RC_{int}} = \frac{kT}{C_{int}}\ [V^2]$$

$$\boxed{\sigma^2_{q_{kTC}} = kT\,C_{int}\ \left[C^2\right]}$$

KTC NOISE

QUANTIZATION NOISE

REFERRED TO THE ADC

$$\sigma_{quant,V} = \frac{LSB}{\sqrt{12}} = \frac{V_{REF}/2^{Nbit}}{\sqrt{12}}$$

$$\boxed{\sigma_{quant,Q} = \frac{V_{REF}/2^{Nbit}}{\sqrt{12}}\, C_{diode}}$$

QUANTIZATION NOISE

↳ to choose ADC we can force:

$$\sigma_{Q_{quant}} < \sigma_{Q_{reset\ kTC}}$$

SNR - SIGNAL TO NOISE RATIO (3T)

$$\boxed{SNR = 20\log_{10} \frac{N_{ph}}{\sqrt{\sigma^2_{shot_Q} + \sigma^2_{Q_{kTC}} + \sigma^2_{Q_{quant}}}}}$$

SIGNAL TO NOISE RATIO

MAXIMUM SNR

$$SNR_{max} = \frac{N_{ph\,max}}{\sqrt{\underbrace{\sigma^2_{Q\,shot}}_{DARK+PH} + \sigma^2_{Q\,kTC}}} = \sqrt{FWC}\quad dB$$

TRADE-OFF DR-SNR on t_{int} — $t_{int}\uparrow$ DR$\downarrow$ SNR$\uparrow$

OTHER NEGLIGIBLE NOISE

- 1/f NOISE
- thermal noise of source-follower

REDUCE TEMPORAL NOISE

- by DOUBLE SAMPLING
- by AVERAGING: take k consecutive images for a selected exposure time and calculate the average of the series for each pixel

DR - DYNAMIC RANGE

$$DR = \frac{\text{MAXIMUM DETECTABLE SIGNAL}}{\text{MINIMUM DETECTABLE SIGNAL}}$$

saturation of the pixel

set by noise

NEQ

MINIMUM DETECTABLE SIGNAL $i_d \gg i_{ph}$

$$N_{ph_{min}} \cong \frac{1}{q}\sqrt{\sigma^2_{Q_{shot\,DARK}} + \sigma_{Q_{RESET}}}$$

(quantiz. made negl.)

MAXIMUM DETECTABLE SIGNAL $i_{ph} \gg i_d$

$$N_{ph_{max}} \cong \frac{1}{q}\,(V_{DD}\, C_{PD})$$

(otherwise use real Q_{ph}^{max})

FPN SPATIAL NOISE

OPTICAL RESPONSES NONUNIFORMITIES CAUSES DIFFERENT PHOTO-SIGNALS FOR NOMINALLY IDENTICAL ILLUMINATED PIXELS

↳ UNWANTED GAIN CHANGES FROM PIXEL TO PIXEL DUE TO
- SOURCE FOLLOWER V_{gs} VARIATIONS
- DARK CURRENT

→ we used to store in a memory the correction values for offset and gain for each pixel

$$\boxed{\Delta V_{PIXOUT} = \underbrace{\frac{\Phi(\lambda)\,\eta(\lambda)\,q\,A_{pix}\,FF\,T_{CFA}(\lambda)\,T_{OPT}(\lambda)\,t_{int}}{C_{dep}+C_g}}_{\text{PHOTOGENERATED CURRENT TERM}}\,G_{SF} + \underbrace{\left(\frac{i_d\,t_{int}}{C_{dep}+C_g}\,G_{SF} + \Delta V_{\substack{OFFSET\\ELN}}\right)}_{\text{DARK CURRENT TERM}}}$$

↓

$$\boxed{\Delta V_{PIXOUT} = I_{\substack{PIX\\IN}} \cdot G_{PIXEL}\left(1+\frac{\sigma_{PRNU\%}}{100}\right) + O_{\substack{PIX\\DARK}}\left(1+\frac{\sigma_{DSNU\%}}{100}\right)}$$

DSNU - DARK SIGNAL NON UNIFORMITIES

IT'S A MEASURE OF THE DISTRIBUTION OF DARK OUTPUT VOLTAGES FOR ALL THE PIXELS MEASURABLE BY TAKING PHOTO IN DARK CONDITIONS.

- to model:

1) take M consecutive images and remove temporal noise by averaging

$$V_{out,d_{m,n}} = \frac{1}{M}\sum_i V_{out,d_{m,n}}^{\,i}$$

2) DSNU is the standard DEVIATION of all the values across the sensor

3) % DSNU is found by normalizing to mean value

$$\sigma_d^2 = \frac{1}{m \cdot M}\sum\left(V_{out,d_{m,n}} - \overline{V_{out,d_{m,n}}}\right)^2$$

$$\boxed{\sigma_{DSNU\%} = \frac{\sigma_d}{\overline{V_{out,d_{m,n}}}}\cdot 100 \longrightarrow \sigma_{DSNU_q} = i_d\,t_{int}\,\frac{\sigma_{DSNU\%}}{100}}$$

PRNU - PHOTOCURRENT NON UNIFORMITIES

IT'S MEASURED AS THE DEVIATION OF THE COLOR COORDINATES OF A PIXEL FROM THE AVERAGE DUE TO GAIN FLUCTUATIONS

① CAPTURE A COLOR WITH ALL PIXEL (BY AVERAGING TO REMOVE TEMPORAL NOISE)

② DIVIDE THE RESULTS BY THE AVERAGE RGB OF THAT COLOR FOR ALL PIXELS

③ WE EVALUATE FOR EACH PIXEL THE DEVIATION FROM COLOR VALUE AVERAGE IS $\sigma_{PRNU\%}$

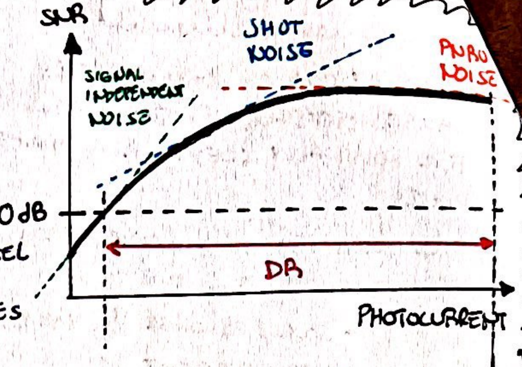
④ REPEAT FOR MORE COLORS

$$\boxed{\sigma_{PRNU_q} = i_{ph}\,t_{int}\,\frac{\sigma_{PRNU\%}}{100}}$$

OVERALL NOISE

$$\boxed{\sigma_{TOT} = \sqrt{\underbrace{\sigma^2_{READ}}_{} + \underbrace{\sigma^2_{SHOT}}_{} + \underbrace{\sigma^2_{PRNU}}_{}}}$$

- σ^2_{READ}: ALL NOISE SOURCES SIGNAL INDEPENDENT (KTC, QUANTIZATION)
- σ^2_{SHOT}: RELATED TO SIGNAL (CURRENT)
- σ^2_{PRNU}: ASSOCIATED WITH PIXEL-TO-PIXEL RESPONSE DIFFERENCES



PTC - PHOTON TRANSFER CURVE

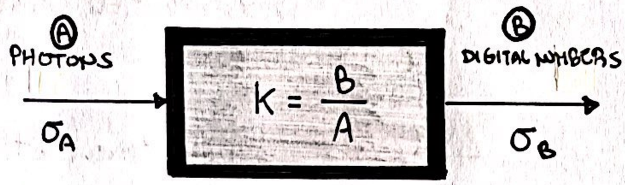
IT'S A TESTING METHODOLOGY EMPLOYED IN THE CHARACTERIZATION, OPTIMIZATION, CALIBRATION AND APPLICATION.

↓

WE CAN EXTRACT QUANTUM EFFICIENCY, NOISE, CHARGE COLLECTION EFFICIENCY, SENSE NODE CAPACITANCE, FULL CHARGE, DYNAMIC RANGE, PIXEL RESPONSIVITY

↓

PTC TREATS A CAMERA SYSTEM AS A BLACK BOX. WE NEED ONLY TO EXPOSE THE CAMERA TO A LIGHT SOURCE AND MEASURE THE SIGNAL AND NOISE OUTPUT RESPONSES



B : MEASURED OUTPUT MEAN SIGNAL LEVEL

σ_B : NOISE STANDARD DEVIATION

$$\sigma_B^2 = (k\sigma_A)^2 + \sigma_{add}^2$$

A : MEAN INPUT SIGNAL LEVEL

σ_A : INPUT NOISE STANDARD DEVIATION

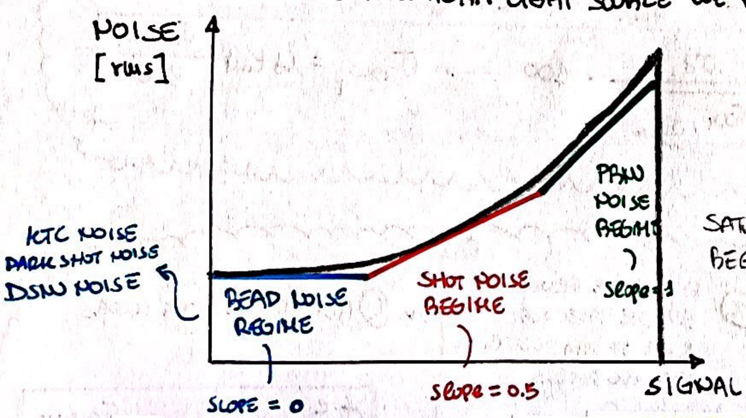
$$\sigma_A = \sqrt{A}$$

σ_{add} : STANDARD DEVIATION OF ALL OTHER NOISE CONTRIBUTIONS DUE TO NON-IDEALITIES OF REAL IMAGE SENSOR (usually negligible)

↓

ACTUALLY THE SENSITIVITY OF AN IMAGE SENSOR IS A FUNCTION OF WAVELENGTH. IN THIS CONTEXT A LINEAR GAIN k (instead of an integral law over the visible spectrum) can be defined only if radiation spectrum is monochromatic. IF IT'S NOT WE CAN ASSUME A DOMINANT WAVELENGTH

BY EXPOSING TO A UNIFORM LIGHT SOURCE WE PLOT THE NOISE OVER AVERAGE OUTPUT :



- we can calculate the AVERAGE OUTPUT SIGNAL DN, its standard deviation σ_{DN}. We can plot (for various photon fluxes) average signal on x-axis ^{B} and its standard deviation on y-axis σ_B

[D.N. = DIGITAL NUMBERS]

from PTC curve (DN - B x-axis, σ_B - DN_{rms} y-axis) we can extract:

- the GAIN:

$$\boxed{K = \left[\frac{\sigma_B^2}{B}\right]_{\sigma_B=1} = \frac{1}{DN_{shot}}}$$

→ DN_{shot} is where 1/2 shot noise line intercept the $1DN_{rms}$ noise

from theory:

$$\boxed{K = \frac{DN}{N_{ph}} = \eta \frac{2^{N_{BIT}}}{\underbrace{FWC}_{\text{full-well charge}}}}$$

- the PNRU:

$$\boxed{\%_{PNRU} = \left[\frac{\sigma_{PNRU}}{DN}\right]_{\sigma_{PNRU}=1} = \frac{1}{DN_{PNRU}}}$$

→ DN_{PNRU} is where PNRU line intercept the $1DN_{rms}$ line

- the MAXIMUM NUMBER OF ELECTRONS:

$$\boxed{N_{sat} = \frac{DN_{sat}\,\eta}{K}}$$ [number of e^-]

- the READ NOISE:

$$\boxed{\sigma_{READ,N} = \frac{\sigma_{READ,DN}\,\eta}{K}}$$

→ maximum slope = 0 line intercept y-axis [number of e^-]

- the DYNAMIC RANGE DR:

$$\boxed{DR = \frac{N_{sat}}{\sigma_{READ,N}}}$$ [dB]