Valutazione Prodotti Finanziari

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Anno scolastico 2022/2023

POLIMI GRADUATE SCHOOL OF MANAGEMENT

Master's Degree in Quantitative Finance

CDS

Group 5

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Summary

1 Introduction

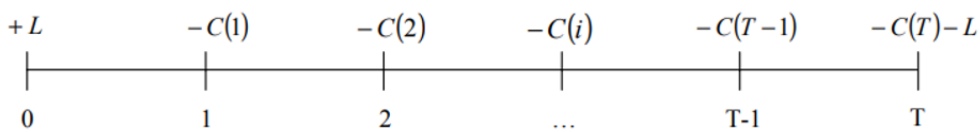
The Credit Default Swap (CDS) is a type of contract in which the protection buyer, who holds the credit, agrees to pay a fixed and periodic amount (Premium Leg: Cj at time t ¡T) determined in basis points relative to a notional principal, to the counterparty, the protection seller, who takes on the credit risk associated with the possibility that a future credit event will occur on that asset, and subsequently pays the Protection Leg (LGD at Default Time). The predetermined amount is usually commensurate with both the probability of the credit event and the default risk of the debtor. The peculiarity of this financial instrument lies in the fact that both parties to the contract may be unrelated to the third party, as the CDS is independent of its presence. The underlying asset is merely the creditworthiness and not the actual credit.

- Refers to a specific time period

- PB pays a premium

- PS accepts a possible LGD payment if the RE occurs within the contract's expiration date (Default Time)

JECI – JP Morgan



The typical cash flows - structure of JP Morgan's JECI instrument - of the contract involve the issuance of a security with a nominal amount equal to L by the protection buyer on credit-risky assets, on which periodic coupons C(t) are paid.

2 Valuation

Ex-Ante the protection period, the present value of the premium leg PV^C(t) is defined as follows:

$PV^C(t) = P(t, t_0)UF + \sum_{i=1}^{n} E_t[D(t, t_i)C_i 1_{t>t_i}] + Accr^d(t)$

where UF is the upfront premium that may be paid on the settlement date at time t0, while the present value of the amount due to be paid at default time by the PB is given by:

$Accr^d(t) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} E_t[D(t, u)(C_i \frac{u-t_{i-1}}{t_i-t_{i-1}})1_{t\in[u,u+du]}]$

Similarly, the present value of the protection leg PVd(t) is:

$PV^C(t) = \int_{t_0}^{t_n} E_t[D(t, u)LGD 1_{t\in[u,u+du]}]$

$= \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} E_t[D(t, u)LGD 1_{t\in[u,u+du]}]$

3 Exercise

Considering the trading date as 20/12/2022, two standardized CDS contracts (rolling on 'credit IMM' dates) are taken: the first with a maturity of 2Y (20/Dec/2024), the second with a maturity of 4Y (20/Dec/2026). Both CDS contracts have a contractual coupon equal to 1% of the notional, and a conventional recovery rate of 40% is assumed. It is also assumed that the interest rate curve is deterministic with zero-coupon rates equal to 3%. Knowing that the upfront premium quoted by the market is 1.26% of the notional for the first CDS and 2.08% of the notional for the second:

1. Determine two levels of default intensity $\lambda 1$ and $\lambda 2$, valid on the intervals [0,2y] and [2y,4y] respectively, compatible with these quotations (for this purpose, an 'end-of-period' rule can be assumed for the default leg payments).

2. Calculate the survival probabilities at 1y, 2y, 3y, 4y.

   Definitions of probabilities of events:

- Probability of default during the period [t, t + dt] in case of default not yet occurred:

  $\text{Prob}_t[\tau \in [u, u + du]] = \lambda(t)$

- Probability of survival during the period [t, T] in case of default not yet occurred.

  $\text{Surv}_t(T) = \text{Prob}_{t_{\tau > T}} = e^{-\int_t^u \lambda(8)ds}$

- Probability of survival during the period [u, u + du] in case of default not yet occurred:

  $\text{Prob}_t[\tau \in [u, u + du]] = \text{Prob}_{\tau > u}\text{Prob}_u[\tau \in [u, u + du]] = \lambda(u)du e^{-\int_t^u \lambda(8)ds}$

  Under the condition of no-arbitrage, independence between credit events and interest rates is assumed, resulting in:

  $\text{UF}^{2Y} = \text{LGD}\sum_{i=1}^{n_{2Y}} P(t,t_i)[e^{-\lambda_{2Y}(t_{i-1}-t)} - e^{-\lambda_{2Y}(t_i-t)}] - \bar{s}\sum_{i=1}^{n_{2Y}} \alpha_i P(t,t_i) e^{-\lambda_{2Y}(t_i-t)}$

  $\text{UF}^{4Y} = \text{LGD}\sum_{i=1}^{n_{2Y}} P(t,t_i)[e^{-\lambda_{2Y}(t_{i-1}-t)} - e^{-\lambda_{2Y}(t_i-t)}] - \bar{s}\sum_{i=1}^{n_{2Y}} \alpha_i P(t,t_i) e^{-\lambda_{2Y}(t_i-t)}$

  $+\text{LGD}\sum_{i>n_{2Y}}^{n_{4Y}} P(t,t_i)[e^{-\lambda_{4Y}(t_{i-1}-t)} - e^{-\lambda_{4Y}(t_i-t)}] - \bar{s}\sum_{i>n_{2Y}}^{n_{4Y}} \alpha_i P(t,t_i) e^{-\lambda_{4Y}(t_i-t)}$

  Where (1 - Rec) is referred to as the loss given default, or LGD.

  The exercise involves implementing the calculation of lambda intensity rates at 2 years and 4 years for a CDS covering a bond. Using Excel calculation functions, we obtain the values of lambda intensities that make the expected cash flows from the protection offered by the CDS buyer and the premium paid by the buyer equal. The cash flow value is calculated using the Black and Cox formula for pricing credit options.
  The spreadsheet uses the bond maturity date and premium payment dates to calculate the maturity times for each premium payment and to calculate

the zero-coupon discount factors for the different times. Additionally, the Upfront2 function is implemented to calculate the value of cash flows at 2 years and the Upfront4 function to calculate the value of cash flows at 4 years.
Comment on the results:

Legend
Solver's variable cells
Solver's objective cells

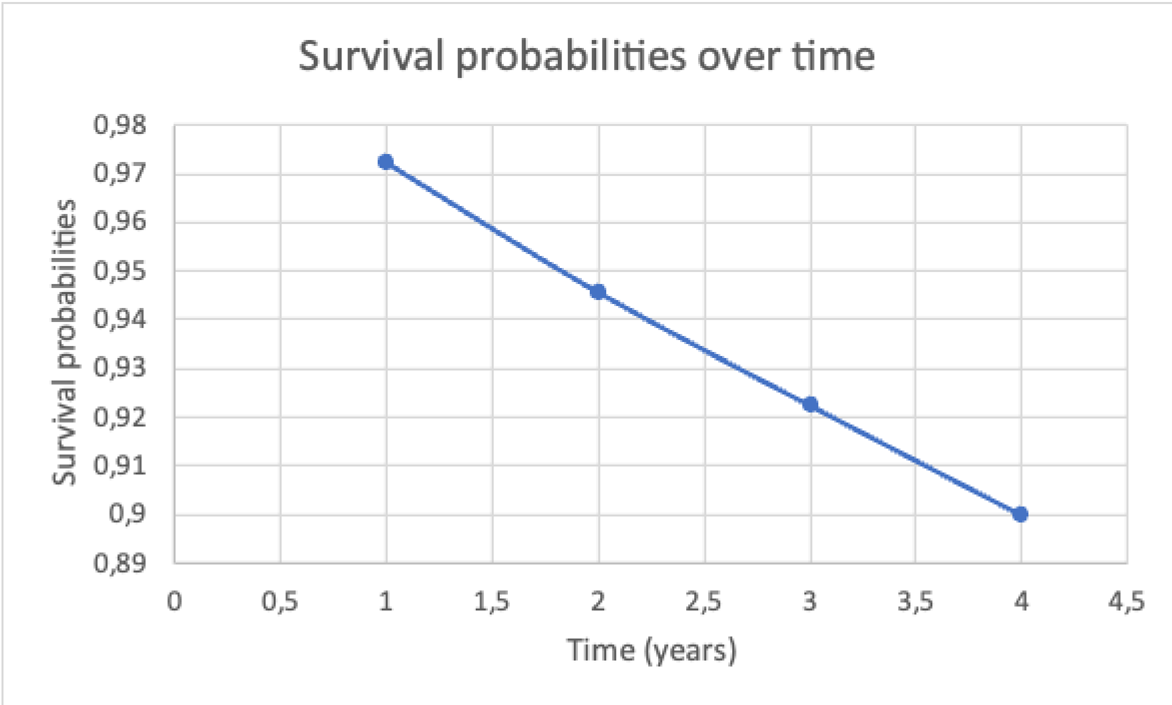
Constraints
0< λ<1
Upfront = Market Upfront

CDS 1							α		
Notional	C	zc	Recovery rate	LGD	T (1° CDS)	UF target	i	P(t,t_i)	1
1,00 €	1%	3%	0,4	0,6	2	1,26%	act/360	0,941764534	2,79%
		α							
Data	Cedola	i	ti-1 - t	ti - t	e - 2y(ti-1 - t)	e - 2y(ti - t)	protection leg	premium leg	Upfront
20/12/2022	- €	0,00	0,0000	0,0000	1,0000	1,0000	0,000000	0,000000	0,000000
20/03/2023	0,0025 €	0,25	0,0000	0,2500	1,0000	0,9930	0,003928	0,002338	0,001590
20/06/2023	0,0026 €	0,26	0,2500	0,5056	0,9930	0,9860	0,003987	0,002373	0,001614
20/09/2023	0,0026 €	0,26	0,5056	0,7611	0,9860	0,9790	0,003959	0,002356	0,001603
20/12/2023	0,0025 €	0,25	0,7611	1,0139	0,9790	0,9721	0,003888	0,002314	0,001574
20/03/2024	0,0025 €	0,25	1,0139	1,2667	0,9721	0,9653	0,003861	0,002298	0,001563
20/06/2024	0,0026 €	0,26	1,2667	1,5222	0,9653	0,9584	0,003876	0,002307	0,001569
20/09/2024	0,0026 €	0,26	1,5222	1,7778	0,9584	0,9516	0,003848	0,002290	0,001558
20/12/2024	0,0025 €	0,25	1,7778	2,0306	0,9516	0,9449	0,003779	0,002249	0,001530
									1,26%

CDS 2							α		
Notional	C	zc	Recovery rate	LGD	T (2° CDS)	UF target	i	P(t,t_i)	4
1,00 €	1%	3%	0,4	0,6	4	2,08%	act/360	0,886920437	2,49%
		α							
Data	Cedola	i	ti-1 - t	ti - t	e - 4y(ti-1 - t)	e - 4y(ti - t)	protection leg	premium leg	Upfront
20/12/2024									
20/03/2025	0,0025 €	0,2500	0,0000	0,2500	1,0000	0,9938	0,003297	0,002204	0,0011
20/06/2025	0,0026 €	0,2556	0,2500	0,5056	0,9938	0,9875	0,003349	0,002238	0,0011
20/09/2025	0,0026 €	0,2556	0,5056	0,7611	0,9875	0,9813	0,003328	0,002224	0,0011
20/12/2025	0,0025 €	0,2528	0,7611	1,0139	0,9813	0,9751	0,003271	0,002186	0,0011
20/03/2026	0,0025 €	0,2500	1,0139	1,2639	0,9751	0,9691	0,003215	0,002149	0,0011
20/06/2026	0,0026 €	0,2556	1,2639	1,5194	0,9691	0,9629	0,003266	0,002183	0,0011
20/09/2026	0,0026 €	0,2556	1,5194	1,7750	0,9629	0,9568	0,003245	0,002169	0,0011
20/12/2026	0,0025 €	0,2528	1,7750	2,0278	0,9568	0,9508	0,003190	0,002132	0,0011
									2,08%

Once the equation to equate the value of the buyer leg and the seller leg, as previously mentioned, is set up and solved using the Excel solver, we find the value of lambda that solves equation to be 2.79
As for exercise 2 regarding the calculation of survival, we set up the following formula for the calculation of survival probabilities:
$\text{Prob}_t[\tau \in [u, u+du]] = \text{Prob}_{\tau>u}\text{Prob}_u[\tau \in [u, u+du]] = \lambda du e^{-\lambda(u-t)}$
From which, given the previously calculated lambda values, the following results can be obtained:

Survival Probabilities	
Years	
1	**0,972482073**
2	**0,945721383**
3	**0,922500386**
4	**0,899849551**



From the graph, it is apparent that, as one would expect, survival probabilities decrease with an increase in time, while default probabilities increase correspondingly.