Valutazione Prodotti Finanziari

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POLIMI GRADUATE SCHOOL OF MANAGEMENT

Master's Degree in Quantitative Finance

Vasicek Short Rate Model

Group 5

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Summary

1 Short Rates Models

Consider a Vasicek model with parameters r(0) = 0.015, k = 0.25, θ = 0.02, σ = 0.1 Price a caplet with fixing date = 2Y, payment date = 2.5Y, and strike = 0.5% Use both the analytical formula and Monte Carlo simulation. Choose between risk neutral and forward measure.
Consider Vasicek model for short rates:

$$\begin{cases} dr(t) = k[\theta - r(t)]dt + \sigma dW^Q(t) \\ \\ r(0) = r_0 \\ \\ \sigma \in R^+; k, \theta, r_0 R \end{cases}$$

where: $\theta = 0.02, r_0 = 0.015, k = 0.25, \sigma = 0.5\%$

In order to price a caplet with fixing date = 2y, payment date = 2.5y and strike 0.5%, we have used to differents methods:

a Montecarlo simulation

b Analytical formula

2 Montecarlo Simulation

In order to price with Montecarlo simulation a caplet we've done the following procedure:

1. a short rate simulation for t+2y date with a single step using the rate distribution obtainable from the model with T-forward measure:
   $r(t) \sim N(r(s)e^{-k(t-s)} + M^T(s,t), \frac{\sigma^2}{2k}[1 - e^{-2k(t-s)}])$
   where $M^T(s,t) = (\theta - \frac{\sigma^2}{k^2})(1 - e^{s-t}) + \frac{\sigma^2}{2k^2}[e^{-k(T-t)} - e^{-2k(T+t-2s}]$
   We've considered as time instants t=0, S=2y, T=2.5y. The rate simulation has happened due to the following relation:
   $r_i(S) = r_t e^{-k(S-t} + M^T(t,S) + \frac{\sigma}{\sqrt{2k}}\sqrt{1 - e^{-2k(S-t)}}w_i$
   dove $w_i \sim N(0,1)$

2. for every state of world we've simulated the entire term structure, that is to calculate zero coupon bond prices associated to simulated short rates (for each of the i simulations, i=1,...,N) through the following generic result:
   $P_i(t,S) = A(t,S)e^{-B(t,S,k)r_i(t)}$
   where $A(t,S) = exp(\theta - \frac{\sigma^2}{2k^2}[B(t,S,k) - (S-t)] - \frac{\sigma^2}{4k}B(t,S,k)^2$,
   $B(t,S,k) = \frac{1}{k}[1 - e^{-k(S-t)}]$,
   We consider the same time instants as before, and the zero coupon bond prices are calculated as in the next formula:

$P_i(S,T) = A(S,T)e^{-B(S,T,k)r_i(S)}$

3. For every single simulation we evaluate Libor rates as following:
   $L_{\tau,i} = \frac{1}{\tau}(\frac{1}{P_i(S,T)} - 1)$
   where τ year is the fraction between fixing date and payment date

4. Evaluation of simulated payoff:
   $\text{Payoff}_i = \max L_{\tau,i} - K, 0\tau$

5. Caplet price determined as discounted payoff mean:
   $V_t = \frac{1}{N}\sum_i \text{Payoff}_i P(t,T)$
   In order to price the caplet with the Vasicek model we've chosen T-forward measure instead of risk neutral one because with the first one it is possible to run only one step of Montecarlo simulation between 0 and S, avoiding instant spot rate simulation at every single time instant

3 Analytical Formula

In order to price a caplet ω=1 with analytical formula we've used the following procedure:

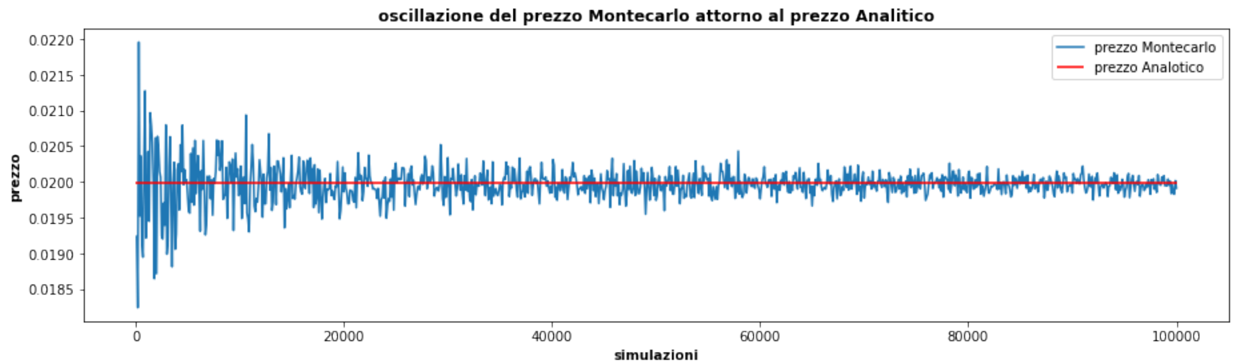
$cf(t;S,T,K,\omega) = \ (1+K\tau)ZCBO(t,S,T,K',\omega')$
Where $K' = \frac{1}{1+K\tau}, \omega' = -\omega$ and
$ZCBO(t,S,T,K',\omega') = \quad NP(t,S)\omega' P(S,T)\phi(\omega d_B) - K\phi(\omega'(d_B - \sigma_B))\}$
$d_B = \frac{\ln\frac{P(S,T)}{K} + \frac{1}{2}\sigma_B^2}{\sigma_B}$
$\sigma_B = \sigma_B B(S,T,k)\sqrt{B(t,S,2k)}$

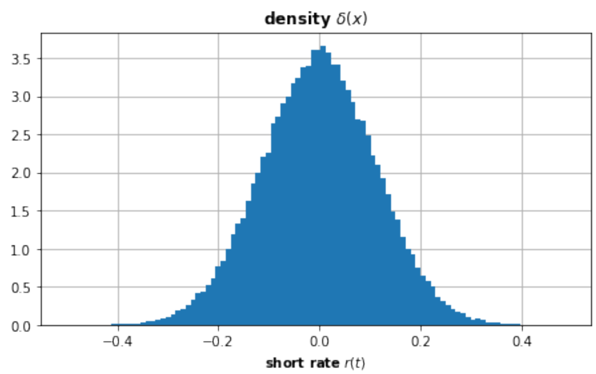
To find P(S,T) we've used the following step:
$P(S,T) = \frac{P(t,T)}{P(t,S)}$
derived by no-arbitrage relation in single curve world

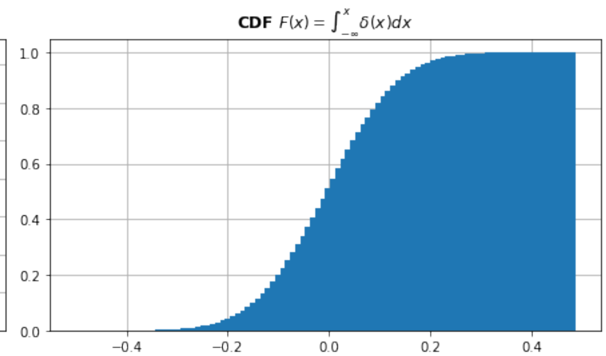
The prices obtained with Montecarlo simulation have been compared with Caplet prices obtained with analytical formula. As scenarios change we obtain this results

N-Montecarlo	MC Caplet prices	Caplet exact prices	Absolute errors
10000	0.0200252	0.0199751	5.01827e-05
50000	0.0200093	0.0199751	3.42786e-05
100000	0.0199545	0.0199750500915890	2.05518e-05

As the number of simulations grows up Montecarlo price approaches close formula's price. The error is evaluated as in the next formula:
$err_N = |p^{ext} - p^{MC}|$
With the goal of analyze simulated price's convergence to exact price, we've made between 1000 and 100000 simulations. From the plot we can observe that with the increase of number of simulations the price tends to fluctuate around the exact value and to stabilize around it







simulazioni: 100000

Errore: 2.05518e-5
prezzo Analytic: 0.0199751
prezzo Montecarlo: 0.0199545
Analytic > Montecarlo: 0.103%