Third SPICE Exercise

Fundamentals Of Electronics - a.a. 2018-2019 - University of Padua (Italy)

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Contents

Chapter 1

Differential amplifier with MOS current source

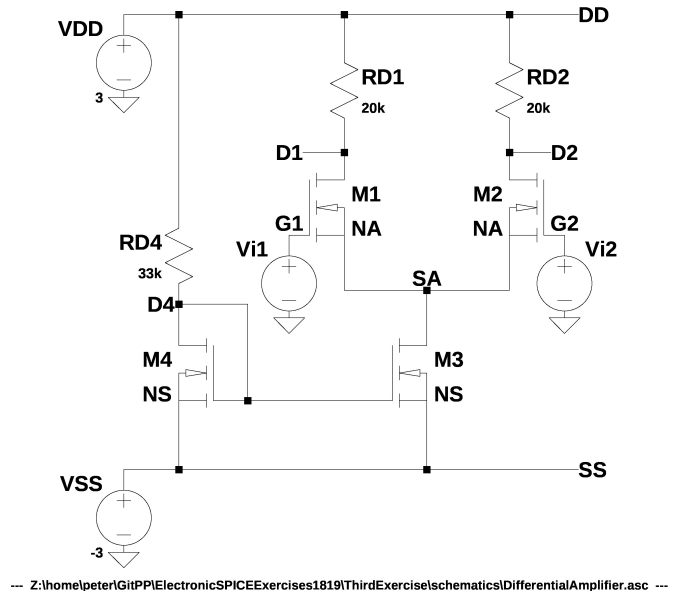

Figure 1.1: Differential amplifier with MOS current source

Initial data:

$$V_t = 0.5V \tag{1.1}$$

$$K'_n = \mu_n C_{ox} = 200\frac{\mu A}{V^2} \tag{1.2}$$

$$\lambda = 0 \tag{1.3}$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 20 \tag{1.4}$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 5 \tag{1.5}$$

$$R_{D_1} = R_{D_2} = 20k\Omega \tag{1.6}$$

$$R_{D_4} = \frac{30}{1000} \cdot 1097752\Omega = 32.93k\Omega \simeq 33k\Omega \tag{1.7}$$

$$V_{DD} = 3V \tag{1.8}$$

$$V_{SS} = -3V \tag{1.9}$$

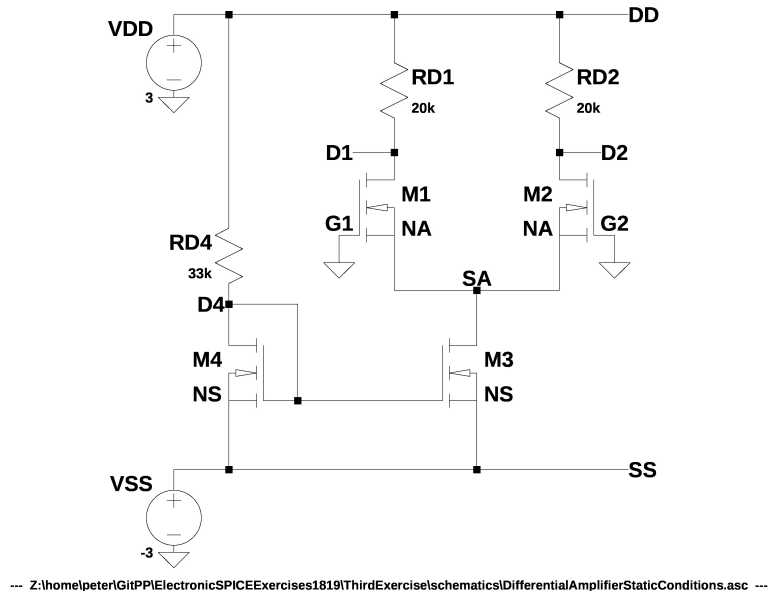

Figure 1.2: Differential amplifier with MOS current source - Static conditions

1.1 Static conditions - Analytic solution

On static conditions it's considered the input signals V_{i_1} and V_{i_2} turned off.
The equivalent circuit is on figure 1.2.

1.1.1 MOSFET M_4

Saturation mode checks

The transistor M_4 has a short circuit between its drain and its gate, so the transistor works in saturation mode and the voltage between the drain and the gate are the same of the voltage between the gate and the source:

$$V_{D_4SS} > V_{G_4SS} - V_t \xrightarrow{V_{G_4SS}=V_{D_4SS}} V_{D_4SS} > V_{D_4SS} - V_t \tag{1.10}$$

$$0 > -V_t \quad \text{(Always true: } V_t > 0 \text{)} \tag{1.11}$$

It's requested to pay attention to the another check to confirm the work on saturation mode:

$$V_{G_4SS} > V_t \xrightarrow{V_{G4SS}=V_{D_4SS}} V_{D_4SS} > V_t \tag{1.12}$$

$V_{D_4SS} \quad (= V_{G_4SS})$

Supposing that the transistor M_4 works on the saturation mode (see the section 1.1.1 for details), the current I_{D_4} could be calculated as:

$$I_{D_4} = \frac{1}{2} K'_n \left(\frac{W}{L}\right)_4 (V_{D_4SS} - V_t)^2 \tag{1.13}$$

Other expression of the current I_{D_4} could be calculated using the LKT:

$$V_{DD} - R_{D_4} I_{D_4} - V_{D4SS} - V_{SS} = 0 \implies I_{D_4} = \frac{V_{DD} - V_{D_4SS} - V_{SS}}{R_{D_4}} \tag{1.14}$$

Using the equations 1.13 and 1.14 it's possible calculating V_{D_4SS}:

$$\frac{1}{2} K'_n \left(\frac{W}{L}\right)_4 (V_{D_4SS} - V_t)^2 = \frac{V_{DD} - V_{D_4SS} - V_{SS}}{R_{D_4}} \tag{1.15}$$

$$\frac{1}{2}\cdot 200\frac{\mu A}{V^2}\cdot 5\frac{\mu m}{\mu m}(V_{D_4SS}-0.5V)^2=\frac{3V-V_{D_4SS}-(-3V)}{33k\Omega} \tag{1.16}$$

$$500\frac{\mu A}{V^2}(V_{D_4SS}-0.5V)^2=\frac{6}{33}mA-\frac{1}{33k\Omega}V_{D_4SS} \tag{1.17}$$

$$500\frac{\mu A}{V^2}(V_{D_4SS}^2-V_{D_4SS}\cdot V+0.25V^2)=\frac{6}{33}mA-\frac{1}{33k\Omega}V_{D_4SS} \tag{1.18}$$

$$500\frac{\mu A}{V^2}\cdot V_{D_4SS}^2+\left(-500\frac{\mu A}{V^2}V+\frac{1}{33k\Omega}\right)V_{D_4SS}+500\frac{\mu A}{V^2}\cdot 0.25V^2-\frac{6}{33}mA=0 \tag{1.19}$$

$$0.5\frac{mA}{V^2}\cdot V_{D_4SS}^2+\left(-0.5\frac{mA}{V^2}V+\frac{1}{33k\Omega}\right)V_{D_4SS}+0.5\frac{mA}{V^2}\cdot 0.25V^2-\frac{6}{33}mA=0 \tag{1.20}$$

$$\left(0.5\frac{mA}{V^2}\right)V_{D_4SS}^2+\left(-\frac{31}{66}\frac{mA}{V}\right)V_{D_4SS}+\left(-\frac{5}{88}mA\right)=0 \tag{1.21}$$

$$V_{D_4SS_{1,2}}=\frac{-\left(-\frac{31}{66}\frac{mA}{V}\right)\pm\sqrt{\left(-\frac{31}{66}\frac{mA}{V}\right)^2-4\cdot\left(0.5\frac{mA}{V^2}\right)\cdot\left(-\frac{5}{88}mA\right)}}{2\cdot 0.5\frac{mA}{V^2}}=\begin{cases}1.04784V\\ -0.10845V & \text{Not possible: } < \text{ of } V_t\end{cases} \tag{1.22}$$

Now it's possible to check the last equation that can confirm the work on saturation mode of the MOSFET M_4 (equation 1.12):

$$1.04784V>0.5V \quad M_4 \text{ works on saturation mode.} \tag{1.23}$$

I_{D_4}

Using the equation 1.13 and the result of the equation 1.22:

$$I_{D_4}=\frac{1}{2}\cdot 200\mu A/V^2\cdot 5\frac{\mu m}{\mu m}\cdot(1.04784V-0.5V)^2=150.06433\mu A \tag{1.24}$$

1.1.2 MOSFET M_3

V_{G_3SS}

Observing the circuit represented on the figure 1.2 it's clear that the voltage V_{G_3SS} is equal to the voltage V_{D_4SS} calculated in the equation 1.22.

$$V_{G_3SS}=V_{D_4SS} \tag{1.25}$$

I_{S_A}

As agree with the consideration of the section 1.1.2 and supposing the work of the MOSFET M_3 on the saturation mode, it's possible calculating the drain current of the MOSFET M_3:

$$I_{SA}=\frac{1}{2}K'_n\left(\frac{W}{L}\right)_3(V_{D_4SS}-V_t)^2 \tag{1.26}$$

$$I_{SA}=\frac{1}{2}\cdot 200\mu A/V^2\cdot 5\frac{\mu m}{\mu m}\cdot(1.04784V-0.5V)^2=150.06433\mu A \tag{1.27}$$

Saturation mode checks

For obtaining the confirm of the work of the MOSFET M_3 on saturation mode the next two equations have to be satisfied:

$$V_{D_3SS}>V_{G_3SS}-V_t \xrightarrow{V_{D_3SS}=V_{S_ASS},V_{G_3SS}=V_{D_4SS}} V_{S_ASS}>V_{D_4SS}-V_t \tag{1.28}$$

$$V_{G_3SS}>V_t \xrightarrow{V_{G_3SS}=V_{D_4SS}} V_{D_4SS}>V_t \tag{1.29}$$

The equation 1.28 hasn't to be checked, V_{S_ASS} isn't calculated yet.
The equation 1.29 is satisfied (see the equation 1.23).

1.1.3 MOSFET M_1 and MOSFET M_2

The MOSFET M_1 and the MOSFET M_2 have the same dimension and the same constructive parameters (see initial data at the start of this chapter 1).
They also have the same voltage applied to every their pins (see figure 1.2).
So, additionally supposing the work on the saturation mode of M_1 and M_2, it's possible to confirm the next equations:

$$I_{D_1} = I_{D_2} \tag{1.30}$$
$$V_{G_1S_A} = V_{G_2S_A} \tag{1.31}$$
$$V_{D_1S_A} = V_{D_2S_A} \tag{1.32}$$

$I_{D_1}(= I_{D_2})$

$$\text{LKC node } S_A\text{: } I_{D_1} + I_{D_2} - I_{S_A} = 0 \xrightarrow{eq.1.30} 2I_{D_1} - I_{S_A} = 0 \implies I_{D_1} = \frac{I_{S_A}}{2} \tag{1.33}$$

$$I_{D_1} = \frac{150.06433\mu A}{2} = 75.03217\mu A \tag{1.34}$$

$V_{G_1S_A}(= V_{G_2S_A})$

Supposing the work of the MOSFET M_1 (equally M_2) on the saturation mode, the drain current could be calculated as:

$$I_{D_1} = \frac{1}{2}K'_n\left(\frac{W}{L}\right)_1(V_{G_1S_A} - V_t)^2 \tag{1.35}$$

It's possible using the equation 1.35 to calculate $V_{G_1S_A}$:

$$I_{D_1} = \frac{1}{2}K'_n\left(\frac{W}{L}\right)_1(V_{G_1S_A} - V_t)^2 \tag{1.36}$$
$$\sqrt{I_{D_1}} = \sqrt{\frac{1}{2}K'_n\left(\frac{W}{L}\right)_1}(V_{G_1S_A} - V_t) \tag{1.37}$$
$$\sqrt{\frac{I_{D_1}}{\frac{1}{2}K'_n\left(\frac{W}{L}\right)_1}} = V_{G_1S_A} - V_t \tag{1.38}$$
$$V_{G_1S_A} = \sqrt{\frac{2I_{D_1}}{K'_n\left(\frac{W}{L}\right)_1}} + V_t \tag{1.39}$$
$$V_{G_1S_A} = \sqrt{\frac{2 \cdot 75.03217\mu A}{200\mu A \cdot 20\frac{\mu A}{\mu A}}} + 0.5V \tag{1.40}$$
$$V_{G_1S_A} = 0.69369V \tag{1.41}$$

V_{S_A} Now it's possible calculating the voltage on the node S_A:

$$V_{G_1S_A} = V_{G_1} - V_{S_A} \implies V_{S_A} = V_{G_1} - V_{G_1S_A} \xrightarrow{V_{G_1}=0, eq.1.41} V_{S_A} = -0.69369V \tag{1.42}$$

Saturation mode checks

In order to check the mode of the M_1 (and M_2) the equations to respect are 1.43 and 1.48.

$$V_{D_1S_A} > V_{G_1S_A} - Vt \tag{1.43}$$
$$V_{D_1} - V_{S_A} > V_{G_1S_A} - Vt \tag{1.44}$$
$$I_{D_1}R_{D_1} - V_{S_A} > V_{G_1S_A} - Vt \tag{1.45}$$
$$75.03217\mu A \cdot 20k\Omega - (-0.69369V) > 0.69369V - 0.5V \tag{1.46}$$
$$2.19433V > 0.19369V \quad \text{True.} \tag{1.47}$$

$$V_{G_1S_A} > V_t \tag{1.48}$$
$$0.69369V > 0.5V \quad \text{True.} \tag{1.49}$$

Checking the mode of the $M3$'s work (see equation 1.28):

$$V_{S_ASS} > V_{D_4SS} - V_t \tag{1.50}$$
$$V_{S_A} - V_{SS} > V_{D_4SS} - V_t \tag{1.51}$$
$$-0.69369V - (-3V) > 1.04784V - 0.5V \tag{1.52}$$
$$2.30631V > 0.54784V \quad M_3 \text{ works on the saturation mode.} \tag{1.53}$$

1.1.4 MOSFET V_{DS_Q}, V_{GS_Q}, I_{D_Q} - Resuming

MOSFET	V_{DS_Q}	V_{GS_Q}	I_{D_Q}
$M1$	$V_{D_1S_A} = 2.19433V$	$V_{G_1S_A} = 0.69369V$	$I_{D_1} = 75.03217\mu A$
$M2$	$V_{D_2S_A} = 2.19433V$	$V_{G_2S_A} = 0.69369V$	$I_{D_2} = 75.03217\mu A$
$M3$	$V_{S_ASS} = 2.30631V$	$V_{D_4SS} = 1.04784V$	$I_{S_A} = 150.06433\mu A$
$M4$	$V_{D_4SS} = 1.04784V$	$V_{D_4SS} = 1.04784V$	$I_{D_4} = 150.06433\mu A$

1.1.5 g_m

$$g_{m_1} = g_{m_2} = K'_n \left(\frac{W}{L}\right)_1 (V_{G_1S_A} - V_t) \tag{1.54}$$
$$= 200\frac{\mu A}{V^2} \cdot 20\frac{\mu A}{\mu A} \cdot (0.69369V - 0.5V) \tag{1.55}$$
$$= 774.76\mu A/V \tag{1.56}$$

1.1.6 MOSFET M_3 with $\lambda = 0.02$

From now the MOSFET M_3 is considered with $\lambda = 0.02$.

On this way there are some changes of the voltages and the currents of the circuit but they could be considered negligible, so it's considered true the past result from now too.

With $\lambda = 0.02$, the r_0 has a finite value:

$$r_0 = \frac{1}{\lambda I_{S_A}} = \frac{1}{0.02 \cdot 150.06433\mu A} \simeq 333.2k\Omega \tag{1.57}$$

It's possible see the r_0 resitances in the small signal circuit by using the PI model of the transistor MOSFET (figure 1.3).

In the small signal circuit are considered only the alternate sources, so the V_{DD} and the V_{SS} voltage sources are substituted by a short circuit.
In this way the mirror current source composed by the MOSFET M_4 and the MOSFET M_5 are off because the drain, the source and the gate of these MOSFETs have no voltage applied. So we can simplify the small signal circuit of the figure 1.3 to the small signal circuit of the figure 1.4.

Now it could be clear the effect of the $\lambda = 0.02$ of the MOSFET M_3 because its resistance r_0 has an effect to the gain of the circuit if the voltage of the node S_A is different than null.

1.1.7 Gain single ended

Differential pure signal

Now it's applied small alternate voltage signal to the circuit of the figure 1.4:

$$V_{i_1} = +\frac{V_{id}}{2} \tag{1.58}$$
$$V_{i_2} = -\frac{V_{id}}{2} \tag{1.59}$$

Applying a small alternate voltage signal to the circuit of the figure 1.4 there aren't current on the resistance r_0 so it could be treated as a short circuit.

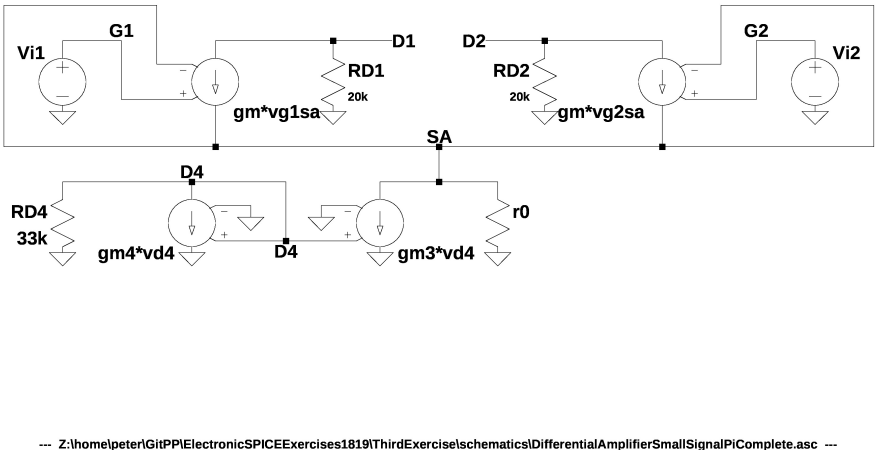

Figure 1.3: Small signal circuit by using the PI model of the Transistor MOSFET

Given that the circuit of the figure 1.4 is symmetrical it's possible analyse the single ended behaviour of the circuit on figure 1.5.

The single ended gain for a small differential signal is:

$$V_{D_2} = -g_m V_{G_2} R_{D_2} \xrightarrow{(eq.1.59)} V_{D_2} = -g_m \left(-\frac{V_{id}}{2}\right) R_{D_2} \implies \frac{V_{D_2}}{V_{id}} = g_m R_{D_2} = A_{d_{single \cdot ended}} \tag{1.60}$$

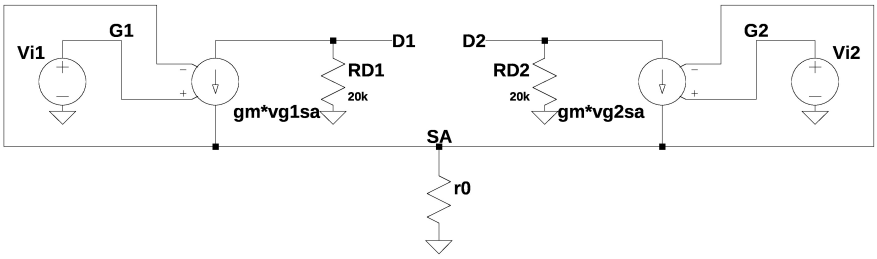

Figure 1.4: Small signal circuit by using the PI model of the Transistor MOSFET - simplifications

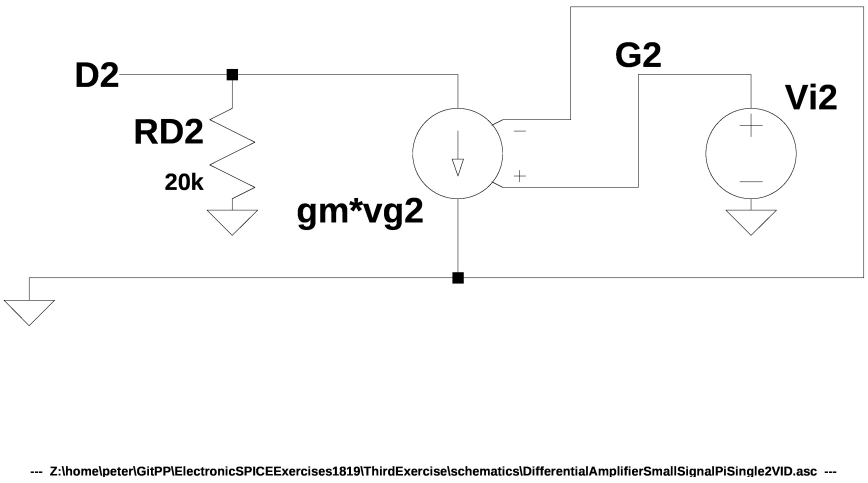

Figure 1.5: Small signal circuit single ended for a differential pure signal

1.2 SPICE analysis

1.2.1 Operating Point on static conditions

```
* Differential Amplifier - Static conditions
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*********************************************************************************

* Parameters
.param RD = 20k

* NMOS models
.model NA NMOS VT0=0.5 KP=200u LAMBDA=0 W=4.00u L=0.20u
.model NS NMOS VT0=0.5 KP=200u LAMBDA=0 W=1.25u L=0.25u

* Resistances
RD1 DD D1 {RD}
RD2 DD D2 {RD}
RD4 DD D4 33k

* Transistors
M1 D1 G1 SA SA NA
M2 D2 G2 SA SA NA
M3 SA D4 SS SS NS
M4 D4 D4 SS SS NS

* Voltage sources
VDD DD 0 3
VSS SS 0 -3
Vi1 G1 0 0
Vi2 G2 0 0

* Analysis
.op

.END
```

```
      --- Operating Point ---

V(dd):     3                voltage
V(d1):     1.49935          voltage
V(d2):     1.49935          voltage
V(d4):     -1.95216         voltage
V(g1):     0                voltage
V(sa):     -0.693691        voltage
V(g2):     0                voltage
V(ss):     -3               voltage
Id(M4):    0.000150065      device_current
Ig(M4):    0                device_current
Ib(M4):    -1.05784e-012    device_current
Is(M4):    -0.000150065     device_current
Id(M3):    0.000150065      device_current
Ig(M3):    0                device_current
Ib(M3):    -2.31631e-012    device_current
```

```
Is(M3):    -0.000150065    device_current
Id(M2):    7.50327e-005    device_current
Ig(M2):    0               device_current
Ib(M2):    -2.20304e-012   device_current
Is(M2):    -7.50327e-005   device_current
Id(M1):    7.50327e-005    device_current
Ig(M1):    0               device_current
Ib(M1):    -2.20304e-012   device_current
Is(M1):    -7.50327e-005   device_current
I(Rd4):    0.000150065     device_current
I(Rd2):    7.50327e-005    device_current
I(Rd1):    7.50327e-005    device_current
I(Vi2):    0               device_current
I(Vi1):    0               device_current
I(Vss):    0.000300131     device_current
I(Vdd):    -0.000300131    device_current
```

1.2.2 Operating Point - common mode signal

```
* Differential Amplifier - Common Mode signals
*******************************************************************************
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*******************************************************************************

* Parameters
.param RD = 20k

* NMOS models
.model NA NMOS VT0=0.5 KP=200u LAMBDA=0 W=4.00u L=0.20u
.model NS NMOS VT0=0.5 KP=200u LAMBDA=0 W=1.25u L=0.25u

* Resistances
RD1 DD D1 {RD}
RD2 DD D2 {RD}
RD4 DD D4 33k

* Transistors
M1 D1 G1 SA SA NA
M2 D2 G2 SA SA NA
M3 SA D4 SS SS NS
M4 D4 D4 SS SS NS

* Voltage sources
VDD DD 0 3
VSS SS 0 -3
Vi1 G1 0 sine(0 10m 10k 0 0 0)
Vi2 G2 0 sine(0 10m 10k 0 0 0)

* Analysis
.op

.END
```

```
        --- Operating Point ---

V(dd):     3                voltage
V(d1):     1.49935          voltage
V(d2):     1.49935          voltage
V(d4):     -1.95216         voltage
V(g1):     0                voltage
V(sa):     -0.693691        voltage
V(g2):     0                voltage
V(ss):     -3               voltage
Id(M4):    0.000150065      device_current
Ig(M4):    0                device_current
Ib(M4):    -1.05784e-012    device_current
Is(M4):    -0.000150065     device_current
Id(M3):    0.000150065      device_current
Ig(M3):    0                device_current
Ib(M3):    -2.31631e-012    device_current
Is(M3):    -0.000150065     device_current
Id(M2):    7.50327e-005     device_current
Ig(M2):    0                device_current
Ib(M2):    -2.20304e-012    device_current
Is(M2):    -7.50327e-005    device_current
Id(M1):    7.50327e-005     device_current
Ig(M1):    0                device_current
Ib(M1):    -2.20304e-012    device_current
Is(M1):    -7.50327e-005    device_current
I(Rd4):    0.000150065      device_current
I(Rd2):    7.50327e-005     device_current
I(Rd1):    7.50327e-005     device_current
I(Vi2):    0                device_current
I(Vi1):    0                device_current
I(Vss):    0.000300131      device_current
I(Vdd):    -0.000300131     device_current
```

1.2.3 Operating Point - differential signals

```
* Differential Amplifier - Differential signals
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********************************************************************************

* Parameters
.param RD = 20k

* NMOS models
.model NA NMOS VT0=0.5 KP=200u LAMBDA=0 W=4.00u L=0.20u
.model NS NMOS VT0=0.5 KP=200u LAMBDA=0 W=1.25u L=0.25u
.model NS3 NMOS VT0=0.5 KP=200u LAMBDA=0.02 W=1.25u L=0.25u

* Resistances
RD1 DD D1 {RD}
RD2 DD D2 {RD}
RD4 DD D4 33k
```

```
* Transistors
M1 D1 G1 SA SA NA
M2 D2 G2 SA SA NA
M3 SA D4 SS SS NS3
M4 D4 D4 SS SS NS

* Voltage sources
VDD DD 0 3
VSS SS 0 -3
Vi1 G1 0 sine(0 10m 10k 0 0 0)
Vi2 0 G2 sine(0 10m 10k 0 0 0)

* Analysis
.TRAN 0u 250u 0 1u

.END
```

```
      --- Operating Point ---

V(dd):     3                voltage
V(d1):     1.49935          voltage
V(d2):     1.49935          voltage
V(d4):     -1.95216         voltage
V(g1):     0                voltage
V(sa):     -0.693691        voltage
V(g2):     0                voltage
V(ss):     -3               voltage
Id(M4):    0.000150065      device_current
Ig(M4):    0                device_current
Ib(M4):    -1.05784e-012    device_current
Is(M4):    -0.000150065     device_current
Id(M3):    0.000150065      device_current
Ig(M3):    0                device_current
Ib(M3):    -2.31631e-012    device_current
Is(M3):    -0.000150065     device_current
Id(M2):    7.50327e-005     device_current
Ig(M2):    0                device_current
Ib(M2):    -2.20304e-012    device_current
Is(M2):    -7.50327e-005    device_current
Id(M1):    7.50327e-005     device_current
Ig(M1):    0                device_current
Ib(M1):    -2.20304e-012    device_current
Is(M1):    -7.50327e-005    device_current
I(Rd4):    0.000150065      device_current
I(Rd2):    7.50327e-005     device_current
I(Rd1):    7.50327e-005     device_current
I(Vi2):    0                device_current
I(Vi1):    0                device_current
I(Vss):    0.000300131      device_current
I(Vdd):    -0.000300131     device_current
```