

# Third SPICE Exercise

Fundamentals Of Electronics - a.a. 2018-2019 - University of Padua (Italy)

Pietro Prandini (mat. 1097752)

June 10, 2019



# Contents

<b>1</b>	<b>Differential amplifier with MOS current source</b>	<b>5</b>
1.1	Static conditions - Analytic solution	6
1.1.1	MOSFET $M_4$	6
	Saturation mode checks	6
	$V_{D_4SS} (= V_{G_4SS})$	6
	$I_{D_4}$	7
1.1.2	MOSFET $M_3$	7
	$V_{G_3SS}$	7
	$I_{S_A}$	7
	Saturation mode checks	7
1.1.3	MOSFET $M_1$ and MOSFET $M_2$	8
	$I_{D_1} (= I_{D_2})$	8
	$V_{G_1S_A} (= V_{G_2S_A})$	8
	Saturation mode checks	8
1.1.4	MOSFET $V_{DSQ}$ , $V_{GSQ}$ , $I_{DQ}$ - Resuming	9
1.1.5	$g_m$	9
1.1.6	MOSFET $M_3$ with $\lambda = 0.02$	9
1.1.7	Differential gain	9
1.2	SPICE analysis	9
1.2.1	Operating Point on static conditions	9
1.2.2	Operating Point - common mode signal	11
1.2.3	Operating Point - differential signals	12



# Chapter 1

## Differential amplifier with MOS current source

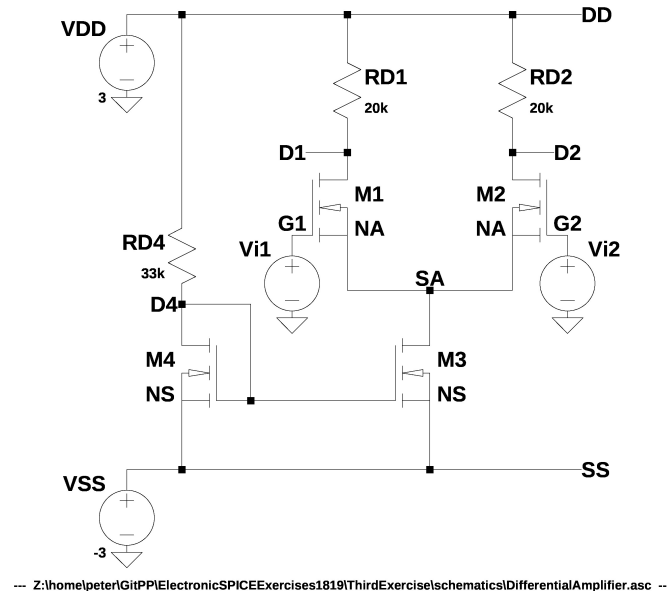


Figure 1.1: Differential amplifier with MOS current source

Initial data:

$$V_t = 0.5V \quad (1.1)$$

$$K'_n = \mu_n C_{ox} = 200 \frac{\mu A}{V^2} \quad (1.2)$$

$$\lambda = 0 \quad (1.3)$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 20 \quad (1.4)$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 5 \quad (1.5)$$

$$R_{D1} = R_{D2} = 20k\Omega \quad (1.6)$$

$$R_{D4} = \frac{30}{1000} \cdot 1097752\Omega = 32.93k\Omega \simeq 33k\Omega \quad (1.7)$$

$$V_{DD} = 3V \quad (1.8)$$

$$V_{SS} = -3V \quad (1.9)$$

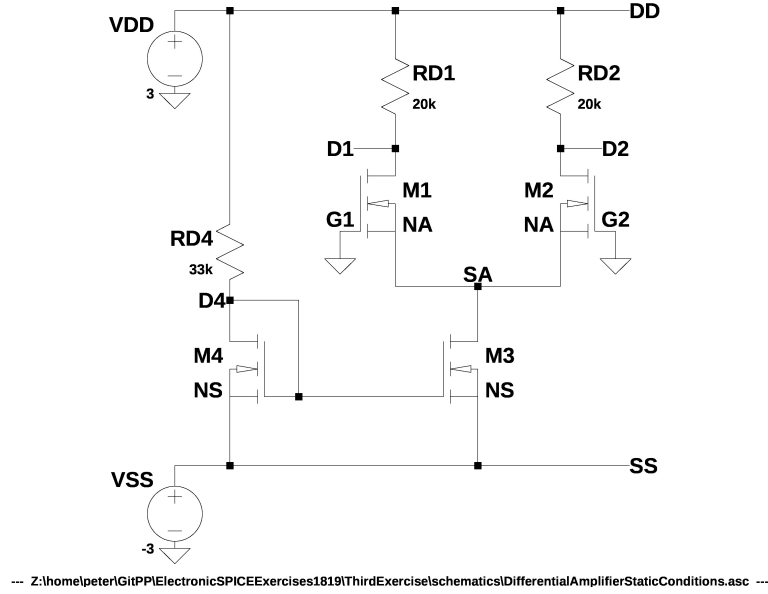


Figure 1.2: Differential amplifier with MOS current source - Static conditions

## 1.1 Static conditions - Analytic solution

On static conditions it's considered the input signals  $V_{i1}$  and  $V_{i2}$  turned off. The equivalent circuit is on figure 1.2.

### 1.1.1 MOSFET $M_4$

#### Saturation mode checks

The transistor  $M_4$  has a short circuit between its drain and its gate, so the transistor works in saturation mode and the voltage between the drain and the gate are the same of the voltage between the gate and the source:

$$V_{D_4SS} > V_{G_4SS} - V_t \xrightarrow{V_{G_4SS}=V_{D_4SS}} V_{D_4SS} > V_{D_4SS} - V_t \quad (1.10)$$

$$0 > -V_t \quad (\text{Always true: } V_t > 0) \quad (1.11)$$

It's requested to pay attention to the another check to confirm the work on saturation mode:

$$V_{G_4SS} > V_t \xrightarrow{V_{G_4SS}=V_{D_4SS}} V_{D_4SS} > V_t \quad (1.12)$$

$$V_{D_4SS} \quad (= V_{G_4SS})$$

Supposing that the transistor  $M_4$  works on the saturation mode (see the section 1.1.1 for details), the current  $I_{D_4}$  could be calculated as:

$$I_{D_4} = \frac{1}{2} K'_n \left( \frac{W}{L} \right)_4 (V_{D_4SS} - V_t)^2 \quad (1.13)$$

Other expression of the current  $I_{D_4}$  could be calculated using the LKT:

$$V_{DD} - R_{D_4} I_{D_4} - V_{D_4SS} - V_{SS} = 0 \implies I_{D_4} = \frac{V_{DD} - V_{D_4SS} - V_{SS}}{R_{D_4}} \quad (1.14)$$

Using the equations 1.13 and 1.14 it's possible calculating  $V_{D_4SS}$ :

$$\frac{1}{2} K'_n \left( \frac{W}{L} \right)_4 (V_{D_4SS} - V_t)^2 = \frac{V_{DD} - V_{D_4SS} - V_{SS}}{R_{D_4}} \quad (1.15)$$

$$\frac{1}{2} \cdot 200 \frac{\mu A}{V^2} \cdot 5 \frac{\mu m}{\mu m} (V_{D_4SS} - 0.5V)^2 = \frac{3V - V_{D_4SS} - (-3V)}{33k\Omega} \quad (1.16)$$

$$500 \frac{\mu A}{V^2} (V_{D_4SS} - 0.5V)^2 = \frac{6}{33} mA - \frac{1}{33k\Omega} V_{D_4SS} \quad (1.17)$$

$$500 \frac{\mu A}{V^2} (V_{D_4SS}^2 - V_{D_4SS} \cdot V + 0.25V^2) = \frac{6}{33} mA - \frac{1}{33k\Omega} V_{D_4SS} \quad (1.18)$$

$$500 \frac{\mu A}{V^2} \cdot V_{D_4SS}^2 + \left( -500 \frac{\mu A}{V^2} V + \frac{1}{33k\Omega} \right) V_{D_4SS} + 500 \frac{\mu A}{V^2} \cdot 0.25V^2 - \frac{6}{33} mA = 0 \quad (1.19)$$

$$0.5 \frac{mA}{V^2} \cdot V_{D_4SS}^2 + \left( -0.5 \frac{mA}{V^2} V + \frac{1}{33k\Omega} \right) V_{D_4SS} + 0.5 \frac{mA}{V^2} \cdot 0.25V^2 - \frac{6}{33} mA = 0 \quad (1.20)$$

$$\left( 0.5 \frac{mA}{V^2} \right) V_{D_4SS}^2 + \left( -\frac{31}{66} \frac{mA}{V} \right) V_{D_4SS} + \left( -\frac{5}{88} mA \right) = 0 \quad (1.21)$$

$$V_{D_4SS_{1,2}} = \frac{-\left(-\frac{31}{66} \frac{mA}{V}\right) \pm \sqrt{\left(-\frac{31}{66} \frac{mA}{V}\right)^2 - 4 \cdot \left(0.5 \frac{mA}{V^2}\right) \cdot \left(-\frac{5}{88} mA\right)}}{2 \cdot 0.5 \frac{mA}{V^2}} = \begin{cases} 1.04784V \\ -0.10845V \end{cases} \quad \text{Not possible: } < \text{ of } V_t \quad (1.22)$$

Now it's possible to check the last equation that can confirm the work on saturation mode of the MOSFET  $M_4$  (equation 1.12):

$$1.04784V > 0.5V \quad M_4 \text{ works on saturation mode.} \quad (1.23)$$

$I_{D_4}$

Using the equation 1.13 and the result of the equation 1.22:

$$I_{D_4} = \frac{1}{2} \cdot 200 \mu A / V^2 \cdot 5 \frac{\mu m}{\mu m} \cdot (1.04784V - 0.5V)^2 = 150.06433 \mu A \quad (1.24)$$

### 1.1.2 MOSFET $M_3$

$V_{G_3SS}$

Observing the circuit represented on the figure 1.2 it's clear that the voltage  $V_{G_3SS}$  is equal to the voltage  $V_{D_4SS}$  calculated in the equation 1.22.

$$V_{G_3SS} = V_{D_4SS} \quad (1.25)$$

$I_{S_A}$

As agree with the consideration of the section 1.1.2 and supposing the work of the MOSFET  $M_3$  on the saturation mode, it's possible calculating the drain current of the MOSFET  $M_3$ :

$$I_{S_A} = \frac{1}{2} K'_n \left( \frac{W}{L} \right)_3 (V_{D_4SS} - V_t)^2 \quad (1.26)$$

$$I_{S_A} = \frac{1}{2} \cdot 200 \mu A / V^2 \cdot 5 \frac{\mu m}{\mu m} \cdot (1.04784V - 0.5V)^2 = 150.06433 \mu A \quad (1.27)$$

### Saturation mode checks

For obtaining the confirm of the work of the MOSFET  $M_3$  on saturation mode the next two equations have to be satisfied:

$$V_{D_3SS} > V_{G_3SS} - V_t \xrightarrow{V_{D_3SS}=V_{S_ASS}, V_{G_3SS}=V_{D_4SS}} V_{S_ASS} > V_{D_4SS} - V_t \quad (1.28)$$

$$V_{G_3SS} > V_t \xrightarrow{V_{G_3SS}=V_{D_4SS}} V_{D_4SS} > V_t \quad (1.29)$$

The equation 1.28 hasn't to be checked,  $V_{S_ASS}$  isn't calculated yet.

The equation 1.29 is satisfied (see the equation 1.23).

### 1.1.3 MOSFET $M_1$ and MOSFET $M_2$

The MOSFET  $M_1$  and the MOSFET  $M_2$  have the same dimension and the same constructive parameters (see initial data at the start of this chapter 1).

They also have the same voltage applied to every their pins (see figure 1.2).

So, additionally supposing the work on the saturation mode of  $M_1$  and  $M_2$ , it's possible to confirm the next equations:

$$I_{D_1} = I_{D_2} \quad (1.30)$$

$$V_{G_1 S_A} = V_{G_2 S_A} \quad (1.31)$$

$$V_{D_1 S_A} = V_{D_2 S_A} \quad (1.32)$$

$$I_{D_1} (= I_{D_2})$$

$$\text{LKC node } S_A: I_{D_1} + I_{D_2} - I_{S_A} = 0 \xrightarrow{\text{eq.1.30}} 2I_{D_1} - I_{S_A} = 0 \implies I_{D_1} = \frac{I_{S_A}}{2} \quad (1.33)$$

$$I_{D_1} = \frac{150.06433\mu A}{2} = 75.03217\mu A \quad (1.34)$$

$$V_{G_1 S_A} (= V_{G_2 S_A})$$

Supposing the work of the MOSFET  $M_1$  (equally  $M_2$ ) on the saturation mode, the drain current could be calculated as:

$$I_{D_1} = \frac{1}{2} K'_n \left( \frac{W}{L} \right)_1 (V_{G_1 S_A} - V_t)^2 \quad (1.35)$$

It's possible using the equation 1.35 to calculate  $V_{G_1 S_A}$ :

$$I_{D_1} = \frac{1}{2} K'_n \left( \frac{W}{L} \right)_1 (V_{G_1 S_A} - V_t)^2 \quad (1.36)$$

$$\sqrt{I_{D_1}} = \sqrt{\frac{1}{2} K'_n \left( \frac{W}{L} \right)_1} (V_{G_1 S_A} - V_t) \quad (1.37)$$

$$\sqrt{\frac{I_{D_1}}{\frac{1}{2} K'_n \left( \frac{W}{L} \right)_1}} = V_{G_1 S_A} - V_t \quad (1.38)$$

$$V_{G_1 S_A} = \sqrt{\frac{2I_{D_1}}{K'_n \left( \frac{W}{L} \right)_1}} + V_t \quad (1.39)$$

$$V_{G_1 S_A} = \sqrt{\frac{2 \cdot 75.03217\mu A}{200\mu A \cdot 20 \frac{\mu A}{\mu A}}} + 0.5V \quad (1.40)$$

$$V_{G_1 S_A} = 0.69369V \quad (1.41)$$

$V_{S_A}$  Now it's possible calculating the voltage on the node  $S_A$ :

$$V_{G_1 S_A} = V_{G_1} - V_{S_A} \implies V_{S_A} = V_{G_1} - V_{G_1 S_A} \xrightarrow{V_{G_1}=0, \text{eq.1.41}} V_{S_A} = -0.69369V \quad (1.42)$$

#### Saturation mode checks

In order to check the mode of the  $M_1$  (and  $M_2$ ) the equations to respect are 1.43 and 1.48.

$$V_{D_1 S_A} > V_{G_1 S_A} - V_t \quad (1.43)$$

$$V_{D_1} - V_{S_A} > V_{G_1 S_A} - V_t \quad (1.44)$$

$$I_{D_1} R_{D_1} - V_{S_A} > V_{G_1 S_A} - V_t \quad (1.45)$$

$$75.03217\mu A \cdot 20k\Omega - (-0.69369V) > 0.69369V - 0.5V \quad (1.46)$$

$$2.19433V > 0.19369V \quad \text{True.} \quad (1.47)$$



$$V_{G_1S_A} > V_t \quad (1.48)$$

$$0.69369V > 0.5V \quad \text{True.} \quad (1.49)$$

Checking the mode of the  $M_3$ 's work (see equation 1.28):

$$V_{S_ASS} > V_{D_4SS} - V_t \quad (1.50)$$

$$V_{S_A} - V_{SS} > V_{D_4SS} - V_t \quad (1.51)$$

$$-0.69369V - (-3V) > 1.04784V - 0.5V \quad (1.52)$$

$$2.30631V > 0.54784V \quad M_3 \text{ works on the saturation mode.} \quad (1.53)$$

### 1.1.4 MOSFET $V_{DSQ}$ , $V_{GSQ}$ , $I_{DQ}$ - Resuming

MOSFET	$V_{DSQ}$	$V_{GSQ}$	$I_{DQ}$
$M_1$	$V_{D_1S_A} = 2.19433V$	$V_{G_1S_A} = 0.69369V$	$I_{D_1} = 75.03217\mu A$
$M_2$	$V_{D_2S_A} = 2.19433V$	$V_{G_2S_A} = 0.69369V$	$I_{D_2} = 75.03217\mu A$
$M_3$	$V_{S_ASS} = 2.30631V$	$V_{D_4SS} = 1.04784V$	$I_{S_A} = 150.06433\mu A$
$M_4$	$V_{D_4SS} = 1.04784V$	$V_{D_4SS} = 1.04784V$	$I_{D_4} = 150.06433\mu A$

### 1.1.5 $g_m$

$$g_{m_1} = g_{m_2} = K'_n \left( \frac{W}{L} \right)_1 (V_{G_1S_A} - V_t) \quad (1.54)$$

$$= 200 \frac{\mu A}{V^2} \cdot 20 \frac{\mu A}{\mu A} \cdot (0.69369V - 0.5V) \quad (1.55)$$

$$= 774.76\mu A/V \quad (1.56)$$

$$g_{m_3} = g_{m_4} = K'_n \left( \frac{W}{L} \right)_3 (V_{D_4SS} - V_t) \quad (1.57)$$

$$= 200 \frac{\mu A}{V^2} \cdot 5 \frac{\mu A}{\mu A} \cdot (1.04784V - 0.5V) \quad (1.58)$$

$$= 547.80\mu A/V \quad (1.59)$$

### 1.1.6 MOSFET $M_3$ with $\lambda = 0.02$

From now the MOSFET  $M_3$  is considered with  $\lambda = 0.02$ .

On this way there are some changes of the voltages and the currents of the circuit but they could be considered negligible, so it's considered true the past result from now too.

With  $\lambda = 0.02$ , the  $r_0$  has a finite value:

$$r_0 = \frac{1}{\lambda I_{S_A}} = \frac{1}{0.02 \cdot 150.06433\mu A} \simeq 333.2k\Omega \quad (1.60)$$

### 1.1.7 Differential gain

## 1.2 SPICE analysis

### 1.2.1 Operating Point on static conditions

```
* Differential Amplifier – Static conditions
*****
* 3st Exercise – Fundamentals Of Electronics – a.a. 2018–2019 – UniPD – Italy *
*                               Pietro Prandini – mat. 1097752                *
*                                                                           *
* This work is licensed under the Creative Commons Attribution–ShareAlike 4.0 *
* International License. To view a copy of this license, visit              *
* http://creativecommons.org/licenses/by-sa/4.0/ or send a letter to Creative *
* Commons, PO Box 1866, Mountain View, CA 94042, USA.                      *
```

```

*****

* Parameters
.param RD = 20k

* NMOS models
.model NA NMOS VT0=0.5 KP=200u LAMBDA=0 W=4.00u L=0.20u
.model NS NMOS VT0=0.5 KP=200u LAMBDA=0 W=1.25u L=0.25u

* Resistances
RD1 DD D1 {RD}
RD2 DD D2 {RD}
RD4 DD D4 33k

* Transistors
M1 D1 G1 SA SA NA
M2 D2 G2 SA SA NA
M3 SA D4 SS SS NS
M4 D4 D4 SS SS NS

* Voltage sources
VDD DD 0 3
VSS SS 0 -3
Vi1 G1 0 0
Vi2 G2 0 0

* Analysis
.op

.END

```

—— Operating Point ——

V(dd):	3	voltage
V(d1):	1.49935	voltage
V(d2):	1.49935	voltage
V(d4):	-1.95216	voltage
V(g1):	0	voltage
V(sa):	-0.693691	voltage
V(g2):	0	voltage
V(ss):	-3	voltage
Id(M4):	0.000150065	device_current
Ig(M4):	0	device_current
Ib(M4):	-1.05784e-012	device_current
Is(M4):	-0.000150065	device_current
Id(M3):	0.000150065	device_current
Ig(M3):	0	device_current
Ib(M3):	-2.31631e-012	device_current
Is(M3):	-0.000150065	device_current
Id(M2):	7.50327e-005	device_current
Ig(M2):	0	device_current
Ib(M2):	-2.20304e-012	device_current
Is(M2):	-7.50327e-005	device_current
Id(M1):	7.50327e-005	device_current
Ig(M1):	0	device_current
Ib(M1):	-2.20304e-012	device_current
Is(M1):	-7.50327e-005	device_current
I(Rd4):	0.000150065	device_current
I(Rd2):	7.50327e-005	device_current
I(Rd1):	7.50327e-005	device_current
I(Vi2):	0	device_current

I(Vi1):	0	device_current
I(Vss):	0.000300131	device_current
I(Vdd):	-0.000300131	device_current

### 1.2.2 Operating Point - common mode signal

```

* Differential Amplifier – Common Mode signals
*****
* 3st Exercise – Fundamentals Of Electronics – a.a. 2018–2019 – UniPD – Italy *
*                               Pietro Prandini – mat. 1097752                *
*                                                                           *
* This work is licensed under the Creative Commons Attribution–ShareAlike 4.0 *
* International License. To view a copy of this license, visit              *
* http://creativecommons.org/licenses/by-sa/4.0/ or send a letter to Creative *
* Commons, PO Box 1866, Mountain View, CA 94042, USA.                      *
*****

* Parameters
.param RD = 20k

* NMOS models
.model NA NMOS VT0=0.5 KP=200u LAMBDA=0 W=4.00u L=0.20u
.model NS NMOS VT0=0.5 KP=200u LAMBDA=0 W=1.25u L=0.25u

* Resistances
RD1 DD D1 {RD}
RD2 DD D2 {RD}
RD4 DD D4 33k

* Transistors
M1 D1 G1 SA SA NA
M2 D2 G2 SA SA NA
M3 SA D4 SS SS NS
M4 D4 D4 SS SS NS

* Voltage sources
VDD DD 0 3
VSS SS 0 -3
Vi1 G1 0 sine(0 10m 10k 0 0 0)
Vi2 G2 0 sine(0 10m 10k 0 0 0)

* Analysis
.op

.END

```

#### —— Operating Point ——

V(dd):	3	voltage
V(d1):	1.49935	voltage
V(d2):	1.49935	voltage
V(d4):	-1.95216	voltage
V(g1):	0	voltage
V(sa):	-0.693691	voltage
V(g2):	0	voltage
V(ss):	-3	voltage
Id(M4):	0.000150065	device_current
Ig(M4):	0	device_current
Ib(M4):	-1.05784e-012	device_current

Is(M4):	-0.000150065	device_current
Id(M3):	0.000150065	device_current
Ig(M3):	0	device_current
Ib(M3):	-2.31631e-012	device_current
Is(M3):	-0.000150065	device_current
Id(M2):	7.50327e-005	device_current
Ig(M2):	0	device_current
Ib(M2):	-2.20304e-012	device_current
Is(M2):	-7.50327e-005	device_current
Id(M1):	7.50327e-005	device_current
Ig(M1):	0	device_current
Ib(M1):	-2.20304e-012	device_current
Is(M1):	-7.50327e-005	device_current
I(Rd4):	0.000150065	device_current
I(Rd2):	7.50327e-005	device_current
I(Rd1):	7.50327e-005	device_current
I(Vi2):	0	device_current
I(Vi1):	0	device_current
I(Vss):	0.000300131	device_current
I(Vdd):	-0.000300131	device_current

### 1.2.3 Operating Point - differential signals

```

* Differential Amplifier – Differential signals
*****
* 3st Exercise – Fundamentals Of Electronics – a.a. 2018–2019 – UniPD – Italy *
*                               Pietro Prandini – mat. 1097752                *
*                                                                           *
* This work is licensed under the Creative Commons Attribution–ShareAlike 4.0 *
* International License. To view a copy of this license, visit              *
* http://creativecommons.org/licenses/by-sa/4.0/ or send a letter to Creative *
* Commons, PO Box 1866, Mountain View, CA 94042, USA.                      *
*****

* Parameters
.param RD = 20k

* NMOS models
.model NA NMOS VT0=0.5 KP=200u LAMBDA=0 W=4.00u L=0.20u
.model NS NMOS VT0=0.5 KP=200u LAMBDA=0 W=1.25u L=0.25u
.model NS3 NMOS VT0=0.5 KP=200u LAMBDA=0.02 W=1.25u L=0.25u

* Resistances
RD1 DD D1 {RD}
RD2 DD D2 {RD}
RD4 DD D4 33k

* Transistors
M1 D1 G1 SA SA NA
M2 D2 G2 SA SA NA
M3 SA D4 SS SS NS3
M4 D4 D4 SS SS NS

* Voltage sources
VDD DD 0 0
VSS SS 0 0
Vi1 G1 0 sine(0 10m 10k 0 0 0)
Vi2 G2 0 sine(0 10m 10k 0 0 0)

* Analysis

```

```
.TRAN 0u 250u 0 1u
```

```
.END
```

—— Operating Point ——

V(dd):	3	voltage
V(d1):	1.49935	voltage
V(d2):	1.49935	voltage
V(d4):	-1.95216	voltage
V(g1):	0	voltage
V(sa):	-0.693691	voltage
V(g2):	0	voltage
V(ss):	-3	voltage
Id(M4):	0.000150065	device_current
Ig(M4):	0	device_current
Ib(M4):	-1.05784e-012	device_current
Is(M4):	-0.000150065	device_current
Id(M3):	0.000150065	device_current
Ig(M3):	0	device_current
Ib(M3):	-2.31631e-012	device_current
Is(M3):	-0.000150065	device_current
Id(M2):	7.50327e-005	device_current
Ig(M2):	0	device_current
Ib(M2):	-2.20304e-012	device_current
Is(M2):	-7.50327e-005	device_current
Id(M1):	7.50327e-005	device_current
Ig(M1):	0	device_current
Ib(M1):	-2.20304e-012	device_current
Is(M1):	-7.50327e-005	device_current
I(Rd4):	0.000150065	device_current
I(Rd2):	7.50327e-005	device_current
I(Rd1):	7.50327e-005	device_current
I(Vi2):	0	device_current
I(Vi1):	0	device_current
I(Vss):	0.000300131	device_current
I(Vdd):	-0.000300131	device_current