파이썬 수치해석

Chapter 3. 수치 적분

박형묵

명신여자고등학교

Update 20250514

강의 자료 다운로드

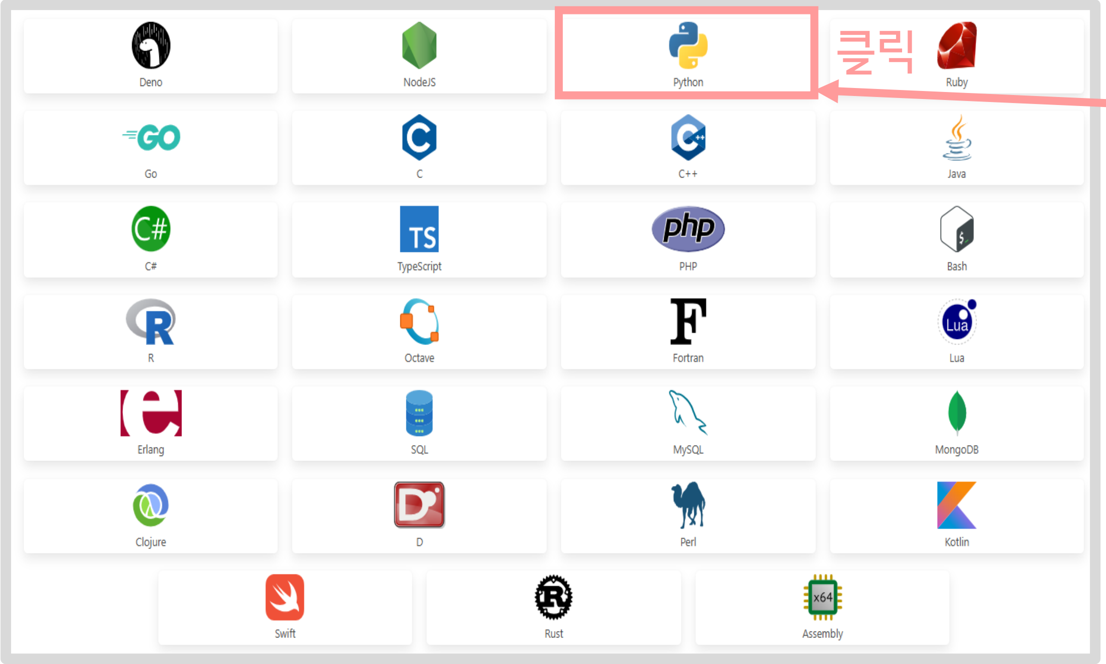
파이썬 수치해석 강의 자료

https://github.com/PigeonDove/PythonNumericalAnalysis

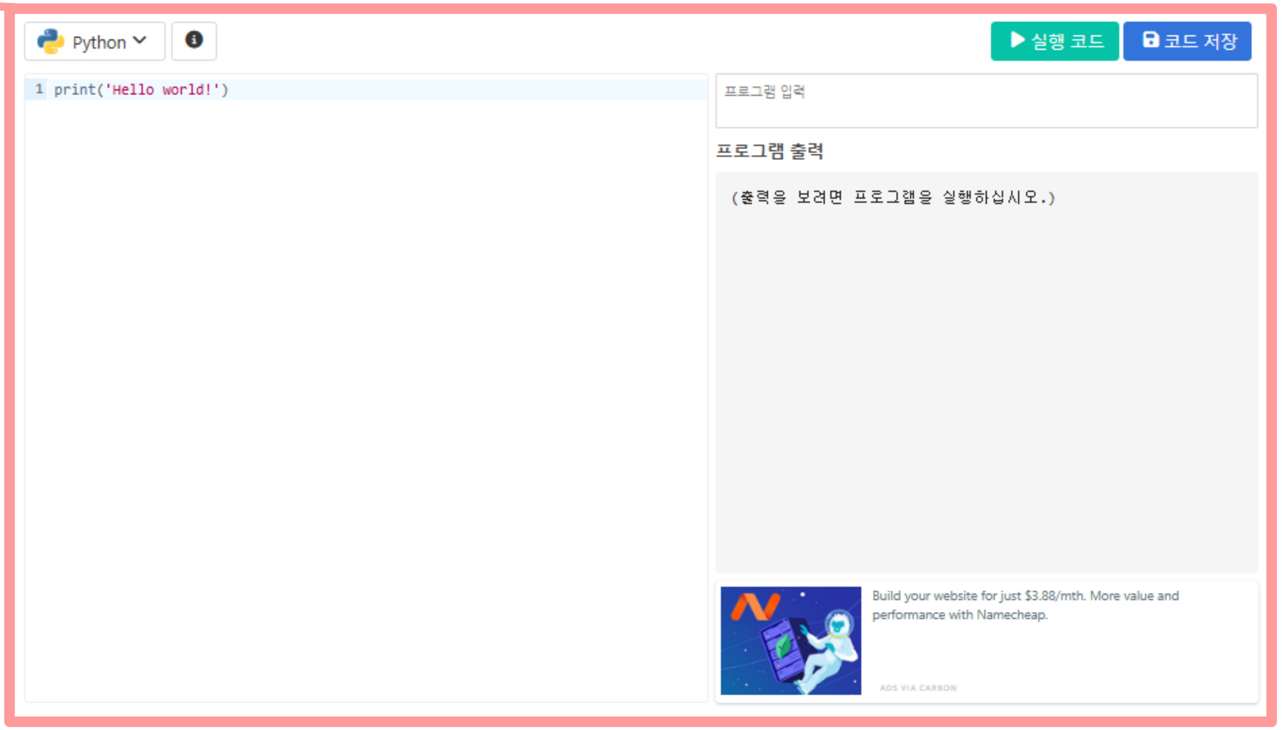
개발 환경

myCompiler 의 Python

파이썬 코딩 웹 사이트



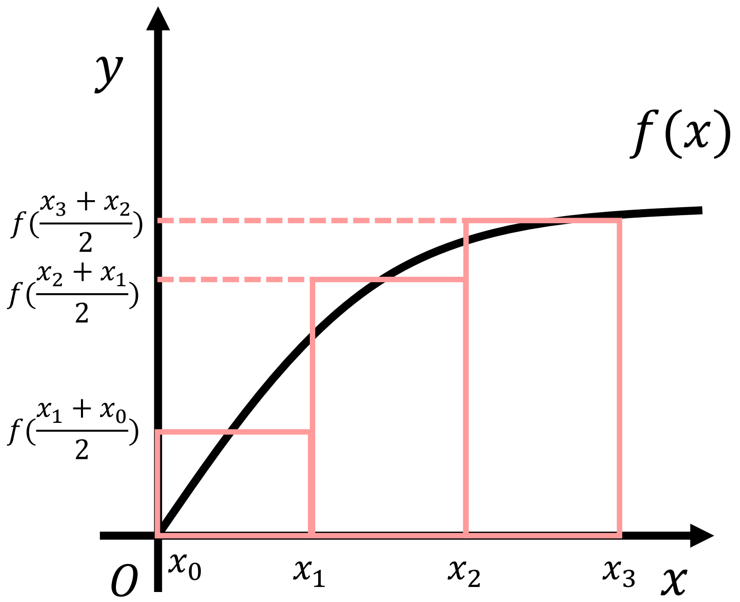
https://www.mycompiler.io/ko/new/python



수치 적분

직사각형법

기본 개념 학습



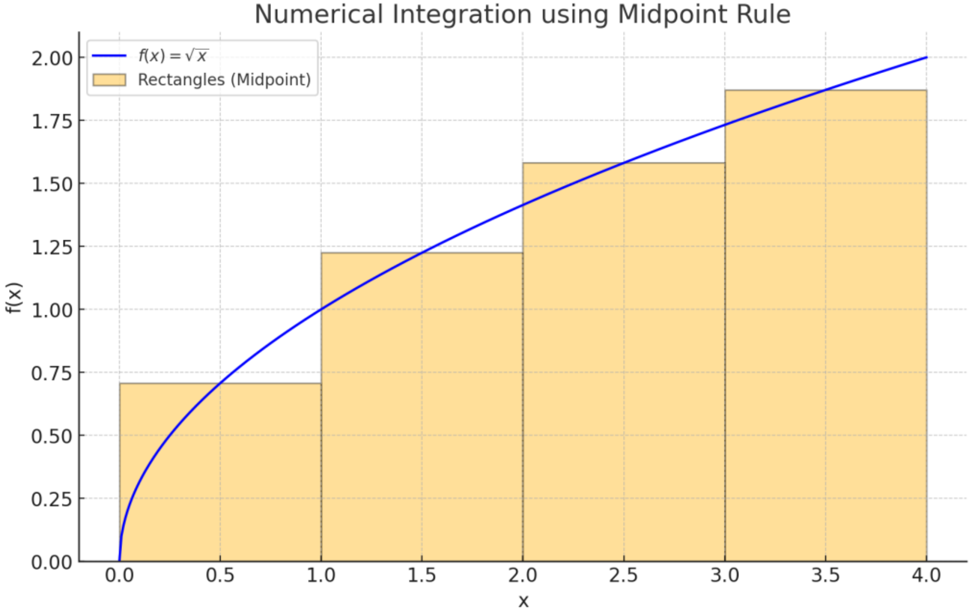
면적을 직사각형으로 근사하여 구하는 방법

$$\int_{x_0}^{x_n} f(x)dx \approx \sum_{i=1}^{n} [(x_i - x_{i-1}) \cdot f\left(\frac{x_i + x_{i-1}}{2}\right)]$$

수치 적분

직사각형법

기본 개념 적용



$\int_0^4 \sqrt{x}dx$ 계산하기

1) 정확한 적분 값

$$\int_0^4 \sqrt{x}dx = \int_0^4 x^{\frac{1}{2}}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 = \frac{2}{3}\cdot 4^{\frac{3}{2}} = \frac{16}{3} = 5.3333$$

2) 직사각형법 근사 값

$$\int_0^4 \sqrt{x}dx \approx \sum_{i=1}^{4}\left[(x_i - x_{i-1})\cdot f\left(\frac{x_i + x_{i-1}}{2}\right)\right]$$

$$= 0.7071 + 1.2247 + 1.5811 + 1.8708 = 5.3838$$

$$Error = \frac{|\text{실제 값} - \text{계산 값}|}{\text{실제 값}}\cdot 100$$

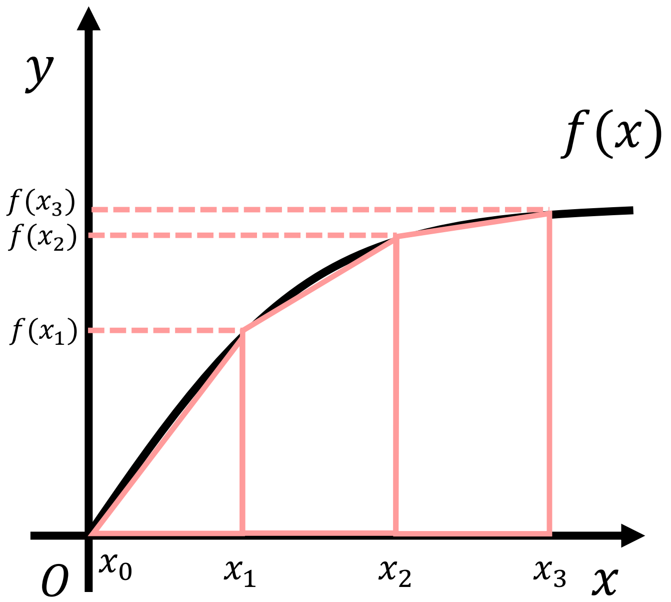
$$Error = \frac{|5.3333 - 5.3838|}{5.3333}\cdot 100 = 0.95\%$$

소스코드 3-1.py, 그래프는 3-2.py

수치 적분

사다리꼴법

기본 개념 학습



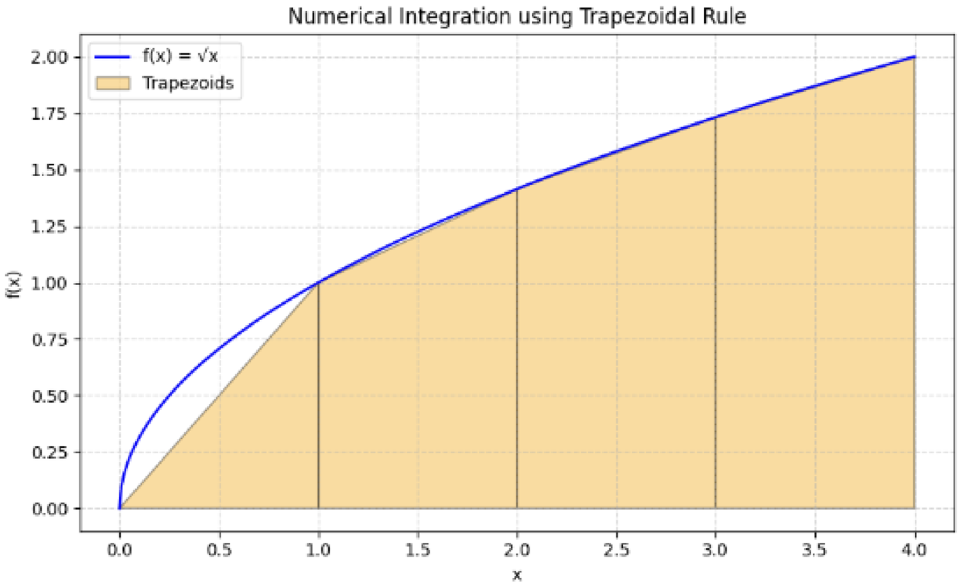
면적을 사다리꼴로 근사하여 구하는 방법

$$\int_{x_0}^{x_n} f(x)dx \approx \sum_{i=1}^{n}\left[\frac{(x_i - x_{i-1})\cdot(f(x_i) + f(x_{i-1}))}{2}\right]$$

수치 적분

사다리꼴법

기본 개념 적용



$\int_0^4 \sqrt{x}dx$ 계산하기

1) 정확한 적분 값

$$\int_0^4 \sqrt{x}dx = \int_0^4 x^{\frac{1}{2}}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 = \frac{2}{3}\cdot 4^{\frac{3}{2}} = \frac{16}{3} = 5.3333$$

2) 사다리꼴법 근사 값

$$\int_0^4 \sqrt{x}dx \approx \sum_{i=1}^{4}\left[\frac{(x_i - x_{i-1})\cdot(f(x_i) + f(x_{i-1}))}{2}\right]$$

$$= 0.5000 + 1.2071 + 1.5731 + 1.8660 = 5.1463$$

$$Error = \frac{|\text{실제 값} - \text{계산 값}|}{\text{실제 값}}\cdot 100$$

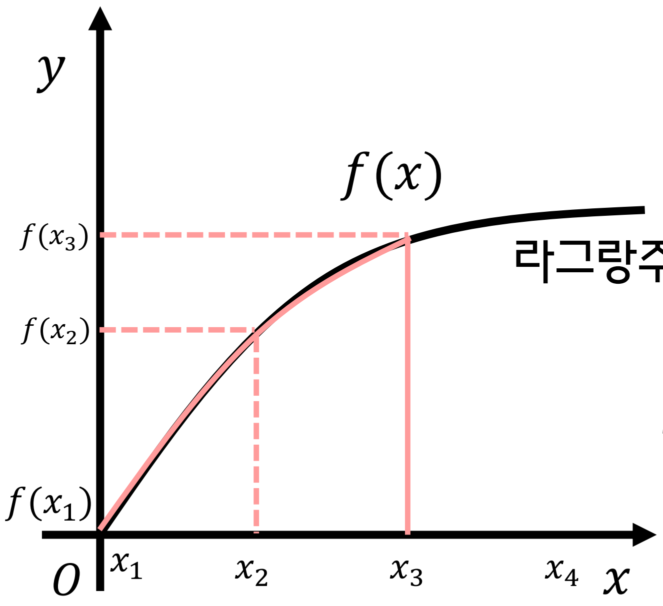
$$Error = \frac{|5.3333 - 5.1463|}{5.3333}\cdot 100 = 3.51\%$$

소스코드 3-3.py, 그래프는 3-4.py

수치 적분

심슨법

기본 개념 학습



면적을 2차 라그랑주 보간법으로 근사하여 구하는 방법

$$y = y_1 \frac{(x - x_2)(x - x_3)\cdots(x - x_{n+1})}{(x_1 - x_2)(x_1 - x_3)\cdots(x_1 - x_{n+1})} + y_2 \frac{(x - x_1)(x - x_3)\cdots(x - x_{n+1})}{(x_2 - x_1)(x_2 - x_3)\cdots(x_2 - x_{n+1})} + \cdots + y_{\mathrm{n+1}} \frac{(x - x_1)(x - x_2)\cdots(x - x_n)}{(x_{n+1} - x_1)(x_{n+1} - x_2)\cdots(x_{n+1} - x_n)}$$

라그랑주 2차식 $\Rightarrow\ P(x) = f(x_1)\dfrac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} + f(x_2)\dfrac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} + f(x_3)\dfrac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}$

$$\int_{x_1}^{x_3} P(x)dx = f(x_1)\int_{x_1}^{x_3} \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}dx + f(x_2)\int_{x_1}^{x_3} \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}dx + f(x_3)\int_{x_1}^{x_3} \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}dx$$

구간 $x_1 \sim x_3$은 같은 간격이므로, $x_1 = -h, x_2 = 0, x_3 = h$이라고 하면,

$$\int_{x_1}^{x_3} P(x)dx = f(-h)\int_{-h}^{h} \frac{(x-0)(x-h)}{(-h-0)(-h-h)}dx + f(0)\int_{-h}^{h} \frac{(x+h)(x-h)}{(0+h)(0-h)}dx + f(h)\int_{-h}^{h} \frac{(x+h)(x-0)}{(h+h)(h-0)}dx$$

$$= f(-h)\int_{-h}^{h} \frac{x^2-hx}{2h^2}dx + f(0)\int_{-h}^{h} \frac{x^2-h^2}{-h^2}dx + f(h)\int_{-h}^{h} \frac{x^2+hx}{2h^2}dx$$

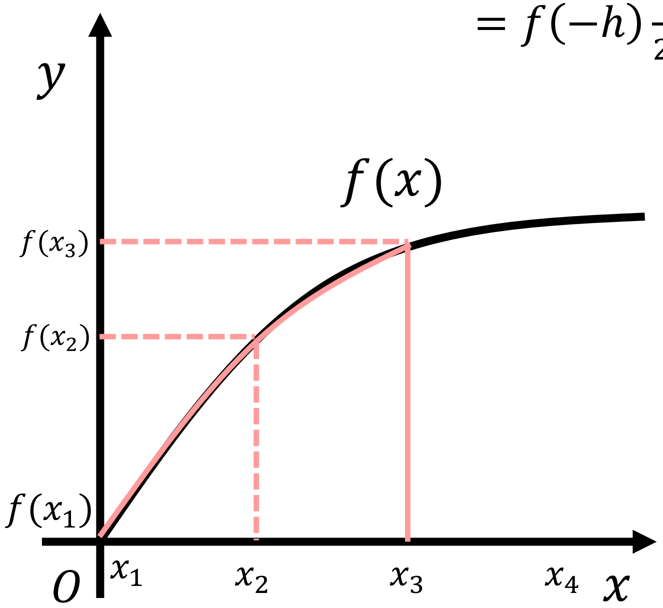
$$= f(-h)\int_{-h}^{h} \frac{x^2-hx}{2h^2}dx + f(0)\int_{-h}^{h} \frac{x^2-h^2}{-h^2}dx + f(h)\int_{-h}^{h} \frac{x^2+hx}{2h^2}dx$$

$$= f(-h)\frac{1}{2h^2}\left[\int_{-h}^{h} x^2\,dx - \int_{-h}^{h} hx\,dx\right] - f(0)\frac{1}{h^2}[\int_{-h}^{h} x^2dx - \int_{-h}^{h} h^2dx] + f(h)\frac{1}{2h^2}[\int_{-h}^{h} x^2dx + \int_{-h}^{h} hxdx]$$

수치 적분

심슨법

기본 개념 학습



$$= f(-h)\frac{1}{2h^2}\left[\int_{-h}^{h} x^2\,dx - \int_{-h}^{h} hx\,dx\right] - f(0)\frac{1}{h^2}\left[\int_{-h}^{h} x^2 dx - \int_{-h}^{h} h^2 dx\right] + f(h)\frac{1}{2h^2}\left[\int_{-h}^{h} x^2 dx + \int_{-h}^{h} hxdx\right]$$

$$= f(-h)\frac{1}{2h^2}\left[\frac{2}{3}h^3 - 0\right] - f(0)\frac{1}{h^2}\left[\frac{2}{3}h^3 - 2h^3\right] + f(h)\frac{1}{2h^2}\left[\frac{2}{3}h^3 + 0\right]$$

$$= f(-h)\frac{h}{3} + f(0)\frac{4h}{3} + f(h)\frac{h}{3} = \frac{h}{3}[f(-h) + 4f(0) + f(h)]$$

$$= f(-h)\frac{h}{3} + f(0)\frac{4h}{3} + f(h)\frac{h}{3} = \frac{h}{3}[f(-h) + 4f(0) + f(h)]$$

$-h, 0, h$를 x_1, x_2, x_3로 복구하면

$$= \frac{(x_3-x_1)}{2}\frac{1}{3}[f(x_1) + 4f(x_2) + f(x_3)]$$

$$\therefore \int_{x_1}^{x_3} P(x)dx = \frac{(x_3 - x_1)}{6}[f(x_1) + 4f(x_2) + f(x_3)]$$

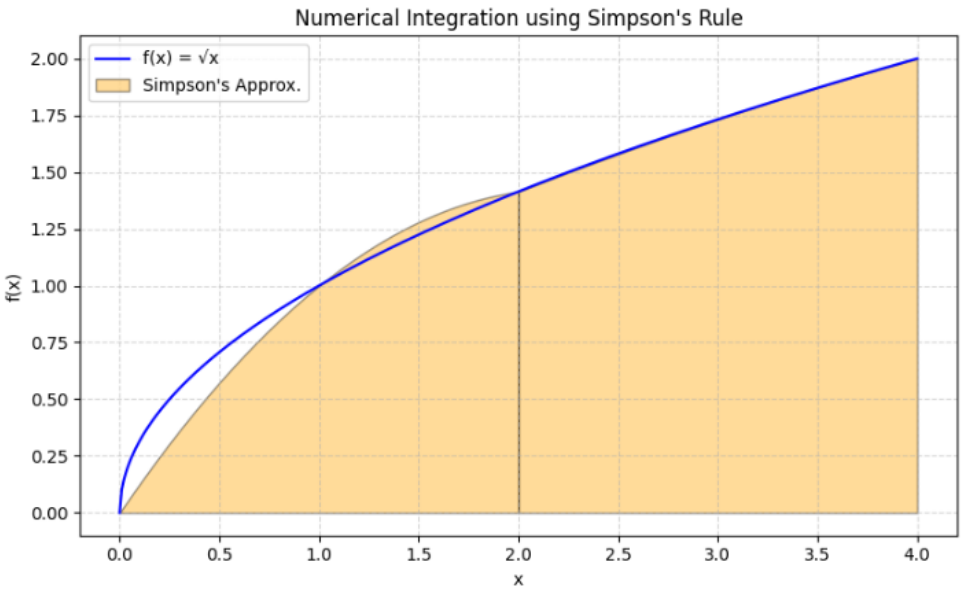
모든 구간에 대해서 세 점을 이용한 $\int_{x_1}^{x_3} P(x)dx$를 계산하여 더하면 된다.

$$\Rightarrow \int_{x_1}^{x_n} f(x)dx \approx \sum_{i=1}^{\frac{m}{2}} \frac{(x_{2i} - x_{2i-2})}{6}[f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})] \quad \because m = \text{구간 수}$$

수치 적분

심슨법

기본 개념 적용



$\int_0^4 \sqrt{x}dx$ 계산하기

1) 정확한 적분 값

$$\int_0^4 \sqrt{x}dx = \int_0^4 x^{\frac{1}{2}}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 = \frac{2}{3}\cdot 4^{\frac{3}{2}} = \frac{16}{3} = 5.3333$$

2) 심슨법 근사 값

$$\int_0^4 \sqrt{x}dx \approx \sum_{i=1}^{\frac{4}{2}} \frac{(x_{2i} - x_{2i-2})}{6}[f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})]$$

$$= 1.8047 + 3.4475 = 5.2522$$

$$Error = \frac{|\text{실제 값} - \text{계산 값}|}{\text{실제 값}} \cdot 100$$

$$Error = \frac{|5.3333 - 5.2522|}{5.3333} \cdot 100 = 1.52\%$$

소스코드 3-5.py, 그래프는 3-6.py

감사합니다

박형묵

명신여자고등학교