파이썬 물리 코딩

Chapter 4. 2차원 운동 - 원

박형묵

명신여자고등학교

Update 20250304

강의 자료 다운로드

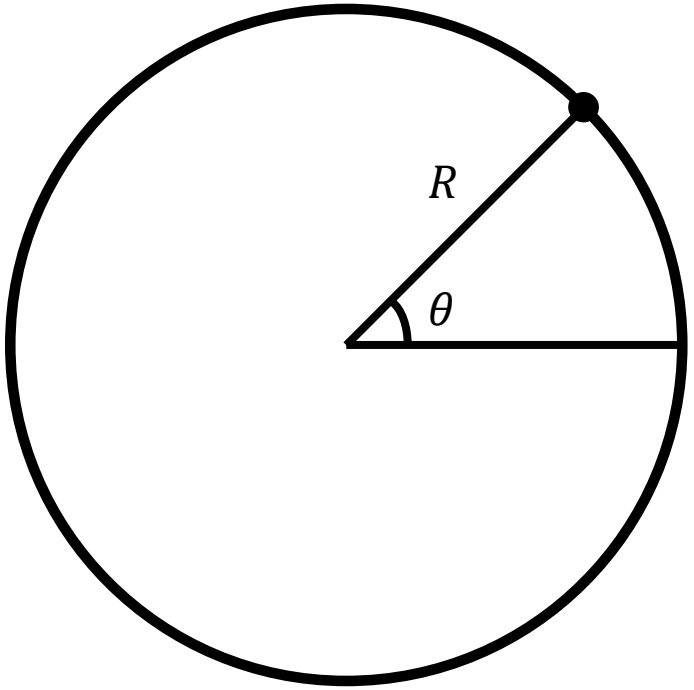
파이썬 물리학 강의 자료

https://github.com/PigeonDove/PythonPhysics

2차원 운동

원 운동

$$(RCos\theta, RSin\theta) = (RCos\omega t, RSin\omega t)$$



θ

$$\frac{d\theta}{dt} = \omega$$

2차원 운동

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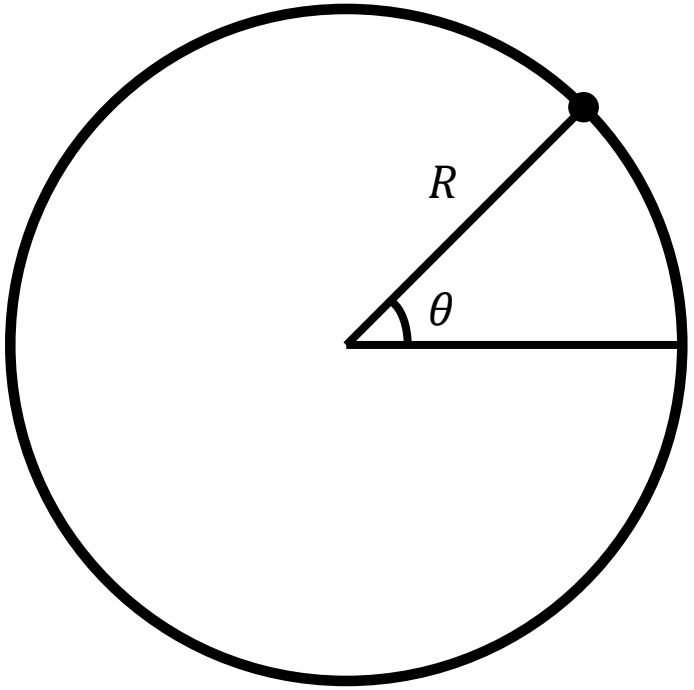

$$\theta$$

$$\frac{d\theta}{dt} = \omega$$

```
Web VPython 3.2


ball = sphere()


radius = 10

omega = pi  # 각속도

t = 0

dt = 0.01 # 시간 간격


while True:

  sleep(dt)

  ball.pos.x = radius * cos(omega * t)

  ball.pos.y = radius * sin(omega * t)

  t = t + dt
```

2차원 운동

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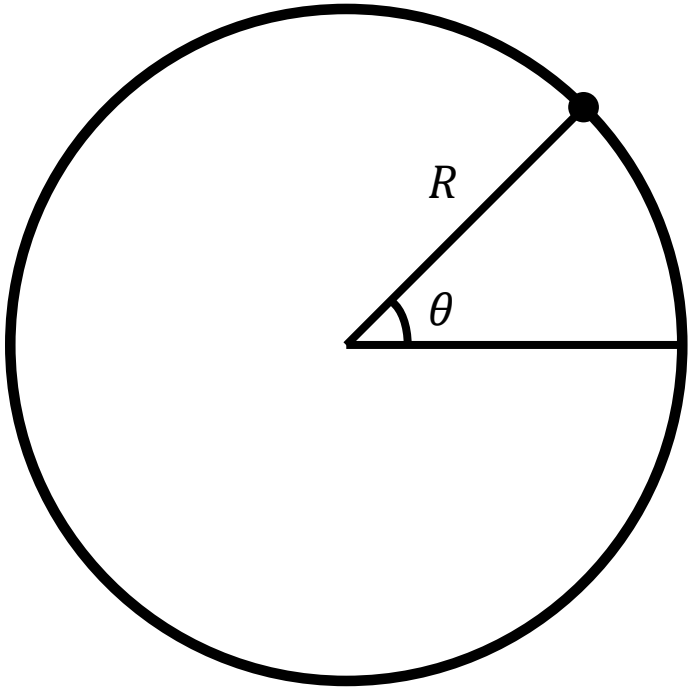

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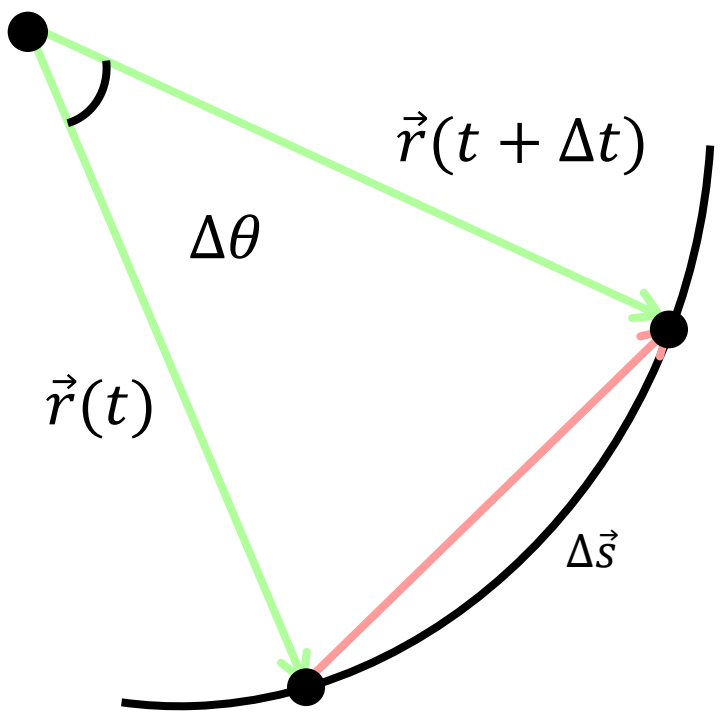
2차원 운동

구심가속도와 접선 가속도

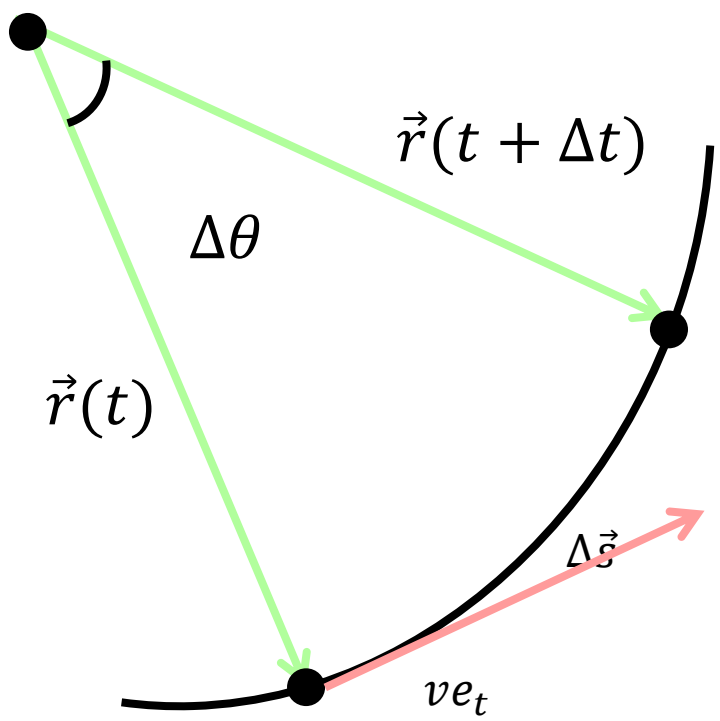


$$\Delta\vec{s} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

$$\lim_{\Delta t\to 0}\frac{\Delta\vec{s}}{\Delta t} = \frac{d\vec{s}}{dt} = \lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t} = \vec{v}$$

2차원 운동

구심가속도와 접선 가속도



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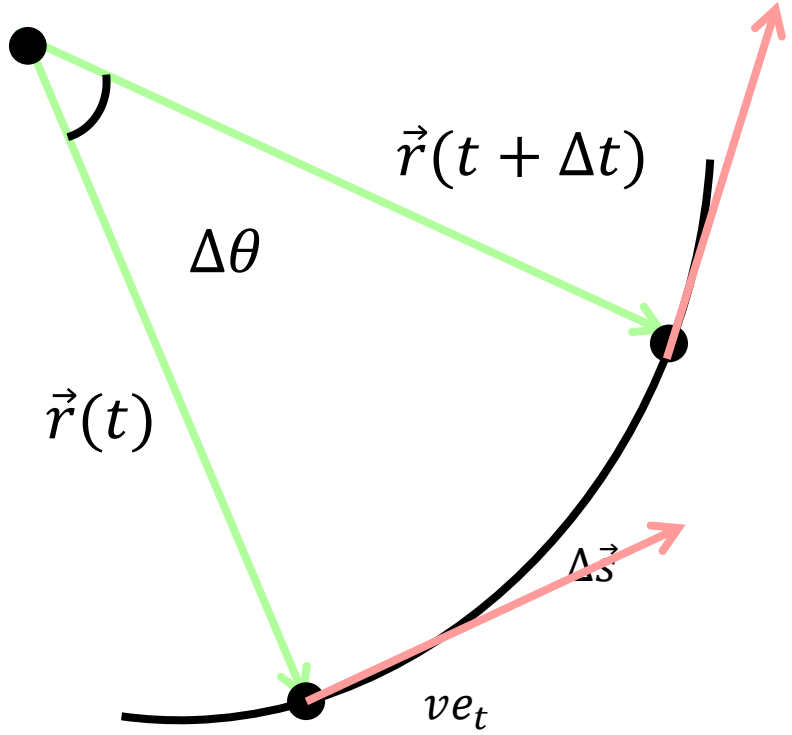
구심가속도와 접선 가속도



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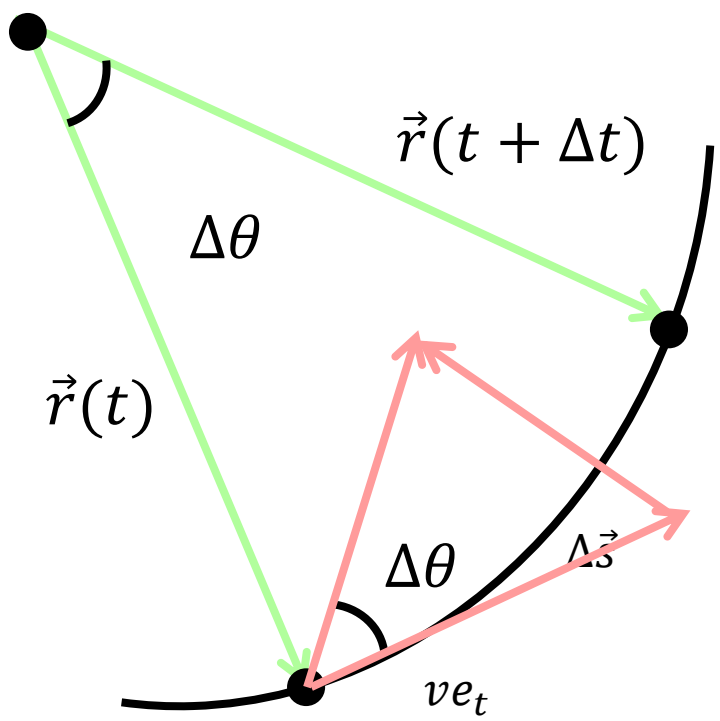
2차원 운동

구심가속도와 접선 가속도



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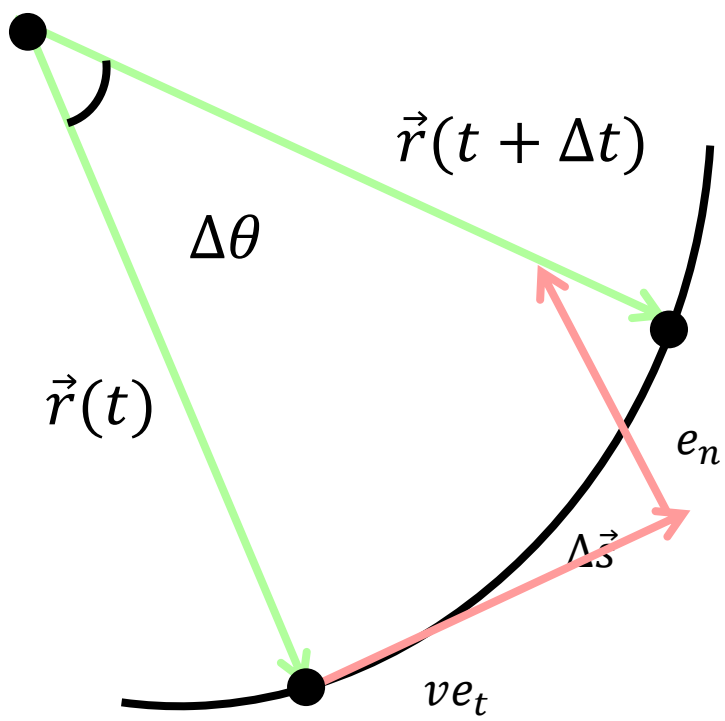
$$\lim_{\Delta t\to 0}\frac{\Delta e_t}{\Delta t} = \frac{de_t}{dt} = \lim_{\Delta t\to 0}\frac{e_t(t+\Delta t)-e_t(t)}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{2Sin(\frac{\Delta\theta}{2})}{\Delta t} = \lim_{\Delta t\to 0}\frac{2Sin(\frac{\Delta\theta}{2})}{\Delta\theta}\cdot\frac{\Delta\theta}{\Delta t} = \lim_{\Delta t\to 0}\frac{Sin(\frac{\Delta\theta}{2})}{\frac{\Delta\theta}{2}}\cdot\frac{\Delta\theta}{\Delta t}$$

2차원 운동

구심가속도와 접선 가속도



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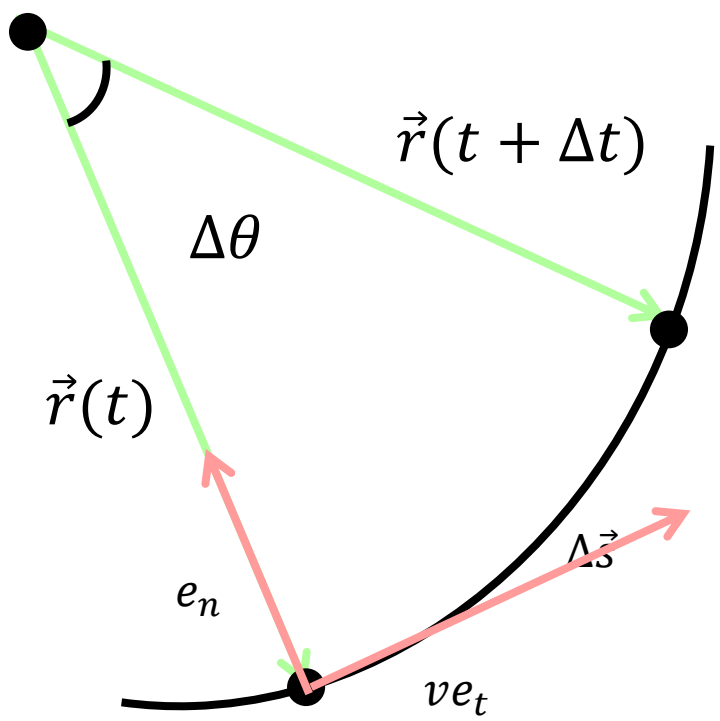
$$= \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}e_n$$

2차원 운동

구심가속도와 접선 가속도



$$\Delta\vec{s} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

$$\lim_{\Delta t\to 0}\frac{\Delta\vec{s}}{\Delta t} = \frac{d\vec{s}}{dt} = \lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t} = ve_t$$

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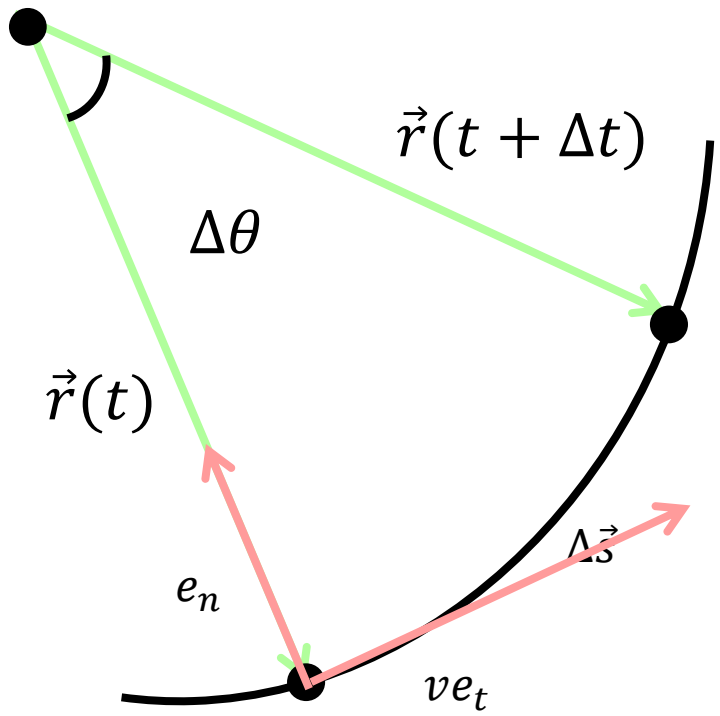
$$\vec{a} = \frac{dv}{dt}e_t + v\frac{d\theta}{dt}e_n$$

2차원 운동

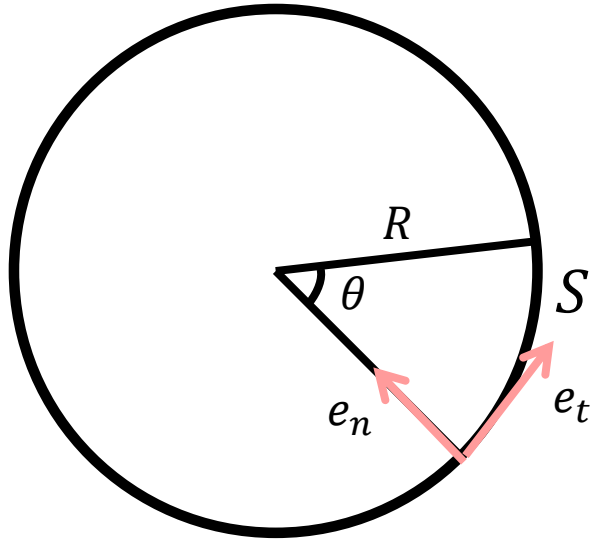
구심가속도와 접선 가속도



$$\Delta\vec{s} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

$$\vec{v} = ve_t = R\omega e_t$$

$$\vec{a} = \frac{dv}{dt}e_t + v\frac{d\theta}{dt}e_n$$



$$S = R\theta$$

$$\Rightarrow \frac{dS}{dt} = \frac{d(R\theta)}{dt} \Rightarrow v = R\frac{d\theta}{dt} \Rightarrow v = R\omega$$

$$a = \frac{dv}{dt} = \frac{d^2(R\theta)}{dt^2} = R\frac{d^2\theta}{dt^2} = R\frac{d\omega}{dt} = R\alpha$$

접선가속도 = $R\alpha$ 접선방향 속도 = $R\omega$

구심가속도 = $\frac{v^2}{R}, R\omega^2$

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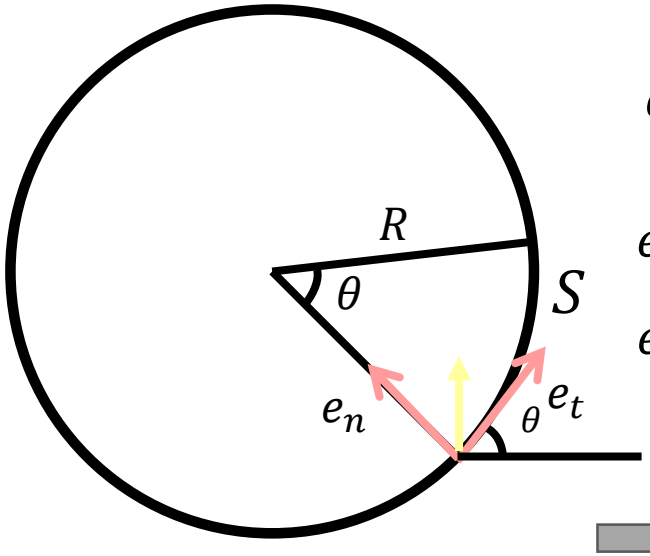

$$\vec{a} = R\alpha e_t + R\omega^2 e_n \qquad |\vec{a}| = \sqrt{(R\alpha)^2 + (R\omega^2)^2}$$

$$e_t = Cos\theta\hat{r}_x + Sin\theta\hat{r}_y \qquad e_n = -Sin\theta\hat{r}_x + Cos\theta\hat{r}_y$$

$$\Rightarrow \quad e_t = Cos\omega t\hat{r}_x + Sin\omega t\hat{r}_y \qquad e_n = -Sin\omega t\hat{r}_x + Cos\omega t\hat{r}_y$$

$$\Rightarrow \quad \vec{a} = R\alpha\left(Cos\omega t\hat{r}_x + Sin\omega t\hat{r}_y\right) + R\omega^2\left(-Sin\omega t\hat{r}_x + Cos\omega t\hat{r}_y\right)$$
$$= R(\alpha - \omega^2)(Cos\omega t\hat{r}_x - Sin\omega t)\hat{r}_x + R(\alpha + \omega^2)(Sin\omega t + Cos\omega t)\hat{r}_y$$

감사합니다

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