

## Maths Exam: Sample Paper

Full marks: 70 marks

1. Let  $f: \mathbb{R} \rightarrow \mathbb{Q}$  be a continuous function.

(a) Prove that  $f$  is a constant function.

(b) Let  $f(x) = \sqrt[3]{2}$ . Prove that  $f(x)$  is an irrational number by using Fermat's Last Theorem and contradiction.

(c) Let  $g: \mathbb{C} \rightarrow \mathbb{C}$  and  $g(x) = \frac{ax+b}{cx+d}$ ,  $bd \neq 0$

Find the necessary and sufficient condition for  $g(x) = f(x)$

(d) Can  $f(x)$  be an odd function? Explain graphically,

(e) Prove that  $f(x)$  is Riemann integrable.

(f) Prove that  $f(x)$  is measurable.

(g) Prove that  $f(x)$  between topological spaces is continuous.

2. Let  $f: [0, 1] \rightarrow \mathbb{R}$  is a continuous function. Prove that

$$\lim_{n \rightarrow \infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} dx = \frac{\pi}{2} f(0)$$

3. Factorize

(a)  $x^{13} + x^{11} + 1$

(b)  $x^5 + x^4 + 1$

4. Consider  $f(x) = \cosh(x)x^2$

(a) Find the real root of  $f(x) = 10$  and correct your answer to 5 significant figures.

(b) Find the Taylor's series of  $f(x)$  at  $x = 0$ .