Predator-prey Simulation

Contents

1 Introduction

In this assessment you are given a program that implements a scientific algorithm. This document explains the background of the algorithm and the supplied implementation. Details of the assessment requirements will be provided separately.

2 Modelling foxes and field mice in a landscape

The program models the interaction of foxes (predator) and field mice (prey) in a two-dimensional landscape. Foxes eat field mice. Without field mice, foxes decline and die out through lack of food. Without foxes, field mice breed and increase in numbers forever. Both animals tend to spread out (or diffuse) to fill parts of the landscape where there are fewer of their own kind. Their landscape contains areas of water where neither foxes nor field mice can live.

Partial differential equations can be used to model the behaviour of foxes and field mice within a landscape:

$$\frac{\partial M}{\partial t} = rM - aMF + k\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right)$$

$$\frac{\partial F}{\partial t} = bMF - mF + l\left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2}\right)$$

where:

- `M` is the density of field mice (prey).
- `F` is the density of foxes (predators).
- `r` is the birth rate of field mice.
- `a` is the predation rate at which foxes eat field mice.
- `b` is the birth rate of foxes per one field mouse eaten.
- `m` is the fox mortality rate.
- `k` is the diffusion rates for field mice.
- `l` is the diffusion rates for foxes.

Do not worry if you do not understand these equations, as discrete versions that are used in the program will be introduced shortly. What the equations say is that:

- The change in field mice over time is affected by the birth rate of field mice, the rate at which field mice are killed by foxes and the diffusion of field mice throughout the landscape.
- The change in foxes over time is affected by the birth rate of foxes, which also depends on the field mice available as food, the natural death rate of foxes, and the diffusion of foxes throughout the landscape.

2.1 Numerical approximation

The mathematical model above treats time and space as continuous quantities, but continuous quantities cannot be handled directly by a computer. We need to use a discrete approximation. To do this, we can divide space and time up into discrete units. The landscape can be modelled as a rectangular grid, of dimension $N_x \times N_y$, where N_x is the number of columns and N_y is the number of rows, and where each grid square can be either land or water. Within each grid square there will be field mice, M_{xy}, and foxes, F_{xy}, living in it, where M_{xy} and F_{xy} are >= 0.

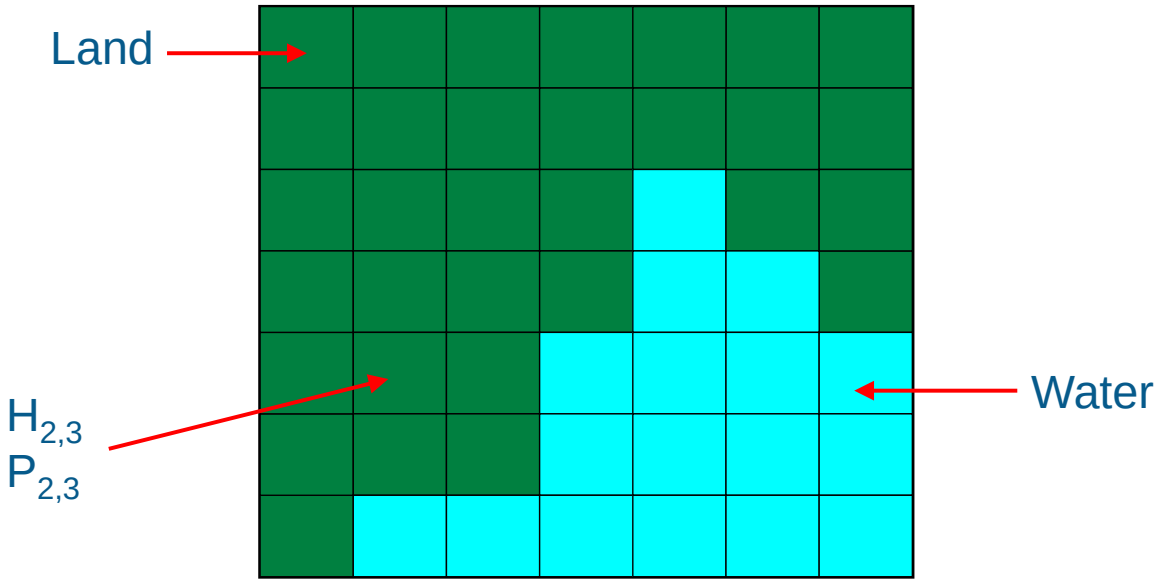

Figure 1: 2-dimensional landscape grid. Squares are indexed by column then by row from the bottom-left.

Time can be modelled by successive, discrete, time-steps, of size Δt. The numbers of field mice and foxes in the landscape at next time step depend on the numbers present in the current time-step. So, the new number of field mice, M, at time $t + \Delta t$, in a specific grid square, is given by:

$$M_{ij}^{new} = M_{ij}^{old} + \Delta t\left(rM_{ij}^{old} - aM_{ij}^{old}F_{ij}^{old} + k\left(\left(M_{i-1j}^{old} + M_{i+1j}^{old} + M_{ij-1}^{old} + M_{ij+1}^{old}\right) - N_{ij}M_{ij}^{old}\right)\right)$$

The number of field mice in a grid square at the next time step equals the number of field mice there at present adjusted by the change in population of field mice over the timestep. The population change is a combination of the number of field mice born in that square (where r is the field mouse birth rate), the number of field mice killed in that square (which depends on number of foxes in the square, where a is the fox predation rate) and the number of field mice that move into or out of that square from the neighbours (where k is the diffusion/migration rate).

N_{ij} is the number of "dry" (i.e., non-water) grid squares out of the four squares neighbouring the square (*i, j*) (to the north, south, east, and west). In our approximation, we assume that field mice cannot move diagonally, nor can they swim.

The new number of foxes, *F*, at time $t + \Delta t$, in a specific grid square, is given by:

$$F_{ij}^{new} = F_{ij}^{old} + \Delta t\left(bM_{ij}^{old}F_{ij}^{old} - mF_{ij}^{old} + l\left(\left(F_{i-1j}^{old} + F_{i+1j}^{old} + F_{ij-1}^{old} + F_{ij+1}^{old}\right) - N_{ij}F_{ij}^{old}\right)\right)$$

The number of foxes in a grid square at the next time step equals the number of foxes there at present adjusted by the change in population of foxes over the timestep. The population change is a combination of the number of foxes that die in that square (where *m* is the fox mortality rate), the number of foxes born in that square (which depends on number of field mice in the square where *b* is the fox birth rate) and the number of foxes that move into or out of that square from the neighbours (where *l* is the diffusion/migration rate). Foxes, like field mice, cannot move diagonally, nor can they swim.

We can assume that the distance between grid squares is 1 (i.e., $\Delta x = \Delta y = 1$), as scaling can be absorbed into the diffusion terms, *k* and *l.*

2.2 More on the birth and death of field mice and foxes

Let us look at the birth of field mice:

$$M_{ij}^{old} + \Delta t\left(rM_{ij}^{old} \ldots\right)$$

As an example, let us take the basic unit of time as 1 month, and let's assume that every pair of field mice breeds 3 times a year (once every 4 months) and each produces a group (called mischief or nest) of 5 baby mice (called pups or pinkies). So, in 4 months, 2 field mice produce 5 more field mice, so each field mouse produces 0.625 field mice a month, so the field mouse birth rate, *r*, is 0.625. To check that this is correct, let's substitute into our equation:

$$M_{ij}^{new} = 2 + 4(0.625 \times 2)$$

Which equals 7 (the 2 parents and the 5 offspring).

As we could have thousands of field mice breeding continuously, we can use a much smaller value of Δt. A value for *r* can depend upon the units chosen (e.g. months, days etc). Likewise, it can improve the scalability of our model if, instead of recording the field mouse, and fox, populations as integers, we record them as real numbers in terms of densities per grid square (e.g. 3.53 field mice per hectare).

Looking at the death of field mice:

$$M_{ij}^{old} + \Delta t\left(rM_{ij}^{old} - aM_{ij}^{old}F_{ij}^{old} \ldots\right)$$

Field mice only die through being eaten by foxes, not naturally, so more foxes mean more field mice are killed. More field mice also mean more are killed as they are easier to find.

Looking at the birth of foxes:

$$F_{ij}^{old} + \Delta t\left(bM_{ij}^{old}F_{ij}^{old} \ldots\right)$$

Unlike field mice, the birth rate of foxes (*b*) depends not only on the number of their own kind but also on their available food, the field mice.

Looking at the death of foxes:

$$F_{ij}^{old} + \Delta t\left(bM_{ij}^{old}F_{ij}^{old} - mF_{ij}^{old} \ldots\right)$$

Foxes differ slightly in that, unlike field mice, they have no predators, so die at a natural rate (*m*).

2.3 More on movement of field mice and foxes across the landscape

We could model a single population over the whole landscape, but this is not very realistic. We want a finer-grained model which allows for different densities of foxes and field mice at different points in the landscape. Our model will be more realistic if we model how animals move around the landscape. For field mice, we have a diffusion term, *k*:

$$k\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right)$$

In the discrete form of our equation, this is represented as a "random walk", defined as:

$$k\left(\left(M_{i-1j}^{old} + M_{i+1j}^{old} + M_{ij-1}^{old} + M_{ij+1}^{old}\right) - N_{ij}M_{ij}^{old}\right)$$

The number of field mice in surrounding squares is summed. From this number is subtracted the number of field mice in the current grid square multiplied by the number of surrounding land-only grid squares (N_{ij}). The total value is then multiplied by a field mouse diffusion term, *k*.

For foxes, a similar calculation is used, with a fox diffusion term, *l*.

3 A predator-prey simulation

Implementations of the simulation in multiple languages are available online at the same location as this handout. The program implements the above approach to simulating predators and prey in a landscape.

The program is in files called:

- `predator-prey-python.zip`
- `predator-prey-python.tar.gz` (same contents as above in a different format)
- `predator-prey-c.zip`
- `predator-prey-c.tar.gz` (same contents as above in a different format)

Download one of these files and unpack it.

Unpacking the bundle will produce a directory called `predator-prey-python` or `predator-prey-c` as appropriate.

3.1 Running the program

The `README.md` file within the above directory explains how to compile (if necessary) and run the program.

3.2 What the program does

The program loads a landscape file, initialises the field mouse and fox densities, then runs the simulation for a discrete number of timesteps. At certain intervals, the simulation outputs the average density of field mice and foxes across the land-only squares (these are both printed and appended to a file of comma-separated values) and also outputs a "Plain PPM" image file representing the density of field mouse and foxes at that point in time (for information on the PPM file format, run man ppm or see http://netpbm.sourceforge.net/doc/ppm.html).

3.2.1 Using seeds to initialise field mouse and fox densities

Seeds are used in random number generators to ensure that the same sequence of random numbers is created each time a component is run. It can be very useful for testing software that includes random numbers if we can generate the same sequence of random numbers on repeated runs. Seeds give us this repeatability as they allow us to control the randomness – if we want to change the randomness, we change the seed.

In the simulation, a field mouse seed parameter is used to configure Python's random number generator prior to generating random values for the initial field mouse density on each land square. Two runs of the simulation on the same landscape with the same field

mouse seed value will produce exactly the same set of initial field mouse densities on the same land squares in both runs. If the user wants a different random assignment of field mouse densities, then they can change the value of the seed. Similarly, a fox seed parameter is used for generating random values for the initial fox density on each land square.

The simulation has logic to implement a special case: if the field mouse seed is 0 then the random value generator is not used and, instead, a density of zero field mice is used across all land squares. Similarly, if the fox seed is 0, then a density of zero foxes is used across all land squares.

3.2.2 Using haloes to avoid boundary checks

The simulation updates each square using the values of its four neighbours (to the north, south, east, and west). What about squares at the edges of the grid? In general, we do not want to write a lot of code to handle these as a special case, so a common solution to this problem is to define a "halo" around the grid. So, to run our simulation for an L × L grid we use a (L+2) × (L+2) grid and set the halo squares to be empty (i.e. water). On these halo squares $M = F = 0$.

When the internal arrays representing the landscape are created by the code, this halo of “water” squares is added around all four edges of the landscape. The halo is not output into the “Plain PPM” image files as its use is an internal implementation detail.