Priority Queues

Computing 2 COMP1927 13s2

Priority

- Some applications of queues require items processed in order of "key" or priority
- rather than in order of entry (FIFO)
- Priority Queues (PQueues or PQs) provide this via:
  - insert item into PQ
  - leave: remove item with highest priority key
    - Highest priority key may be one with smallest or largest value depending on the application
- Plus generic ADT operations: new, drop, empty, ...

Priority Queues

- Characteristic priority queue (PQ) operations:
  - insert item
  - delete item with highest priority key
    - Highest priority key may be the smallest or largest key depending on the application
- Along with generic ADT operations, such as:
  - create new queue
  - test if queue is empty
  - destroy existing queue (reclaim memory)

PRIORITY QUEUE INTERFACE

```
typedef struct priQ * PriQ;
//We assume we have a more complex Item type that has
//a key and a value, where the key is the priority and the
//value is the data being stored

// Core operations
PriQ initPriQ(void);
void insert(PriQ q, Item i);
//retrieve and delete Item with highest priority
Item delete(PriQ q);

// Useful operations
int sizePriQ(PriQ q);
void changePriority(PriQ q, Key k, Item i);
void deleteKey(PriQ q, Key k);
int maxSize(PriQ q);
```

Heap Sort : Sorting by PQs

- PQs do partial sorting. Can we use them for full-sorting?
- Two-stage process:
  - put all elements from array into priority queue
  - one-at-a-time, delete maximum and add to sorted array
- Since each PQ operation (when implemented via a heap) is $log_2 n$
  - overall cost is $O(nlog_2 n)$

Comparison of Possible Implementations

Implementation	**insert**	**delete**
ordered array/list	O(N)	O(1)
unordered array/list	O(1)	O(N)

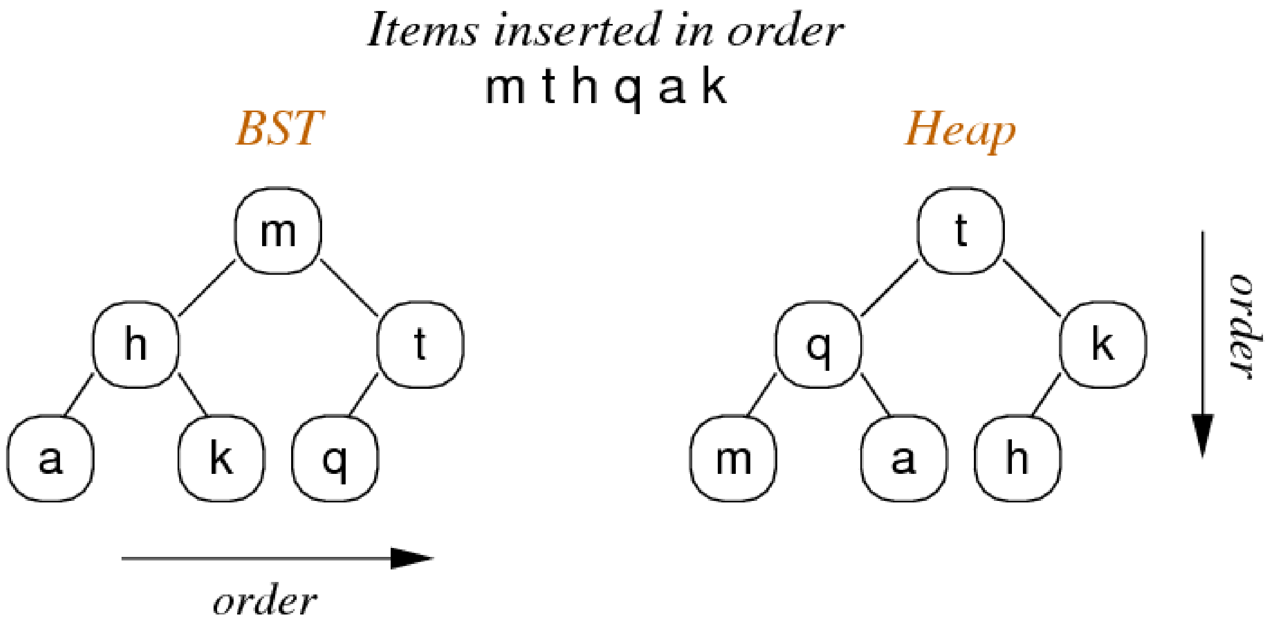
- Can we implement BOTH operations efficiently?
  - Heap
    - O(log N) for insert and delete

Priority Queue Sort Algorithm

```
void pqSort(Item a[], int lo, int hi) {
    int i;
    PriQ q = initPriQ();
    // put items into PQ
    for (i = lo; i <= hi; i++){
        insert(q, a[i]);
    }
    // remove items from PQ
    for (i = hi; i >= lo; i--){
        a[i] = delete(q);
    }
}
```

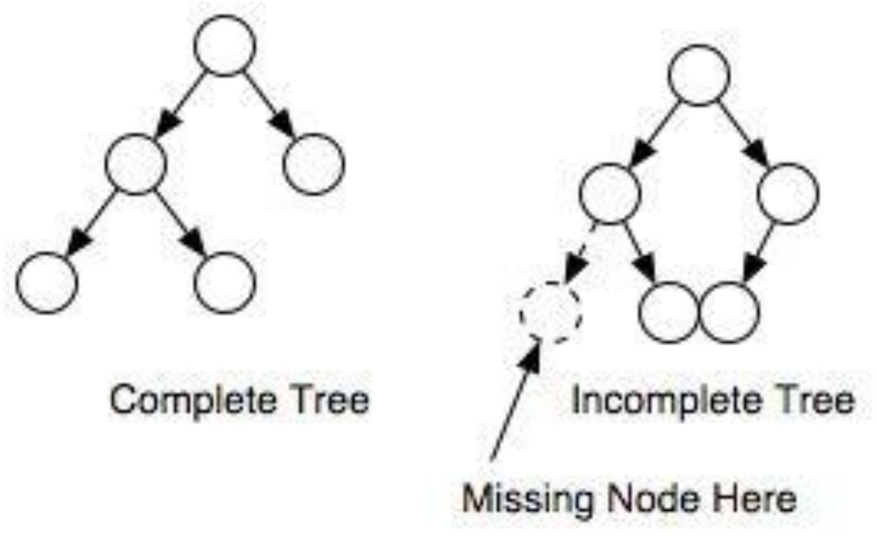
HEAPS

- Heaps can be viewed as trees with top-to-bottom ordering
  - Heap-ordered trees
    - for all keys both subtrees are ≤ root
    - property applies to all nodes in tree (i.e. root contains largest value in that subtree)



Complete Tree Property

- Heaps are "complete trees“
  - every level is filled in before adding a node to the next level
  - the nodes in a given level are filled in from left to right, with no breaks.

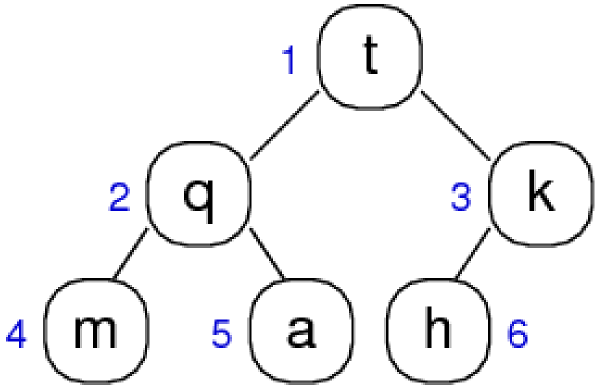


Heap Implementations

- BSTs are typically implemented as linked data structures
- Heaps CAN be implemented as linked data structures
  - Heaps are TYPICALLY implemented via arrays.
  - The property of being **complete** makes array implementations suitable

Array based heap implementation

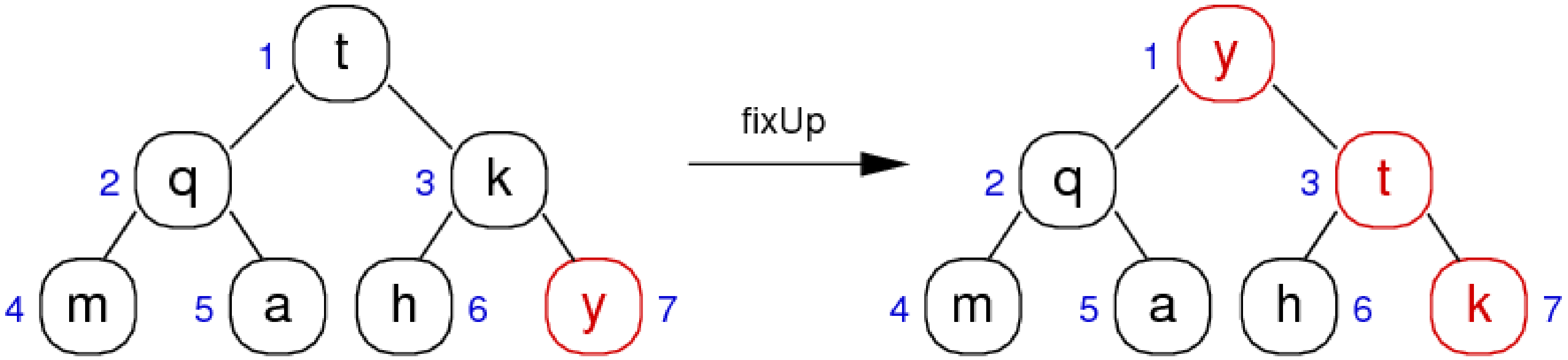
- Simple index calculations allow navigation through the tree:
  - left child of node at index i is located at 2i
  - right child of node at index i is located at 2i+1
  - parent of node at index i is located at i/2



0	1	2	3	4	5	6
	t	q	k	m	a	h

Heap Insertion

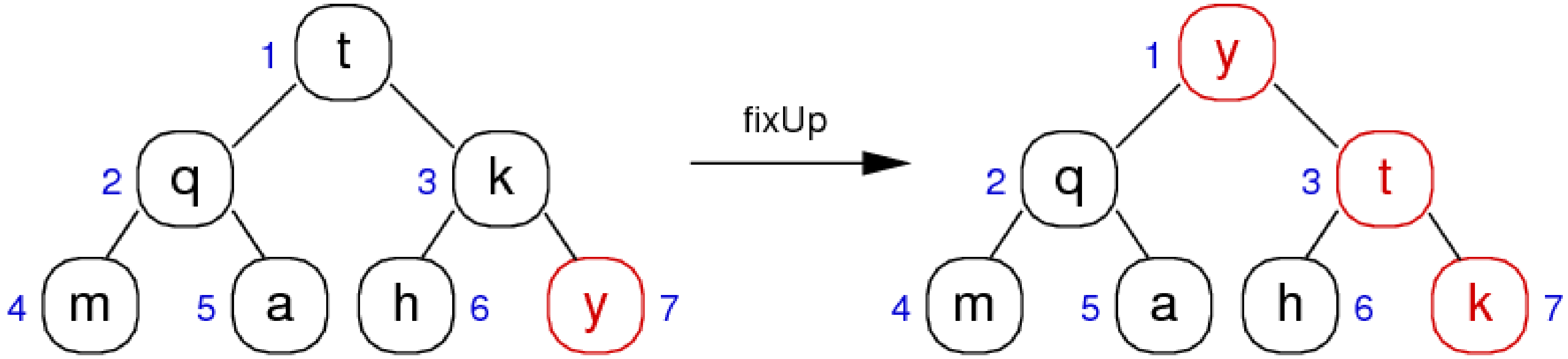
- Insertion is a two-step process
  - add new element at bottom-most, rightmost position
  - reorganise values along path to root to restore heap property



HEAP INSERTION FIX-UP CODE

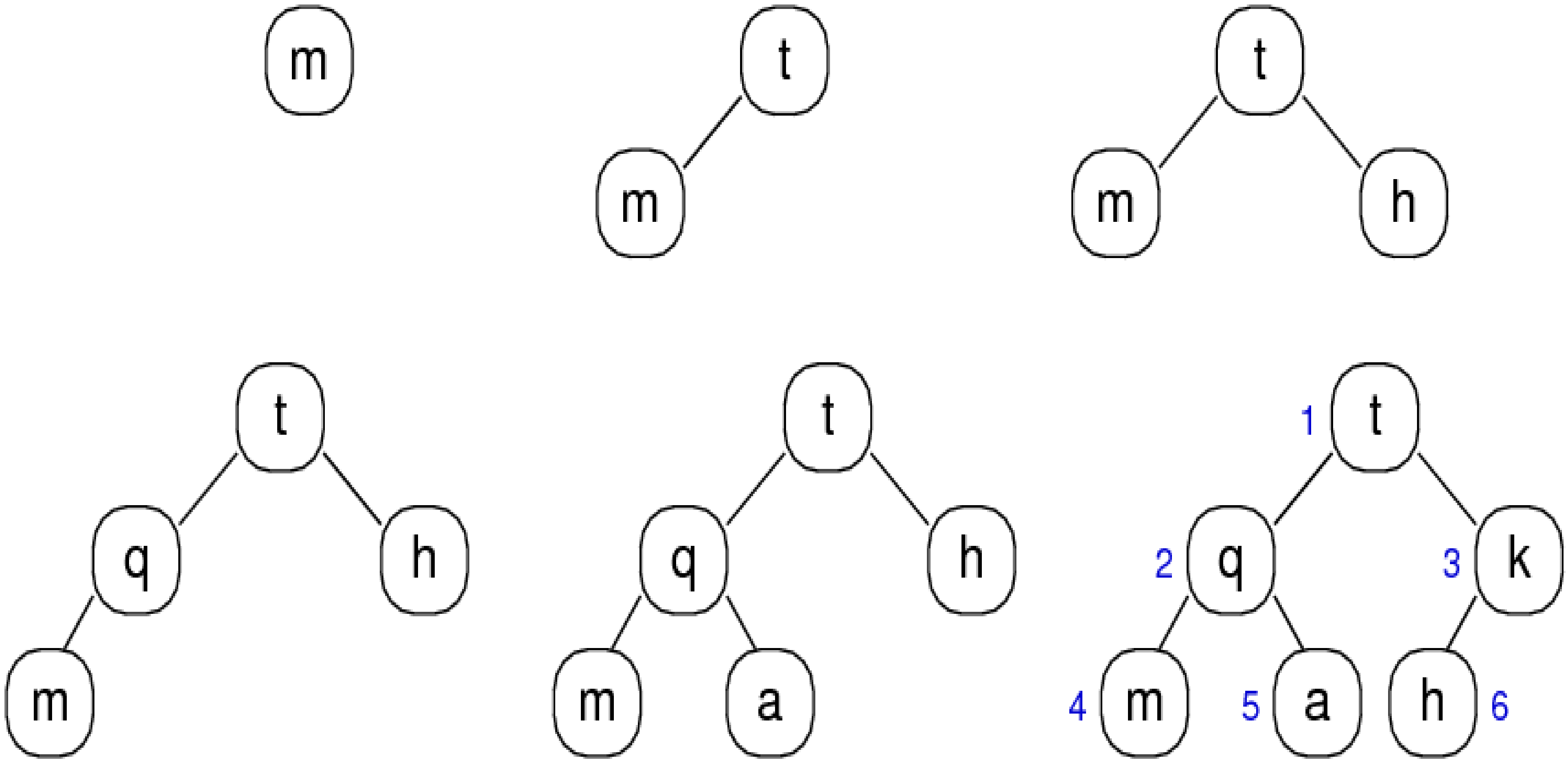
- Bottom-up heapify:

```
// force value at a[k] into correct position
void fixUp(Item a[], int k) {
    while (k > 1 && less(a[k/2],a[k])) {
        swap(a, k, k/2);
        k = k/2; // integer division
    }
}
```



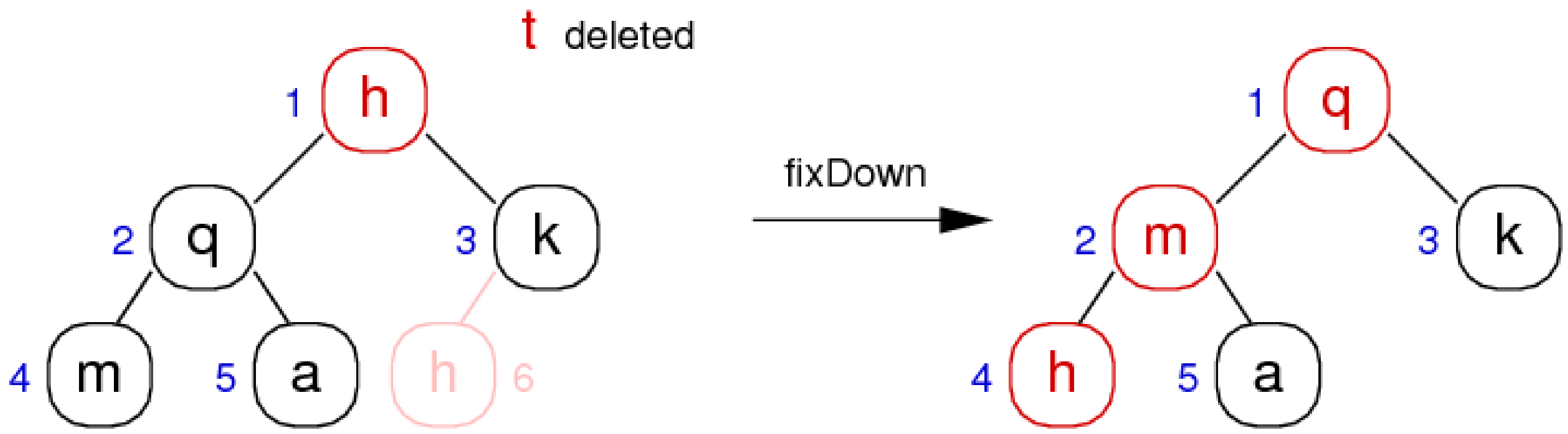
Heap Insertion

Items inserted in order m t h q a k



Deletion with Heaps

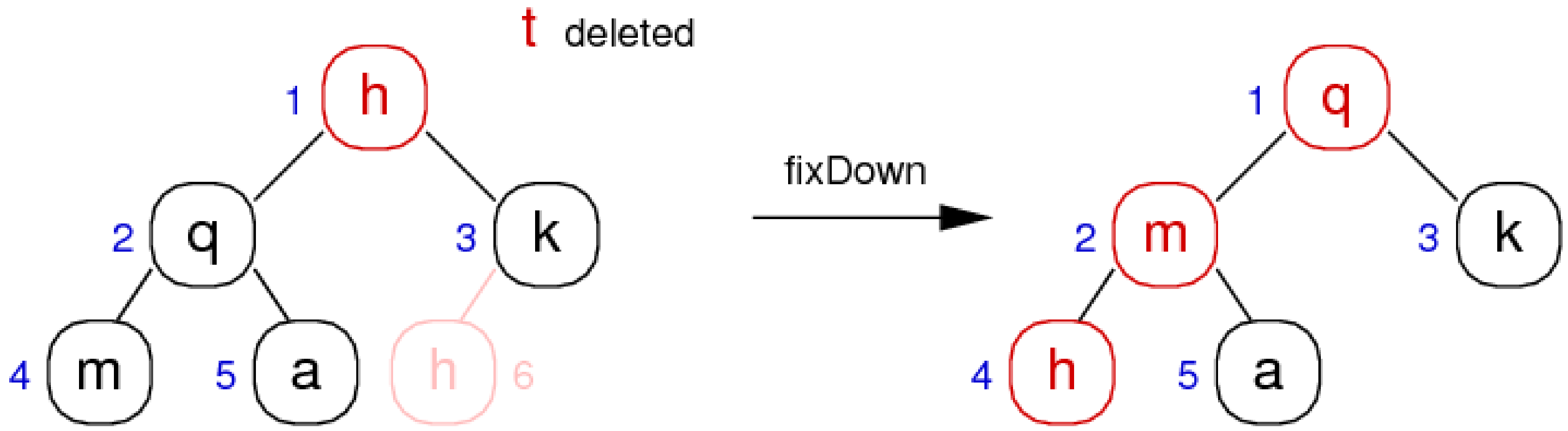
- Deletion is a three-step process
  - replace root value by bottom-most, rightmost value
  - remove bottom-most, rightmost value
  - reorganise values along path from root to restore heap



Heap deletion Fix-down code

- Top-down heapify:

```
void fixDown(Item a[], int k) {
   int done = 0;
   while (2*k <= N && !done) {
       int j = 2*k; //choose larger of two children
       if (j < N && less(a[j], a[j+1])){
           j++;
       }
       if (!less(a[k], a[j])){
           done =1;
       }else{
           swap(a, k, j);
           k = j;
       }
   }
}
```



EXERCISE:

- Show the construction of the heap produced by inserting
  - H E A P S F U N
- Show the heap after an item is deleted.
- Show the heap after another item is deleted.