ST 512 Homework 2 Key

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Problem 9.10. Solution: When the number of contrasts to be estimated is large, Bonferroni could be too conservative, leading to a high rate of false negative (type II errors). Although this error is not usually considered the worst, we still do not want to make it.

Problem 9.13. Solution: Consider the one-way ANOVA (means) model:

$$Y_{ij} = \mu_i + \epsilon_{ij}$$

with $i = 1, \ldots, 5, j = 1, \ldots, 10$, where for $i = 1, \ldots, 4$
μ_i represents the true weight loss mean of the i-th agent
μ_5 represents true weight loss mean of the standard agent
ϵ_{ij} are **assumed** to be identically independent distributed as normal distribution with common variance σ^2.

Alternatively, effect model can be written as follows:

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

with $i = 1, \ldots, 5, j = 1, \ldots, 10$, where
μ represents the true population mean
α_i represents the true weight loss effect of the i-th agent for $i = 1, \ldots, 4$ and weight loss effect of the standard agent for $i = 5$
ϵ_{ij} are **assumed** to be identically independent distributed as normal distribution with common variance σ^2.

Construct the relevant hypothesis test: H_0 : All weight loss means are equal among the five agents

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_a : At least one weight loss means differ from the other (H_a : At least one equal sign does not hold).
Equivalently, for effect model,

$$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$$

H_a : At least one weight loss effects not equal to 0.

From the SAS output, our test statistic is $F = 15.68$ with p-value $= 0.0001$ which is less than 0.05. Hence there is enough evidence to reject the null. At the 5% significance level, there is enough evidence to conclude that at least one weight reducing agent's mean differs.

Problem 9.14. Solution:

(a) The significant different pairs of means under Fisher's LSD procedure: 4 vs 2, 4 vs 3, 4 vs S, 1 vs 2, 1 vs 3, 1 vs S, 2 vs S, and 3 vs S;

(b) Under Tukey's procedure: 4 vs 3, 4 vs S, 1 vs 3, 1 vs S and 2 vs S are significant different.

Problem 9.17. Solution:

(a) $\theta_a = \mu_5 - \frac{1}{4}(\mu_1 + \mu_2 + \mu_3 + \mu_4)$

(b) $\theta_b = \frac{1}{2}(\mu_1 + \mu_3) - \frac{1}{2}(\mu_2 + \mu_4)$

(c) $\theta_c = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{2}(\mu_3 + \mu_4)$

(d) $\theta_d = \frac{1}{2}(\mu_1 + \mu_3) - \mu_5$

Problem 9.18. Solution: We have $k = 4$, $\alpha' = 0.05/k = 0.0125$ $t_{\alpha'/2,45} = 2.60$ and $MS(E) = 0.982$, hence the margin of error of Bonferroni correction for each one of the four contrasts is MOE $= t_{\alpha'/2,45}\sqrt{MS(E)\sum_{i=1}^{5}\frac{a_i^2}{n_i}}$.

(a) $\hat{\theta}_a = -2.125$, $\sum_{i=1}^{5}\frac{a_i^2}{n_i} = \frac{1}{10}(1 + 4 \cdot (\frac{1}{4})^2) = 0.125$ and margin of error MOR $= 0.91$. Since $|\hat{\theta}_a|$ is larger than the margin of error, there is enough evidence to reject that the weight loss means of the standard agent and the average of the four new agents are equal. We may conclude that the average of the new weight-reducing agents is significantly different from that of the standard agent.
Also, the (simultaneous with the other estimates) 95% CI is

$$-2.125 \pm 0.91 = (-3.035, -1.215)$$

and does not contain 0.

(b) $\hat{\theta}_b = -0.47$, $\sum_{i=1}^{5}\frac{a_i^2}{n_i} = \frac{1}{10}(4 \cdot (\frac{1}{2})^2) = 0.1$ and margin of error MOR $= 0.82$. Since $|\hat{\theta}_b|$ is smaller than the margin of error, there is not enough evidence to reject that the weight loss means of the agents with and without counseling are equal. We may conclude with or without counseling has no significant difference on average weight loss.
Also, the (simultaneous with the other estimates) 95% CI is

$$-0.47 \pm 0.82 = (-1.56, 0.08)$$

and does contain 0.

(c) $\hat{\theta}_c = 0.28$, $\sum_{i=1}^{5}\frac{a_i^2}{n_i} = \frac{1}{10}(4 \cdot (\frac{1}{2})^2) = 0.1$ and margin of error MOR $= 0.82$. Since $|\hat{\theta}_c|$ is smaller than the margin of error, there is not enough evidence to reject that the weight loss means of agents with and without exercise are equal. We may conclude with or without exercise has no significant difference on average weight loss.
Also, the (simultaneous with the other estimates) 95% CI is

$$0.28 \pm 0.82 = (-0.51, 1.1)$$

and does contain 0.

(d) $\hat{\theta}_d = 1.89$, $\sum_{i=1}^{5} \frac{a_i^2}{n_i} = \frac{1}{10}(2 \cdot (\frac{1}{2})^2 + 1) = 0.15$ and margin of error MOR $= 0.10$. Since $|\hat{\theta}_d|$ is larger than the margin of error, there is enough evidence to reject that the weight loss means of the standard agent and the average of the agents with counseling are equal. We may conclude agent with counseling on average is significantly different than the standard agent.
Also, the (simultaneous with the other estimates) 95% CI is

$$1.89 \pm 0.10 = (1.79, 1.99)$$

and does not contain 0.