Recap

Example 9.5

- Two-factor, random effects, nested design
- Inference:
  1. Individual F tests, if necessary
  2. Estimate variance components
  3. Estimate response correlations
  4. Estimate mean response
- Plan:
  1. Fit model
  2. Check assumptions
  3. Determine correct tests
  4. Carry out inference procedures
  5. Make appropriate contextual conclusions

F Tests!

Example 9.5

- **Plant Effect:** $H_0 : \sigma^2_{Plant} = 0$ vs. $H_1 : \sigma^2_{Plant} > 0$
  $F = 4.88$ and $p = 0.0324$
- **Leaf(Plant) Effect:** $H_0 : \sigma^2_{Leaf(Plant)} = 0$ vs. $H_1 : \sigma^2_{Leaf(Plant)} > 0$
  $F = 185.39$ and $p < 0.0001$
- **Conclusion:** There is sufficient evidence at the 5% level that there is an effect on the variance in plant acid both from the randomly selected plants, and from the randomly selected leaves within each plant.

Estimating Variance Components

Example 9.5

- From our SAS output we know $MSA = 114.392963$, $MSB(A) = 23.431667$, and $MSE = 0.126389$.
- Estimating via our Type III EMS (A = Plant, B = Leaf) we have

$$
\begin{aligned}
MSA &= \hat{\sigma}^2 + 9\sigma_A^2 + 3\sigma_{B(A)}^2 \\
MSB(A) &= \hat{\sigma}^2 + 3\hat{\sigma}_{B(A)}^2 \\
MSE &= \hat{\sigma}^2
\end{aligned}
$$

- Substitution yields

$$
\begin{array}{cccccccc}
\hat{\sigma}^2 & = & MSE & = & & = & 0.1264 \\
\hat{\sigma}_{B(A)}^2 & = & \frac{MSB(A)-MSE}{n} & = & \frac{23.4317-0.1264}{3} & = & 0.7.8 \\
\hat{\sigma}_A^2 & = & \frac{MSA-MSB(A)}{nb} & = & \frac{114.3930-23.4317}{12} & = & 10.1
\end{array}
$$

Or just use PROC MIXED!

Implied Correlations

- Recall that $y_{(i)jk} = \mu + \alpha_i + \beta_{(i)j} + \epsilon_{(i)jk}$
- Also recall that
    - $\alpha_i \overset{iid}{\sim} N\left(0, \sigma_A^2\right)$
    - $\beta_{(i)j} \overset{iid}{\sim} N\left(0, \sigma_{B(A)}^2\right)$
    - $\epsilon_{(i)jk} \overset{iid}{\sim} N\left(0, \sigma^2\right)$
    - and that each random effect is independent of other random effects
- Recall the total variance for any observation is $\sigma_y^2 = \sigma_A^2 + \sigma_{B(A)}^2 + \sigma^2$

Correlations

- What about same leaves of the same plant?

$$\frac{\sigma_A^2 + \sigma_{B(A)}^2}{\sigma_A^2 + \sigma_{B(A)}^2 + \sigma^2}$$

- What about different leaves on the same plant?

$$\frac{\sigma_A^2}{\sigma_A^2 + \sigma_{B(A)}^2 + \sigma^2}$$

- What about observations from different plants?

$$Corr = 0$$

- **Conclusion:** Observations taken from within the same leaf are more correlated than observations within a plant (but from different leaves) which are more correlated than observations from different plants.

Estimating and Using Correlations

- For the Plant Acid Example this yields:
  - **Same Leaf, Same Plant:** $\frac{10.1+7.8}{10.1+7.8+0.13} = \frac{17.9}{18.0} = 0.99$
  - **Different Leaf, Same Plant:** $\frac{10.1}{10.1+7.8+0.13} = \frac{10.1}{18.0} = 0.56$
- Observations extremely correlated - almost all variation in plant acid is explained by plant and leaf(plant) effects.
- What would the correlation (or covariance) matrix of $\mathbf{y}$ look like?

Estimating the Mean Response

- From SAS we get our estimated mean acid concentration to be 14.2611
- Also from SAS our CI is (8.5882, 19.9341)
- We are 95% confident that the true mean concentration of plant acid for all plants is between 8.6 and 19.9.

Summary: Chicken Processing Plant Example

- Significant plant-to-plant and leaf-to-leaf variability
- Extremely correlated responses
- Hey, no Satterthwaite this time!