

ST 512 HW 3 Solution

Cai Li and Qian Guan

Spring 2016

14.8

(a)

There are two factors: age(A_1, A_2, A_3), product type(P_1, P_2);

This is a 3×2 factorial experiment (or 2×3).

(b)

$$\begin{aligned} Y_{ijk} &= \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk} & \epsilon_{ijk} &\stackrel{i.i.d}{\sim} N(0, \sigma^2) \\ i &= 1, 2, 3 \\ j &= 1, 2 \\ k &= 1, \dots, 10 \end{aligned}$$

- Y_{ijk} : the response for replicate k at level i of age and level j of type.
- μ : overall mean of responses.
- α_i : the main effect for being at level i of age. $\sum \alpha_i = 0$. (sum not needed for credit)
- β_j : the main effect for being at level j of type. $\sum \beta_j = 0$. (sum not needed for credit)
- $(\alpha\beta)_{ij}$: the interaction effect for being at level i of age and level j of type. $\sum_i (\alpha\beta)_{i.} = 0$, $\sum_j (\alpha\beta)_{.j} = 0$. (sums not needed for credit)

- ϵ_{ijk} : random error for replicate k at level i of age and level j of type.
- σ^2 : variance of random error - common variance for each treatment group.
- i=1,2,3 corresponds to age 5-6 years, 7-8 years and 9-10 years; j=1,2 corresponds to product type breakfast cereals and video games.

14.11

(a)

There are three factors: sweetener (12%, 16%), milk fat (10%, 12%, 15%), air (5%, 10%, 15%);

It is a $2 \times 3 \times 3$ factorial experiment.

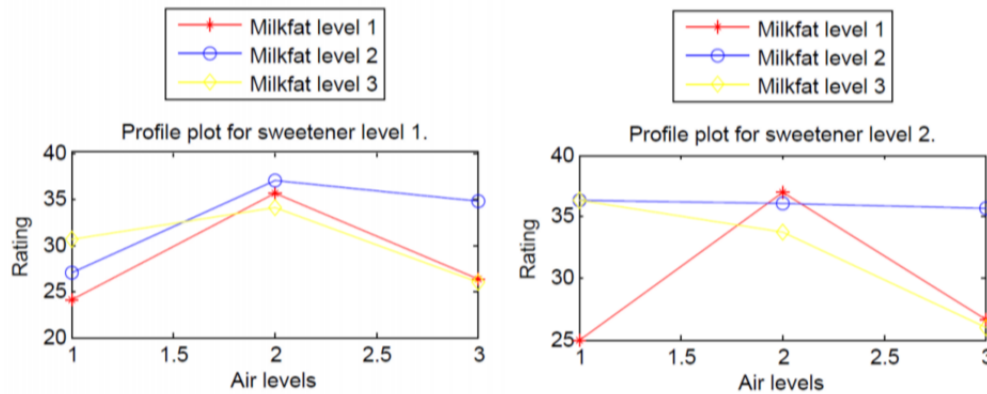
(b)

$$\begin{aligned}
 Y_{ijkl} &= \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijkl} \\
 \epsilon_{ijkl} &\stackrel{i.i.d}{\sim} N(0, \sigma^2) \\
 i &= 1, 2 \\
 j &= 1, 2, 3 \\
 k &= 1, 2, 3 \\
 l &= 1, 2, 3
 \end{aligned}$$

- Y_{ijkl} : the response for replicate l at level i of sweetener, level j of milkfat and level k of air.
- μ : overall mean of ratings.
- α_i : the main effect for being at level i of sweetener. $\sum \alpha_i = 0$. (sum not needed for credit)
- β_j : the main effect for being at level j of milkfat. $\sum \beta_j = 0$. (sum not needed for credit)

- γ_k : the main effect for being at level k of air. $\sum \gamma_k = 0$. (sum not needed for credit)
- $(\alpha\beta)_{ij}$: the interaction effect for being at level i of sweetener and level j of milkfat.
- $(\alpha\gamma)_{ik}$: the interaction effect for being at level i of sweetener and level k of air.
- $(\beta\gamma)_{jk}$: the interaction effect for being at level j of milkfat and level k of air.
- $(\alpha\beta\gamma)_{ijk}$: the interaction effect for being at level i of sweetener, level j of milkfat and level k of air.
- Note: Marginal sum of interaction terms equals to 0. (not needed for credit)
- ϵ_{ijkl} : random error for replicate l at level i of sweetener, level j of milkfat and level k of air.
- σ^2 : variance of random error - common variance for each treatment group.

(c)(d)



Yes, it seems there is a three-way interaction as the two-way interactions manifest themselves differently depending on the level of sweetener.

14.12

(a)

- ϵ_{ijkl} are assumed to be identically independent distributed as normal distribution with common variance σ^2 . From the residual plots, we can see the assumptions are likely met
- First test to conduct is for an interaction effect.
 $H_0 : (\alpha\beta\gamma)_{ijk} = 0$ vs $H_1 : (\alpha\beta\gamma)_{ijk} \neq 0$
- $\epsilon_{ijkl} \stackrel{i.i.d}{\sim} N(0, \sigma^2)$
- $F=3.52$
- $P\text{-val} = 0.016 < 0.05$, reject H_0
- At the 0.05 S.L., there is enough evidence to conclude that the percentage of sweetener, air, and milk fat has an effect on the ratings.
Since this is significant, we do not look at any lower order interactions or main effects

(b)

Yes. The conclusions from the F-test are consistent with our observations from the profile plots.

(c)

Since the three-way interaction term is significant, all the factors are important to the response. We will investigate simple effects. For example, the effect of air 5% vs air 10 % at milkfat 10% and sweetener 12 %.