Answer to 17.9(a):

The experiment is a x b design. The levels for factor A is $[a_1, a_2, \dots, a_a]$, and the levels for factor B is $[b_1, b_2, \dots, b_b]$

If A and B are both fixed: all the levels of $[a_1, a_2, \dots, a_a]$ for A and all levels of $[b_1, b_2, \dots, b_b]$ for B, are included in the design.

If A and B are both random: out of all the levels of $[a_1, a_2, \dots, a_a]$ for A, only a subset of the $[a_1, a_2, \dots, a_a]$ levels for A is randomly selected and included. For B, only a randomly selected subset of all levels of $[b_1, b_2, \dots, b_b]$ for B, are included in the design.

Answer to 17.13(a):

The mixed effects model can be represented as:

$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + \tau\beta_{ij} + \tau\gamma_{ik} + \beta\gamma_{jk} + \tau\beta\gamma_{ijk} + \varepsilon_{l(ijk)}$

here i= 1,2, 3, j =1,2,…, 4, and k=1, 2, … 6,

Furthermore,
y_{ijkl} is the l^{th} observation for the level of i, for factor chemical, the level of j, for factor B, and the level of k, for factor C.
μ = overall mean response which is unknown

τ_i = the random effect corresponding to the i^{th} level of factor A. The τ_i s have independent normal distribution with mean zero and variance σ^2_A

β_j = the fixed effect corresponding to the j^{th} level of factor B.

γ_k = the fixed effect corresponding to the k^{th} level of factor C.

$\tau\beta_{ij}$ = the random effect due to interaction of the i^{th} level of factor A, and the j^{th} level of factor B. The $\tau\beta_{ij}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta}$

$\tau\gamma_{ik}$ = the random effect due to interaction of the i^{th} level of factor A, and the k^{th} level of factor C. The $\tau\gamma_{ik}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\gamma}$

$\beta\gamma_{jk}$ = the random effect due to interaction of the j^{th} level of factor B, and the k^{th} level of factor C. The $\beta\gamma_{jk}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\beta\gamma}$

$\tau\beta\gamma_{ijk}$ = the random effect due to interaction for the i^{th} level of factor A, j^{th} level of factor B, and the k^{th} level of factor C. The $\tau\beta\gamma_{ijk}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta\gamma}$

$\varepsilon_{l(ijk)}$ = the random error associated with the l^{th} observation and has an independent distribution with man zero, and variance σ^2

τ_i, $\tau\beta_{ij}$, $\tau\gamma_{ik}$, and $\tau\beta\gamma_{ijk}$ are mutually independent

Answer to 17.13(b):

Source	Expected Mean Square (EMS)	Degrees of Freedom (df)
A	$\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC} + nc\sigma^2_{AB} + nbc\sigma^2_A$	a-1=2
B	$\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB} + nac\varphi^2_B$	b-1=3
C	$\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC} + nab\varphi^2_C$	c-1=5
AB	$\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB}$	(a-1)(b-1) = 6
AC	$\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC}$	(a-1)(c-1)=10
BC	$\sigma^2 + n\sigma^2_{ABC} + na\varphi^2_{BC}$	(b-1)(c-1)=15
ABC	$\sigma^2 + n\sigma^2_{ABC}$	(a-1)(b-1)(c-1) = 30
Error	σ^2	abc(n-1) = 216

Answer to 17.13(c):

For random effects of A, σ^2_A :
H_0: $\sigma^2_A = 0$, H_A: $\sigma^2_A > 0$
Under H_0 , $\sigma^2_A = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC} + nc\sigma^2_{AB}$
This modified EMS is not in the ANOVA table so we need to approximate the F-test is needed, with Satterwithe's degree of freedom formula. The modified EMS can be obtained from $MS_{AB} + MS_{AC} - MS_{ABC}$

For fixed effects of B, φ^2_B :
H_0: $\varphi^2_B = 0$, H_A: $\varphi^2_B > 0$
Under H_0 , $\varphi^2_B = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB}$
This modified EMS is in the ANOVA table,as well as with the degrees of freedom so we F-test = MS_B / MS_{AB}

For fixed effects of C, φ^2_C :
H_0: $\varphi^2_C = 0$, H_A: $\varphi^2_C > 0$
Under H_0 , $\varphi^2_C = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC}$

This modified EMS is in the ANOVA table, as well as with the degrees of freedom so we F-test = MS_C / MS_{AC}

For random effects of AB, σ^2_{AB} :
H_0: $\sigma^2_{AB} = 0$, H_A: $\sigma^2_{AB} > 0$
Under H_0 , $\sigma^2_{AB} = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB}$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MS_{AB}/MS_{ABC}

For random effects of AC, σ^2_{AC} :
H_0: $\sigma^2_{AC} = 0$, H_A: $\sigma^2_{AC} > 0$
Under H_0 , $\sigma^2_{AC} = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC}$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MS_{AC}/MS_{ABC}

For random effects of BC, σ^2_{BC} :
H_0: $\sigma^2_{BC} = 0$, H_A: $\sigma^2_{BC} > 0$
Under H_0 , $\sigma^2_{BC} = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC}$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MS_{BC}/MS_{ABC}

For random effects of BC, σ^2_{ABC} :
H_0: $\sigma^2_{ABC} = 0$, H_A: $\sigma^2_{ABC} > 0$
Under H_0 , $\sigma^2_{ABC} = 0$, modified EMS = σ^2
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MS_{ABC}/MSE

Answer to 17.10(a):

The mixed effects model can be represented as:

$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{k(ij)}$

here i= 1,2, 3, 4, and j =1,2,…, 5

Furthermore,
y_{ijk} is the k^{th} observation for the level of i, for factor chemical, the level of j, for factor location.

μ = overall mean response which is unknown

τ_i = the fixed effect corresponding to the i^{th} level of factor ‘chemical’.

β_j = the random effect corresponding to the j^{th} level of factor ‘location’. The β_j s have independent normal distribution with mean zero and variance $\sigma^2_{location}$

$\tau\beta_{ij}$ = the random effect due to interaction of the i^{th} level of factor chemical, and the j^{th} level of factor location. The $\tau\beta_{ij}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta}$

$\varepsilon_{k(ij)}$ = the random error associated with the k^{th} observation and has an independent distribution with mean zero, and variance σ^2

$\varepsilon_{k(ij)}$, $\tau\beta_{ij}$, and β_j are mutually independent

Answer to 17.10(b):

Source	df	Sum of Squares	Mean Square	Expected mean Square	Error term	Error DF	F Value	p-value
Chemical	3	180.123	60.044	Var(Residual) + 2 Var (chemical*location) + Q(chemical)	MS (chemical*location)	12	44.59	< 0.0001
Location	4	3.811	0.952	Var(Residual) + 2 Var (chemical*location) + 8 Var (location)	MS (chemical*location)	12	0.71	0.6020
Chemical*location	12	16.158	1.346	Var(Residual) + 2 Var (chemical*location)	MS(Residual)	20	3.89	0.0037
Error	20	6.925	0.346	Var(Residual)				

Answer to 17.11:

Fixed effects for chemical:

H_0: $\varphi^2_{chemical} = 0$, H_A : $\varphi^2_{chemical} > 0$
Decision: With 5% significance level, we reject the null hypothesis
Conclusion: There is significantly sufficient evidence that there is variability amongst the i^{th} levels of chemical.

Random effects for location:

H_0: $\sigma^2_{location} = 0$, H_A: $\sigma^2_{location} > 0$
Decision: With 5% significance level, we fail to reject the null hypothesis

Conclusion: There is not significantly sufficient evidence that there is variability amongst the j^{th} levels of location.

Random effects for interaction due to chemical*location:

H_0: $\sigma^2_{chemical * location} = 0$, H_A: $\sigma^2_{chemical * location} > 0$
Decision: With 5% significance level, we reject the null hypothesis
Conclusion: There is significantly sufficient evidence that there is variability amongst the interaction due to j^{th} levels of location and i^{th} levels of chemical.

Answer to 17.27(a):

The mixed effects model can be represented as:

$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{k(ij)}$

here i= 1,2, 3, 4, and j =1,2, 3

Furthermore,
y_{ijk} is the k^{th} observation for the level of i, for factor machine, the level of j, for factor operator.

μ = overall mean response which is unknown

τ_i = the random effect corresponding to the i^{th} level of factor 'machine'. The τ_i s have independent normal distribution with mean zero and variance $\sigma^2_{machine}$

β_j = the random effect corresponding to the j^{th} level of factor 'operator'. The β_j s have independent normal distribution with mean zero and variance $\sigma^2_{operator}$

$\tau\beta_{ij}$ = the random effect due to interaction of the i^{th} level of factor machine, and the j^{th} level of factor operator. The $\tau\beta_{ij}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta}$

$\varepsilon_{k(ij)}$ = the random error associated with the k^{th} observation and has an independent distribution with mean zero, and variance σ^2

$\varepsilon_{k(ij)}$, $\tau\beta_{ij}$, and τ_i, β_j are mutually independent

Answer to 17.27(b):

Source	df	Sum of Squares	Mean Square	Expected mean Square	Error term	Error DF	F Value	p-value
Machine	3	12.4583	60.044	Var(Residual) + 2 Var (machine*operator) + 6 Var (machine)	MS (machine*operator)	6	0.56	0.6619
Operator	2	160.333	0.952	Var(Residual) + 2 Var (machine*operator) + 8 Var (operator)	MS (machine*operator)	6	10.77	0.01103
Machine*Operator	6	44.6667	1.346	Var(Residual) + 2 Var (machine*operator)	MS(Residual)	12	1.96	0.1507
Error	12	45.50	0.346	Var(Residual)				

Answer to 17.27(c):

Random effects for machine:

H_0: $\sigma^2_{machine} = 0$, H_A: $\sigma^2_{machine} > 0$, p-value =0.6619
Decision: With 5% significance level, we fail to reject the null hypothesis
Conclusion: There is not significantly sufficient evidence that there is variability amongst the i^{th} levels of machine.

Random effects for operator:

H_0: $\sigma^2_{operator} = 0$, H_A: $\sigma^2_{operator} > 0$, p-value =0.0103
Decision: With 5% significance level, we reject the null hypothesis
Conclusion: There is not significantly sufficient evidence that there is variability amongst the j^{th} levels of operator.

Random effects for interaction due to machine*operator:

H_0: $\sigma^2_{machine * operator} = 0$, H_A: $\sigma^2_{machine * operator} > 0$, p-value =0.1507
Decision: With 5% significance level, we fail to reject the null hypothesis
Conclusion: There is significantly sufficient evidence that there is variability amongst the interaction due to j^{th} levels of operator and i^{th} levels of machine.

Answer to 17.28(a):

Co-variance estimates from Type III:

Machine = -0.5486
Operator = 9.0903
Machine * operator = 1.8264
Residual = 3.7917

Co-variance estimates from REML:

Machine = 0
Operator = 9.2275
Machine * operator = 1.2777
Residual = 3.7917

REML is preferred as it includes adjustments needed for degree of freedom for estimating the mean squares. We also see from the output that the co-variance estimates for REML is different to that of Type III. For Type III the co-variance estimate is negative for machine.

Answer to 17.28(c):

The confidence interval for average solder strength = (199.43, 215.16)
This value states that with 95% statistical confidence we can state that the true average solder strength lies between 199.43, and 215.16.