**Name: Akond Rahman**
**HW#8**
**Lab 001-B**
**Date: April 17, 2016**

**Answer to 17.9(a):**

The experiment is a x b design. The levels for factor A is [$a_1$, $a_2$, … , $a_a$], and the levels for factor B is [$b_1$, $b_2$, ... , $b_b$]

If A and B are both fixed: all the levels of [$a_1$, $a_2$, … , $a_a$] for A and all levels of [$b_1$, $b_2$, ... , $b_b$] for B, are included in the design.

If A and B are both random: out of all the levels of [$a_1$, $a_2$, … , $a_a$] for A, only a subset of the [$a_1$, $a_2$, … , $a_a$] levels for A is randomly selected and included. For B, only a randomly selected subset of all levels of [$b_1$, $b_2$, ... , $b_b$] for B, are included in the design.

**Answer to 17.13(a):**

The mixed effects model can be represented as:

$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + \tau\beta_{ij} + \tau\gamma_{ik} + \beta\gamma_{jk} + \tau\beta\gamma_{ijk} + \varepsilon_{l(ijk)}$

here i= 1,2, 3, j =1,2,…, 4, and k=1, 2, … 6,

Furthermore,
$y_{ijkl}$ is the $l^{th}$ observation for the level of i, for factor chemical, the level of j, for factor B, and the level of k, for factor C.
$\mu$ = overall mean response which is unknown

$\tau_i$ = the random effect corresponding to the $i^{th}$ level of factor A. The $\tau_i$ s have independent normal distribution with mean zero and variance $\sigma^2_A$

$\beta_j$ = the fixed effect corresponding to the $j^{th}$ level of factor B.

$\gamma_k$ = the fixed effect corresponding to the $k^{th}$ level of factor C.

$\tau\beta_{ij}$ = the random effect due to interaction of the $i^{th}$ level of factor A, and the $j^{th}$ level of factor B. The $\tau\beta_{ij}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta}$

$\tau\gamma_{ik}$ = the random effect due to interaction of the $i^{th}$ level of factor A, and the $k^{th}$ level of factor C. The $\tau\gamma_{ik}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\gamma}$

$\beta\gamma_{jk}$ = the random effect due to interaction of the $j^{th}$ level of factor B, and the $k^{th}$ level of factor C. The $\beta\gamma_{jk}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\beta\gamma}$

$\tau\beta\gamma_{ijk}$ = the random effect due to interaction for the $i^{th}$ level of factor A, $j^{th}$ level of factor B, and the $k^{th}$ level of factor C. The $\tau\beta\gamma_{ijk}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta\gamma}$

$\varepsilon_{l(ijk)}$ = the random error associated with the $l^{th}$ observation and has an independent distribution with man zero, and variance $\sigma^2$

$\tau_i$, $\tau\beta_{ij}$, $\tau\gamma_{ik}$, and $\tau\beta\gamma_{ijk}$ are mutually independent

**Answer to 17.13(b):**

| Source | Expected Mean Square (EMS) | Degrees of Freedom (df) |
|---|---|---|
| A | $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC} + nc\sigma^2_{AB} + nbc\sigma^2_A$ | a-1=2 |
| B | $\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB} + nac\varphi^2_B$ | b-1=3 |
| C | $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC} + nab\varphi^2_C$ | c-1=5 |
| AB | $\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB}$ | (a-1)(b-1) = 6 |
| AC | $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC}$ | (a-1)(c-1)=10 |
| BC | $\sigma^2 + n\sigma^2_{ABC} + na\varphi^2_{BC}$ | (b-1)(c-1)=15 |
| ABC | $\sigma^2 + n\sigma^2_{ABC}$ | (a-1)(b-1)(c-1) = 30 |
| Error | $\sigma^2$ | abc(n-1) = 216 |

**Answer to 17.13(c):**

For random effects of A, $\sigma^2_A$ :
$H_0$: $\sigma^2_A = 0$, $H_A$: $\sigma^2_A > 0$
Under $H_0$ , $\sigma^2_A = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC} + nc\sigma^2_{AB}$
This modified EMS is not in the ANOVA table so we need to approximate the F-test is needed, with Satterwithe's degree of freedom formula. The modified EMS can be obtained from $MS_{AB} + MS_{AC} - MS_{ABC}$

For fixed effects of B, $\varphi^2_B$ :
$H_0$: $\varphi^2_B = 0$, $H_A$: $\varphi^2_B > 0$
Under $H_0$ , $\varphi^2_B = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB}$
This modified EMS is in the ANOVA table,as well as with the degrees of freedom so we F-test = $MS_B / MS_{AB}$

For fixed effects of C, $\varphi^2_C$ :
$H_0$: $\varphi^2_C = 0$, $H_A$: $\varphi^2_C > 0$
Under $H_0$ , $\varphi^2_C = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nb\sigma^2_{AC}$

This modified EMS is in the ANOVA table, as well as with the degrees of freedom so we F-test = $MS_C$ / $MS_{AC}$

For random effects of AB, $\sigma^2_{AB}$ :
$H_0$: $\sigma^2_{AB} = 0$, $H_A$: $\sigma^2_{AB} > 0$
Under $H_0$ , $\sigma^2_{AB} = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC} + nc\sigma^2_{AB}$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = $MS_{AB}/MS_{ABC}$

For random effects of AC, $\sigma^2_{AC}$ :
$H_0$: $\sigma^2_{AC} = 0$, $H_A$: $\sigma^2_{AC} > 0$
Under $H_0$ , $\sigma^2_{AC} = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC}$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = $MS_{AC}/MS_{ABC}$

For random effects of BC, $\sigma^2_{BC}$ :
$H_0$: $\sigma^2_{BC} = 0$, $H_A$: $\sigma^2_{BC} > 0$
Under $H_0$ , $\sigma^2_{BC} = 0$, modified EMS = $\sigma^2 + n\sigma^2_{ABC}$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = $MS_{BC}/MS_{ABC}$

For random effects of BC, $\sigma^2_{ABC}$ :
$H_0$: $\sigma^2_{ABC} = 0$, $H_A$: $\sigma^2_{ABC} > 0$
Under $H_0$ , $\sigma^2_{ABC} = 0$, modified EMS = $\sigma^2$
This modified EMS and degrees of freedom is in the ANOVA table. The F-test = $MS_{ABC}/MSE$

**Answer to 17.10(a):**

The mixed effects model can be represented as:

$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{k(ij)}$

here i= 1,2, 3, 4, and j =1,2,…, 5

Furthermore,
$y_{ijk}$ is the $k^{th}$ observation for the level of i, for factor chemical, the level of j, for factor location.

$\mu$ = overall mean response which is unknown

$\tau_i$ = the fixed effect corresponding to the $i^{th}$ level of factor ‘chemical’.

$\beta_j$ = the random effect corresponding to the $j^{th}$ level of factor ‘location’. The $\beta_j$ s have independent normal distribution with mean zero and variance $\sigma^2_{location}$

$\tau\beta_{ij}$ = the random effect due to interaction of the $i^{th}$ level of factor chemical, and the $j^{th}$ level of factor location. The $\tau\beta_{ij}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta}$

$\varepsilon_{k(ij)}$ = the random error associated with the $k^{th}$ observation and has an independent distribution with mean zero, and variance $\sigma^2$

$\varepsilon_{k(ij)}$ , $\tau\beta_{ij}$ , and $\beta_j$ are mutually independent

**Answer to 17.10(b):**

| Source | df | Sum of Squares | Mean Square | Expected mean Square | Error term | Error DF | F Value | p-value |
|---|---|---|---|---|---|---|---|---|
| Chemical | 3 | 180.123 | 60.044 | Var(Residual) + 2 Var (chemical*location) + Q(chemical) | MS (chemical*location) | 12 | 44.59 | < 0.0001 |
| Location | 4 | 3.811 | 0.952 | Var(Residual) + 2 Var (chemical*location) + 8 Var (location) | MS (chemical*location) | 12 | 0.71 | 0.6020 |
| Chemical*location | 12 | 16.158 | 1.346 | Var(Residual) + 2 Var (chemical*location) | MS(Residual) | 20 | 3.89 | 0.0037 |
| Error | 20 | 6.925 | 0.346 | Var(Residual) | | | | |

**Answer to 17.11:**

Fixed effects for chemical:

$H_0$: $\varphi^2_{chemical} = 0$, $H_A$ : $\varphi^2_{chemical} > 0$
Decision: With 5% significance level, we reject the null hypothesis
Conclusion: There is significantly sufficient evidence that there is variability amongst the $i^{th}$ levels of chemical.

Random effects for location:

$H_0$: $\sigma^2_{location} = 0$, $H_A$: $\sigma^2_{location} > 0$
Decision: With 5% significance level, we fail to reject the null hypothesis

Conclusion: There is not significantly sufficient evidence that there is variability amongst the $j^{th}$ levels of location.

Random effects for interaction due to chemical*location:

$H_0$: $\sigma^2_{chemical * location} = 0$, $H_A$: $\sigma^2_{chemical * location} > 0$
Decision: With 5% significance level, we reject the null hypothesis
Conclusion: There is significantly sufficient evidence that there is variability amongst the interaction due to $j^{th}$ levels of location and $i^{th}$ levels of chemical.

**Answer to 17.27(a):**

The mixed effects model can be represented as:

$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{k(ij)}$

here i= 1,2, 3, 4, and j =1,2, 3

Furthermore,
$y_{ijk}$ is the $k^{th}$ observation for the level of i, for factor machine, the level of j, for factor operator.

$\mu$ = overall mean response which is unknown

$\tau_i$ = the random effect corresponding to the $i^{th}$ level of factor 'machine'. The $\tau_i$ s have independent normal distribution with mean zero and variance $\sigma^2_{machine}$

$\beta_j$ = the random effect corresponding to the $j^{th}$ level of factor 'operator'. The $\beta_j$ s have independent normal distribution with mean zero and variance $\sigma^2_{operator}$

$\tau\beta_{ij}$ = the random effect due to interaction of the $i^{th}$ level of factor machine, and the $j^{th}$ level of factor operator. The $\tau\beta_{ij}$ s have independent normal distribution with mean zero and variance $\sigma^2_{\tau\beta}$

$\varepsilon_{k(ij)}$ = the random error associated with the $k^{th}$ observation and has an independent distribution with mean zero, and variance $\sigma^2$

$\varepsilon_{k(ij)}$ , $\tau\beta_{ij}$ , and $\tau_i$, $\beta_j$ are mutually independent

**Answer to 17.27(b):**

| Source | df | Sum of Squares | Mean Square | Expected mean Square | Error term | Error DF | F Value | p-value |
|---|---|---|---|---|---|---|---|---|
| Machine | 3 | 12.4583 | 60.044 | Var(Residual) + 2 Var (machine*operator) + 6 Var (machine) | MS (machine*operator) | 6 | 0.56 | 0.6619 |
| Operator | 2 | 160.333 | 0.952 | Var(Residual) + 2 Var (machine*operator) + 8 Var (operator) | MS (machine*operator) | 6 | 10.77 | 0.01103 |
| Machine*Operator | 6 | 44.6667 | 1.346 | Var(Residual) + 2 Var (machine*operator) | MS(Residual) | 12 | 1.96 | 0.1507 |
| Error | 12 | 45.50 | 0.346 | Var(Residual) | | | | |

**Answer to 17.27(c):**

Random effects for machine:

$H_0$: $\sigma^2_{machine} = 0$, $H_A$: $\sigma^2_{machine} > 0$, p-value =0.6619
Decision: With 5% significance level, we fail to reject the null hypothesis
Conclusion: There is not significantly sufficient evidence that there is variability amongst the $i^{th}$ levels of machine.

Random effects for operator:

$H_0$: $\sigma^2_{operator} = 0$, $H_A$: $\sigma^2_{operator} > 0$, p-value =0.0103
Decision: With 5% significance level, we reject the null hypothesis
Conclusion: There is not significantly sufficient evidence that there is variability amongst the $j^{th}$ levels of operator.

Random effects for interaction due to machine*operator:

$H_0$: $\sigma^2_{machine * operator} = 0$, $H_A$: $\sigma^2_{machine * operator} > 0$, p-value =0.1507
Decision: With 5% significance level, we fail to reject the null hypothesis
Conclusion: There is significantly sufficient evidence that there is variability amongst the interaction due to $j^{th}$ levels of operator and $i^{th}$ levels of machine.

**Answer to 17.28(a):**

Co-variance estimates from Type III:

Machine = -0.5486
Operator = 9.0903
Machine * operator = 1.8264
Residual = 3.7917

Co-variance estimates from REML:

Machine = 0
Operator = 9.2275
Machine * operator = 1.2777
Residual = 3.7917

REML is preferred as it includes adjustments needed for degree of freedom for estimating the mean squares. We also see from the output that the co-variance estimates for REML is different to that of Type III. For Type III the co-variance estimate is negative for machine.

**Answer to 17.28(c):**

The confidence interval for average solder strength = (199.43, 215.16)
This value states that with 95% statistical confidence we can state that the true average solder strength lies between 199.43, and 215.16.

```
/*
Akond Rahman
Date: April 14, 2016
Section 001B
Title: 17.10 and 17.11 problem for HW#8
*/
/*Loading data*/
data fire_ant_killer;
   input chemical location @ ;
   do k = 1 to 2;
      input kill_count @;
      output;
   end;
   cards;
   1 1 7.2 9.6
   1 2 8.5 9.6
   1 3 9.1 8.6
   1 4 8.2 9.0
   1 5 7.8 8.0
   2 1 4.2 3.5
   2 2 2.9 3.3
   2 3 1.8 2.4
   2 4 3.6 4.4
   2 5 3.7 3.9
   3 1 9.5 9.3
   3 2 8.8 9.2
   3 3 7.6 7.1
   3 4 7.3 7.0
   3 5 9.2 8.3
   4 1 5.4 3.9
   4 2 6.3 6.0
   4 3 6.1 5.6
   4 4 5.0 5.4
   4 5 6.5 6.9
;
run;
quit;
proc mixed data = fire_ant_killer method = type3 ;
   class  chemical location; /* chemical and location are the two factors*/
   model kill_count = chemical ; /*chemical is the fixed factor, kill_count is the response*/
   random  location location*chemical; /* location*chemical, and location are the random factors*/
run;
```

# The SAS System

## The Mixed Procedure

| Model Information | |
|---|---|
| Data Set | WORK.FIRE_ANT_KILLER |
| Dependent Variable | kill_count |
| Covariance Structure | Variance Components |
| Estimation Method | Type 3 |
| Residual Variance Method | Factor |
| Fixed Effects SE Method | Model-Based |
| Degrees of Freedom Method | Containment |

**Class Level Information**

| Class | Levels | Values |
|---|---|---|
| chemical | 4 | 1 2 3 4 |
| location | 5 | 1 2 3 4 5 |

| Dimensions | |
|---|---|
| Covariance Parameters | 3 |
| Columns in X | 5 |
| Columns in Z | 25 |
| Subjects | 1 |
| Max Obs Per Subject | 40 |

| Number of Observations | |
|---|---|
| Number of Observations Read | 40 |
| Number of Observations Used | 40 |
| Number of Observations Not Used | 0 |

**Type 3 Analysis of Variance**

| Source | DF | Sum of Squares | Mean Square | Expected Mean Square | Error Term | Error DF | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| chemical | 3 | 180.132750 | 60.044250 | Var(Residual) + 2 Var (chemical*location) + Q(chemical) | MS (chemical*location) | 12 | 44.59 | <.0001 |
| location | 4 | 3.811500 | 0.952875 | Var(Residual) + 2 Var (chemical*location) + 8 Var (location) | MS (chemical*location) | 12 | 0.71 | 0.6020 |
| chemical*location | 12 | 16.158500 | 1.346542 | Var(Residual) + 2 Var (chemical*location) | MS(Residual) | 20 | 3.89 | 0.0037 |
| Residual | 20 | 6.925000 | 0.346250 | Var(Residual) | . | . | . | . |

**Covariance Parameter Estimates**

| Cov Parm | Estimate |
|---|---|
| location | -0.04921 |
| chemical*location | 0.5001 |

| Covariance Parameter Estimates | |
|---|---|
| Cov Parm | Estimate |
| Residual | 0.3462 |

| Fit Statistics | |
|---|---|
| -2 Res Log Likelihood | 93.5 |
| AIC (smaller is better) | 99.5 |
| AICC (smaller is better) | 100.3 |
| BIC (smaller is better) | 98.4 |

| Type 3 Tests of Fixed Effects | | | | |
|---|---|---|---|---|
| Effect | Num DF | Den DF | F Value | Pr > F |
| chemical | 3 | 12 | 44.59 | <.0001 |

```
/*
Akond Rahman
Date: April 14, 2016
Section 001B
Title: 17.27 and 17.28 problem for HW#8
*/
/*Loading data*/
data strength_data;
   input machine operator @ ;
   do k = 1 to 2;
      input strength @;
      output;
   end;
   cards;
   1 1 204 205
   1 2 205 207
   1 3 211 209
   2 1 205 210
   2 2 205 206
   2 3 207 210
   3 1 203 204
   3 2 206 204
   3 3 209 214
   4 1 205 203
   4 2 209 207
   4 3 215 212
;
run;
quit;
/*Using Type III method*/
proc mixed data = strength_data method = type3 ;
   class  machine operator; /* machine and operator are the two factors*/
   model strength =;   /* strength is the response*/
   random  machine operator machine*operator; /* machine*operator, machine and operator are the randon
run;
/* Ans.to 17.28 (a)  */
/*Using REML method*/
proc mixed data = strength_data method = reml  ;
   class  machine operator; /* machine and operator are the two factors*/
   model strength = / ddfm = satterth cl  ; /* strength is the response, used Satterwieth's adjustment
   random  machine operator machine*operator; /* machine*operator, machine and operator are the randon
   estimate 'Avg. solder strength' intercept 1 / cl; * get CI of the estimate of the avg. solder strer
run;
```

# The SAS System

## The Mixed Procedure

| Model Information | |
|---|---|
| Data Set | WORK.STRENGTH_DATA |
| Dependent Variable | strength |
| Covariance Structure | Variance Components |
| Estimation Method | Type 3 |
| Residual Variance Method | Factor |
| Fixed Effects SE Method | Model-Based |
| Degrees of Freedom Method | Containment |

| Class Level Information | | |
|---|---|---|
| Class | Levels | Values |
| machine | 4 | 1 2 3 4 |
| operator | 3 | 1 2 3 |

| Dimensions | |
|---|---|
| Covariance Parameters | 4 |
| Columns in X | 1 |
| Columns in Z | 19 |
| Subjects | 1 |
| Max Obs Per Subject | 24 |

| Number of Observations | |
|---|---|
| Number of Observations Read | 24 |
| Number of Observations Used | 24 |
| Number of Observations Not Used | 0 |

Type 3 Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | Expected Mean Square | Error Term | Error DF | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| machine | 3 | 12.458333 | 4.152778 | Var(Residual) + 2 Var (machine*operator) + 6 Var (machine) | MS (machine*operator) | 6 | 0.56 | 0.6619 |
| operator | 2 | 160.333333 | 80.166667 | Var(Residual) + 2 Var (machine*operator) + 8 Var (operator) | MS (machine*operator) | 6 | 10.77 | 0.0103 |
| machine*operator | 6 | 44.666667 | 7.444444 | Var(Residual) + 2 Var (machine*operator) | MS(Residual) | 12 | 1.96 | 0.1507 |
| Residual | 12 | 45.500000 | 3.791667 | Var(Residual) | . | . | . | . |

| Covariance Parameter Estimates | |
|---|---|
| Cov Parm | Estimate |
| machine | -0.5486 |
| operator | 9.0903 |

| Covariance Parameter Estimates | |
|---|---|
| **Cov Parm** | **Estimate** |
| **machine*operator** | 1.8264 |
| **Residual** | 3.7917 |

| Fit Statistics | |
|---|---|
| **-2 Res Log Likelihood** | 109.5 |
| **AIC (smaller is better)** | 117.5 |
| **AICC (smaller is better)** | 119.7 |
| **BIC (smaller is better)** | 115.1 |

# The SAS System

## The Mixed Procedure

| Model Information | |
|---|---|
| Data Set | WORK.STRENGTH_DATA |
| Dependent Variable | strength |
| Covariance Structure | Variance Components |
| Estimation Method | REML |
| Residual Variance Method | Profile |
| Fixed Effects SE Method | Model-Based |
| Degrees of Freedom Method | Satterthwaite |

| Class Level Information | | |
|---|---|---|
| Class | Levels | Values |
| machine | 4 | 1 2 3 4 |
| operator | 3 | 1 2 3 |

| Dimensions | |
|---|---|
| Covariance Parameters | 4 |
| Columns in X | 1 |
| Columns in Z | 19 |
| Subjects | 1 |
| Max Obs Per Subject | 24 |

| Number of Observations | |
|---|---|
| Number of Observations Read | 24 |
| Number of Observations Used | 24 |
| Number of Observations Not Used | 0 |

| Iteration History | | | |
|---|---|---|---|
| Iteration | Evaluations | -2 Res Log Like | Criterion |
| 0 | 1 | 124.48875799 | |
| 1 | 3 | 109.85258730 | 0.00025935 |
| 2 | 1 | 109.84333612 | 0.00000208 |
| 3 | 1 | 109.84326553 | 0.00000000 |

Convergence criteria met.

| Covariance Parameter Estimates | |
|---|---|
| Cov Parm | Estimate |
| machine | 0 |
| operator | 9.2275 |
| machine*operator | 1.2777 |

| Covariance Parameter Estimates | |
|---|---|
| Cov Parm | Estimate |
| Residual | 3.7917 |

| Fit Statistics | |
|---|---|
| -2 Res Log Likelihood | 109.8 |
| AIC (smaller is better) | 115.8 |
| AICC (smaller is better) | 117.1 |
| BIC (smaller is better) | 114.0 |

**Solution for Fixed Effects**

| Effect | Estimate | Standard Error | DF | t Value | Pr > \|t\| | Alpha | Lower | Upper |
|---|---|---|---|---|---|---|---|---|
| Intercept | 207.29 | 1.8277 | 2 | 113.42 | <.0001 | 0.05 | 199.43 | 215.16 |

**Estimates**

| Label | Estimate | Standard Error | DF | t Value | Pr > \|t\| | Alpha | Lower | Upper |
|---|---|---|---|---|---|---|---|---|
| Avg. solder strength | 207.29 | 1.8277 | 2 | 113.42 | <.0001 | 0.05 | 199.43 | 215.16 |