**Answer to 17.1(a):**

The random effects model can be represented as:

Y\_ij = mu + tau\_i + epsilon\_ij,

with mu\_i=E(y\_ij) = mu ,

here i= 1,2, …, 10, for the ten batches and j =1,2,…, 5, for each of the average percentage of the fade protection ingredient

Furthermore,

Y\_ij is the jth observation of average percentage of the faded protection ingredient in a randomly selected container of the paint, for batch i

epsilon\_ij is the random error associated with the jth observation for average percentage of the fade protection ingredient for ith batch

epsilon\_ij is normally distributed with mean zero, and variance σ2ε

epsilon\_ij s are independent

tau\_i is the random effect due to the ith batch with mean zero and variance σ2ε

tau\_i is independent for i =1,2, …, 10

random components epsilon\_ij and tau\_i are independent of ach other

mu is the overall mean average percentage of the faded protection ingredient in a randomly selected container of the paint, which is an unknown constant

as it is a random effects model, the samples tau\_i’s are identical

**Answer to 17.1(b):**

H0: σ2tau = 0

HA: σ2tau ≠ 0

As it is a random effects model, the samples tau\_i’s are identical. The null hypothesis states that the taus or the random effects for different batches are the same, whereas the alternative hypothesis states that at least one of the batches are different from another and has a significant effect on the average percentage of the faded protection ingredient in a randomly selected container of the paint

We will use a F-test:

F = MSbatch / MSE = 5.738/4.558 =1.2588

The p-value for F9,40(1.2588) = 0.2889.

We fail to reject the null hypothesis at

At a 5% significance level there is no sufficient sample that suggests one of the tau’s are different to the others i.e. there is no sufficient sample that states at least one of the batches have a significant random effect on the average percentage of the faded protection ingredient in a randomly selected container of the paint

**Answer to 17.1(c):**

Variation within containers from one one batch to another = σ2ε

Variation due to one batch to another = σ2a

We estimate σ2ε, σ2ε-hat = MSE = 4.558

We estimate σ2a, σ2a-hat = (MST-MSE)/n = (5.738-4.558)/5 = 0.236, this value matches with the output from co-variance estimates of mixed methods using Type III method.

Total variance = σ2total = σ2a + σ2ε = 4.794

Proportion of variance due to batch-to-batch variation = 0.236/4.794 = 0.04922 = 4.922%

**Answer to 17.2 (a):**

Point estimate of average percentage of fade protection ingredient in randomly selected container = y.. = 5.242

**Answer to 17.2 (b):**

For the 95% confidence interval the formula is

= y.. ± t0.025,9 \* sqrt(MSbatch/(t \* n))

= 5.242 ± 2.262 \* sqrt(5.738/(5 \* 10))

= (5.242-0.7663, 5.242+0.7663)

=(4.4757, 6.0083)

**Answer to 17.2 (c):**

With 95% statistical confidence we can state that the true average percentage of the faded protection ingredient in a randomly selected container of the paint is between 4.4757 and 6.0083.

**Answer to 17.3 (a):**

**Scenario (a):**

y\_ij = mu + tau\_i + epsilon\_ij,

with mu\_i=E(y\_ij) = mu + tau\_i ,

here i= 1,2, …, 5, for the five bulls and j =1,2,…, 6, for each of the average daily gain in weight for bull i

Furthermore,

epsilon\_ij is the random error associated with the jth observation for the average daily weight gain i weight for ith bull

epsilon\_ij is normally distributed with mean zero, and variance σ2

epsilon\_ij s are independent

tau\_i is the fixed effect due to the ith bull with mean zero and variance σ2ε

tau\_i is independent for i =1,2, …,5

random components epsilon\_ij and tau\_i are independent of ach other

mu is the overall mean of the average daily gain in weight for bull i which is an unknown constant

**Scenario (b):**

The random effects model can be represented as:

Y\_ij = mu + tau\_i + epsilon\_ij,

with mu\_i=E(y\_ij) = mu ,

here i= 1,2, …, 5, for the five bulls and j =1,2,…, 6, for each of the average daily weight gain of calves

Furthermore,

Y\_ij is the jth observation of average daily weight gain of calves, for bull i

epsilon\_ij is the random error associated with the jth observation for average daily weight gain of calves for ith bull

epsilon\_ij is normally distributed with mean zero, and variance σ2ε

epsilon\_ij s are independent

tau\_i is the random effect due to the ith bull with mean zero and variance σ2ε

tau\_i is independent for i =1,2, …,5

random components epsilon\_ij and tau\_i are independent of each other

mu is the overall mean average daily weight gain of calves for randomly selected calf, which is an unknown constant

as it is a random effects model, the samples tau\_i’s are identical

**Answer to 17.3 (b):**

**Scenario (a):**

H0 = tau\_1 = tau\_2 =tau\_3 =tau\_4 =tau\_5 = 0

HA = at least one of tau\_i’s is different from zero

The null hypotheses state that the five tau’s are in the fixed model are identically zero. The alternative hypothesis states that at least one of the tau’s are different from the other tau’s.

**Scenario (b):**

H0: σ2tau = 0

HA: σ2tau ≠ 0

As it is a random effects model, the samples tau\_i’s are identical. The null hypothesis states that the taus or the random effects for different bulls are the same, whereas the alternative hypothesis states that at least one of the bulls are different from another and has a significant effect on the average daily weight gain of calves.

**Answer to 17.4(a):**

Test Statistic = F = MSbull / MSE = 0.122287/0.010219 = 11.97

The p-value for F4,25(11.97) < 0.0001

Reject the null hypothesis at 5% statistical significance level

At 5% significance level there is sufficient evidence that not all the tau’s in the population are the same. At least one of the levels of the bulls has a significant random effect on average daily weight gain of calves.

**Answer to 17.4(b):**

Variation within average daily weight gain for individual calves within one bull to bull another = σ2ε = 0.01022

Variation due to one bull to another = σ2a = 0.01868

Total variance = σ2total = σ2a + σ2ε

= 0.0289

Variance due to bull to bull variation = 0.01868/0.0289 = 0.646366 = 64.6366%

**Answer to 17.4(c):**

For the 95% confidence interval the formula is

= y.. ± t0.025,4 \* sqrt(MSbull/(t \* n))

= 1.087 ± 2.777 \* sqrt(0.122287/(6 \* 5))

= (1.087 – 0.1773, 1.087 + 0.1773)

=(0.9097, 1.2643)

With 95% statistical confidence we can state that the true average daily weight gain for individual calves is between 0.9097 and 1.2643.

**Answer to NBP (a):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | DF | Sum of Squares | Mean Square | F Value | p-value |
| A | 2 | 611.4444 | 305.7222 | 50.95 | < 0.0001 |
| B(A) | 3 | 72.1667 | 24.05555 | 4.01 | 0.0344 |
|  |  |  |  |  |  |

We reject the null hypothesis with significance level 0.05 for nested effects B(A), as the p-value is 0.0344 < 0.05

We can state that with a 0.05 significance level there is sufficient evidence that nested effects have a significant impact on fasting blood sugar, i.e. for different levels of variability for B on fasting blood sugar.

**Answer to NBP (b):**

As there is sufficient evidence that shows the levels of B, nested within a certain levels of A, we should investigate the simple effects of B that are nested within for a certain level of A.

**Answer to NBP (c):**

Bonferroni correction with 90% confidence limits, the individual significance level for each confidence interval is 0.10/3 = 0.03333

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Parameter | Estimate | Std. Error | T Value | Pr > |t| | 96.667% confidence limits | |
| Lower | Upper |
| B-1 vs. B-2 within A=1 | -4.00 | 2.00 | -2.00 | 0.0687 | -8.8061896 | 0.80618965 |
| B-1 vs. B-2 within A=2 | 2.66666667 | 2.00 | 1.33 | 0.2072 | -2.1395229 | 7.47285631 |
| B-1 vs. B-2 within A=3 | -5.00 | 2.00 | -2.50 | 0.0279 | -9.8061896 | -0.1938103 |

**Answer to NBP (d):**

Using Tukey’s correction we can observe the difference of how average blood sugar levels vary with three different drugs

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| i | j | Difference between means | Simultaneous 95% confidence limits | |
| Lower | Upper |
| 1 | 2 | -1.0 | -4.772885 | 2.772885 |
| 1 | 3 | -12.8333 | -16.60621 | -9.06044 |
| 3 | 3 | -11.8333 | -15.606219 | -8.06044 |

We observe that for two cases, both, difference between level 1 and level 3 of factor B, also difference between level 2 and level 3 of factor B, has a significant effect on average blood sugar, as the confidence intervals do not include zero. This finding implies that the levels of factor A i.e. the three tablets can have a significant effect on average blood sugar.