**Answer to 17.9(a):**

The experiment is a x b design. The levels for factor A is [a1, a2, … , aa], and the levels for factor B is [b1, b2, ... , bb]

If A and B are both fixed: all the levels of [a1, a2, … , aa] for A and all levels of [b1, b2, ... , bb] for B, are included in the design.

If A and B are both random: out of all the levels of [a1, a2, … , aa] for A, only a subset of the [a1, a2, … , aa] levels for A is randomly selected and included. For B, only a randomly selected subset of all levels of [b1, b2, ... , bb] for B, are included in the design.

**Answer to 17.13(a):**

The mixed effects model can be represented as:

yijkl = μ + τi + βj + γk + τβij + τγik + βγjk + τβγijk + εl(ijk)

here i= 1,2, 3, j =1,2,…, 4, and k=1, 2, … 6,

Furthermore,

yijkl is the lth observation for the level of i, for factor chemical, the level of j, for factor B, and the level of k, for factor C.

μ = overall mean response which is unknown

τi = the random effect corresponding to the ith level of factor A. The τi s have independent normal distribution with mean zero and variance σ2A

βj = the fixed effect corresponding to the jth level of factor B.

γk = the fixed effect corresponding to the kth level of factor C.

τβij = the random effect due to interaction of the ith level of factor A, and the jth level of factor B. The τβij s have independent normal distribution with mean zero and variance σ2τβ

τγik = the random effect due to interaction of the ith level of factor A, and the kth level of factor C. The τγik s have independent normal distribution with mean zero and variance σ2τγ

βγjk = the random effect due to interaction of the jth level of factor B, and the kth level of factor C. The βγjk s have independent normal distribution with mean zero and variance σ2βγ

τβγijk = the random effect due to interaction for the ith level of factor A, jth level of factor B, and the kth level of factor C. The τβγijk s have independent normal distribution with mean zero and variance σ2τβγ

εl(ijk) = the random error associated with the lth observation and has an independent distribution with man zero, and variance σ2

τi , τβij , τγik , and τβγijk are mutually independent

**Answer to 17.13(b):**

|  |  |  |
| --- | --- | --- |
| Source | Expected Mean Square (EMS) | Degrees of Freedom (df) |
| A | σ2 + nσ2ABC + nbσ2AC + ncσ2AB + nbcσ2A | a-1=2 |
| B | σ2 + nσ2ABC + ncσ2AB + nacφ2B | b-1=3 |
| C | σ2 + nσ2ABC + nbσ2AC + nabφ2C | c-1=5 |
| AB | σ2 + nσ2ABC + ncσ2AB | (a-1)(b-1) = 6 |
| AC | σ2 + nσ2ABC + nbσ2AC | (a-1)(c-1)=10 |
| BC | σ2 + nσ2ABC + naφ2BC | (b-1)(c-1)=15 |
| ABC | σ2 + nσ2ABC | (a-1)(b-1)(c-1) = 30 |
| Error | σ2 | abc(n-1) = 216 |

**Answer to 17.13(c):**

For random effects of A, σ2A :

H0: σ2A = 0, HA: σ2A > 0

Under H0 , σ2A = 0, modified EMS = σ2 + nσ2ABC + nbσ2AC + ncσ2AB

This modified EMS is not in the ANOVA table so we need to approximate the F-test is needed, with Satterwithe’s degree of freedom formula. The modified EMS can be obtained from MSAB + MSAC - MSABC

For fixed effects of B, φ2B :

H0: φ2B = 0, HA: φ2B > 0

Under H0 , φ2B = 0, modified EMS = σ2 + nσ2ABC + ncσ2AB

This modified EMS is in the ANOVA table,as well as with the degrees of freedom so we F-test = MSB / MSAB

For fixed effects of C, φ2C :

H0: φ2C = 0, HA: φ2C > 0

Under H0 , φ2C = 0, modified EMS = σ2 + nσ2ABC + nbσ2AC

This modified EMS is in the ANOVA table, as well as with the degrees of freedom so we F-test = MSC / MSAC

For random effects of AB, σ2AB :

H0: σ2AB = 0, HA: σ2AB > 0

Under H0 , σ2AB = 0, modified EMS = σ2 + nσ2ABC + ncσ2AB

This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MSAB/MSABC

For random effects of AC, σ2AC :

H0: σ2AC = 0, HA: σ2AC > 0

Under H0 , σ2AC = 0, modified EMS = σ2 + nσ2ABC

This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MSAC/MSABC

For random effects of BC, σ2BC :

H0: σ2BC = 0, HA: σ2BC > 0

Under H0 , σ2BC = 0, modified EMS = σ2 + nσ2ABC

This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MSBC/MSABC

For random effects of BC, σ2ABC :

H0: σ2ABC = 0, HA: σ2ABC > 0

Under H0 , σ2ABC = 0, modified EMS = σ2

This modified EMS and degrees of freedom is in the ANOVA table. The F-test = MSABC/MSE

**Answer to 17.10(a):**

The mixed effects model can be represented as:

yijk = μ + τi + βj + τβij + εk(ij)

here i= 1,2, 3, 4, and j =1,2,…, 5

Furthermore,

yijk is the kth observation for the level of i, for factor chemical, the level of j, for factor location.

μ = overall mean response which is unknown

τi = the fixed effect corresponding to the ith level of factor ‘chemical’.

βj = the random effect corresponding to the jth level of factor ‘location’. The βj s have independent normal distribution with mean zero and variance σ2location

τβij = the random effect due to interaction of the ith level of factor chemical, and the jth level of factor location. The τβij s have independent normal distribution with mean zero and variance σ2τβ

εk(ij) = the random error associated with the kth observation and has an independent distribution with mean zero, and variance σ2

εk(ij) , τβij , and βj are mutually independent

**Answer to 17.10(b):**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Source | df | Sum of Squares | Mean Square | Expected mean Square | Error term | Error DF | F Value | p-value |
| Chemical | 3 | 180.123 | 60.044 | Var(Residual) + 2 Var (chemical\*location) + Q(chemical) | MS (chemical\*location) | 12 | 44.59 | < 0.0001 |
| Location | 4 | 3.811 | 0.952 | Var(Residual) + 2 Var (chemical\*location) + 8 Var (location) | MS (chemical\*location) | 12 | 0.71 | 0.6020 |
| Chemical\*location | 12 | 16.158 | 1.346 | Var(Residual) + 2 Var (chemical\*location) | MS(Residual) | 20 | 3.89 | 0.0037 |
| Error | 20 | 6.925 | 0.346 | Var(Residual) |  |  |  |  |

**Answer to 17.11:**

Fixed effects for chemical:

H0: φ2chemical = 0, HA :φ2chemical > 0

Decision: With 5% significance level, we reject the null hypothesis

Conclusion: There is significantly sufficient evidence that there is variability amongst the ith levels of chemical.

Random effects for location:

H0: σ2location = 0, HA: σ2location > 0

Decision: With 5% significance level, we fail to reject the null hypothesis

Conclusion: There is not significantly sufficient evidence that there is variability amongst the jth levels of location.

Random effects for interaction due to chemical\*location:

H0: σ2chemical \* location = 0, HA: σ2chemical \* location > 0

Decision: With 5% significance level, we reject the null hypothesis

Conclusion: There is significantly sufficient evidence that there is variability amongst the interaction due to jth levels of location and ith levels of chemical.

**Answer to 17.27(a):**

The mixed effects model can be represented as:

yijk = μ + τi + βj + τβij + εk(ij)

here i= 1,2, 3, 4, and j =1,2, 3

Furthermore,

yijk is the kth observation for the level of i, for factor machine, the level of j, for factor operator.

μ = overall mean response which is unknown

τi = the random effect corresponding to the ith level of factor ‘machine’. The τi s have independent normal distribution with mean zero and variance σ2machine

βj = the random effect corresponding to the jth level of factor ‘operator’. The βj s have independent normal distribution with mean zero and variance σ2operator

τβij = the random effect due to interaction of the ith level of factor machine, and the jth level of factor operator. The τβij s have independent normal distribution with mean zero and variance σ2τβ

εk(ij) = the random error associated with the kth observation and has an independent distribution with mean zero, and variance σ2

εk(ij) , τβij , and τi, βj are mutually independent

**Answer to 17.27(b):**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Source | df | Sum of Squares | Mean Square | Expected mean Square | Error term | Error DF | F Value | p-value |
| Machine | 3 | 12.4583 | 60.044 | Var(Residual) + 2 Var (machine\*operator) + 6 Var (machine) | MS (machine\*operator) | 6 | 0.56 | 0.6619 |
| Operator | 2 | 160.333 | 0.952 | Var(Residual) + 2 Var (machine\*operator) + 8 Var (operator) | MS (machine\*operator) | 6 | 10.77 | 0.01103 |
| Machine\*Operator | 6 | 44.6667 | 1.346 | Var(Residual) + 2 Var (machine\*operator) | MS(Residual) | 12 | 1.96 | 0.1507 |
| Error | 12 | 45.50 | 0.346 | Var(Residual) |  |  |  |  |

**Answer to 17.27(c):**

Random effects for machine:

H0: σ2machine = 0, HA: σ2machine > 0, p-value =0.6619

Decision: With 5% significance level, we fail to reject the null hypothesis

Conclusion: There is not significantly sufficient evidence that there is variability amongst the ith levels of machine.

Random effects for operator:

H0: σ2operator = 0, HA: σ2operator > 0, p-value =0.0103

Decision: With 5% significance level, we reject the null hypothesis

Conclusion: There is not significantly sufficient evidence that there is variability amongst the jth levels of operator.

Random effects for interaction due to machine\*operator:

H0: σ2machine \* operator = 0, HA: σ2machine \* operator > 0, p-value =0.1507

Decision: With 5% significance level, we fail to reject the null hypothesis

Conclusion: There is significantly sufficient evidence that there is variability amongst the interaction due to jth levels of operator and ith levels of machine.

**Answer to 17.28(a):**

Co-variance estimates from Type III:

Machine = -0.5486

Operator = 9.0903

Machine \* operator = 1.8264

Residual = 3.7917

Co-variance estimates from REML:

Machine = 0

Operator = 9.2275

Machine \* operator = 1.2777

Residual = 3.7917

REML is preferred as it includes adjustments needed for degree of freedom for estimating the mean squares. We also see from the output that the co-variance estimates for REML is different to that of Type III. For Type III the co-variance estimate is negative for machine.

**Answer to 17.28(c):**

The confidence interval for average solder strength = (199.43, 215.16)

This value states that with 95% statistical confidence we can state that the true average solder strength lies between 199.43, and 215.16.