CMPUT 350 F2021
A2 Problem 2 Solutions

a) Claim: $h^*(n)$ is consistent.

Proof:

Need to show:

1) $h^*(v) \le h^*(n) + cost(v,n)$ for arbitrary node v with neighbour n

If $h^*(v) > h^*(n) + cost(v,n)$, then there is a path
v->n->...G which with length < $h^*(v)$. This means that
$h^*(v)$ is not minimal - a contradiction.

2) $h^*(G) = 0$ for goal node G

This is true because no steps need to be taken to arrive at a goal

QED

b) We need to show: $\max(h1(n),\ldots,hk(n)) \le h^*(n)$
L.H.S. $\le \max(h^*(n),\ldots,h^*(n)) = h^*(n) \le h(*)$
hi(n) admissible

QED

c) For node v and neighbour n we have:

$h(v) := \max(h1(v),\ldots,hk(v)) \le \max(h1(n)+cost(v,n),\ldots,hk(n)+cost(v,n))$
hi consistent

$= \max(h1(n),\ldots,hk(n)) + cost(v,n)$
$= h(n) + cost(v,n)$

=> h(n) is consistent

QED

d) True for k=1, but false for k >1. For instance $hi(n)=h^*(n)$, which is admissible.
Then $h1(n)+\ldots+hk(n) = k \cdot h^*(n)$ which is not $\le h^*(n)$ for $k > 1$ QED

e) True. $(h1(n)+\ldots hk(n)/k \le (h^*(n)+\ldots+h^*(n))/k = h^*(n)$
hi(n) admissible

QED