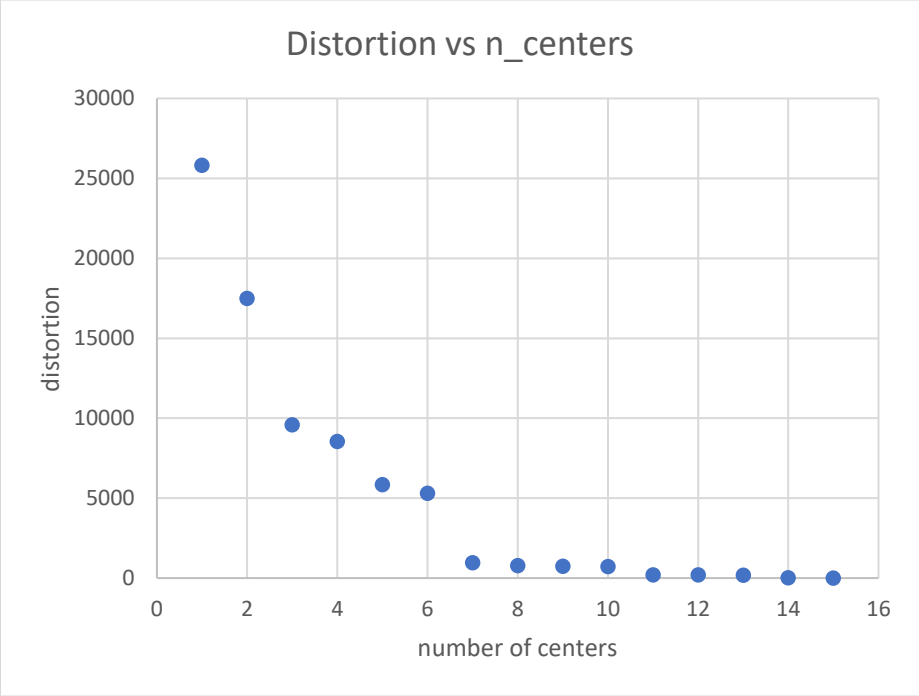
Ans1. The maximum possible clusters for n points are n clusters. The minimum possible clusters for n points is 1 cluster.

Ans2.

Using the algorithm for a custom data set of 15 points



As expected, the graph looks like it has a shape similar to the graph of y=1/n in the top right quadrant.

Ans3. The lowest possible distortion is 0. This will happen when there are n centers for n points. The reason this will happen is because the k-means clusters algorithm will assign each point to itself as a cluster. Thus the distance between all points and their closest centers will be 0.

Ans4. From the above graph it looks like the optimal number of centers is n/3. However there isn't an entirely correct way to determine the optimal number of centers from this graph. In a program, we could use the gaussian mixture model. In this model the program tries different number of centers between 1 and n, and chooses the number that assigns points to centers such that it looks like it has a normal distribution.