Stat 607: HW1

(1) For $u_1, v_1, u_2, v_2 \in \mathbb{R}^m$, show that $(u_1 v_1')(u_2 v_2') = u_2' v_1 (u_1 v_2')$ and that $(u_1 v_1')u_2 = (v_1' u_2)u_1$.

(2) If $A \in \mathbb{R}^{m \times n}$, show that $\|A\|_{\mathsf{F}} \leq \sqrt{\mathsf{Rank}(A)}\|A\|_2$. Show also that if A is of rank n, then $\|A(A'A)^{-1}A'\|_2 = 1$.

(3) Show that for any matrix $A \in \mathbb{R}^{m \times m}$,

$$\|A|_1 \leq \sqrt{m}\|A\|_2 \leq n\|A\|_1, \quad \text{and} \quad \|A\|_\infty \leq \sqrt{m}\|A\|_2 \leq m\|A\|_\infty.$$

(4) In computing, the number of floating-point operations per second (FLOPS) is the most important measure of computer performance. Consider the following resource.

`http://en.community.dell.com/techcenter`
`/high-performance-computing/w/wiki/2329`

   (a) Read that paper and other online resources to understand how to compute the FLOPS of a computer.

   (b) Given a matrix $X \in \mathbb{R}^{m \times m}$, what in the number of floating-point operations needed to compute X^2. Use this result to devise a simple simulation study to approximate the (double precision) FLOPS of your computer. Compare the result with the theoretical results you may find online.