STAT 406: HW8

- All computer code should be written using the language R. Type ALL your code into one PLAIN Text format file. Plain text format is available by default in R. Please do not use Microsoft Word .doc format or .rtf format of .pdf format. Inside your plain text file, make sure you identify each problem in a comment placed at the beginning of the problem. The file name should match your name as in 'JohnDoe.R'.
- Submit your R code file online (under Assignments) at or before the due date, and hand in a hard copy of the code as well as a printed copy of your answers to the questions. The hard copy is due at the beginning of your respective lab sessions.
- I recommend that before submitting your homework, you also create a new directory and run your R code, to make sure that it is self-contained and runs as you intended.

1. Consider estimating the integral $I = \int_0^1 e^{-x^2} dx$. Write the integral as an expectation against the uniform density. Use this to implement a basic Monte Carlo approach to approximate I. Use a Monte Carlo sample size $N = 1000$, and return the estimate and its Monte Carlo error.

2. Consider the vector

$$m = (2.16,\ 0.74,\ 1.87,\ 3.03,\ 3.11,\ 2.74,\ 1.23,\ 3.64,\ 1.57,\ 2.12)\,,$$

and the density

$$f(x) = \begin{cases} \frac{x^{\alpha-1}\exp\left(-\beta x - \frac{1}{2}\sum_{i=1}^{10}(x-m_i)^2\right)}{\int_0^\infty x^{\alpha-1}\exp\left(-\beta x - \sum_{i=1}^{10}(x-m_i)^2\right)dx}, & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

where $\alpha = 2$, and $\beta = 0.5$.

(a) Describe a rejection method to sample from f using the instrument density g given by

$$g(x) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

where $\Gamma(x)$ is the Gamma function.

(b) Write a function that takes one argument n and return n values generated from density f according to the rejection algorithm presented above.

(c) Use your algorithm to estimate the integral $I = \int_0^\infty x f(x) dx$ by Monte Carlo, with $n = 1000$. Report the estimate and a Monte Carlo confidence interval for I.