Statistics 700 Homework 3

Importance Sampling

Due date: 6:00 pm (EST) Oct. 17, 2017

ESS for importance sampling. In class, we talked about the effective sample (ESS) size of an importance sampling with m samples defined as

$$ESS(m) = \frac{m}{1 + var_g[w(x)]}.$$

1. Suppose $\pi(x) \propto$ standard Gaussian density, $g(x) \propto$ student t distribution with 2 degrees of freedom, $h(x) = x$. Implement an importance sampler and calculate $ESS(m)$ for $m = 50, 100, 200, 500, 1000$.

2. Suppose $g(x) \propto$ standard Gaussian density, $\pi(x) \propto$ student t distribution with 2 degrees of freedom, $h(x) = x$. Implement an importance sampler and calculate $ESS(m)$ for $m = 50, 100, 200, 500, 1000$.

3. What do you find by comparing the results above?

4. Suppose the target density is

$$\pi(\mu, \sigma^2) \propto \sigma^{-5} \exp\left[-\frac{(\mu - 1)^2 + 4}{2\sigma^2}\right],$$

   where $(\mu, \sigma^2) \in [-3, 5] \times [0.01, 50]$.

   (a) Make a contour plot of the target density in the specified range.

   (b) Given the target density and the contour plot, how can you choose a good importance function? Design an importance sampling procedure and estimate the ESS for several different sample sizes. Compare your samples with that obtained from grid sampling.

(c) Optional (bonus points ≤ 5). If μ is the only quantity of interest, does marginalization help with obtaining better importance samples? If so, can you verify it?

Rejection control algorithm. In classed, we talked about the rejection control algorithm which has the following two steps:

1. For $j = 1, \ldots, m$, accept $x^{(j)}$ with probability

$$r^{(j)} = \min\left\{1, \frac{w^{(j)}}{c}\right\},$$

where $w^{(j)} = \pi(x^{(j)})/g(x^{(j)})$.

2. If the jth sample $x^{(j)}$ is accepted, its weight is updated to $w^{(*j)} = q_c w^{(j)}/r^{(j)}$, where

$$q_c = \int \min\left\{1, \frac{w(x)}{c}\right\} g(x)dx,$$

where $w(x) = \pi(x)/g(x)$.

Suppose $\pi(x) \propto$ Gaussian density with mean 0 and standard deviation 0.3, $g(x) \propto$ student t distribution with 2 degrees of freedom.

1. Implement the importance sampling with and without rejection control.
2. Do you see improvement by using rejection control with different c?
3. What value of c do you choose to use finally and why?
4. Repeat the above procedures when $\pi(x) \propto$ Gaussian density with mean 0 and standard deviation 3.
5. Optional (bonus points ≤ 5). Explain intuitively how to choose a c such that the rejection control importance sampling gives satisfactory results.