STAT 406 Lab 4, 10/06/2015

1. Generate Random Variable

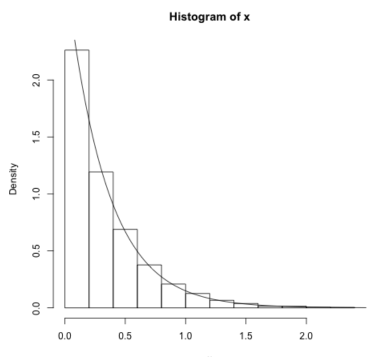
We learned from lecture that random variables from a particular distribution can be gen- erated from uniform random variables by inverting their cumulative distribution function (cdf). That is, if you create samples $U_1, \cdots, U_n$ from $Uniform(0,1)$ distribution and compute $F^{-1}(U_1), \cdots, F^{-1}(U_n)$, then what you get is samples from a distribution with cdf F . Using this property, we'll generate exponential random variables and gamma random variables.

(a) The Exponential(λ) distribution has cdf:

$$F(x) = 1 - e^{-\lambda x}, \lambda > 0$$

Using $runif$ function, generate 100 samples from $Exponential(3)$ distribution using the inversion method. Graph the density histogram for the sample with the true density superimposed for comparison.

```
n = 100
lambda = 3
x = -(1/lambda)*log(runif(n))
hist(x, prob = TRUE)
y = seq(0,10,length = 1000)
lines(y,dexp(y,3))
```



(b) $Gamma(k, \theta)$, $k > 0$, $\theta > 0$ distribution has density:

$$f(x) = \frac{\theta^k}{(k-1)!} x^{k-1} e^{-\theta x}$$

where $k \ and \ \theta$ are shape and rate parameters. What are $k \ and \ \theta$ for $Exponential(\lambda)$ distribution in terms of Gamma distribution?

Gamma distribution also has the property that if $X \sim Gamma(a, \theta)$ and $Y \sim Gamma(b, \theta)$, and X and Y are independent, then

$$X + Y \sim Gamma(a + b, \theta).$$

Using this property, create 10 samples from Gamma(5, 3) distribution.

```
n = 10
k = 5
lambda = 3
x = matrix(-(1/lambda)*log(runif(n*k)), ncol=k)
g = apply(x, 1, sum)
```

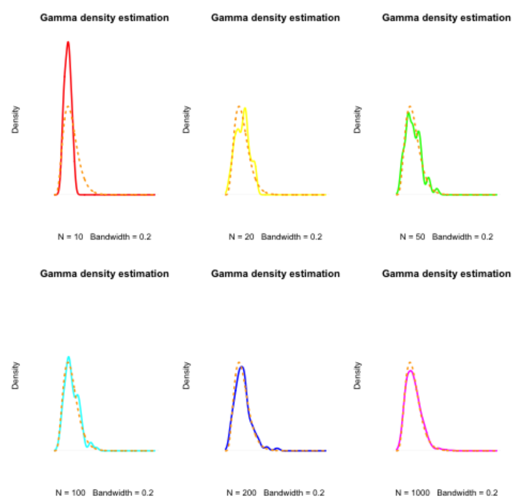
(c) Take an n-sample from the Gamma(5, 3) using your algorithm from part (b), with $n = 10, 20, 50, 100, 200, 1000$.

```
n = c(10, 20, 50, 100, 200, 1000)
g = list()
k = 5
lambda = 3
g = lapply(n, function(i) matrix(-(1/lambda)*log(runif(i*k)), ncol=k))
class(g) #to see what is the data structure used for g
sapply(g, dim) #check the dimension of each matrix in g
## the following will get the gamma random numbers of different sizes n. ##
gamma = lapply(g, function(M) matrix(-(1/lambda)*log(runif(i*k)), ncol=k))
class(gamma) #to see what is the data structure
sapply(gamma, length) #check the dimension of each vector in g
```

(d) Now, use R function **density** to obtain an estimate of the underlying density. Show how the estimate gets better as n increases, and graph the estimated densities and the true density superimposed for comparison.

```
#continue with gamma from the output of part (c)
grid = 1000
y = seq(0,10,length = grid)
plot(0,xlim=c(0,10),ylim=c(0,1),xlab="x",ylab="Density",main="Gamma density estimation", type='n')
par(mfrow=c(2,3))
for(i in 1:6){
    di = density(gamma[[i]],from=0,to=10, n = grid, bw=0.2)
    plot(di, xlim=c(0,10), ylim=c(0,1), axes=F, col=rainbow(6)[i], lwd = 2, main="Gamma density estimation")
    lines(y,dgamma(y,5,3), col = 'orange', lty = 3, lwd = 2)
}

names(di) #check to see what are in the density estimator output di, notice di$x and di$y
##multiple plot in R : http://cran.r-project.org/doc/contrib/Lemon-kickstart/kr_addat.html
```



2. More of LLN - Empirical Distribution Functions

Write a function which calculates the empirical distribution function of $X_1, \cdots, X_n \overset{\text{iid}}{\sim} N(0,1)$. The empirical distribution function is defined as

$$\Phi_m(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \le x)$$

where $I(X \le x)$ is an indicator function, taking value 1 if $X \le x$ and 0 otherwise. Plot this function for $x \in [-4, 4]$. Let $m = 5, 50, 500$ respectively and plot everything on the same figure, with the true Normal cdf superimposed for comparison.

```
Phi.m = function(m,xseq){
# xseq can be either a scalar or a vector
X = rnorm(m)
Phi.m.x = sapply(xseq,function(x){mean(X<x)})
return(Phi.n.x)
}

m = c(5,50,500)
xseq = seq(-4,4,by=0.01)
plot(0,xlim=c(-4,4),ylim=c(0,1),xlab="x",ylab="Phi.m(x)",main="Empirical CDF", type='n')
for (i in 1:length(m)){
par(new=T)
plot(xseq,Phi.n(m[i],xseq),col=i,axes=F,type="l",xlim=c(-4,4),ylim=c(0,1),xlab="",ylab="")
}
lines(xseq,pnorm(xseq,mean=0,sd=1),col="blue",lty=3,lwd=2)
legend("topleft",legend=paste(m,'points'),col=1:length(m),lty=1)
```