STAT 406 Fall 2015 Lab 8

Consider the following model. Given the data tuples $(y_i, x_i), i = 1, 2, \cdots, n$, we model the distribution of Y_i's such that

$$Y_i | x_i, \lambda \sim \text{Exponential}(\lambda x_i)$$

with parameter λ. Notice that we treat x_i's as fixed input data values (as one does for linear regressions in usual case which is called "fixed design") without modeling them as realizations of a random variable with a distribution.

Given the maximum likelihood estimator

$$\widehat{\lambda}_{\text{MLE}} = \frac{n}{\sum_{i=1}^{n} x_i y_i}$$

We are interested in the bias, variance and MSE of the maximum likelihood estimator $\widehat{\lambda}_{\text{MLE}}$:

Now please download `lab8.Rdata` from CTools, and load the data using `load("lab8.Rdata")`. The name of the dataset is `sampledata`. We will explore and solve the interested question in the sequel.

1. Suppose the parameter λ is given apriori, estimate bias, variance and MSE of the maximum likelihood estimator $\widehat{\lambda}_{\text{MLE}}$ with Monte Carlo methods (this is a review and warm up question).
2. Use parametric bootstrap to evaluate the bias and variance of the MLE estimate $\widehat{\lambda}_{\text{MLE}}$ with Monte Carlo methods.
3. Suppose that you still use the same estimator for λ as in (2). Apply nonparametric bootstrap technique to evaluate the bias and variance of the point estimate applied to the given data set.