

4. Collaborative Filtering: the Naive Approach

Collaborative Filtering: the Naive Approach

The video shows a chalkboard with the following content:

- Top Left:** A matrix Y with rows for users and columns for movies. The matrix is:

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| 1 | 5 | 5 | 5 | 5 | 5 |
| 2 | 5 | 5 | 5 | 5 | 5 |
| 3 | 5 | 5 | 5 | 5 | 5 |
| 4 | 5 | 5 | 5 | 5 | 5 |
| 5 | 5 | 5 | 5 | 5 | 5 |
- Top Right:** Formulas for K-nearest neighbors and matrix factorization.

$$\hat{Y}_{ai} = \frac{\sum_{b \in KNN(a,i)} Y_{bi}}{K}$$

$$\hat{Y}_{ai} = \sum_{b \in KNN(a,i)} \frac{Y_{bi}}{K}$$
- Bottom Left:** Two matrices Y and X with dimensions n and m . Y is labeled "given" and X is labeled "output". A set $D = \{(a,i) | Y_{ai} \text{ is given}\}$ is defined.
- Bottom Right:** The objective function $J(X)$ and its derivative.

$$J(X) = \sum_{(a,i) \in D} \frac{(Y_{ai} - X_{ai})^2}{2} + \frac{\lambda}{2} \sum_{(a,i)} X_{ai}^2$$

$$\frac{\partial J(X)}{\partial X_{ai}} = \frac{\partial}{\partial X_{ai}} \left(\frac{(Y_{ai} - X_{ai})^2}{2} + \frac{\lambda}{2} X_{ai}^2 \right) = 0 \implies X_{ai} = \frac{Y_{ai}}{1 + \lambda}$$

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Compute the Derivative of the Regression Objective

2.0/2 points (graded)

Recall that each user a has a set of movies that (s)he has already rated. Let Y be a matrix with n row and m columns whose $(a, i)^{\text{th}}$ entry Y_{ai} is the rating by user a of movie i if this rating has already been given, and blank if not. Our goal is to come up with a matrix X that has no blank entries and whose $(a, i)^{\text{th}}$ entry X_{ai} is the prediction of the rating user a will give to movie i .

Let D be the set of all (a, i) 's for which a user rating Y_{ai} exists, i.e. $(a, i) \in D$ if and only if the rating of user a to movie i exists.

A naive approach to solve this problem would be to minimize the following objective:

$$J(X) = \sum_{a,i \in D} \frac{(Y_{ai} - X_{ai})^2}{2} + \frac{\lambda}{2} \sum_{(a,i)} X_{ai}^2$$

where the first term is the sum of the squared errors for entries with observed rating, and the second term is a regularization term roughly to prevent the predictions to become extremely large, and the parameter λ controls the balance between these two terms.

Compute the derivative $\frac{\partial J}{\partial X_{ai}}$ of the objective function $J(X)$. (Note that $J(X)$ can be viewed as a function of the variables X_{ai} .)

(Type $x_{\{ai\}}$ for matrix entries X_{ai} , $y_{\{ai\}}$ for matrix entries Y_{ai} and "lambda" for λ . Note that X and Y are capital letters to represent matrices.)

For (any fixed) $(a, i) \in D$,

$$\frac{\partial J}{\partial X_{ai}} = \boxed{-(Y_{ai} - X_{ai}) + \lambda X_{ai}} \quad \checkmark \text{ Answer: } X_{ai} - Y_{ai} + \lambda X_{ai}$$

For (any fixed) $(a, i) \notin D$:

$$\frac{\partial J}{\partial X_{ai}} = \boxed{\lambda X_{ai}} \quad \checkmark \text{ Answer: } \lambda X_{ai}$$

STANDARD NOTATION

Solution:

Derive the objective and remember to treat any entry in the matrix that is not the one that we are deriving by as a constant. Hence, the derivative of all components of the sum that are not (a, i) will be zero.

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You have used 1 of 2 attempts

i Answers are displayed within the problem

Performance of the Naive Approach

2.0/2 points (graded)

Let us now check the quality of the solution when using this wrong approach. Recall the naive approach assumes independence between all entries of the matrix.

What value of the matrix X will minimize the loss $J(X) = \sum_{a,i \in D} \frac{(Y_{ai} - X_{ai})^2}{2} + \frac{\lambda}{2} \sum_{(a,i)} X_{ai}^2$? That is, for each (a, i) , solve the following equation for X_{ai} :

$$\frac{\partial J}{\partial X_{ai}} = 0.$$

We will denote the argmin as \widehat{X} and its components as \widehat{X}_{ai} .

For $(a, i) \in D$:

$$\widehat{X}_{ai} = \boxed{Y_{ai}/(1+\lambda)} \quad \checkmark \text{ Answer: } Y_{ai}/(1+\lambda)$$

For $(a, i) \notin D$:

$$\widehat{X}_{ai} = \boxed{0} \quad \checkmark \text{ Answer: } 0$$

STANDARD NOTATION

Solution:

Derive the objective (see previous question) and compare to zero to find the values at the minima. Using the results from the problem above, we have For $(a, i) \in D$:

$$\frac{\partial J}{\partial X_{ai}} = X_{ai} - Y_{ai} + \lambda X_{ai} = 0 \iff X_{ai} = \frac{Y_{ai}}{1 + \lambda}$$

For $(a, i) \notin D$:

$$\frac{\partial J}{\partial X_{ai}} = \lambda X_{ai} = 0 \iff X_{ai} = 0.$$

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You have used 1 of 3 attempts

i Answers are displayed within the problem

Discussion

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|--|----------------------|
| <p>🗨️ [Staff] Lambda not accepted as a variable.</p> <p>I wrote the answer to question no. 1, but the grader said lambda not accepted as a variable. After 0 attempts left. I looked the answer, and it was the same as I wrote in the box.</p> | 7 ▾ |
| <p>✅ 5:12 . X . Can someone explain it?</p> | 8 ▾ |
| <p>🗨️ Meaningful symbols continuation</p> | 5 ▾ |
| <p>✅ Does this regularization make sense, even naively?</p> | 2 ▾ |
| <p>✅ [STAFF] Invalid Input: ai not permitted in answer as a variable</p> <p>I am getting Invalid Input: ai not permitted in answer as a variable when answering both questions. Would like to know why.</p> | 1 new_ 3 ▾ |
| <p>✅ About derivative in the Question 1</p> <p>Can anyone tell me for (any fixed) $(a,i) \in D$, why the derivative is not $(\lambda+1) \cdot X_{ai}$?</p> | 4 ▾ |
| <p>✅ [STAFF] Please SEE THIS ASAP !!! I CAN'T SUBMIT AND DUE TIME IS STILL NOT OVER . PLEASE VIEW THE ATTACHED SCREENSHOTS</p> | 11 ▾ |
| <p>🗨️ Derivative of the Regression Exercise - Case-sensitive input</p> | 2 ▾ |
| <p>✅ Can we assign zero to blank rating?</p> <p>Hi. Given that we can estimate from 1 to 5, can we assign 0 to blank entries? Or we should specifically left it blank? Thanks in advance.</p> | 2 ▾ |
| <p>✅ X-Y or Y-X?</p> <p>i would like to ask is there any differences between $Y_{\{ai\}} - X_{\{ai\}}$ and $X_{\{ai\}} - Y_{\{ai\}}$? i know the difference is about +/- but in $J(X)$ we had taken the squared value of it, do we hav...</p> | 3 ▾ |
| <p>✅ Regularization term</p> | 3 ▾ |
| <p>✅ [Staff] Performance of the Naive Approach - Answer not displayed</p> <p>trying to solve the problem but failed. Can you please tell me the answer?</p> | 3 ▾ |
| <p>✅ Compute the Derivative of the Regression Objective</p> <p>How do we write the summation in the answer box?</p> | 2 ▾ |
| <p>✅ Compute the Derivative of the Regression Objective</p> | 2 ▾ |