

QUESTION: Provide your comments on the effectiveness of a *median filter* and a *Gaussian filter* for the example above? Explain why?

Your answer:

In the example above, the median filter works better than the Gaussian filter as it is able to remove most of the salt and pepper noise from the original image, whereas much of the noise still exists when a Gaussian filter is used instead. Median filter will work better because it replaces the value of a pixel by the median of nearby pixels. The Gaussian filter on the other hand replaces the value of a pixel by the mean value of an area centered at the pixel. Hence, the median filter will be less affected by noisy data in a given area of pixels.

```
In [23]: def myMedianBlur(img, size):
    """
    Implementation of median blur filter.
    """
    out = img.copy()
    W,H = img.shape[0],img.shape[1]
    s = (size - 1)/2
    s = int(s)
    # TODO: Implement the median blur.
    # NOTE: Your implementation is NOT necessary to provide the identical
    # output as OpenCV built-in function. However, it should be visually very
    # similar.
    #####
    for row in range(H):
        for col in range(W):
            img_pad = np.lib.pad(img, (s, s), (s, s), "constant", constant_values=0)
            #####
            out[row,col] = findMedian(img_pad[row-row*W+1:row*W+1+2*s,col-col*W+1:col*W+1+2*s])
            #####
    #####
    return out

In [24]: img = cv2.imread('imgs/SaltAndPepperNoise.jpg', 0)
myMedian = myMedianBlur(img,5)
median = cv2.medianBlur(img,5)

# Note that your implementation is NOT necessary to provide
# the identical output as OpenCV built-in function. However,
# it should visually very similar.
plt.figure(figsize=(16,8))
plt.subplot(121),plt.imshow(median, 'gray')
plt.title('OpenCV Median Blur'),plt.xticks([],),plt.yticks([])
plt.subplot(122),plt.imshow(myMedian, 'gray')
plt.title('My Median Blur'),plt.xticks([],),plt.yticks([])
plt.show()
```



Image gradient

For 1-D continuous function $f(x)$, the gradient is given as:

$$D_x[f](x) = \frac{d}{dx}f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}, \quad \text{or} \quad \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

For 1-D discrete function $f[n]$, the gradient becomes difference.

$$D_x[f][n] = f[n + 1] - f[n], \quad \text{or} \quad \frac{f[n + 1] - f[n - 1]}{2}$$

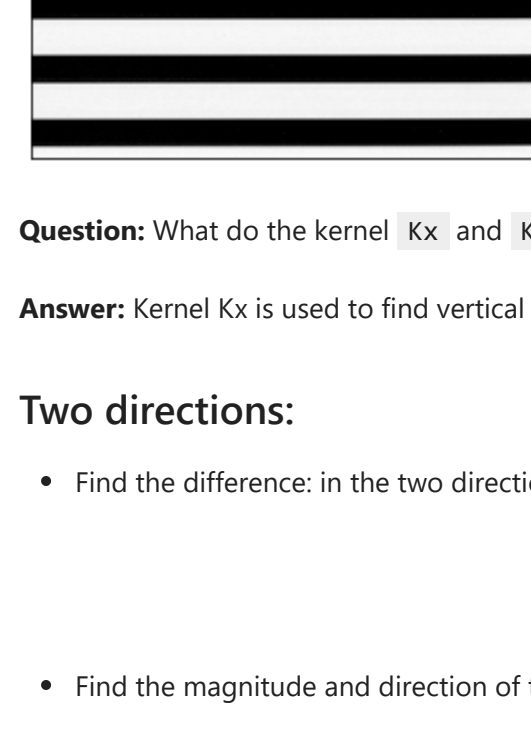
The kernel to find gradient in 1-D discrete function is $[1, 0, -1]$.

```
In [25]: img = cv2.imread('imgs/banded_vertical.jpg', 0).astype(np.float32)

#####
# TODO: Create a 3x3 kernel, Kx, to find the gradient in x-axis of an image.#
#####
Kx = np.float32([[1, 0, -1],
                [2, 0, -2],
                [1, 0, -1]])

#####
#                                     #
#####
dstx = cv2.filter2D(img,-1,Kx)

plt.figure(figsize=(10,5))
plt.subplot(121),plt.imshow(img, cmap='gray')
plt.title('Original'),plt.xticks([],),plt.yticks([])
plt.subplot(122),plt.imshow(np.abs(dstx), cmap='gray')
plt.title('Output 1'),plt.xticks([],),plt.yticks([])
plt.show()
```

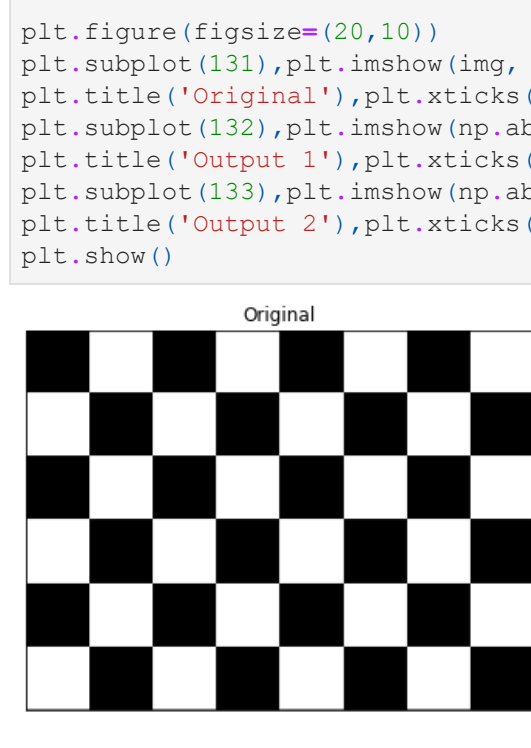


```
In [26]: img = cv2.imread('imgs/banded_horizontal.jpg', 0).astype(np.float32)

#####
# TODO: Create a 3x3 kernel, Ky, to find the gradient in y-axis of an image.#
#####
Ky = np.float32([[0, 1, 0],
                [-1, -2, -1]])

#####
#                                     #
#####
dsty = cv2.filter2D(img,-1,Ky)

plt.figure(figsize=(10,5))
plt.subplot(121),plt.imshow(img, 'gray')
plt.title('Original'),plt.xticks([],),plt.yticks([])
plt.subplot(122),plt.imshow(np.abs(dsty), 'gray')
plt.title('Output'),plt.xticks([],),plt.yticks([])
plt.show()
```



Question: What do the kernel K_x and K_y do in image processing?

Answer: Kernel K_x is used to find vertical edges in images while Kernel K_y is used to find horizontal edges in images.

Two directions:

- Find the difference in the two directions:

$$g_x[m,n] = f[m + 1, n] - f[m - 1, n]$$
$$g_y[m,n] = f[m, n + 1] - f[m, n - 1]$$

- Find the magnitude and direction of the gradient vector:

```
In [27]: img = cv2.imread('imgs/chequered.jpg', 0).astype(np.float32)

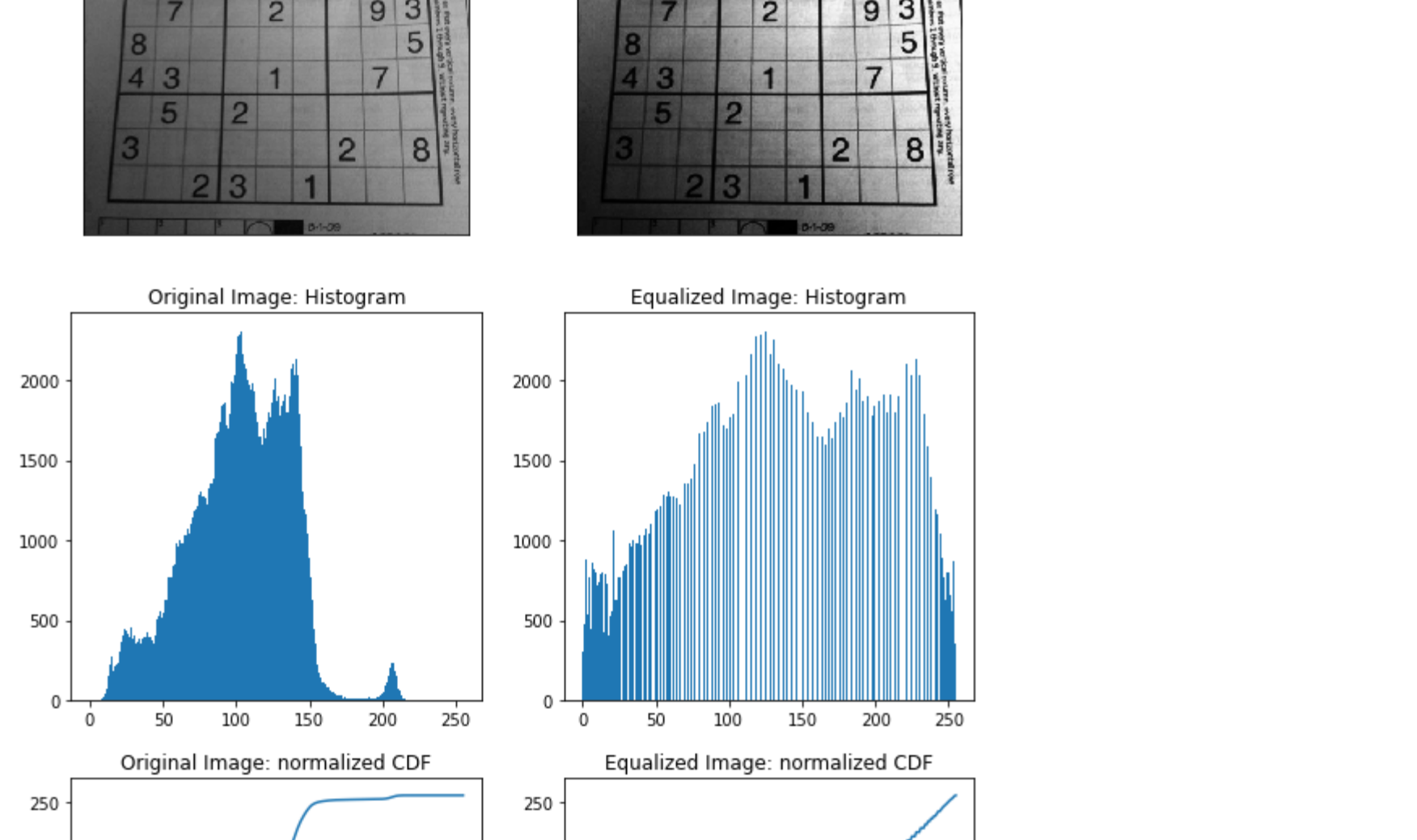
#####
# TODO: Using the theory provided above, compute the magnitude of 2 #
#####
Kx = np.float32([[1, 0, -1],
                [2, 0, -2],
                [1, 0, -1]])

Ky = np.float32([[0, 1, 1],
                [0, 0, 0],
                [-1, -2, -1]])

dstx = cv2.filter2D(img,-1,Kx)
dsty = cv2.filter2D(img,-1,Ky)

dst1 = np.sqrt(dstx**2 + dsty**2)
#####
# You can achieve a similar (NOT identical) output with the following code line.
K = np.array([[0, 1, 0],
              [1,-4,1],
              [0, 0, 0]], dtype=np.float32)
dst2 = cv2.filter2D(img,-1,K)

plt.figure(figsize=(20,10))
plt.subplot(131),plt.imshow(img, 'gray')
plt.subplot(132),plt.imshow(img_eq, cmap='gray'),plt.title('Equalized Image'),plt.xticks([],),plt.yticks([])
plt.subplot(133),plt.hist(img_eq.ravel(), bins=256, range=(0.0, 255.0)),plt.title('Original Image: Histogram')
plt.subplot(134),plt.hist(img_eq.ravel(), bins=256, range=(0.0, 255.0)),plt.title('Equalized Image: Histogram')
plt.subplot(135),plt.plot(range(0,256),np.cumsum(hist_eq[0]*255/(W*H))),plt.title('Original Image: normalized CDF')
plt.subplot(136),plt.plot(range(0,256),np.cumsum(hist_eq[0]*255/(W*H))),plt.title('Equalized Image: normalized CDF')
plt.show()
```



Histogram

- It is a graphical representation of the intensity distribution of an image.
- It quantifies the number of pixels for each intensity value considered.

Histogram equalization

- Equalization implies mapping one distribution (the given histogram) to another distribution (a wider and more uniform distribution of intensity values) so the intensity values are spreaded over the whole range.
- To accomplish the equalization effect, the remapping should be the cumulative distribution function (cdf) (more details, refer to Learning OpenCV). For the histogram $H(i)$, its cumulative distribution $H'(i)$ is:

$$H'(i) = \sum_{0 \leq j < i} H(j)$$

To use this as a remapping function, we have to normalize $H'(i)$ such that the maximum value is 255 (or the maximum value for the intensity of the image). From the example above, the cumulative function is:

Cumulative distribution function

- Finally, we use a simple remapping procedure to obtain the intensity values of the equalized image:

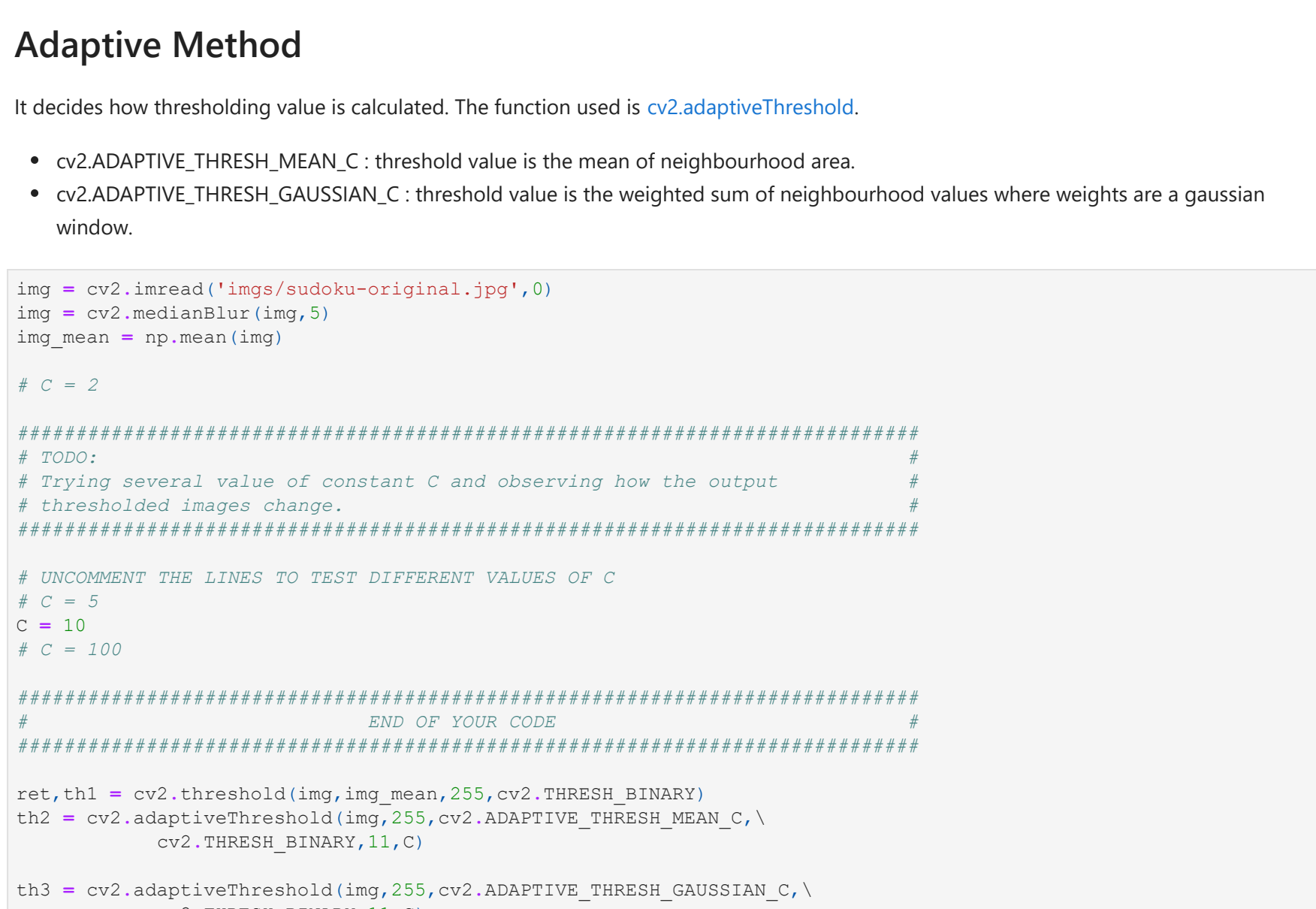
Histogram Equalization

$$equalized(x, y) = H'(\text{vec}(x, y))$$

```
In [28]: img = cv2.imread('imgs/sudoku-original.jpg',0)
W,H = img.shape
img_eq = cv2.equalizeHist(img)

hist = np.histogram(img, bins=256, range=(0.0, 255.0))
hist_eq = np.histogram(img_eq, bins=256, range=(0.0, 255.0))

plt.figure(figsize=(10,15))
plt.subplot(131),plt.imshow(img, cmap='gray'),plt.title('Original Image'),plt.xticks([],),plt.yticks([])
plt.subplot(132),plt.imshow(img_eq, cmap='gray'),plt.title('Equalized Image'),plt.xticks([],),plt.yticks([])
plt.subplot(133),plt.hist(img_eq.ravel(), bins=256, range=(0.0, 255.0)),plt.title('Original Image: Histogram')
plt.subplot(134),plt.hist(img_eq.ravel(), bins=256, range=(0.0, 255.0)),plt.title('Equalized Image: Histogram')
plt.subplot(135),plt.plot(range(0,256),np.cumsum(hist_eq[0]*255/(W*H))),plt.title('Original Image: normalized CDF')
plt.subplot(136),plt.plot(range(0,256),np.cumsum(hist_eq[0]*255/(W*H))),plt.title('Equalized Image: normalized CDF')
plt.show()
```



QUIZ: Is histogram equalization reversible?

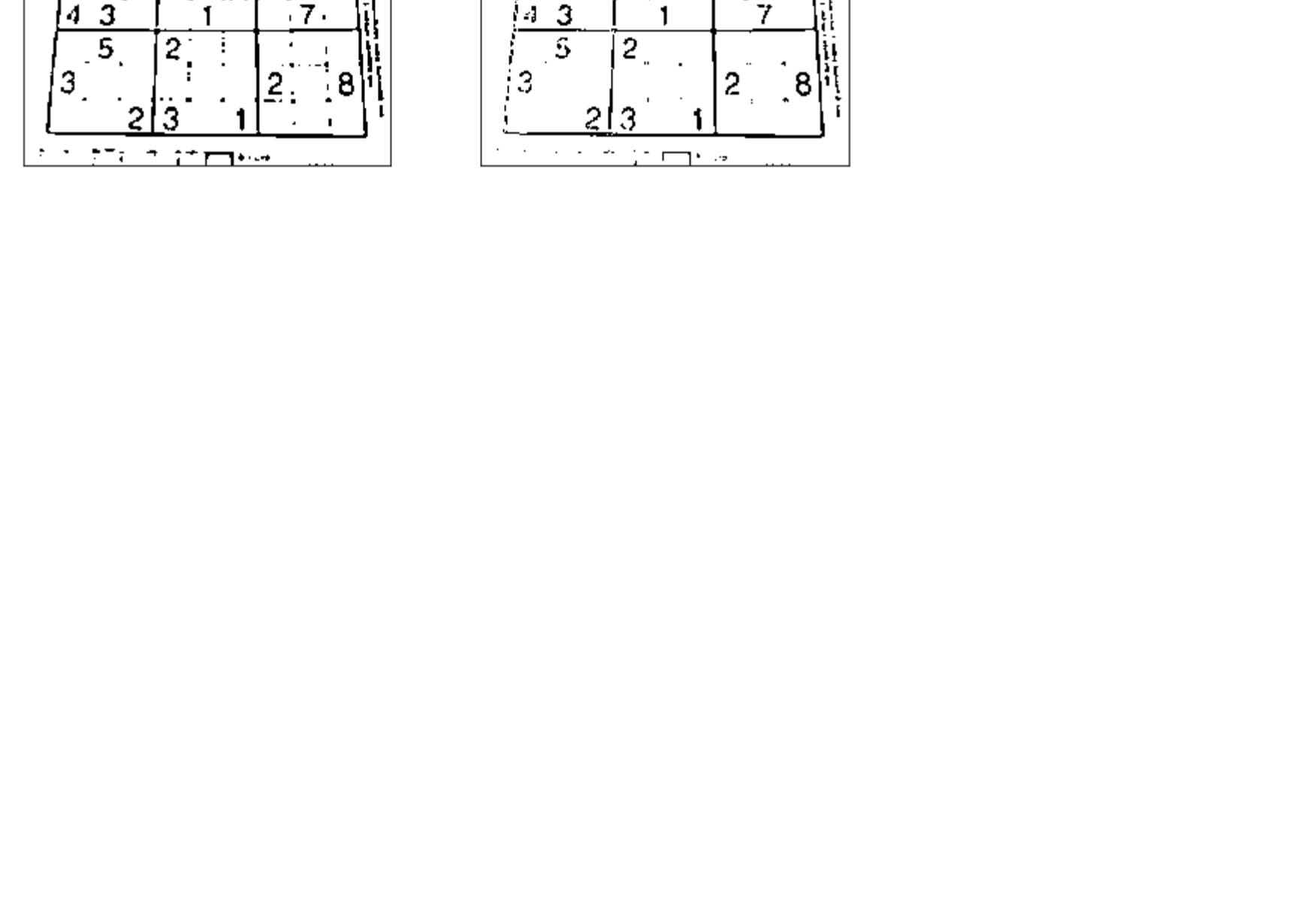
Your answer:

No, histogram equalization is not reversible. Given an equalized image and its initial histogram, the original image cannot be recovered.

```
In [29]: def myEqualizeHist(img):
    """
    A implementation of a histogram equalization for image of 'uint8' data type.
    """
    out = img
    # TODO: Implement the histogram equalization function.
    hist, bins = np.histogram(img, bins=256, range=(0.0, 255.0))
    cdf = hist.cumsum()
    cdf_normalized = cdf * float(hist.max()) / cdf.max()
    cdf_m = np.ma.masked_equal(cdf,0)
    cdf_m = (cdf_m - cdf_m.min())*255/(cdf_m.max()-cdf_m.min())
    cdf = np.ma.filled(cdf_m,0).astype('uint8')
    out = cdf[img]
    #####
    return out

In [30]: # Verify the correctness of your implementation by plotting the
# normalized CDF of equalized image
img = cv2.imread('imgs/sudoku-original.jpg',0)
W,H = img.shape
img_myeq = myEqualizeHist(img)

# Your implementation may NOT need to return an image that is
# exactly the same as the one OpenCV built-in function does.
# However, the normalized CDF should make sense.
hist_myeq = np.histogram(img_myeq, bins=256, range=(0.0, 255.0))
plt.figure(figsize=(5,5))
plt.plot(range(0,256),np.cumsum(hist_myeq[0]*255/(W*H)))
plt.title('Equalized Image: normalized CDF')
plt.show()
```



QUIZ: Is histogram equalization reversible?

Your answer:

No, histogram equalization is not reversible. Given an equalized image and its initial histogram, the original image cannot be recovered.

```
In [31]: # Get list of available flags for thresholding styles
flags = [i for i in dir(cv2) if i.startswith('THRESH_')]
print(flags)

['THRESH_BINARY', 'THRESH_BINARY_INV', 'THRESH_MASK', 'THRESH_OTSU', 'THRESH_TOZERO', 'THRESH_TOZERO_INV', 'THRESH_TRIANGLE', 'THRESH_TRUNC']
```

Adaptive Method

- It decides how thresholding value is calculated. The function used is `cv2.adaptiveThreshold`.
- `cv2.ADAPTIVE_THRESH_MEAN_C`: threshold value is the mean of neighbourhood area.
- `cv2.ADAPTIVE_THRESH_GAUSSIAN_C`: threshold value is the weighted sum of neighbourhood values where weights are a gaussian window.

```
In [37]: img = cv2.imread('imgs/sudoku-original.jpg',0)
img = cv2.medianBlur(img,3)
img_mean = np.mean(img)

# C = 2

#####
# TODO:
# Trying several value of constant C and observing how the output #
# thresholded images change. #
#####
# UNCOMMENT THE LINES TO TEST DIFFERENT VALUES OF C
C = 10
C = 100

#####
#                                     #
#####
END OF YOUR CODE

ret,th1 = cv2.threshold(img,img_mean,255,cv2.THRESH_BINARY)
th2 = cv2.adaptiveThreshold(img,255,cv2.ADAPTIVE_THRESH_MEAN_C,\
                           cv2.THRESH_BINARY,11,C)
th3 = cv2.adaptiveThreshold(img,255,cv2.ADAPTIVE_THRESH_GAUSSIAN_C,\
                           cv2.THRESH_BINARY,11,C)

titles = ['Original Image', 'Global Thresholding (v = 103.69)',
          'Adaptive Mean Thresholding', 'Adaptive Gaussian Thresholding']
images = [img, th1, th2, th3]

fig = plt.figure(figsize=(10, 10))

for i in range(4):
    plt.subplot(2,2,i+1)
    plt.imshow(images[i], 'gray')
    plt.title(titles[i])
    plt.xticks([])
    plt.yticks([])

plt.show()
```

