slr

Models Optimization

Machine Learning

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Regression Techniques

Simple Linear Regression (SLR)

Rect Line

$y = mx + b$

Regression Model

$y = \beta_0 + \beta_1 x + \epsilon$

Regression Rect

$\hat{y} = \beta_0 + \beta_1 x + \epsilon$

$\hat{y}_i = \beta_0 + \beta_1 x_i + \epsilon_i$

The error Eq: Ordinary Least Squares

$\mathcal{E} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

The Loss function (OLS)

$\mathcal{E} = \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2$

To minimize the $\mathcal{E}$

1. $\frac{\partial}{\partial \beta_0} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 = 0$

2. $\frac{\partial}{\partial \beta_1} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 = 0$

To minimize $\mathcal{E}$ w.r.t β_0

$$f(x)^n = nf(x)^{n-1} f'(x)$$

$$\frac{\partial}{\partial \beta_0} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 = 0$$

$$\frac{\partial \mathcal{E}}{\partial \beta_0} = 2 \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]$$

$$\frac{\partial \mathcal{E}}{\partial \beta_0} = 2 \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)](-1)$$

$$\rightarrow 2/2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)(-1) = 0/2$$

$$\rightarrow \sum_{i=1}^{n} (-y_i + \beta_0 + \beta_1 x_i) = 0$$

$$\rightarrow -\sum_{i=1}^{n} y_i + \beta_0 \sum_{i=1}^{n} + \beta_1 \sum_{i=1}^{n} x_i = 0$$

$$\rightarrow \beta_0 \sum_{i=1}^{n} + \beta_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

To minimize $\mathcal{E}$ w.r.t β_1

$$\frac{\partial}{\partial \beta_1} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 = 0$$

$$\frac{\partial \mathcal{E}}{\partial \beta_1} = 2 \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]$$

$$\frac{\partial \mathcal{E}}{\partial \beta_1} = 2\sum_{i=1}^{n}[y_i - (\beta_0 + \beta_1 x_i)](-x_i)$$

$$\rightarrow 2/2\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0/2$$

$$\rightarrow \sum_{i=1}^{n}(-x_i y_i + \beta_0 x_i + \beta_1 x_i^2) = 0$$

$$\rightarrow \sum_{i=1}^{n} x_i y_i + \beta_0 \sum_{i=1}^{n} x_i + \beta_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\beta_0 \sum_{i=1}^{n} x_i + \beta_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$$

Normal equations

1. $n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$
2. $\beta_0 \sum_{i=1}^{n} x_i + \beta_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$

Matrix notation

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

To solve the SoLE for β_1

- $D_s = ?$
- $D_{\beta_1} = ?$
- $\beta_1 = \frac{D_{\beta_1}}{D_s}$

$$D_s = n\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i$$

β_1 Determinant

$$\begin{bmatrix} n & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

$$D_{\beta_1} = n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i$$

$$\beta_1 = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i}$$

To solve the SoLE for β_0

Given the following eq(1):

1. $n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$

let’s solve β_0:

$$\beta_0 = \frac{\sum_{i=1}^{n} y_i - \beta_1 \sum_{i=1}^{n} x_i}{n}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

Simple Linear Regression for Predictive Analysis

$$\beta_0 = \frac{\sum_{i=1}^{n} y_i - \beta_1 \sum_{i=1}^{n} x_i}{n}$$

$$\beta_1 = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i}$$