(1) Consider the vectors

$$u = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}; \qquad v = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

(a) Find the projection of u onto v
(b) Find the projection of v onto u

(2) Apply the Gram-Schmidt process to transform the given basis into an orthonormal basis.

$$\left\{ \begin{bmatrix} 4 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$$

In order to make it easier to match up with the solutions that will be on the submission page on Canvas, please apply Gram-Schmidt in the order the original vectors are in.

(3) Apply the Gram-Schmidt process to transform the given basis into an orthonormal basis.

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

In order to make it easier to match up with the solutions that will be on the submission page on Canvas, please apply Gram-Schmidt in the order the original vectors are in.

(4) Find the least squares solution, $\hat{x}$ to

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

(5) Find the least squares solution, $\hat{x}$ to

$$\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

(6) Find a quadratic fit for data points $(-2, 0)$, $(-1, 0)$, $(0, 1)$, $(1, 2)$, $(2, 5)$. This is similar to finding a linear fit, which I showed in the lectures. For a linear fit we use the equation $y = mx + b$ to write down our matrix equation. For a quadratic fit we will use the same idea, except with the equation $y = ax^2 + bx + c$. Notice that the matrix for the quadratic fit will have three columns.

I didn't show this in the lecture or notes because I want you to practice going from something we did (linear fit) to something we didn't (quadratic fit) using the ideas of the linear fit. I will put a problem on the Final that goes a few steps beyond, but you can again use the same ideas.

(7) Solve the eigenvalue problem for the matrix. Please find the eigenvalues and eigenvectors.

$$\begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

(8) Solve the eigenvalue problem for the matrix. Please find the eigenvalues and eigenvectors.

$$\begin{bmatrix} 0 & -3 & 5 \\ -4 & 4 & -10 \\ 0 & 0 & 4 \end{bmatrix}$$

Please refrain from exchanging rows as that will give you the wrong answer because with any row exchange you would be solving the problem $PAx = \lambda x$, which is not equivalent to the problem $Ax = \lambda x$.

(9) If a diagonalization exists, find the matrices S and Λ from $A = S\Lambda S^{-1}$. If a diagonalization does not exist, state the reasoning.

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

(10) If a diagonalization exists, find the matrices S and Λ from $A = S\Lambda S^{-1}$. If a diagonalization does not exist, state the reasoning.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$