```
import numpy as np
import scipy.linalg
import matplotlib.pyplot as plt
```

Week 10 Lecture 2: Finite Differences for PDEs

Rahman notes:

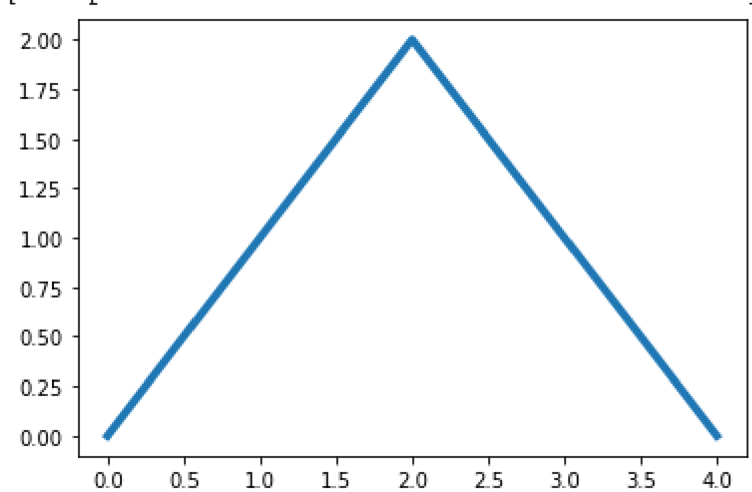
Consider the heat equation problem in the theory lecture:

$$u_t = 2u_{xx}; \qquad u(0,t) = u(4,t) = 0; \qquad u(x,0) = 2 - |x-2|$$

We can plot the initial condition

```
x = np.arange(0, 4+0.1, 0.1)
y = 2 - np.abs(x-2)
plt.plot(x, y, linewidth = 4)
```

```
[<matplotlib.lines.Line2D at 0x7f4ff102e590>]
```



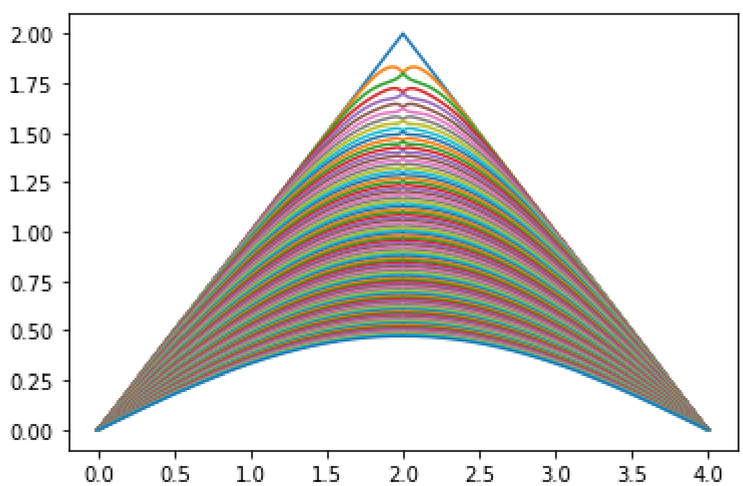
Now lets use the Crank-Nicolson scheme (the gold standard for finite differences for the heat equation) to solve this. We saw in the theory lecture that our finite difference scheme simplifies to

$$-\mu u_{i+1}^{n+1} + (1+2\mu)u_i^{n+1} - \mu u_{i-1}^{n+1} = \mu u_{i+1}^n + (1-2\mu)u_i^n + \mu u_{i-1}^n$$

this indicates we have a tridiagonal system on both sides of the equation, but since we know we simply do the matrix multiplication to give us a vector on the right hand side. For more detail take a look at the theory lecture.

```
dt = 0.01
dx = 0.01
x = np.arange(0, 4+dx, dx)
X = len(x)
```

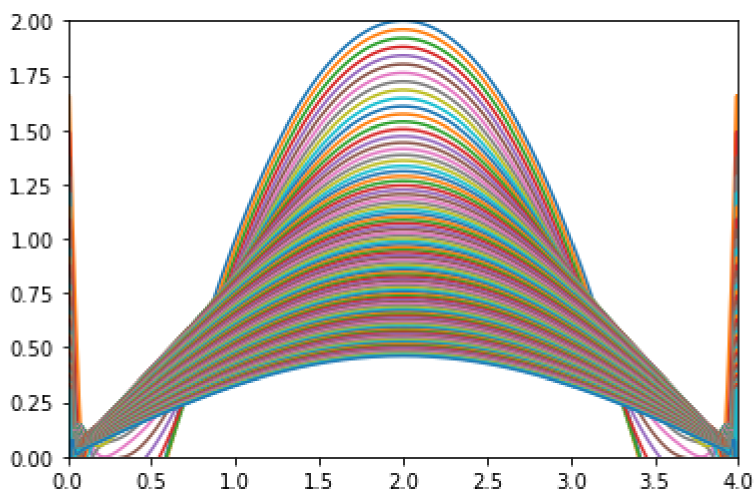
```
X = len(x)
k = 2

u = 2 - np.abs(x-2)
mu = k*dt/(2*dx**2)

u_a = 0 #Dirichlet boundary conditions
u_b = 0 #Dirichlet boundary conditions

main_diag = (1+2*mu)*np.ones(X-2)
second_diag = -mu*np.ones(X-3)
A = np.diag(main_diag) + np.diag(second_diag, 1) + np.diag(second_diag, -1)

main_diag = (1-2*mu)*np.ones(X-2)
B = np.diag(main_diag) - np.diag(second_diag, 1) - np.diag(second_diag, -1)

t = np.arange(dt, 1+dt, dt)

plt.plot(x,u)

for t in range(len(t)):
    b = B@u[1:-1]
    b[0] = b[0] + mu*u_a
    b[-1] = b[-1] + mu*u_b
    u[1:-1] = scipy.linalg.solve(A, b)
    plt.plot(x,u)
```



Lets try it with an initial condition that doesn't have a kink.

```
dt = 0.01
dx = 0.01
x = np.arange(0, 4+dx, dx)
X = len(x)
k = 2

u = 2 - (x-2)**2
mu = k*dt/(2*dx**2)
```

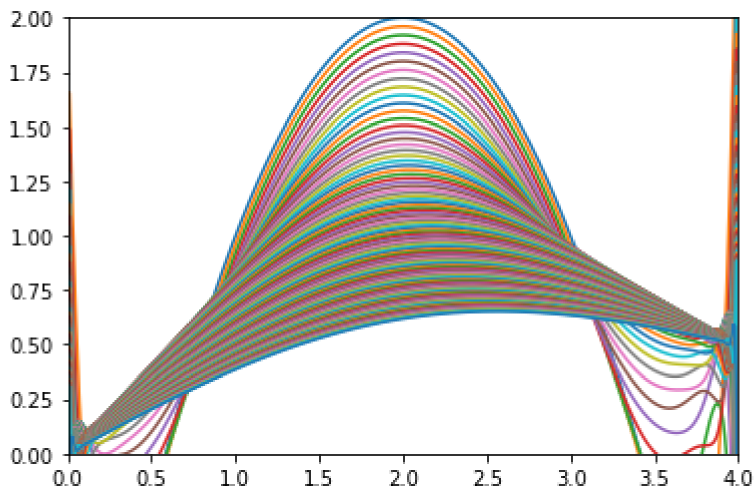
```
mu = k*dt/(2*dx**2)

u_a = 0 #Dirichlet boundary conditions
u_b = 0 #Dirichlet boundary conditions

main_diag = (1+2*mu)*np.ones(X-2)
second_diag = -mu*np.ones(X-3)
A = np.diag(main_diag) + np.diag(second_diag, 1) + np.diag(second_diag, -1)

main_diag = (1-2*mu)*np.ones(X-2)
B = np.diag(main_diag) - np.diag(second_diag, 1) - np.diag(second_diag, -1)

t = np.arange(dt, 1+dt, dt)

plt.plot(x,u)

for t in range(len(t)):
    b = B@u[1:-1]
    b[0] = b[0] + mu*u_a
    b[-1] = b[-1] + mu*u_b
    u[1:-1] = scipy.linalg.solve(A, b)
    plt.plot(x,u)
    plt.axis([0, 4, 0, 2])
```



Lets try it with different boundary conditions.

```
dt = 0.01
dx = 0.01
x = np.arange(0, 4+dx, dx)
X = len(x)
k = 2

u = 2 - (x-2)**2
mu = k*dt/(2*dx**2)

u_a = 0 #Dirichlet boundary conditions
u_b = 1 #Dirichlet boundary conditions
```

```
u_b = 1 #Dirichlet boundary conditions

main_diag = (1+2*mu)*np.ones(X-2)
second_diag = -mu*np.ones(X-3)
A = np.diag(main_diag) + np.diag(second_diag, 1) + np.diag(second_diag, -1)

main_diag = (1-2*mu)*np.ones(X-2)
B = np.diag(main_diag) - np.diag(second_diag, 1) - np.diag(second_diag, -1)

t = np.arange(dt, 1+dt, dt)

plt.plot(x,u)

for t in range(len(t)):
    b = B@u[1:-1]
    b[0] = b[0] + mu*u_a
    b[-1] = b[-1] + mu*u_b
    u[1:-1] = scipy.linalg.solve(A, b)
    plt.plot(x,u)
    plt.axis([0, 4, 0, 2])
```



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