CORRECTLY SOLVED AND SUPERIOR WORK

Determine the convergence or divergence of the series: $\sum_{n=2}^{\infty}\left[\frac{2n-1}{n^2+n}\right]$ **(10 points)**

Compare to $\sum_{N=2}^{\infty} \frac{2N}{N^2} = \sum_{N=2}^{\infty} \frac{2}{N}$ diverges by harmonic series ✓

$\frac{10}{10}$

$\sum_{N=2}^{\infty} \frac{2N-1}{N^2+N} \leq \sum_{N=2}^{\infty} \frac{2}{N}$ so DCT fails ✓

LCT

$$\lim_{N\to\infty} \frac{\frac{2N-1}{N^2+N}}{\frac{2}{N}} = \lim_{N\to\infty} \frac{(2N-1)N}{(N^2+N)2} = \lim_{N\to\infty} \frac{2N^2-N}{2N^2+2N}\left(\frac{\frac{1}{N^2}}{\frac{1}{N^2}}\right) = \lim_{N\to\infty} \frac{2-\frac{1}{N}}{2+\frac{2}{N}} = \frac{2-\cancel{\frac{1}{\infty}}^{0}}{2+\cancel{\frac{2}{\infty}}^{0}} = 1$$

✓ L

since $0 < L < \infty$ both series behave the same way ✓

$\therefore \sum_{N=2}^{\infty} \frac{2N-1}{N^2+N}$ diverges by LCT

The quality of the notation and organization on this problem (circle one):

-1 point (unacceptably messy, illogical, or poor mathematical notation)

0 points (acceptable work)

+1 point (superior neatness rigorous notation and mathematical logic) → $\boxed{\frac{11}{10} \text{ total}}$

INCORRECTLY SOLVED BUT STILL SUPERIOR WORK

Determine the convergence or divergence of the series: $\sum_{n=2}^{\infty}\left[\frac{2n-1}{n^2+n}\right]$ **(10 points)**

Compare to $\sum_{N=2}^{\infty} \frac{1}{N^2}$ converges by p-series $p=2 \geq 1$ not a good comparison

$\sum_{N=2}^{\infty} \frac{2N-1}{N^2+N}$?? $\sum_{N=2}^{\infty} \frac{1}{N^2}$ so DCT fails

$\frac{3}{10}$

LCT

$$\lim_{N\to\infty} \frac{\frac{2N-1}{N^2+N}}{\frac{1}{N^2}} = \lim_{N\to\infty} \frac{2N^3-N^2}{N^2+N}\,\frac{\frac{1}{N^3}}{\frac{1}{N^3}} = \lim_{N\to\infty} \frac{2-\frac{1}{N}}{\frac{1}{N}+\frac{1}{N^2}} = \frac{2-\cancel{\frac{1}{\infty}}^{0}}{\cancel{\frac{1}{\infty}}^{0}+\cancel{\frac{1}{\infty}}^{0}} = \infty = L$$

since $L=\infty$ the series both diverge by LCT

Wrong conclusion for LCT; $L=\infty$ is inconclusive!

The quality of the notation and organization on this problem (circle one):

-1 point (unacceptably messy, illogical, or poor mathematical notation)

0 points (acceptable work)

+1 point (superior neatness rigorous notation and mathematical logic) → $\boxed{\frac{4}{10} \text{ total}}$

CORRECTLY SOLVED AND ACCEPTABLE WORK

Determine the convergence or divergence of the series: $\sum_{n=2}^{\infty}\left[\frac{2n-1}{n^2+n}\right]$ **(10 points)**

Compare $\sum_{N=2}^{\infty} \frac{2N}{N} = \sum \frac{2}{N} \geq \sum \frac{2N-1}{N^2+N}$

↑ diverges harmonic

$$\lim_{N\to\infty} \frac{\frac{2}{N}}{\frac{2N-1}{N^2+N}} = \lim \frac{2N^2+2N}{2N^2-N} = \lim \frac{2+\cancel{\frac{2}{N}}^{\to 0}}{2-\cancel{\frac{1}{N}}^{\to 0}} = 1$$

$0 < 1 < \infty$

So original series diverges LCT

10/10

The quality of the notation and organization on this problem (circle one):
-1 point (unacceptably messy, illogical, or poor mathematical notation)
0 points (acceptable work)
+1 point (superior neatness rigorous notation and mathematical logic)

INCORRECTLY SOLVED BUT ACCEPTABLE WORK

Determine the convergence or divergence of the series: $\sum_{n=2}^{\infty}\left[\frac{2n-1}{n^2+n}\right]$ **(10 points)**

$$\lim_{N\to\infty}\left|\frac{a_{N+1}}{a_N}\right| = \lim_{N\to\infty}\left|\frac{\frac{2(N+1)-1}{(N+1)^2+N+1}}{\frac{2N-1}{N^2+N}}\right| = \lim_{N\to\infty}\left|\frac{(2N+1)(N^2+N)}{(N^2+2N+1+N+1)(2N-1)}\right|$$

Wrong method

0/10

$$= \lim_{N\to\infty} \frac{2N^3+N^2+2N^2+N}{(N^2+3N+2)(2N-1)} = \lim_{N\to\infty} \frac{2N^3+3N^2+N}{2N^3+5N^2+N-2} \cdot \frac{\frac{1}{N^3}}{\frac{1}{N^3}}$$

$$= \lim_{N\to\infty} \frac{2+3/N+1/N^2}{2+5/N+1/N^2-2/N^2} = \frac{2}{2} = 1 \rightarrow$$ L=1 Converges by Ratio Test

L=1 means conclude nothing!

The quality of the notation and organization on this problem (circle one):
-1 point (unacceptably messy, illogical, or poor mathematical notation)
0 points (acceptable work)
+1 point (superior neatness rigorous notation and mathematical logic)

CORRECTLY SOLVED BUT UNACCEPTABLE WORK

Determine the convergence or divergence of the series: $\sum_{n=2}^{\infty}\left[\frac{2n-1}{n^2+n}\right]$ **(10 points)**

$$\sum \frac{2N-1}{N^2+N} = \frac{2N}{N^2} = \frac{2}{N}$$ Notation

→ diverges harmonic series

$$\frac{2}{N} \geq \frac{2N-1}{N^2+N}$$

$$\frac{8}{10}$$

$$\lim_{N\to\infty} \frac{\frac{2}{N}}{\frac{2N-1}{N^2+N}} = \frac{2N^2+2N}{2N^2-N} = \frac{2+\cancel{2/N}\to 0}{2-\cancel{1/N}} = 1$$

limit notation

$0 < L < \infty$ so diverges

what is L? what diverges?

name of test?? −2

The quality of the notation and organization on this problem (circle one):
-1 point (unacceptably messy, illogical, or poor mathematical notation)
0 points (acceptable work)
+1 point (superior neatness rigorous notation and mathematical logic)

$\boxed{-1 \text{ so } \frac{7}{10} \text{ total}}$

INCORRECTLY SOLVED AND UNACCEPTABLE WORK

Determine the convergence or divergence of the series: $\sum_{n=2}^{\infty}\left[\frac{2n-1}{n^2+n}\right]$ **(10 points)**

$$\sum \frac{2N-1}{N^2+N} \qquad \sum \frac{2}{N^2} \quad a=2, \quad r=\frac{N}{2}>1$$ Converges geometric

Not geometric!

$$\frac{a}{1-r} = \frac{2}{1-2} = -2$$??

$$\sum \frac{1}{N}$$ ~~diverges harmonic~~

algebra error

$$\frac{0}{10}$$

$$-2+\infty = \infty$$

$$\lim \frac{2N-1}{N^2+N} = 0?$$

a_N is not defined

So a_N diverges ??? ; Comparison test of sums

Not a thing

The quality of the notation and organization on this problem (circle one):
-1 point (unacceptably messy, illogical, or poor mathematical notation)
0 points (acceptable work)
+1 point (superior neatness rigorous notation and mathematical logic)

$\boxed{-1 \text{ so } \frac{-1}{10} \text{ total}}$