Delft University of Technology

Quantitative Methods for Logistics
ME44206

# Assignment Q2

**Roxana de Nie (4665716)**
**Floor van Lunen (4686462)**
**Ruben Siemerink (4703669)**
**Job Oldenhuis (4602781)**
**Jolien Meulepas (4532201)**

**MSc Transport, Infrastructure and Logistics**

**MSc Mechanical Engineering - Multi-machine Engineering**

**September 21, 2022**

# A. Pickups only

In the first question, the mathematical model is determined and explained.

## Indices and sets

| | | |
|---|---|---|
| $i$ | current location ID | $[0...N]$ |
| $j$ | next location ID | $[0...N]$ |
| $k$ | vehicle | $[1...K]$ |

The locations are divided into a set and two sub sets. The set containing all nodes is divided into the two sub sets: Clients and Depots. Furthermore, the first question only considers pickups. Therefore, a set of pickups is defined that only includes entries with a demand greater than 0, and a set of deliveries that only includes entries with a demand below zero.

$Clients = [2, ..., N]$
$Depots = [0, 1]$
$Nodes = [0, ..., N]$
$Pickups = [0, ..., P]$ (= all entries $i$ where $PID_i > 0$)
$Deliveries = [0, ..., G]$ (= all entries $i$ where $DID_i > 0$)

## Parameters

In this vehicle routing problem, certain parameters are considered. Firstly, all nodes have an associated x- and y-coordinate. Using these by applying the Pythagorean theorem, Euclidean distances between the nodes are calculated. One distance unit corresponds to one time unit. All customers have an associated demand which shows the parcel volume for a specific task. This demand is 0 for the depots. All positive demand quantities indicate pickups, while negative quantities indicate deliveries. All locations have an associated ready time, due time and service time. The ready time provides the earliest time for a task, due time the latest time for a task and the service time the time needed for a task at a customer. Ready and due time at a depot are considered as their opening and closing time. Pickup ID's indicate the origin location for a delivery, and Delivery ID's indicate the destination location for the pickup. All vehicles have a capacity which is the maximum weight of parcels that can be carried by that vehicle. Big M is a big number used in order to keep linearity in the provided mathematical model. Finally, V indicates the total number of vehicles used in a problem.

| | |
|---|---|
| $Xco_i$ / $Xco_j$ | x coordinate of location $i$ or $j$ |
| $Yco_i$ / $Yco_j$ | y coordinate of location $i$ or $j$ |
| $d_{ij}$* | distance between location $i$ and $j$ |
| $D_i$ | demand of customer $i$ (= 0 for depots) |
| $RT_i$ | ready time of location $i$ (= 0 for depots) |
| $DT_i$ | due time of location $i$ |
| $ST_i$ | service time at location $i$ |
| $PID_i$ | pickup ID at location $i$ |
| $DID_i$ | deliver ID at location $i$ |
| $q_k$ | capacity of vehicle $k$ |
| $M$ | Big number |
| $V_k$ | number of vehicles $k$ |

$$^*d_{ij} = \sqrt{(Xco_j - Xco_i)^2 + (Yco_j - Yco_i)^2}$$

## Decision variables

There are three decision variables in this mathematical model. The first decision variable $x_{ijk}$ is a binary decision variable that indicates weather a vehicle visits node j directly after node i. Decision variable $T_{ik}$ and $Q_{ik}$ indicate the arrival time and the capacity of vehicle k at node i.

| | |
|---|---|
| $x_{ijk}$ | binary decision variable indicating if vehicle $k$ visits location $j$ immediately after location $i$ |
| $T_{ik}$ | arrival time of vehicle $k$ at location $i$ |
| $Q_{ik}$ | Used capacity of vehicle $k$ at location $i$ |

## Variables

In this mathematical model, a variable for waiting time indicates how long a vehicle k has to wait at node i before it can start its task. This is dependent upon the ready time of node i, and the arrival time of vehicle k at node i.

$$WT_{ik} = RT_i - T_{ik} \qquad \forall i \in Nodes, \forall k \in K$$

## Objective function

The goal is to minimize the total distance traveled. In this first exercise only pickups are considered. The objective function describes the distance travelled from node (either a depot or a customer) i to j (another node), multiplied by the binary decision variable $x_{ijk}$, which describes if vehicle k is travelling directly from node i to j. This results in the objective function Z. The objective function Z is minimized in order to minimize the total distance traveled.

$$Z = min \sum_{i \in Nodes} \sum_{j \in Nodes} \sum_{k \in K} d_{ij} * x_{ijk}$$

## Functional Constraints

The objective function stated above is subjected to functional constraints, which will be explained below.

The first constraint indicates that all pickup nodes must be visited exactly once.

$$\sum_{k \in K} \sum_{j \in Nodes} x_{ijk} = 1 \qquad \forall i \in Pickups \tag{1}$$

Constraint 2 ensures that only pickup nodes will be visited.

$$\sum_{k \in K} \sum_{j \in Nodes} x_{ijk} = 0 \qquad \forall i \in Deliveries \tag{2}$$

The next constraint is a flow constraint, that makes sure each vehicle visiting a client will also leave this client.

$$\sum_{j \in Nodes} x_{jik} - \sum_{j \in Nodes} x_{ijk} = 0 \qquad \forall i \in Clients, \forall k \in K \tag{3}$$

The first of the two time constraints (constraint 4) describes that the arrival time of the node following directly after the current node (node j), must be greater than or equal to the arrival time at the current node (i) plus the service time at node i and the distance between the two nodes. If node j is visited directly after node i, $x_{ijk}$ becomes 1, meaning that big M will not be subtracted from the equation and the constraint will hold. If node j is not visited directly after node i, big M is subtracted and the constraint will not hold (as the right part of the equation will become very large). This time constraint also functions as an eliminator of sub tours, as these time windows will make it impossible to finish different sub tours in the indicicated time windows.

$$T_{jk} \geq T_{ik} + ST_i + d_{ij} - M * (1 - x_{ijk}) \qquad \forall (i,j) \in Nodes, \forall k \in K \tag{4}$$

The next constraint considers that all vehicles must arrive at node i after or at the earliest time for a task (ready time), and before or at the latest time for a task (ready time).

$$RT_i \leq T_{ik} \leq DT_i \qquad \forall i \in Nodes, \forall k \in K \tag{5}$$

The next constraint is a constraint for capacity, formulated in a same way as the first of the two time constraints. It describes that the capacity of the node following directly after the current node (node j), must be greater than or equal to the capacity at the current node (i) plus the demand at the current node. If node j is visited directly after node i, $x_{ijk}$ becomes 1, meaning that big M will be multiplied with 0 and the first constraint will not hold, but the second will. If node j is not visited directly after node i, big M is multiplied with 1 and therefore the first constraint will hold (as the right part of the equation will become very large) and the second will not. Additionally constraint 9 is therefore added to ensure equality.

$$Q_{ik} + D_j - Q_{jk} \leq M * (1 - x_{ijk}) \qquad \forall i \in Nodes, \forall j \in Clients, \forall k \in K \tag{6}$$

$$Q_{ik} + D_j - Q_{jk} \geq -M * (1 - x_{ijk}) \qquad \forall i \in Nodes, \forall j \in Clients, \forall k \in K \tag{7}$$

Constraint 9 states that the capacity of the vehicles can't be exceeded.

$$Q_{ik} \leq q_k \qquad \forall k \in K, i \in Clients \tag{8}$$

The next constraint ensures that the capacity at the depots used is 0.

$$Q_{ik} = 0 \qquad \forall k \in K, i \in Depots \tag{9}$$

Constraint 10 considers that the total number of depots a vehicle leaves is equal to one.

$$\sum_{i \in Depots} \sum_{j \in Nodes} x_{ijk} = 1 \qquad \forall k \in K \tag{10}$$

Constraint 11 describes the binary form of the decision variable $x_{ijk}$.

$$x_{ijk} \epsilon \{0, 1\} \qquad \forall i, j, k \tag{11}$$

Constraint 12 and 13 are the non negativity constraints for decision variables $T_{ik}$ and $Q_{ik}$

$$T_{ik} \geq 0 \qquad \forall i, k \tag{12}$$

$$Q_{ik} \geq 0 \qquad \forall i, k \tag{13}$$

Constraint 14 and 15 indicate that decision variables $T_{ik}$ and $Q_{ik}$ are real numbers

$$T_{ik} = \mathbb{R} \qquad \forall i, k \tag{14}$$

$$Q_{ik} = \mathbb{R} \qquad \forall i, k \tag{15}$$

## B. Results

This section includes the results of the implementation of the mathematical model presented in question A. For the results of question B, one vehicle is considered with capacity of 200. The total distance traveled, the locations visited by the vehicle together with the sequence, the time of visit and the load of the vehicle at each location is presented in Table 1 and Table 2. Figure 1 shows a visualisation of the route that has been executed.

Table 1: Distance travelled model A

| | |
|---|---|
| Distance traveled | 82.947 |

Table 2: Results model A

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 1 | 7 | 12 | 13 | 10 | 5 | 4 | 0 |
| | **Time** | 0 | 30 | 194 | 289 | 475 | 588 | 681 | 790 |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 110 | 130 | 130 |

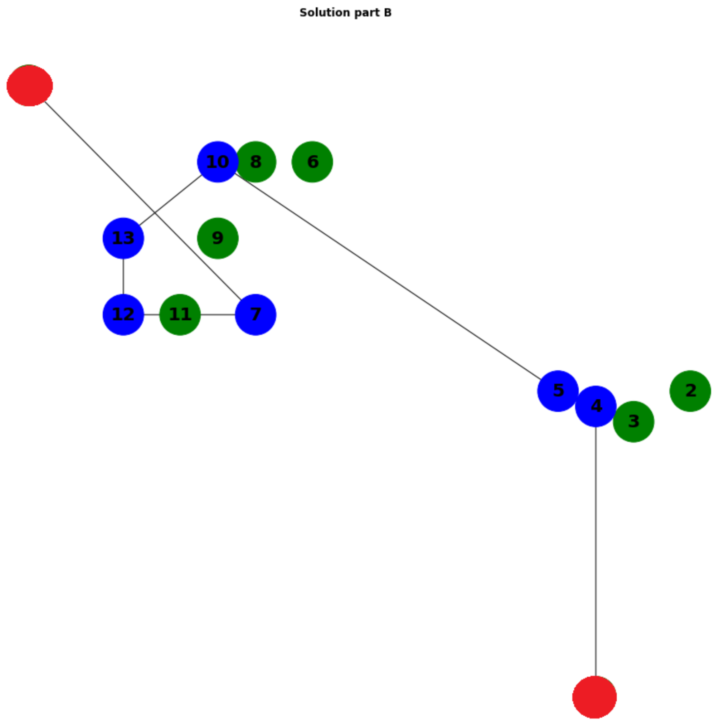



Figure 1: Route visualisation for b

## C. Mathematical model vehicle belongs to own depot

In order to adapt the mathematical model for the exercise where a vehicle that leaves a depot must also come back at that same depot, a few adaptions to the existing mathematical model need to be made. First, new nodes need to be created containing the exact same data as the depots, representing a 'clone' of the two existing depots. These nodes are added as the $n+i$ node in the data list, with $n$ being the size of the nodes list and $i$ being depots. Additionally, two constraints are added to the model. Constraints 16 and 17 ensure that a vehicle leaving a depot, also enters the exact copy depot.

$$\sum_{j \in Nodes} x_{ijk} - \sum_{j \in Nodes} x_{j,n+i,k} = 0 \qquad \forall i \in Depots, \forall k \in K \tag{16}$$

$$\sum_{j \in Nodes} x_{n+i,j,k} - \sum_{j \in Nodes} x_{jik} = 0 \qquad \forall i \in Depots, \forall k \in K \tag{17}$$

# D. Results of the model from C

In this part experiments are executed with different cases for the mathematical model presented in question C. For each of the cases the following outputs are visualised: total distance traveled, the locations visited by the vehicle together with the sequence (i.e., route of the vehicle), the time of visit and the load of the vehicle at each location.

### 1. 1 vehicle, 200 capacity

For the first case, 1 vehicle with a capacity of 200 is considered. The total distance, together with the other mentioned outputs can be seen in Table 3 and Table 4.

Table 3: Distance 1 vehicle, 200 capacity

| | |
|---|---|
| Distance traveled | 94.544 |

Table 4: Results 1 vehicle, 200 capacity

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 0 | 7 | 12 | 13 | 10 | 5 | 4 | 0 |
| | **Time** | 0 | 31 | 194 | 289 | 492 | 605 | 697 | 981 |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 110 | 130 | 130 |

### 2. 2 vehicles, 100 capacity

For the second case, 2 vehicles are considered with a capacity of 100 each. The total distance, together with the other mentioned outputs can be seen in Table 5 and Table 6.

Table 5: Distance 2 vehicles, 100 capacity

| | |
|---|---|
| Distance traveled | 90.797 |

Table 6: Results 2 vehicles, 100 capacity

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 0 | 5 | 4 | 0 | | |
| | **Time** | 0 | 534 | 626 | 735 | | |
| | **Load** | 0 | 10 | 30 | 30 | | |
| **Vehicle 2** | **Location** | 1 | 7 | 12 | 13 | 10 | 1 |
| | **Time** | 0 | 30 | 179 | 278 | 475 | 576 |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 100 |

### 3. 2 vehicles, 75 capacity

Finally, 2 vehicles are used with a capacity of 75 each. The total distance, together with the other mentioned outputs can be seen in Table 7 and Table 8.

Table 7: Distance 2 vehicles, 75 capacity

| | |
|---|---|
| Distance traveled | 107.868 |

Table 8: Results 2 vehicles, 75 capacity

| | | | | | | |
|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 1 | 12 | 13 | 10 | 1 |
| | **Time** | 0 | 179 | 278 | 475 | 576 |
| | **Load** | 0 | 20 | 30 | 70 | 70 |
| **Vehicle 2** | **Location** | 0 | 7 | 5 | 4 | 0 |
| | **Time** | 0 | 31 | 534 | 626 | 735 |
| | **Load** | 0 | 30 | 40 | 60 | 60 |

## Evaluation of the results gained

This part entails evaluation of the results gained for the implemented models A and C. The following comparisons have been made and evaluated:

- The total distance travelled for all the results.
- The travel sequence, arrival time and load for 1 vehicle without an own start and end depot and with an own start and end depot.
- The travel sequence for 1 vehicle in comparison to 2 vehicles.
- The travel sequence, arrival time and load for 2 vehicles with different capacities (100 and 75 each).

### Evaluation of total distance traveled

Table 9 gives an overview of the results of the total distance travelled gained from implementing model A and C. When comparing the results from B, 1 vehicle with 200 capacity, to results from D, with 1 vehicle and 200 capacity, the total distance traveled has increased. This means that when assigning a vehicle to start and end at the same depot, the total distance traveled increases. When implementing 2 vehicles, with a lower capacity each, the total distance travelled decreases, which means that a more efficient route now can be executed with implementing 2 vehicles of 100 capacity, instead of 1 with 200 capacity. However when lowering the capacity to 75 for both vehicles, the total distance increases and reaches the highest value of all the results. This can be explained due to the fact that they can pick up fewer products and therefore longer distances have to be travelled to make sure all pick ups are executed.

Table 9: Overview of the total distance traveled results

| **Results** | **Total distance traveled** |
|---|---|
| Result B, 1 vehicle, 200 capacity | 82.947 |
| Result D, 1 vehicle, 200 capacity | 94.544 |
| Result D, 2 vehicles, 100 capacity | 90.797 |
| Result D, 2 vehicles 75 capacity | 107.868 |

### Evaluation of results without own start and end depot and with own start and end depot

Next, the visited locations, executed travel sequences, time of visit and load of the vehicle at each location from the results from B with 1 vehicle with 200 capacity and the results from D with 1 vehicle and 200 capacity are compared to each other. When looking at Table 10 and Table 11, it can be seen that the travel sequence for both situations almost is equal to each other, except that for Result D, the vehicle starts and ends at the same depot. The load at each locations visited is the same for both situations, however a difference is noticed when looking at the time of visit at each location. In total, result D takes up more time to complete his route, which can be explained due to the fact that the vehicle has to take a route where it begins and ends at the same depot. The locations visited are the same, but the vehicle has to divert at the end of the route to his own start depot, which results in a larger time component.

Table 10: Results B, 1 vehicle 200 capacity

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 1 | 7 | 12 | 13 | 10 | 5 | 4 | 0 |
| | **Time** | 0 | 30 | 194 | 289 | 475 | 588 | 681 | 790 |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 110 | 130 | 130 |

Table 11: Results D, 1 vehicle, 200 capacity

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 0 | 7 | 12 | 13 | 10 | 5 | 4 | 0 |
| | **Time** | 0 | 31 | 194 | 289 | 492 | 605 | 697 | 981 |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 110 | 130 | 130 |

### Travel sequence of 2 vehicles comparing to 1 vehicle

When comparing the situation with 2 vehicles to the situation of 1 vehicle, it can be seen that the travel sequence of 1 vehicle has been 'split' in the middle (7-12-13-10) and (5-4) and each half has been assigned to a vehicle with their own start and end depot. However a different implementation of location 7 in the travel sequence has been executed regarding capacity 100 or capacity 75 for each vehicle, which will be explained next.

### Evaluation of results with 2 vehicles, 100 capacity and 75 capacity

Finally, situation with 2 vehicles with 100 capacity each and 2 vehicles with 75 capacity each are compared to each other. When looking at Table 12 and Table 13, it can be seen that vehicle 1 with 100 capacity almost completes the same route and same time of visit as vehicle 2 for 75 capacity. Vehicle 1 with 100 capacity completes the route 0 - 5 - 4 - 0, and vehicle 2 with 75 capacity completes the route 0 - 7 - 5 - 4 - 0, which has location 7 added in the sequence right after the depot. It can also be seen that vehicle 2 with 100 capacity and vehicle 1 with 75 capacity also almost complete the same route and time of visit as each other. Vehicle 1 with 75 capacity competes the route 1 - 12 - 13 - 10 - 1 and vehicle 2 with 100 capacity completes the route 1 - 7 - 12 - 13 - 10 - 1, which has location 7 added in the sequence after the depot. It can be concluded that location 7 is visited in the opposite travel sequence for these situations. Location 7 could not be added to the travel sequence of vehicle 1 with 75 capacity any more due to the capacity limit of the vehicle. Therefore the pickup at location 7 was transferred to vehicle 2 and added to this travel sequence.

Table 12: Results 2 vehicles, 100 capacity

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 0 | 5 | 4 | 0 | | |
| | **Time** | 0 | 534 | 626 | 735 | | |
| | **Load** | 0 | 10 | 30 | 30 | | |
| **Vehicle 2** | **Location** | 1 | 7 | 12 | 13 | 10 | 1 |
| | **Time** | 0 | 30 | 179 | 278 | 475 | 576 |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 100 |

Table 13: Results 2 vehicles, 75 capacity

| | | | | | | |
|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 1 | 12 | 13 | 10 | 1 |
| | **Time** | 0 | 179 | 278 | 475 | 576 |
| | **Load** | 0 | 20 | 30 | 70 | 70 |
| **Vehicle 2** | **Location** | 0 | 7 | 5 | 4 | 0 |
| | **Time** | 0 | 31 | 534 | 626 | 735 |
| | **Load** | 0 | 30 | 40 | 60 | 60 |

# E. Different costs per depot

For question E a few adaptions to the mathematical model need to be made. First of all a new parameter needs to be included:

$C_i$ depot costs

Thereafter, the objective function needs to be adapted in order to include the minimization of the costs as well. The minimization of the costs consists of the multiplication of the binary decision variable $y_{ik}$ with the costs of

$$Z = min \sum_{i \in Nodes} \sum_{j \in Nodes} \sum_{k \in K} d_{ij} * x_{ijk} + \sum_{i \in depots} \sum_{j \in Nodes} \sum_{k \in K} x_{ijk} * C_i$$

**Multi objective function**

Now, 2 different objective functions are implemented in the model, one for minimizing the total distance traveled, which was already implemented in the model, and now the second one for including the minimization of the costs is implemented as well. The approach for adding the second objective to the model is chosen based on combining all objectives into one overall objective function, combining single objectives with scalar weights. For implementation of the multi objective function, insights have to be gained into the weight assigned to the single objectives. Weights are assigned to the objectives functions to include the relation between costs and distance, and set the ratio of both equal to each other.

$$Z = min \sum_{i \in Nodes} \sum_{j \in Nodes} \sum_{k \in K} d_{ij} * x_{ijk}$$

$$Z = min \sum_{i \in Depots} \sum_{j \in Nodes} \sum_{k \in K} x_{ijk} * C_i$$

**Determining the weight of the objectives**

In order to determine the weight of the objective functions, information is gained about the diesel consumption of vans per kilometer. Vans have an average consumption of 11 liters of diesel per 100 kilometers, resulting in 0.11 L/km [2], with a diesel price of 1,716 eur/L [1]. Multiplying these two components results in 0.18865 eur/km and a km/euro ratio of 5,297 km/euro (Table 14). To scale the relation of both components to each other, a larger weight is given to cost objective in comparison to the distance objective. When rounding the numbers, the ratio eventually comes to **0.2 distance objective : 1 cost objective**.

Table 14: Ratio distance and cost

| Kilometer | Euro |
|---|---|
| 1 | 0.18865 |
| 5,2975 | 1 |

A sensitivity analysis is executed with applying different weights to the objective, to see how our found ratio changes when changing the ratio. Table 15 visualises the results of the experiments. When applying the ratio of distance 0.4 : cost 1, the results are the same as our chosen ratio.

Table 15: Sensitivity analysis with results

| Weight distance objective | Weight costs objective | Distance output | Depot cost output |
|---|---|---|---|
| 0.2 | 1 | 1016.807 | 1150.0 |
| 0.4 | 1 | 1016.807 | 1150.0 |
| 0.6 | 1 | 840.348 | 1250.0 |
| 0.8 | 1 | 840.348 | 1250.0 |
| 1 | 1 | 783.291 | 1300.0 |

# F. Experimenting with different costs

This chapter includes the implementation of the model E and includes experiments executed with the model.

### 1. 10 vehicles, no depot costs

First of all, 10 vehicles with no depot costs are implemented, for which the results can be seen in Table 16 and Table 17.

Table 16: Results of F1

| | |
|---|---|
| Distance traveled | 701.641 |
| Depot costs | 0.0 |

Table 17: Results of 10 vehicles, 200 capacity and no depot costs

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **location** | 0 | 21 | 26 | 30 | 31 | 29 | 24 | 0 | |
| | **Time** | 0 | 73 | 169 | 358 | 453 | 546 | 732 | 1236 | |
| | **Load** | 0 | 10 | 50 | 60 | 70 | 90 | 100 | 100 | |
| **Vehicle 2** | **Location** | 0 | 58 | 55 | 54 | 57 | 65 | 67 | 0 | |
| | **Time** | 0 | 35 | 186 | 286 | 385 | 632 | 875 | 1236 | |
| | **Load** | 0 | 40 | 80 | 100 | 130 | 140 | 150 | 150 | |
| **Vehicle 3** | **Location** | 0 | 82 | 79 | 77 | 72 | 80 | 0 | | |
| | **Time** | 0 | 77 | 170 | 203 | 298 | 668 | 1236 | | |
| | **Load** | 0 | 30 | 50 | 60 | 80 | 90 | 90 | | |
| **Vehicle 4** | **Location** | 0 | 6 | 4 | 9 | 12 | 10 | 7 | 0 | |
| | **Time** | 0 | 15 | 106 | 255 | 448 | 541 | 633 | 742 | |
| | **Load** | 0 | 10 | 20 | 40 | 50 | 60 | 80 | 80 | |
| **Vehicle 5** | **Location** | 0 | 44 | 43 | 45 | 46 | 52 | 51 | 50 | 0 |
| | **Time** | 0 | 17 | 110 | 359 | 541 | 725 | 815 | 1001 | 1110 |
| | **Load** | 0 | 10 | 30 | 40 | 50 | 60 | 70 | 80 | 80 |
| **Vehicle 6** | **Location** | 0 | 33 | 34 | 36 | 39 | 37 | 0 | | |
| | **Time** | 0 | 32 | 124 | 283 | 479 | 665 | 700 | | |
| | **Load** | 0 | 30 | 70 | 80 | 110 | 120 | 120 | | |
| **Vehicle 7** | **Location** | 0 | 68 | 66 | 64 | 63 | 0 | | | |
| | **Time** | 0 | 12 | 103 | 195 | 317 | 425 | | | |
| | **Load** | 0 | 10 | 20 | 70 | 90 | 90 | | | |
| **Vehicle 8** | **Location** | 0 | 91 | 88 | 87 | 83 | 85 | 0 | | |
| | **Time** | 0 | 21 | 116 | 207 | 369 | 523 | 644 | | |
| | **Load** | 0 | 10 | 30 | 40 | 60 | 80 | 80 | | |
| **Vehicle 9** | **Location** | 0 | 99 | 97 | 93 | 98 | 101 | 0 | | |
| | **Time** | 0 | 31 | 126 | 368 | 561 | 647 | 775 | | |
| | **Load** | 0 | 20 | 30 | 50 | 80 | 100 | 100 | | |
| **Vehicle 10** | **Location** | 1 | 14 | 19 | 20 | 17 | 1 | | | |
| | **Time** | 0 | 30 | 179 | 278 | 475 | 572 | | | |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 100 | | | |

### 2. 10 vehicles, depot costs

As a second test, 10 vehicles with depot costs of 150 for depot 1 and 100 for depot 2 are implemented, for which the results can be seen in Table 18 and Table 19.

Table 18: Results of F2

| | |
|---|---|
| Distance traveled | 1016.807 |
| Depot costs | 1150.0 |

Table 19: Results of 10 vehicles, 200 capacity and depot costs of 150 and 100 euros

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 1 | 99 | 97 | 93 | 98 | 101 | 1 | | |
| | **Time** | 0 | 61 | 156 | 441 | 561 | 647 | 777 | | |
| | **Load** | 0 | 20 | 30 | 50 | 80 | 100 | 100 | | |
| **Vehicle 2** | **Location** | 1 | 79 | 82 | 77 | 72 | 80 | 1 | | |
| | **Time** | 0 | 121 | 124 | 260 | 360 | 668 | 852 | | |
| | **Load** | 0 | 20 | 50 | 60 | 80 | 90 | 90 | | |
| **Vehicle 3** | **Location** | 1 | 21 | 26 | 30 | 31 | 29 | 1 | | |
| | **Time** | 0 | 73 | 224 | 358 | 453 | 546 | 672 | | |
| | **Load** | 0 | 10 | 50 | 60 | 70 | 90 | 90 | | |
| **Vehicle 4** | **Location** | 1 | 58 | 55 | 54 | 57 | 65 | 24 | 1 | |
| | **Time** | 0 | 87 | 247 | 342 | 436 | 632 | 732 | 769 | |
| | **Load** | 0 | 40 | 80 | 100 | 130 | 140 | 150 | 150 | |
| **Vehicle 5** | **Location** | 0 | 91 | 88 | 87 | 83 | 85 | 0 | | |
| | **Time** | 0 | 49 | 144 | 238 | 369 | 523 | 1154 | | |
| | **Load** | 0 | 10 | 30 | 40 | 60 | 80 | 80 | | |
| **Vehicle 6** | **Location** | 1 | 33 | 34 | 36 | 39 | 37 | 1 | | |
| | **Time** | 0 | 66 | 158 | 283 | 479 | 665 | 813 | | |
| | **Load** | 0 | 30 | 70 | 80 | 110 | 120 | 120 | | |
| **Vehicle 7** | **Location** | 0 | 68 | 66 | 64 | 63 | 67 | 0 | | |
| | **Time** | 0 | 12 | 126 | 218 | 317 | 826 | 1141 | | |
| | **Load** | 0 | 10 | 20 | 70 | 90 | 100 | 100 | | |
| **Vehicle 8** | **Location** | 0 | 44 | 43 | 45 | 46 | 52 | 51 | 50 | 0 |
| | **Time** | 0 | 19 | 149 | 412 | 541 | 725 | 815 | 1001 | 1110 |
| | **Load** | 0 | 10 | 30 | 40 | 50 | 60 | 70 | 80 | 80 |
| **Vehicle 9** | **Location** | 1 | 6 | 4 | 9 | 12 | 10 | 7 | 1 | |
| | **Time** | 0 | 37 | 146 | 324 | 448 | 541 | 681 | 804 | |
| | **Load** | 0 | 10 | 20 | 40 | 50 | 60 | 80 | 80 | |
| **Vehicle 10** | **Location** | 1 | 14 | 19 | 20 | 17 | 1 | | | |
| | **Time** | 0 | 30 | 250 | 345 | 475 | 572 | | | |
| | **Load** | 0 | 30 | 50 | 60 | 100 | 100 | | | |

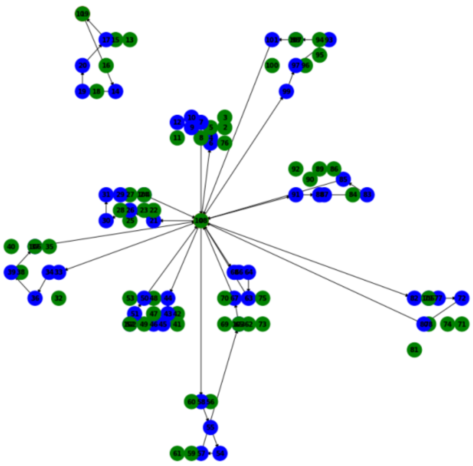
When looking at the differences in results, a few points stand out. Firstly, the distance traveled is way higher. This can be clarified by looking at the depot costs. Depot 1 (node 0) is used in case 1 for 9/10 vehicles. This is due to the fact that starting at depot 1 results in multiple efficient short routes. However, in case 2, departing at depot 1 costs 150 versus 100 if a vehicle starts at depot 2 (node 1). To save costs, depot 2 is used as much as possible and depot 1 is now used only once. This results in a higher distance traveled in case 2, as the depot costs outweigh the travel costs. Secondly, it can be noted that most of the travel sequences are the same in both cases. This can be explained due to the optimal distance between nodes in combination with the load capacity constraint. This shows there is still the same optimal solution to shorten the distance starting from a certain node. Apparently it does not always matter if this starting node is further away from the starting depot. When comparing case 1 and 2, the difference here is in the nodes that are added or removed from these sequences. Some of the optimal sequences are merged together because the vehicle happens to drive by it on the way back to the depot. One can conclude the optimal solution consists of parts of optimal traveling route that need to be merged into the optimal solution.

### 3. Experimenting with different depot costs

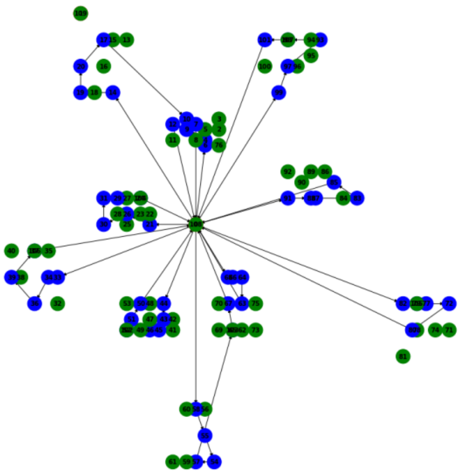
Experiments are executed with different costs of the depots, which are presented in Table 20. For all the different situations, the capacity is set to 200 and with 10 vehicles. For most of the experiments depot cost 1 is set to 100, and tweaking with depot costs 2 was executed. From the results it can be seen that the total distance and depot costs remains the same while keeping the depot 1 cost at 100 and changing the values for the second depot costs. It can be concluded that changing depot 2 costs does not influences the total distance, route executed and depot costs. However, when adapting both depots costs to the same value, the total distance remains the same for changing it to 100, 200 and 300. A different route is then executed, as can be seen in Figure 2.

Table 20: Experimenting with different depot costs

| Depot 1 cost | Depot 2 cost | total distance | total depot cost |
|---:|---:|---:|---:|
| 100 | 150 | 748.738 | 1000 |
| 100 | 125 | 748.738 | 1000 |
| 100 | 200 | 748.738 | 1000 |
| 100 | 225 | 748.738 | 1000 |
| 50 | 150 | 748.738 | 500 |
| 150 | 150 | 701.641 | 1500 |
| 200 | 200 | 701.641 | 2000 |
| 300 | 300 | 701.641 | 3000 |



(a) Route executed for depot costs 100,100 and 200,200 and 300,300



(b) route executed for all the other depot costs experiments

Figure 2: Route visualisation for experiments F3

### 4. 15 vehicles, depot costs

To investigate the effect of changing the capacity of the vehicles, different capacity values have been used, presented in Table 21.

Table 21: Results of different simulations, changing the capacity

| | | |
|---|---|---|
| 15 vehicles, 120 capacity | Distance traveled | 1618.106 |
| | Depot costs | 1500 |
| | Vehicles used | 13 / 15 |
| 15 vehicles, 150 capacity | Distance traveled | 1584.134 |
| | Depot costs | 1500 |
| | Vehicles used | 13 / 15 |
| 15 vehicles, 200 capacity | Distance traveled | 1582.777 |
| | Depot costs | 1500 |
| | Vehicles used | 13 / 15 |
| 15 vehicles, 400 capacity | Distance traveled | 1582.777 |
| | Depot costs | 1500 |
| | Vehicles used | 13 / 15 |
| 15 vehicles, 100 capacity | Simulation can't finish, no result. | |

It's interesting to see how vehicles with 200 and 400 capacity have the same result. This means that 200 is the maximum capacity needed for improvement. While the theoretical bottom capacity is 66, the simulation won't find any solutions (or at least, it takes a excessive long time) from capacities of 100 and lower. Also,

increasing the capacity to 150 makes a good difference on the distance traveled. However, when increasing it with 50 again, this difference is very little.
The number of vehicles used stays the same for all simulations. This means that changing the capacity has little or no effect to using different vehicles. A sidenote is that in this case the depot costs are always the same, because all vehicles are stationed at the same depot with the lowest costs (15 * 100 = 1500). When the depot costs will be triggered only when a vehicles leaves the depot to visit a client, it might be better to use less vehicles and make longer routes.

Table 22: Results of 15 vehicles with 120 capacity.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Vehicle 1** | **Location** | 1 | 43 | 55 | 54 | 57 | 24 | 1 | |
| | **Time** | 0 | 133 | 247 | 342 | 436 | 732 | 769 | |
| | **Load** | 0 | 20 | 60 | 80 | 110 | 120 | 120 | |
| **Vehicle 2** | **Location** | 1 | 66 | 64 | 63 | 65 | 67 | 1 | |
| | **Time** | 0 | 79 | 171 | 317 | 693 | 826 | 980 | |
| | **Load** | 0 | 10 | 60 | 80 | 90 | 100 | 100 | |
| **Vehicle 3** | **Location** | 1 | 88 | 87 | 83 | 85 | 7 | 1 | |
| | **Time** | 0 | 144 | 238 | 374 | 499 | 621 | 744 | |
| | **Load** | 0 | 20 | 30 | 50 | 70 | 90 | 90 | |
| **Vehicle 4** | **Location** | 1 | 68 | 19 | 20 | 1 | | | |
| | **Time** | 0 | 77 | 214 | 309 | 1236 | | | |
| | **Load** | 0 | 10 | 30 | 40 | 40 | | | |
| **Vehicle 5** | **Location** | 1 | 21 | 26 | 29 | 1 | | | |
| | **Time** | 0 | 64 | 224 | 546 | 672 | | | |
| | **Load** | 0 | 10 | 50 | 70 | 70 | | | |
| **Vehicle 6** | **Location** | 1 | 44 | 30 | 31 | 1 | | | |
| | **Time** | 0 | 80 | 358 | 453 | 578 | | | |
| | **Load** | 0 | 10 | 20 | 30 | 30 | | | |
| **Vehicle 7** | **Location** | 1 | 6 | 4 | 9 | 12 | 1 | | |
| | **Time** | 0 | 55 | 146 | 255 | 448 | 567 | | |
| | **Load** | 0 | 10 | 20 | 40 | 50 | 50 | | |
| **Vehicle 8** | **Location** | 1 | 79 | 82 | 77 | 72 | 80 | 1 | |
| | **Time** | 0 | 109 | 124 | 260 | 360 | 668 | 1236 | |
| | **Load** | 0 | 20 | 50 | 60 | 80 | 90 | 90 | |
| **Vehicle 9** | **Location** | 1 | 33 | 34 | 36 | 39 | 37 | 109 | |
| | **Time** | 0 | 66 | 158 | 344 | 522 | 665 | 711 | |
| | **Load** | 0 | 30 | 70 | 80 | 110 | 120 | 120 | |
| **Vehicle 10** | **Location** | 1 | 99 | 97 | 93 | 98 | 101 | 1 | |
| | **Time** | 0 | 61 | 156 | 441 | 622 | 647 | 777 | |
| | **Load** | 0 | 20 | 30 | 50 | 100 | 120 | 120 | |
| **Vehicle 11** | **Location** | 1 | 14 | 17 | 1 | | | | |
| | **Time** | 0 | 32 | 475 | 572 | | | | |
| | **Load** | 0 | 30 | 70 | 70 | | | | |
| **Vehicle 12** | **Location** | 1 | 58 | 45 | 46 | 52 | 51 | 50 | 1 |
| | **Time** | 0 | 87 | 412 | 600 | 786 | 880 | 1001 | 1148 |
| | **Load** | 0 | 40 | 50 | 60 | 70 | 80 | 90 | 90 |
| **Vehicle 13** | **Location** | 1 | 91 | 10 | 1 | | | | |
| | **Time** | 0 | 84 | 534 | 654 | | | | |
| | **Load** | 0 | 10 | 20 | 20 | | | | |
| **Vehicle 14** | **Location** | 1 | | | | | | | |
| | **Time** | 0 | | | | | | | |
| | **Load** | 0 | | | | | | | |
| **Vehicle 15** | **Location** | 1 | | | | | | | |
| | **Time** | 0 | | | | | | | |
| | **Load** | 0 | | | | | | | |

## G. Parcels must be picked up and delivered

Finally, the deliveries are also part of the problem: all parcels must be picked up and delivered at the correct location as indicated in the input. No parcels can be stored at the depot. To achieve this, constraint 1 from the model needs to be adjusted, because not only pickups but all client nodes must be visited exactly once. This resulted in constraint 18.

$$\sum_{k \in K} \sum_{j \in Nodes} x_{ijk} = 1 \qquad \forall i \in Clients \tag{18}$$

Additionally, constraint 2, that ensures that delivery locations can not be visited, must be replaced by a constraint that ensures that a delivery location can only be visited if the corresponding pickup location has been visited by the same vehicle, which is done by constraint 19.

$$\sum_{j \in Nodes} x_{ijk} - x_{j,DID_i,k} = 0 \qquad \forall k \in K, i \in Pickups \tag{19}$$

Lastly the parameter $C_i$ for depot costs is adjusted to four depots with costs of 150, 100, 100 and 125 euros respectively.

## H. Results of G.

This section includes the results of the implementation of the additions to the mathematical model presented in question G, in Table 23 and Table 24. A total number of 10 vehicles is used with a capacity of 200 each. For this question a time limit of 3600 seconds was used.

Table 23: Results of G

| | |
|---|---|
| Distance traveled | 950.152 |
| Total depot costs | 1100 |
| Computational time | 3600 seconds |
| Optimality gap | 2.69% |

Table 24: Results of 10 vehicles with 200 capacity

Vehicle 1

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 46 | 37 | 10 |
| 45 | 130 | 30 |
| 44 | 222 | 20 |
| 43 | 314 | 0 |
| 47 | 407 | 10 |
| 49 | 500 | 0 |
| 48 | 592 | 10 |
| 51 | 684 | 0 |
| 54 | 777 | 10 |
| 104 | 777 | 0 |
| 53 | 869 | 10 |
| 55 | 963 | 0 |
| 2 | 1088 | 0 |

Vehicle 2

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 8 | 23 | 10 |
| 6 | 114 | 20 |
| 10 | 206 | 10 |
| 11 | 298 | 30 |
| 13 | 392 | 10 |
| 14 | 485 | 20 |
| 12 | 578 | 30 |
| 9 | 670 | 50 |
| 7 | 763 | 40 |
| 5 | 856 | 20 |
| 4 | 948 | 10 |
| 78 | 1041 | 0 |
| 2 | 1157 | 0 |

Vehicle 3

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 16 | 30 | 30 |
| 20 | 124 | 0 |
| 21 | 217 | 20 |
| 22 | 312 | 30 |
| 18 | 407 | 20 |
| 19 | 502 | 60 |
| 17 | 594 | 20 |
| 15 | 687 | 0 |
| 2 | 793 | 0 |

Vehicle 4

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 35 | 32 | 30 |
| 36 | 124 | 70 |
| 34 | 219 | 40 |
| 38 | 314 | 50 |
| 40 | 410 | 10 |
| 41 | 502 | 40 |
| 42 | 597 | 30 |
| 39 | 692 | 40 |
| 108 | 692 | 30 |
| 37 | 785 | 0 |
| 2 | 903 | 0 |

Vehicle 5

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 60 | 59 | 40 |
| 58 | 151 | 0 |
| 57 | 246 | 40 |
| 56 | 341 | 60 |
| 59 | 435 | 90 |
| 61 | 527 | 70 |
| 63 | 620 | 30 |
| 62 | 720 | 0 |
| 52 | 1001 | 10 |
| 50 | 1093 | 0 |
| 2 | 1236 | 0 |

Vehicle 6

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 23 | 22 | 10 |
| 27 | 117 | 0 |
| 28 | 209 | 40 |
| 30 | 301 | 0 |
| 32 | 395 | 10 |
| 33 | 499 | 20 |
| 31 | 592 | 40 |
| 29 | 684 | 30 |
| 26 | 777 | 40 |
| 106 | 777 | 30 |
| 25 | 873 | 10 |
| 24 | 965 | 0 |
| 2 | 1077 | 0 |

Vehicle 7

| Location | Time | Load |
|---|---|---|
| 0 | 0 | 0 |
| 70 | 12 | 10 |
| 68 | 103 | 20 |
| 66 | 195 | 70 |
| 65 | 290 | 90 |
| 77 | 383 | 40 |
| 75 | 478 | 30 |
| 64 | 571 | 20 |
| 67 | 684 | 30 |
| 105 | 684 | 20 |
| 71 | 777 | 0 |
| 69 | 872 | 10 |
| 72 | 964 | 0 |
| 0 | 1236 | 0 |

Vehicle 8

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 101 | 38 | 20 |
| 99 | 134 | 30 |
| 98 | 226 | 10 |
| 97 | 319 | 0 |
| 95 | 413 | 20 |
| 96 | 505 | 0 |
| 100 | 600 | 30 |
| 109 | 600 | 0 |
| 103 | 695 | 20 |
| 102 | 790 | 0 |
| 2 | 916 | 0 |

Vehicle 9

| Location | Time | Load |
|---|---|---|
| 2 | 0 | 0 |
| 93 | 43 | 10 |
| 90 | 138 | 30 |
| 89 | 229 | 40 |
| 86 | 325 | 20 |
| 85 | 418 | 40 |
| 87 | 514 | 60 |
| 88 | 606 | 40 |
| 91 | 699 | 30 |
| 92 | 792 | 10 |
| 94 | 886 | 0 |
| 2 | 1017 | 0 |

Vehicle 10

| Location | Time | Load |
|---|---|---|
| 0 | 0 | 0 |
| 84 | 47 | 30 |
| 81 | 140 | 50 |
| 107 | 140 | 30 |
| 79 | 232 | 40 |
| 74 | 327 | 60 |
| 73 | 422 | 30 |
| 76 | 515 | 20 |
| 80 | 609 | 0 |
| 82 | 700 | 10 |
| 83 | 820 | 0 |
| 0 | 1236 | 0 |

# I. Large scale problem

The problem provided for this question is a large scale problem. Using the optimisation tool from Gurobi would result in a huge computational time. Hence adjustments to the model are needed to speed up the optimization process. This can be done by using heuristics. Heuristic algorithms are designed to approximate large scale problems faster and more efficient at the costs of optimality, accuracy and precision.
For this specific problem, first of all an initial solution could be created to use this as a starting point for further iterations. This initial solution can be found through the use of a constructive heuristics could be executed, such as the nearest neighbor heuristics. After the initial solution is found, an improvement heuristic could be applied to improve the initial solution. In this assignment an approach is asked which finds a feasible solution in a maximum computation time of 60 minutes, this asks for a heuristic with a relatively short computation time. Therefore, a local search heuristic could be implemented, which finds a local optimum. The advantage of this heuristic in comparison to a heuristic that searches for a global optimum is that the computation time is shorter. The disadvantage is that it is not certain that the local optimum found is also the global optimum.

# References

[1] United Consumers. *Actuele brandstofprijzen.* 2021. URL: https://www.unitedconsumers.com/tanken/informatie/brandstof-prijzen.asp.

[2] Webfleet Solutions. *Hoeveel diesel verbruikt een vrachtwagen per kilometer?* 2020. URL: https://www.webfleet.com/nl_nl/webfleet/blog/hoeveel-diesel-verbruikt-een-vrachtwagen-per-kilometer/.