Homework 4

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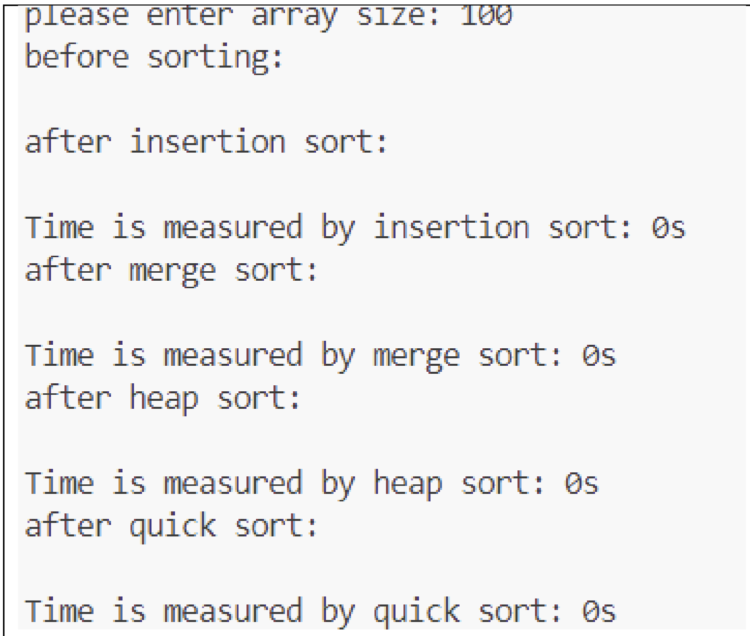
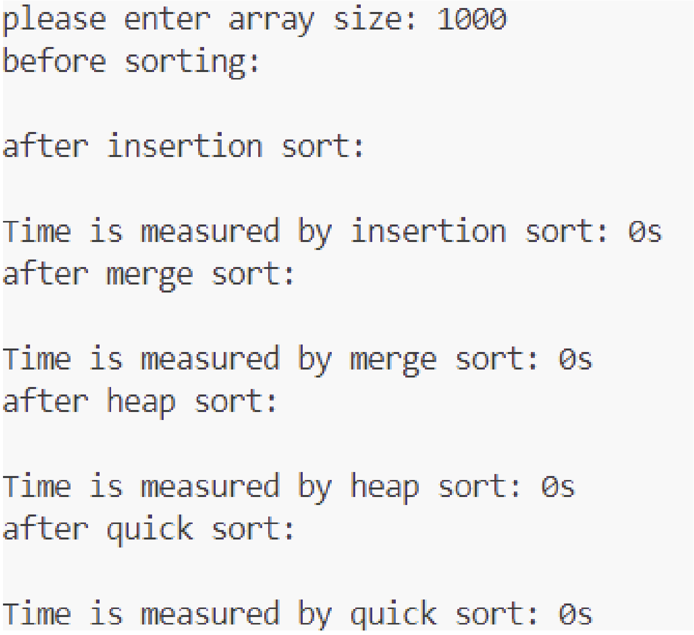
Comparison of insertion/merge/heap sort/quicksort:

Size/time	100	1,000	10,000	100,000	200,000	300,000
Insertion sort	0s	0s	0.039s	2.63s	11.073s	36.037s
Merge sort	0s	0s	0.002s	0.02s	0.034s	0.09s
Heap sort	0s	0s	0s	0.02s	0.047s	0.062s
r-Quicksort	0s	0s	0s	0.017s	0.031s	0.035s



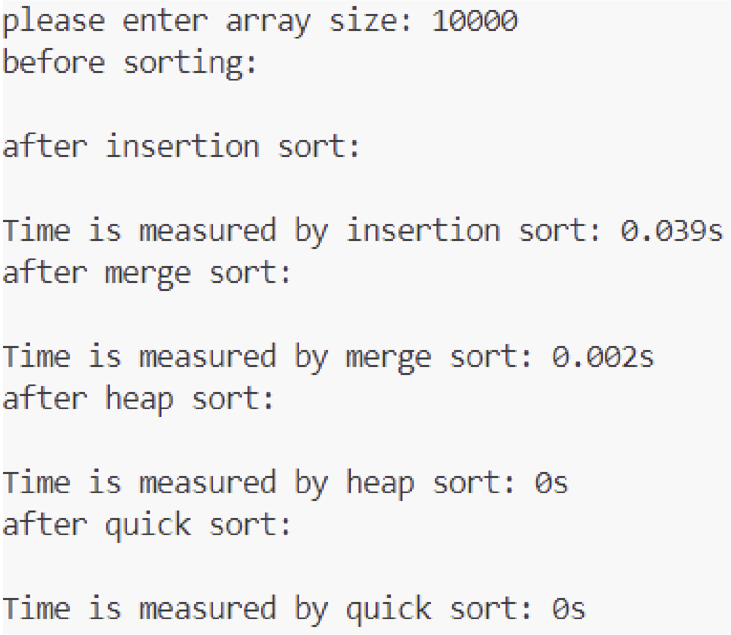
```
please enter array size: 100
before sorting:

after insertion sort:

Time is measured by insertion sort: 0s
after merge sort:

Time is measured by merge sort: 0s
after heap sort:

Time is measured by heap sort: 0s
after quick sort:

Time is measured by quick sort: 0s
```

100



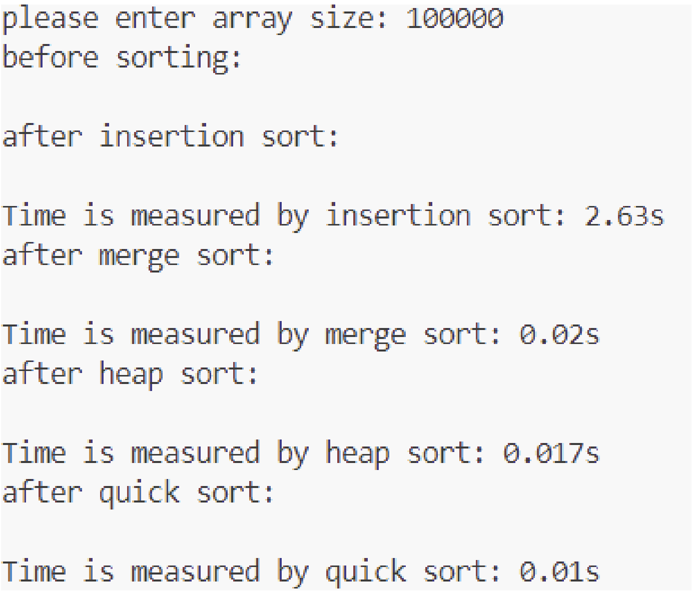
```
please enter array size: 1000
before sorting:

after insertion sort:

Time is measured by insertion sort: 0s
after merge sort:

Time is measured by merge sort: 0s
after heap sort:

Time is measured by heap sort: 0s
after quick sort:

Time is measured by quick sort: 0s
```

1,000



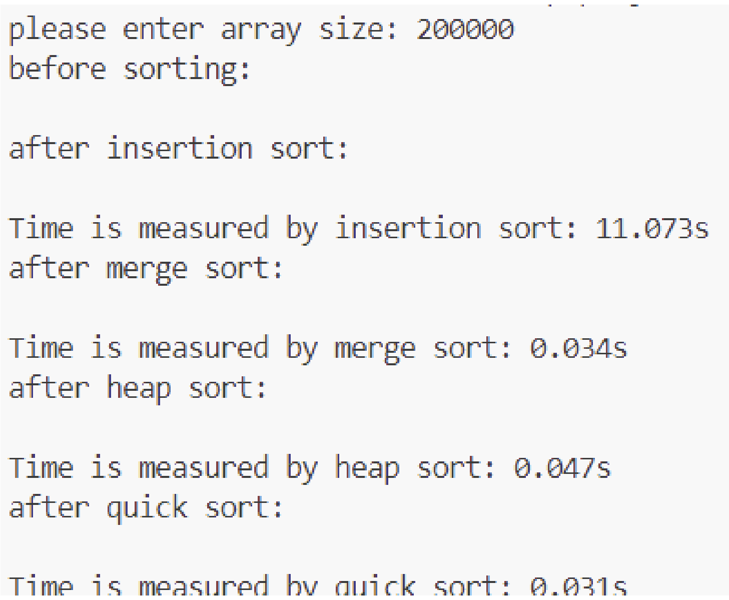
```
please enter array size: 10000
before sorting:

after insertion sort:

Time is measured by insertion sort: 0.039s
after merge sort:

Time is measured by merge sort: 0.002s
after heap sort:

Time is measured by heap sort: 0s
after quick sort:

Time is measured by quick sort: 0s
```

10,000



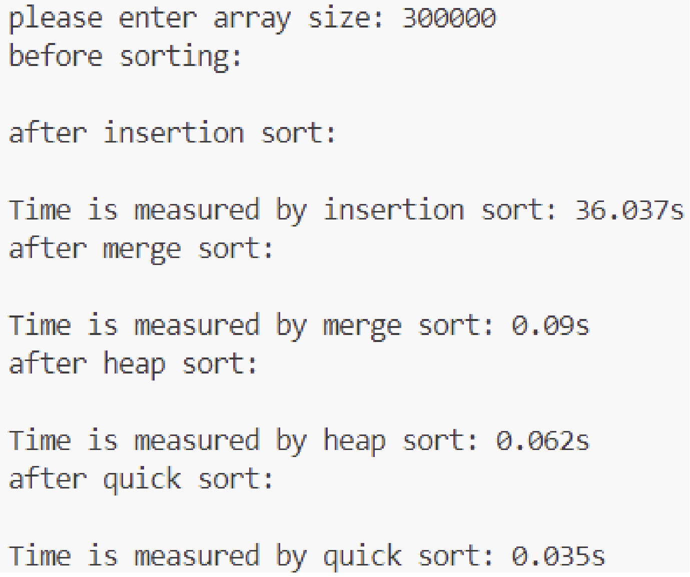
```
please enter array size: 100000
before sorting:

after insertion sort:

Time is measured by insertion sort: 2.63s
after merge sort:

Time is measured by merge sort: 0.02s
after heap sort:

Time is measured by heap sort: 0.017s
after quick sort:

Time is measured by quick sort: 0.01s
```

100,000



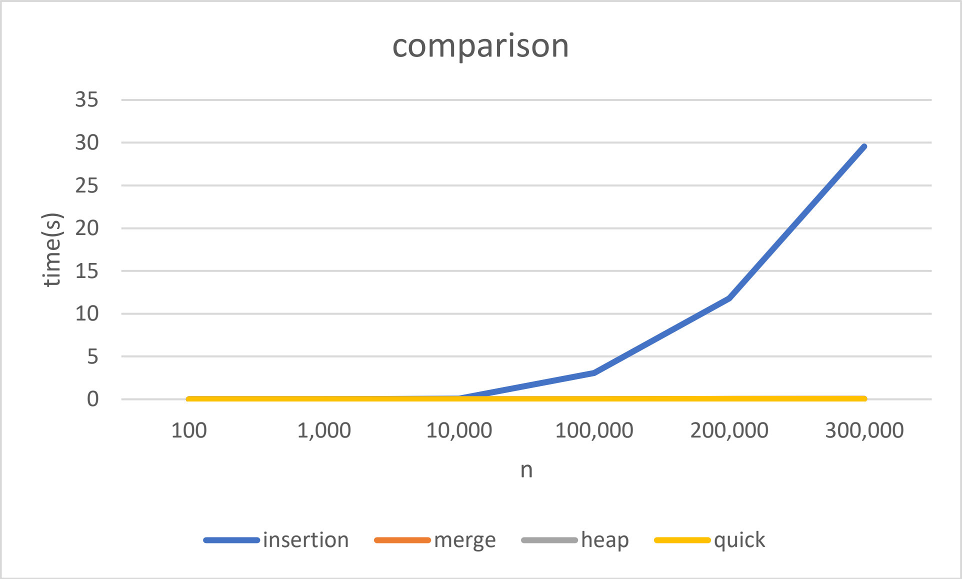
```
please enter array size: 200000
before sorting:

after insertion sort:

Time is measured by insertion sort: 11.073s
after merge sort:

Time is measured by merge sort: 0.034s
after heap sort:

Time is measured by heap sort: 0.047s
after quick sort:

Time is measured by quick sort: 0.031s
```

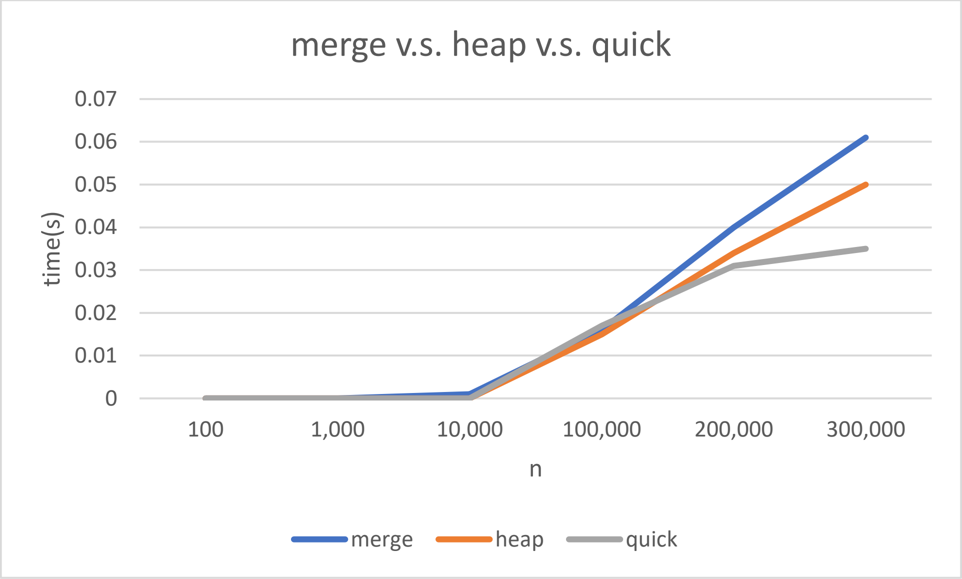
200,000



```
please enter array size: 300000
before sorting:

after insertion sort:

Time is measured by insertion sort: 36.037s
after merge sort:

Time is measured by merge sort: 0.09s
after heap sort:

Time is measured by heap sort: 0.062s
after quick sort:

Time is measured by quick sort: 0.035s
```

300,000





100000 以及 200000:

$$\frac{0.031}{0.017} = 1.823 \rightarrow \frac{200000}{100000} = 2.12(O(nlogn))$$

100000 以及 300000:

$$\frac{0.035}{0.017} = 2.058 \rightarrow \frac{300000}{100000} = 3.286(O(n))$$

說明:

首先，因為數字是在特定範圍亂數產生，某種程度上可以視作已經經過 randomized partition，因此沒有特定再去對 partition 做 randomized。然而，實驗之後發現 quicksort 的表現比較不穩定，沒有達到課堂所述的 O(nlogn)。因為 quicksort 的表現會因為輸入資料的亂度影響，即亂度越高，執行效果越佳。所

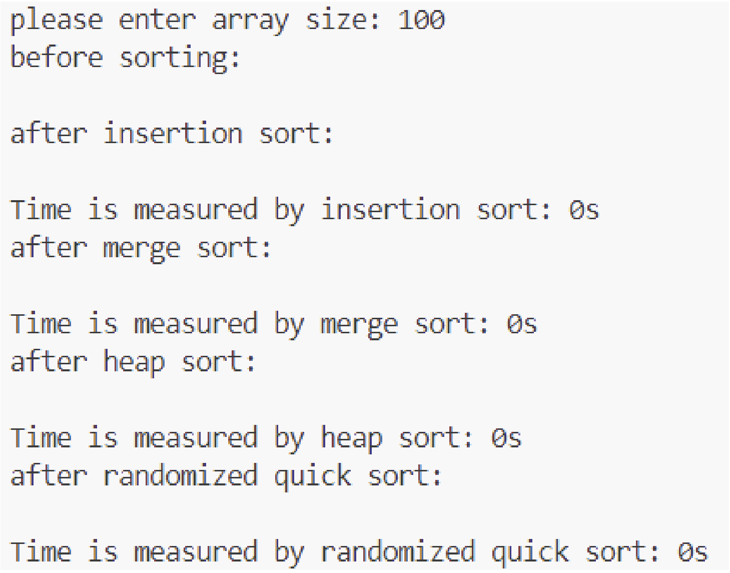
以當我們在討論他的時間複雜度時，其表現可能因為亂度不一，而導致其資料之間的差異這麼的大。因此後來才又針對 partition 再去用亂數處理。透過 randomized partition，的確發現 randomized quicksort 的表現趨於穩定，表現都比 heap sort 以及 merge sort 好。在所有的執行時間當中，可以發現 insertion sort 是最差的，其次 merge sort 以及 heap sort。考慮到其時間複雜度以及條件限制，我們可以合理推論，insertion sort 適合用在資料較少的排序中，以避免資料太大使執行時間過大的狀況出現。Merge sort 則是可以很穩定的表現，隨著資料上升。Heap sort 則是因為不會用到額外空間(即原地排列)，適合對內部記憶體有限制的情況使用。而 quicksort 則要保證其輸入資料的亂度，否則會出現最壞的狀況，即$O(n^2)$。

Algorithms	Advantages	Disadvantages
Insertion	・ 針對小數據或部分有序數列，表現較佳 ・ 實現簡單	・ 對於大型未知數據性能差
Merge	・ 穩定的時間複雜度，保持一致性能(不受資料原本排序影響)	・ 需要額外記憶體空間來儲存中間結果 ・ 需要遞迴跟合併的操作，實現較為複雜
Heap	・ 原地排序，不須額外用到其他記憶體 ・ 性能穩定(最差情況仍然為 nlogn)	・ 演算法牽涉交會元素的位置，可能導致實際效率不如其他演算法。
Randomized quick	・ 實現簡單 ・ 具有 nlogn 的時間複雜度	・ 需要保證輸入資料的亂度，否則時間複雜度變成 n^2

加上 randomized partition 之執行結果

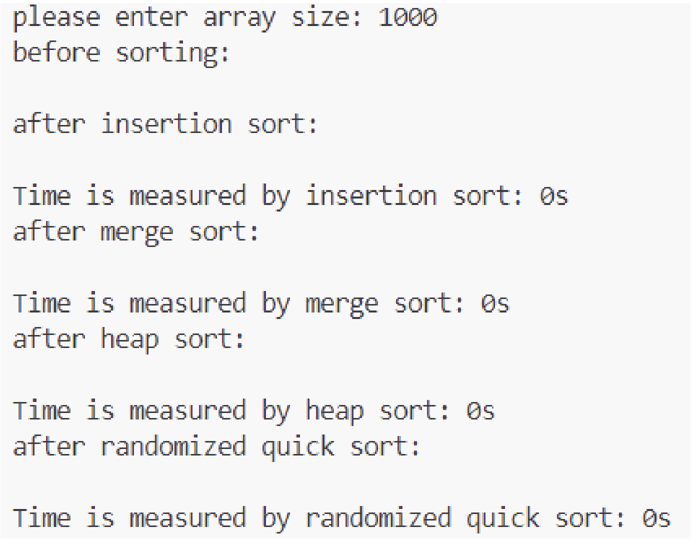
Comparison between insertion sort/merge sort/heap sort/randomized sort:

Size/time	100	1,000	10,000	100,000	200,000	300,000
Insertion sort	0s	0s	0.058s	3.608s	16.511s	34.793s
Merge sort	0s	0s	0.01s	0.031s	0.061s	0.096s
Heap sort	0s	0s	0.003s	0.031s	0.046s	0.079s
r-Quicksort	0s	0s	0s	0.022s	0.026s	0.038s



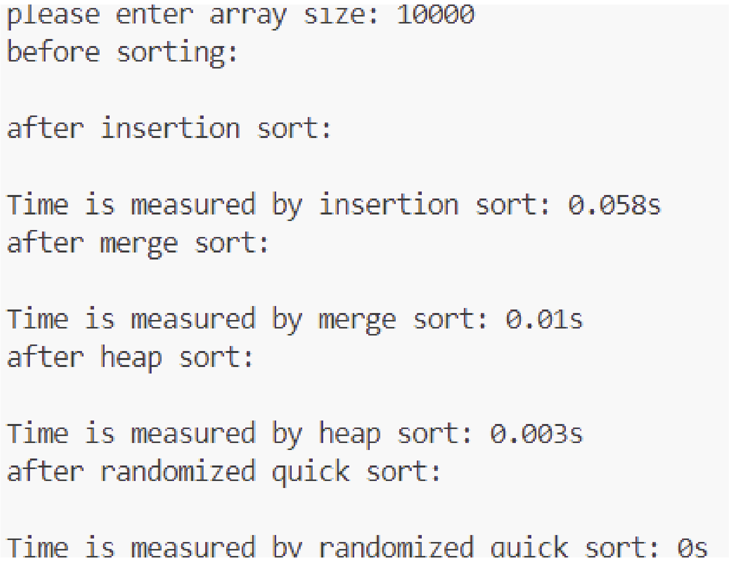
```
please enter array size: 100
before sorting:

after insertion sort:

Time is measured by insertion sort: 0s
after merge sort:

Time is measured by merge sort: 0s
after heap sort:

Time is measured by heap sort: 0s
after randomized quick sort:

Time is measured by randomized quick sort: 0s
```

100



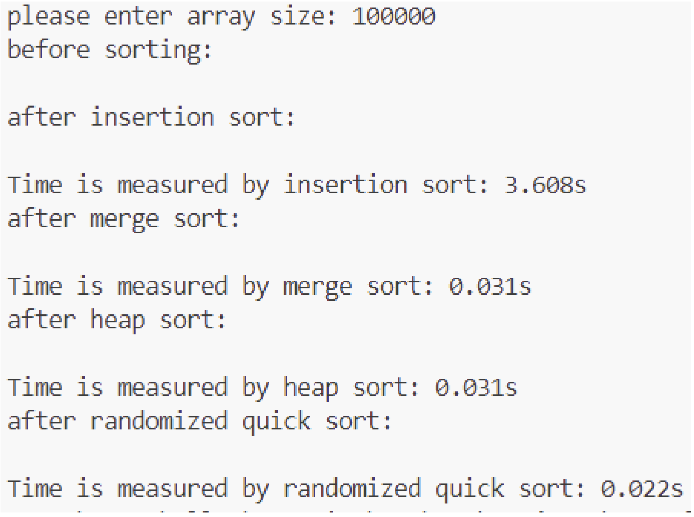
```
please enter array size: 1000
before sorting:

after insertion sort:

Time is measured by insertion sort: 0s
after merge sort:

Time is measured by merge sort: 0s
after heap sort:

Time is measured by heap sort: 0s
after randomized quick sort:

Time is measured by randomized quick sort: 0s
```

1,000



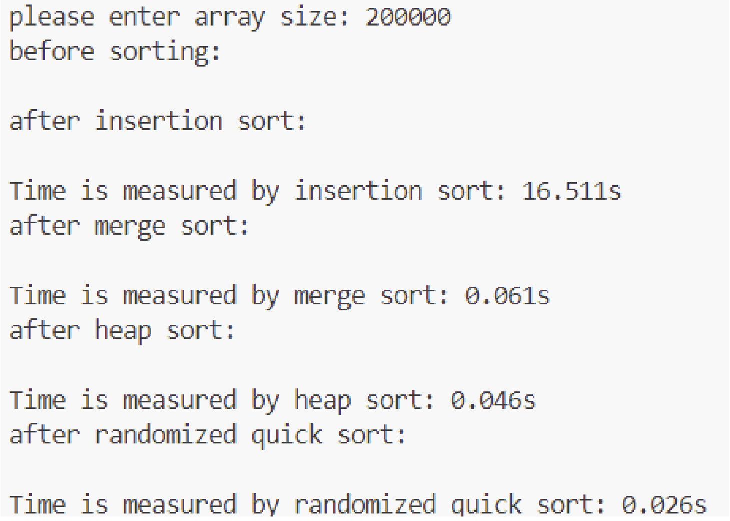
```
please enter array size: 10000
before sorting:

after insertion sort:

Time is measured by insertion sort: 0.058s
after merge sort:

Time is measured by merge sort: 0.01s
after heap sort:

Time is measured by heap sort: 0.003s
after randomized quick sort:

Time is measured by randomized quick sort: 0s
```

10,000



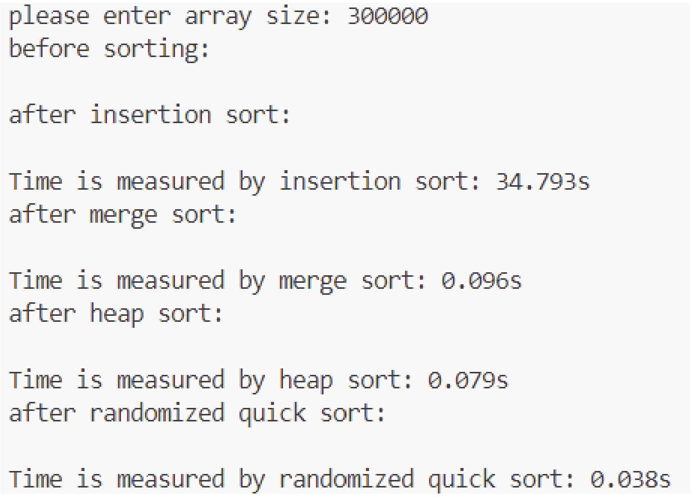
```
please enter array size: 100000
before sorting:

after insertion sort:

Time is measured by insertion sort: 3.608s
after merge sort:

Time is measured by merge sort: 0.031s
after heap sort:

Time is measured by heap sort: 0.031s
after randomized quick sort:

Time is measured by randomized quick sort: 0.022s
```

100,000



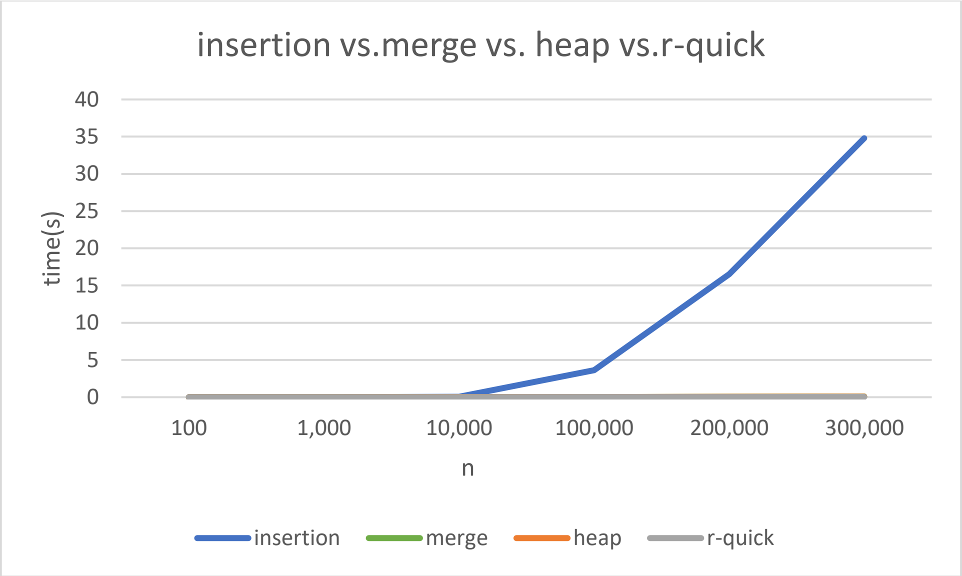
```
please enter array size: 200000
before sorting:

after insertion sort:

Time is measured by insertion sort: 16.511s
after merge sort:

Time is measured by merge sort: 0.061s
after heap sort:

Time is measured by heap sort: 0.046s
after randomized quick sort:

Time is measured by randomized quick sort: 0.026s
```

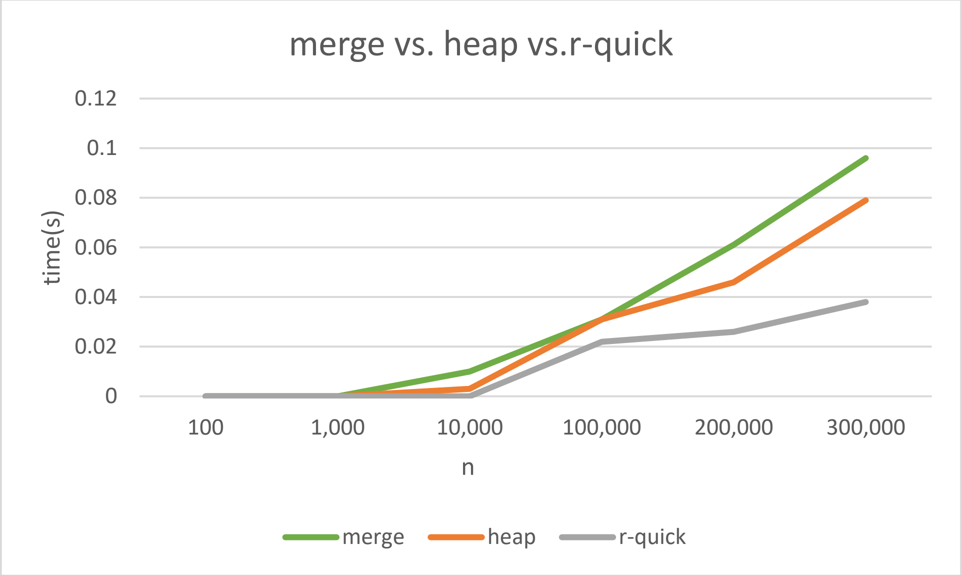
200,000



```
please enter array size: 300000
before sorting:

after insertion sort:

Time is measured by insertion sort: 34.793s
after merge sort:

Time is measured by merge sort: 0.096s
after heap sort:

Time is measured by heap sort: 0.079s
after randomized quick sort:

Time is measured by randomized quick sort: 0.038s
```

300,000





100000 以及 200000:

$$\frac{0.026}{0.022} = 1.18 \rightarrow \frac{200000}{100000} = 2.12(O(nlogn))$$

100000 以及 300000:

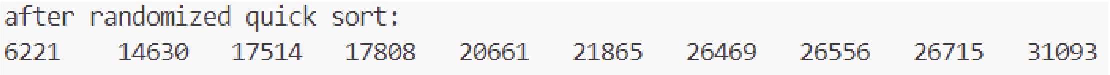
$$\frac{0.038}{0.022} = 1.72 \rightarrow \frac{300000}{100000} = 3.286(O(nlogn))$$

說明:

加上 randomized 的確發現比沒有加上 randomized 的趨勢更加穩定，但不是很明顯。有可能是原數列就是亂數產生的，所以本質上已經是亂的。加上 randomized 是避免輸入資料有排序規則的情況。上方的所有 quicksort 執行結果都比原本 nlogn 小，猜測可能是因為剛好輸入的資料非常亂，所以讓 quicksort 的效率提升。因此使 deterministic 的結果比統計行為(nlogn)佳。

Correctness of randomized quicksort:

以 n=10 為例



```
after randomized quick sort:
6221    14630   17514   17808   20661   21865   26469   26556   26715   31093
```