Pina Kolling
piko0011

Summary: Lecture 10

Summary for the chapters *11.1* up to page 245 and *11.2* (page 258 optional). [5, 1]

Randomized algorithms

Algorithms based on randomization.
(The algorithm employs a degree of randomness as part of its logic or procedure.)

Bipartite matching

Bipartite Graph

A graph $G = (U, V, E)$ is called bipartite if the vertices can be divided into two disjoint and independent sets U and V. (There are no edges between two elements of U or two elements of V).

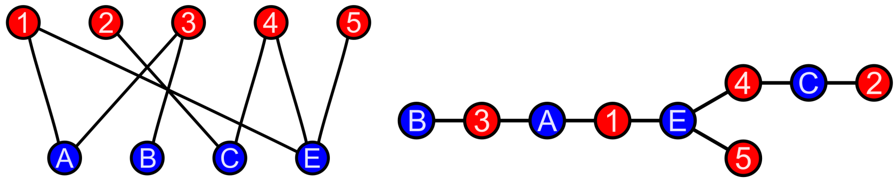


Figure 1: Examples of bipartife graphs with U and V marked in red and blue [2]

Problem: BipartiteMatching

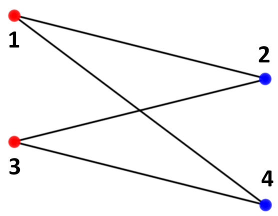
Given: Bipartite graph $G = (U, V, E)$.

Is there a perfect matching $M \subseteq E$ such that for any two edges (u, v) and (u', v') in M $u \neq u'$ and $v \neq v'$.

I other words: A matching in a Bipartite Graph is a set of the edges chosen in such a way that no two edges share an endpoint. The matching M is called perfect if for every node in V there is some edge in M. [3, 5, 4]

- construct bipartite graph with n nodes as $n \times n$ matrix A
- the element $A_{i,j}$ is a variable $x_{i,j}$ if $(i, j) \in E$
- the element $A_{i,j}$ is 0 if $(i, j) \notin E$

Example:



$$A = \begin{pmatrix} 0 & x_{1,2} & 0 & x_{1,4} \\ x_{2,1} & 0 & x_{2,3} & 0 \\ 0 & x_{3,2} & 0 & x_{3,4} \\ x_{4,1} & 0 & x_{4,3} & 0 \end{pmatrix}$$

Questions:
Is EVERY node cotained in the subset M when it is a perfect matching? Does a perfect matching then only exist with an even number of vertices and $|U| = |V|$?

Pina Kolling
piko0011

Determinant calculation

Leibniz-formula:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \qquad |A| = a{\cdot}e{\cdot}i + b{\cdot}f{\cdot}g + c{\cdot}d{\cdot}h - g{\cdot}e{\cdot}c - h{\cdot}f{\cdot}a - i{\cdot}d{\cdot}b$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad |B| = 1{\cdot}5{\cdot}9 + 2{\cdot}6{\cdot}7 + 3{\cdot}4{\cdot}8 - 7{\cdot}5{\cdot}3 - 8{\cdot}6{\cdot}1 - 9{\cdot}4{\cdot}2 = 0$$

$$|A| = \sum_{\pi} \sigma(\pi) \prod_{i=1}^{n} A_{i,\pi(i)}$$

- $\sigma(\pi)$ decides if $+$ or $-$
- leads to $n!$ summands
- Example: $n = 3$
  $3! = 6$ summands
  6 permutations for π

$+1$: (1 2 3) (2 3 1) (3 1 2)

-1: (3 2 1) (2 1 3) (1 3 2)

$\rightarrow |A| = A_{1,1} \cdot A_{2,2} \cdot A_{3,3} + A_{2,1} \cdot A_{3,2} \cdot A_{1,3} + ...$

Gaussian elimination:

- Gauß algorithm for solving LSE (linear systems of equations)
- allowed operations:
  - addition of rwos
  - subtraction of rows
  - multiply row with integer x
  - divide row by integer x
  - switch to rows
- wanted: upper triangular form (all entries below the diagonal 0)
- determinant is the product of the diagonal entries
- Example:

$$\begin{pmatrix} 1 & 3 & 2 & 5 \\ 1 & 7 & -2 & 4 \\ -1 & -3 & -2 & 2 \\ 0 & 1 & 6 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 2 & 5 \\ 0 & 4 & -4 & -1 \\ 0 & 0 & 0 & 7 \\ 0 & 1 & 6 & 2 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 1 & 3 & 2 & 5 \\ 0 & 4 & -4 & -1 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 7 & 2\frac{1}{4} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 2 & 5 \\ 0 & 4 & -4 & -1 \\ 0 & 0 & 7 & 2\frac{1}{4} \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

Figure 2: Examples gaussian elimination [5]

$|A| = 1 \cdot 4 \cdot 7 \cdot 7 = 196$

Pina Kolling
piko0011

Symbolic Determinants

TODO
Questions:

Monte Carlo algorithm

TODO
Questions:

Randomized complexity classes

TODO
Questions:

- some slides about stuff that is not in the book
- Satz von Rice

TODO
Questions:

References

[1] Martin Berglund. *Lecture notes in Computational Complexity.*

[2] *Bipartite graph image source.* `https://en.wikipedia.org/wiki/Bipartite_graph`.

[3] GeeksforGeeks. *Maximum Bipartite Matching.* `https://www.geeksforgeeks.org/maximum-bipartite-matching/`, last opened: 09.12.22.

[4] Swastik Kopparty. *Bipartite Graphs and Matchings.* `https://sites.math.rutgers.edu/~sk1233/courses/graphtheory-F11/matching.pdf`, last opened 09.12.22. 2011.

[5] Christos H. Papadimitriou. *Computational Complexity.* Addison-Wesley Publishing Company, 1994.