

Summary: Lecture 8

Summary for the chapters 9.3 and 9.4. [2, 1]

Undirected graph

An undirected graph G is a pair of sets (V, E) where V is the finite set of nodes and E is a set of unordered pairs in V that are symmetric:

$$\forall i, j \in V, i \neq j : (i, j) \in E \Rightarrow (j, i) \in E$$

IndependentSet

IndependentSet

INDEPENDENTSET

Input: An undirected Graph $G = (V, E)$ and a number k .

Question: Is there a set $I \subseteq V$ of $k = |I|$ nodes with no edges in between? (INDEPENDENTSET)

INDEPENDENTSET is NP-complete.

Proof idea:

- triangle construction: any independent set can contain at most one node of the triangle

$$(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

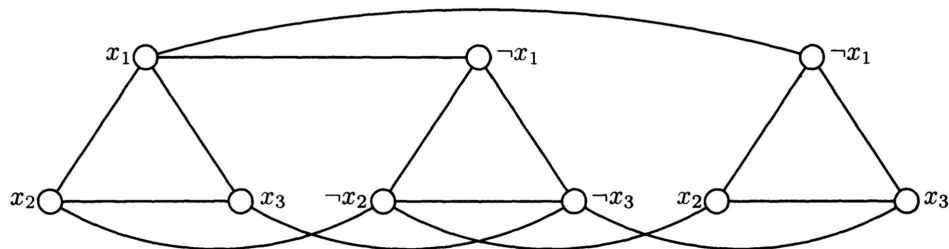


Figure 1: Graph with triangles [2]

- consider only graphs whose nodes can be partitioned in m disjoint triangles
→ independent set can contain at most m nodes (one from each triangle)
- reduction from 3SAT to INDEPENDENTSET
- construct graph of formula ϕ :
 - each literal as a node
 - clauses as triangles
 - edges between nodes in different triangles if they correspond to the same literal (negated)
 - $K = m$ (m clauses)

TODO

Questions:

HamiltonPath is NP-complete

HAMILTONPATH is NP-complete.

Proof idea:

- Another reduction from 3SAT

TODO

Questions:

TSP(D)

TSP(D)

TSP(D) is a decision version of TSP.

Input: A $n \times n$ distance matrix and a bound $B \in \mathbb{N}$

Question: Is there a round tour of length $\leq B$ that visits all *cities*?

TSP(D) is NP-complete.

Proof idea:

- budget of nodes is $B = |V| + 1$

TODO

Questions:

Knapsack

Knapsack

KNAPSACK is NP-complete.

- filled in in one dimensional array on the board
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TODO

Questions:

References

- [1] Martin Berglund. *Lecture notes in Computational Complexity*.
- [2] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.