

## Summary: Lecture 5

Summary for the chapters  $X$  and  $X$ . [2]

### Reduction

#### Examples of NP-problems:

- Travelling Salesman Problem
- SATISFIABLE
- REACHABILITY (in P)
- CIRCUIT VALUE (in P)

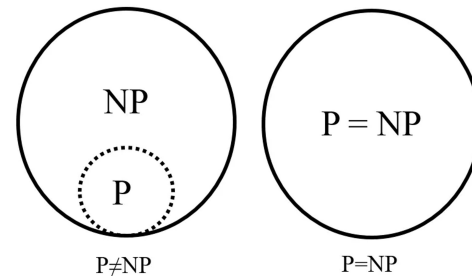


Figure 1: P and NP sets [1]

- reduction: a problem is at least as hard as another
- problem  $A$  is at least as hard as problem  $B$  if  $B$  reduces to  $A$
- $B$  reduces to  $A$  if there is a transformation  $R$ 
  - $R$  produces for every input  $x$  of  $B$  an equivalent input  $R(x)$  of  $A$
  - the answer of input  $x$  on  $B$  and input  $R(x)$  on  $A$  have to be the same
- to solve  $B$  on input  $x$ ,  $A$  can be solved instead with input  $R(x)$

#### Reduction

Problem  $A$  is at least as hard as problem  $B$  if  $B$  reduces to  $A$ .

#### Transformation function:

- transformation function  $R$  should not be too hard to compute  
→  $R$  should be limited

#### Transformation function

fdhjksk

TODO

Questions:

#### Reduction HAMILTONIAN PATH to SATISFIABLE

- instance: Graph  $G$   
question: Is there a path in  $G$  that visits each node one?
- log space reduction from HP to S
- demonstrates HP not significantly harder than SAT
- write a logical formula that only becomes true when it is HP
- 4, 3, 1, 2 as path  
 $x_{1,4} = T, x_{2,3} = T, x_{3,1} = T, x_{4,2} = T,$

- slide is not quite correct
- $(\text{not}x_{1,1} \text{ or } \text{not}x_{2,1}) \text{ and } (\text{not}x_{1,1} \text{ or } \text{not}x_{3,1})$   
 $\text{and}(\text{not}x_{1,1} \text{ or } \text{not}x_{4,1}) \text{ and } (\text{not}x_{2,1} \text{ or } \text{not}x_{3,1})$   
 $\text{and}(\text{not}x_{2,1} \text{ or } \text{not}x_{4,1}) \text{ and } (\text{not}x_{3,1} \text{ or } \text{not}x_{4,1}) \text{ and } \dots$   
first index: step, second: node

TODO

Questions:

## Boolean Circuits

TODO

Questions:

## Reduction REACHABILITY PATH to CIRCUIT VALUE

TODO

Questions:

## Further examples

TODO

Questions:

## Closedness under Composition

TODO

Questions:

## References

- [1] *Image source: P-NP sets.* <https://www.techno-science.net/actualite/np-conjecture-000-000-partie-denouee-N21607.html>.
- [2] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.