

## Summary: Lecture 6

Summary for the chapter 8.2. [1, 3]

### Completeness

Let  $C$  be a complexity class and let  $L$  be a language in  $C$ .  $L$  is called *C-complete* if any language  $L' \in C$  can be reduced to  $L$ .

(Every language of a complexity class can be reduced to  $L$ .)

- reducibility is transitive  $\rightarrow$  problems are ordered by difficulty
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### TODO

Questions:

Which problems can be reduced to a formal language?

SAT can be expressed as formal language. [2]

Because CIRCUIT SAT can be reduced to SAT: CIRCUIT SAT can be reduced to a formal language. (?)

Where to set the line between language decisions and other problems?

### What does completeness do for us?

- a reduction definition is useful because the complexity classes are closed under reduction
- examples look helpful
- $L$  and  $R$  seem to be important:

$$\begin{array}{ll} L' \in P & A \\ L \rightarrow L' & R \end{array}$$

- drawing set circle inclusion thing (P and NP)

### TODO

Questions:

### P-completeness of CIRCUIT VALUE

#### Problem: Circuit Value

The CIRCUIT VALUE Problem is the problem of computing the output of a given Boolean circuit on a given input.

In terms of time complexity, it can be solved in linear time (topological sort).

- P-complete
- limit of power of reductions

- got a little tired and zoned out

TODO

Questions:

## The reduction (?)

### Problem: Circuit Sat

The circuit satisfiability problem (CIRCUIT SAT) is the decision problem of determining whether a given Boolean circuit has an assignment of its inputs that makes the output true.

Input: a Boolean circuit  $C$

Question: Is there a truth assignment which makes  $C$  output the value true?

## CIRCUIT SAT is NP-complete

- circuit decides nondeterministically (?)
- a variable is added in the nondeterministic Turing Machine
- check if one of the variables is true: use this choice (?)
- problem: can we set these variables such that the Turing Machine accepts?
- answer corresponds directly to *is there a choice of decisions such that the Turing machine accepts?*
- extremely direct reduction
- Cook's theorem :)
- SAT is NP-complete

TODO

Questions:

## References

- [1] Martin Berglund. *Lecture notes in Computational Complexity*.
- [2] klaus-joern Lange. “The Boolean Formula Value Problem as Formal Language”. In: (Jan. 2012). DOI: 10.1007/978-3-642-31644-9\_9.
- [3] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.