

Summary: Lecture 9

Summary for the chapter 10.3. [2, 1]

Function problems

Function problem

Finding a specific solution to a problem if possible, else return *no*.

- focus so far: languages deciding decision problems
- give *yes* or *no* as answer
- now: focus on finding a solution:
 - find satisfying truth assignment for a boolean expression
 - find optimal tour for TSP→ function problems
- decision problems are helpful for negative results of function problems
- complexity of the decision problem helps to specify the complexity of the corresponding function problem

SAT and FSAT

SAT

The SAT (satisfiability) problem is the problem of determining if there exists an interpretation that satisfies a given Boolean formula. [3]

FSAT

The FSAT (satisfiability) problem is a function problem.

Given a boolean expression ϕ .

If ϕ is satisfiable, return a satisfying truth assignment and otherwise return *no*.

- for input ϕ there might be no satisfying truth assignment
 - return *no*
- for input ϕ there might be more than one satisfying truth assignment
 - return any satisfying truth assignment
- if SAT can be solved in polynomial time, FSAT can be solved in polynomial time, too

Algorithm for FSAT:

- expression ϕ with variables x_1, \dots, x_n
- ask if ϕ is satisfiable:
 - if *no*: stop and return *no*
 - if *yes*: come up with satisfying truth assignment

- * consider two expressions: $\phi[x_1 = \text{true}]$ and $\phi[x_1 = \text{false}]$
- * check which one is satisfiable
(if both are, chose one)
- * substitute the value of x_1 in ϕ
- * continue with x_2
- * at most $2n$ calls to find the satisfying truth assignment

Self-reducibility:

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TODO

Questions:

Self-reducibility

TSP and TSP(D)

TSP(D)

Given a list of cities and the distances between each pair of cities.

Is there a shortest possible route that visits each city exactly once and returns to the origin city?

TSP

Given a list of cities and the distances between each pair of cities.

What is the shortest possible route that visits each city exactly once and returns to the origin city?

- solve TSP with an algorithm for TSP(D)

TODO

Questions:

FP and FNP

FP

Content

FNP

Content

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Questions:

Reductions between function problems

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- translate answers back to the original problem
- reduction is a pair (R, S) :
 - R translates input x to input x'
 - S translates result y' to result y
- A' is B there (A' does not exist on the slides)

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Questions:

How to prove $FP = FNP$?

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Questions:

Computing a satisfying assignment bit by bit

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- SAT' is a formula φ plus an assignment that satisfies φ
- assignment as clauses that connects the single variables or their negation with \wedge

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Questions:

If $FP=FNP$ optimization problems become easy

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Questions:

Another argument

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- cryptographic argument: if $P=NP$, no safe encoding exists

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Questions:

Total FNP

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TODO
Questions:

References

- [1] Martin Berglund. *Lecture notes in Computational Complexity*.
- [2] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.
- [3] Prof. Dr. Thomas Schwentick. *Lecture notes in Grundbegriffe der theoretischen Informatik*.
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