

## Summary: Lecture 6

Summary for the chapter 8.2. [1, 4]

### Completeness

Let  $C$  be a complexity class and let  $L$  be a language in  $C$ .  $L$  is called  $C$ -complete if any language  $L' \in C$  can be reduced to  $L$ .

(Every language of a complexity class can be reduced to  $L$ .)

- reducibility is transitive  $\rightarrow$  problems are ordered by difficulty
- 

### TODO

Questions:

Which problems can be reduced to a formal language?

SAT can be expressed as formal language. [3]  
 $\Rightarrow$  SAT can be reduced to a formal language. (?)

Because CIRCUIT SAT can be reduced to SAT: CIRCUIT SAT can be reduced to a formal language. (?)

#### Formal language

Formal languages are abstract languages, which define the syntax of the words that get accepted by that language. It is a set of words that get accepted by the language and has a set of symbols that is called alphabet, which contains all the possible characters of the words. Those characters are called nonterminal symbols. [2, 5]

#### Kleene star

The Kleene star  $\Sigma^*$  of an alphabet  $\Sigma$  is the set of all words that can be created through concatenation of the symbols of the alphabet  $\Sigma$ . The empty word  $\epsilon$  is included.

#### Formal language

A formal language  $L$  over an alphabet  $\Sigma$  is a subset of the Kleene star of the alphabet:  
 $L \subseteq \Sigma^*$

Where to set the line between language decisions and other problems? Can every problem be constructed as a formal language?

### What does completeness do for us?

- a reduction definition is useful because the complexity classes are closed under reduction
- examples look helpful

- $L$  and  $R$  seem to be important:

$$\begin{array}{ll} L' \in P & A \\ L \rightarrow L' & R \end{array}$$

- drawing set circle inclusion thing (P and NP)

TODO

Questions:

## P-completeness of CIRCUIT VALUE

### Problem: Circuit Value

The CIRCUIT VALUE Problem is the problem of computing the output of a given Boolean circuit on a given input.

In terms of time complexity, it can be solved in linear time (topological sort).

- P-complete
- limit of power of reductions
- got a little tired and zoned out

TODO

Questions:

## The reduction (?)

### Problem: Circuit Sat

The circuit satisfiability problem (CIRCUIT SAT) is the decision problem of determining whether a given Boolean circuit has an assignment of its inputs that makes the output true.

Input: a Boolean circuit  $C$

Question: Is there a truth assignment which makes  $C$  output the value true?

## CIRCUIT SAT is NP-complete

- circuit decides nondeterministically (?)
- a variable is added in the nondeterministic Turing Machine
- check if one of the variables is true: use this choice (?)
- problem: can we set these variables such that the Turing Machine accepts?
- answer corresponds directly to *is there a choice of decisions such that the Turing machine accepts?*
- extremely direct reduction
- Cook's theorem :)
- SAT is NP-complete

TODO

Questions:

## References

- [1] Martin Berglund. *Lecture notes in Computational Complexity*.
- [2] *Chomsky's Normal Form (CNF)*. Website. <https://www.javatpoint.com/automata-chomskys-normal-form>, opened on 26.09.2022.
- [3] klaus-joern Lange. "The Boolean Formula Value Problem as Formal Language". In: (Jan. 2012). DOI: 10.1007/978-3-642-31644-9\_9.
- [4] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.
- [5] A.J. Kfoury Robert N. Moll Michael A. Arbib. *An Introduction to Formal Language Theory*. Springer-Verlag, 1988.