

Summary: Lecture 10

Summary for the chapters 11.1 up to page 245 and 11.2 (page 258 optional). [5, 1]

Randomized algorithms

Algorithms based on randomization.

(The algorithm employs a degree of randomness as part of its logic or procedure.)

Bipartite matching

Bipartite Graph

A graph $G = (U, V, E)$ is called bipartite if the vertices can be divided into two disjoint and independent sets U and V . (There are no edges between two elements of U or two elements of V).

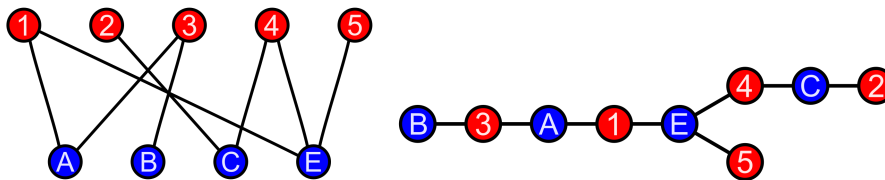


Figure 1: Examples of bipartite graphs with U and V marked in red and blue [2]

Problem: BipartiteMatching

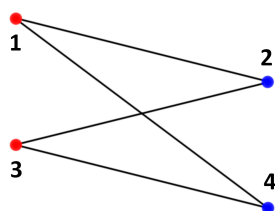
Given: Bipartite graph $G = (U, V, E)$.

Is there a perfect matching $M \subseteq E$ such that for any two edges (u, v) and (u', v') in M $u \neq u'$ and $v \neq v'$.

In other words: A matching in a Bipartite Graph is a set of the edges chosen in such a way that no two edges share an endpoint. The matching M is called perfect if for every node in V there is some edge in M . [3, 5, 4]

- construct bipartite graph with n nodes as $n \times n$ matrix A
- the element $A_{i,j}$ is a variable $x_{i,j}$ if $(i, j) \in E$
- the element $A_{i,j}$ is 0 if $(i, j) \notin E$

Example:



$$A = \begin{pmatrix} 0 & x_{1,2} & 0 & x_{1,4} \\ x_{2,1} & 0 & x_{2,3} & 0 \\ 0 & x_{3,2} & 0 & x_{3,4} \\ x_{4,1} & 0 & x_{4,3} & 0 \end{pmatrix}$$

Questions:

Is EVERY node contained in the subset M when it is a perfect matching? Does a perfect matching then only exist with an even number of vertices and $|U| = |V|$?

Determinant calculation

Leibniz-formula:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \quad |A| = a \cdot e \cdot i + b \cdot f \cdot g + c \cdot d \cdot h - g \cdot e \cdot c - h \cdot f \cdot a - i \cdot d \cdot b$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad |B| = 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2 = 0$$

$$|A| = \sum_{\pi} \sigma(\pi) \prod_{i=1}^n A_{i,\pi(i)}$$

TODO

Questions:

Symbolic Determinants

TODO

Questions:

- some slides about stuff that is not in the book
- Satz von Rice

TODO

Questions:

References

- [1] Martin Berglund. *Lecture notes in Computational Complexity*.
- [2] *Bipartite graph image source*. https://en.wikipedia.org/wiki/Bipartite_graph.
- [3] GeeksforGeeks. *Maximum Bipartite Matching*. <https://www.geeksforgeeks.org/maximum-bipartite-matching/>, last opened: 09.12.22.
- [4] Swastik Kopparty. *Bipartite Graphs and Matchings*. <https://sites.math.rutgers.edu/~sk1233/courses/graphtheory-F11/matching.pdf>, last opened 09.12.22. 2011.
- [5] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.