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Summary: Lecture 4

Summary for the chapter *7.3* from page 150 on. [3]

Nondeterministic Turing Machine

A nondeterministic Turing machine (NTM) has states, which have more than one possible next state for an action. The states are not completely determined by its action and the current symbol it sees, (unlike a deterministic Turing Machine).
NTMs are for example used in thought experiments. One of the most important problems in is the P versus NP problem: How difficult it is to simulate nondeterministic computation with a deterministic computer? [1, 3]

Asymmetry of non-deterministism

Asymmetry of nondeterministic acceptance:

- Example: find out if a formula φ is satisfiable ($\varphi \in SAT$):
  - choose truth values for the variables nondeterministically
  - check if they make φ become true
- this approach seems to be unpractical so check whether φ is not satisfiable ($\varphi \in \overline{SAT}$)
- Question whether NP = coNP is a statement about *all* options

TODO with book

Asymmetry of nondeterministic space:

- Example: REACHABILITY $\in$ NL
  - starting at start node 1
  - algorithm walks through nondeterminstically chosen edges $\leq n$ times
  - only current position is remembered ($\log n$ space)
  - accepts if current node is node n
- this approach seems to be unpractical so check if node n is *not* reachable from node 1

TODO with book

$\log n$ space

A graph algorithm using $O(\log n)$ space stores a fixed number of pointers, independent of n, and manipulates them in some way. [2]

Nondeterministically computing functions

- A nondeterminstic Turing Mashine M computes a function f if the the following hold for every input x:
  - one of the computations of M stops in the halting state $h$with the corret result $f(x)$ on the output tape
  - all computations that do not correctly output $f(x)$ stop instead in a *no*-state (this path failed then)

TODO with book

Questions:
Does this lead to the Haltingproblem?

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Immerman-Szelepscènyi

Theorem:
Given a graph G and a start node x, the number of nodes that are reachable from x in G can nondeterministically be computed in space $\log n$ (where n is the number of nodes of G).

Proof concept:

- nodes $1, ..., n$ with start node 1
- $S(i)$ is the set of nodes which are reachable from the startnode with a pathlength of i
  - $S(0)$ will contain node 1
  - $s(1)$ will contain all neighbours of 1
- Algorithm consitsts out of 3 nested for-loops:
  - outer for loop:
    - computes number of nodes reachable from initial node (for loop with k steps)
    - in each stepthe previous set is overwritten with the next one because the space is limited
  - second for loop:
    - it is computed how far the previous steps got and summed up how far it can get
  - third loop:
    - checking if node belongs to $S(i)$
    - return no when all guesses were correct?
- nodes cant be marked (this would use linear space) but with determinism it gets into $\log n$

Algorithm (2) slides seems to be important
TODO and TODO with book
Questions:

REACHABILITY $\in$ NL

NL = nondeterministic logarithmic space
little bit of stuff between log n and constand but no interesting stuff

TODO
Questions:

NSPACE is closed under complement

TODO
Questions:

References

[1] Jeff Erickson. *Nondeterministic Turing Machine*. `http://jeffe.cs.illinois.edu/teaching/algorithms/models/09-nondeterminism.pdf`. 2016.

[2] *Logarithmic Space and NL-Completeness*. `http://www.cs.toronto.edu/~ashe/logspace_handout.pdf`. 2020.

[3] Christos H. Papadimitriou. *Computational Complexity*. Addison-Wesley Publishing Company, 1994.