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Summary: Lecture 9

Summary for the chapter *10.3.* [2, 1]

Function problems

Function problem

Finding a specific solution to a problem if possible, else return *no*.

- focus so far: languages deciding decision problems
- give *yes* or *no* as answer
- now: focus on finding a solution:
  - find satisfying truth assignment for a boolean expression
  - find optimal tour for TSP

  → function problems

- decision problems are helpful for negative results of function problems
- complexity of the decision problem helps to specify the complexity of the corresponding function problem

SAT and FSAT

SAT

The SAT (satisfiability) problem is the problem of determining if there exists an interpretation that satisfies a given Boolean formula. [3]

FSAT

The FSAT (satisfiability) problem is a function problem.

Given a boolean expression ϕ.
If ϕ is satisfiable, return a satisfying truth assignment and otherwise return *no*.

- for input ϕ there might be no satisying truth assignment
  - return *no*
- for input ϕ there might be more than one satisying truth assignment
  - return any satisfying truth assignment
- if SAT can be solved in polynomial time, FSAT can be solved in polynomial time, too

Algorithm for FSAT:

- expression ϕ with variables $x_1, ..., x_n$
- ask if ϕ is satisfiable:
  - if *no*: stop and return *no*
  - if *yes*: come up with satisying truth assignment

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  - ∗ consider two expressions: $\phi[x_1 = \text{true}]$ and $\phi[x_1 = \text{false}]$
  - ∗ check which one is satisfiable
    (if both are, chose one)
  - ∗ substitute the value of x_1 in ϕ
  - ∗ continue with x_2
  - ∗ at most $2n$ calls to find the satisfying truth assignment

Self-reducibilty:

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TODO
Self-reducibility
Questions:

TSP and TSP(D)

TSP(D)

Given a list of cities and the distances between each pair of cities.

Is there a possible route of length k that visits each city exactly once and returns to the origin city?

TSP

Given a list of cities and the distances between each pair of cities.

What is the shortest possible route that visits each city exactly once and returns to the origin city?

- solve TSP with an algorithm for TSP(D)
- find optimum cost C of the tour with binary search (between 0 and 2^n)
- remove one intercity distance at a time to check if it is part of the optimal tour
- after n^2 calls only entries of the distance matrix are there that are used for the optimum tour

TODO
algorithm TSP (?)
maybe example (?) Questions:

FP and FNP

Lanugage L

$L = \{x : (x, y) \in R \textit{ for some } y\}$
L gets an input x and finds a y with $((x, y) \in R$ and the relation $R \subseteq \Sigma^* \times \Sigma^*$.

NP

The language $L \subseteq \Sigma^*$ is in NP only if there is a polynomially decidable and polynomially balanced relation R such that $L = \{x : (x, y) \in R \text{ for some } y\}$.

Relationship between decision and function problems:

- L is a lanuage in NP
  - **Decision problem:**
    There is a string y with $R(x, y)$ only if $x \in L$.
  - **Function problem:**
    Given x, find a string y such that $R(x, y)$ if it exists, else return *no*.

FP

Content

FNP

Content

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TODO
Questions:

Reductions between function problems

Title

Content

- translate answers back to the original problem
- reduction is a pair (R, S):
  - R translates input x to input x'
  - S translates result y' to result y
- A' is B there (A' does not exist on the slides)

TODO
Questions:

How to prove FP = FNP?

Title

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TODO
Questions:

Computing a satisfying assignment bit by bit

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- SAT′ is a formular φ plus an assignment that satisfies φ
- assignment as clauses that connects the single variables or their negation with $\wedge$

TODO
Questions:

If FP=FNP optimuzation problems become easy

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TODO
Questions:

Another argument

Title
Content

- cryptographic argument: if P=NP, no safe encoding exists

TODO
Questions:

Total FNP

Title
Content

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TODO
Questions:

References

[1] Martin Berglund. *Lecture notes in Computational Complexity.*

[2] Christos H. Papadimitriou. *Computational Complexity.* Addison-Wesley Publishing Company, 1994.

[3] Prof. Dr. Thomas Schwentick. *Lecture notes in Grundbegriffe der theoretischen Informatik.* `https://www.cs.tu-dortmund.de/nps/de/Studium/Ordnungen_Handbuecher_Beschluesse/Modulhandbuecher/Archiv/Bachelor_LA_GyGe_Inf_Modellv/_Module/INF-BfP-GTI/index.html`.