REPORT

Runtime analysis of the Java implementation of the CYK-algorithm

Pina Kolling

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Abstract

Formal languages and grammars can be used in different fields, for example for AI learning methods. The CYK-algorithm is a method to examine if an input word is part of a formal language. In this report, the implementation of different parsing methods for the CYK-algorithm is described. The runtimes of those methods are analysed and compared. For each method experiments with different inputs are documented and evaluated. The results of the experiments and the theoretical runtime analysis in O-notation are compared to show the differences and similarities. The results are also shown in graphs. In addition to this, one method is modified to work with linear grammars and concepts of error correction are described and presented.

Contents

1 Introduction

Parsing in Computer Science is the process of reading, analysing and processing an input string. In this context it is used for analysing a string of characters to examine if the string is built according to the rules of a formal grammar.

A formal grammar describes how to form strings with correct syntax out of characters from a formal language's alphabet (explained in Section 2.2 and 2.1). To examine if such a string follows the rules of a grammar, the *Cocke-Younger-Kasami*-algorithm (short: *CYK*) can be used. This algorithm is described in Section 2.5. To use the *CYK*-algorithm, the grammar needs to be in a specific format, that is called *Chomsky-Normal-Form* (short: *CNF*), which is explained in Section 2.3. [7, 10]

Three different parsing methods are implemented and described of which one executes the *CYK*-algorithm with dynamic programming in *Java*. The different parsing methods are described and presented as pseudo code in Section 3. For the implementation three different classes are implemented: `main.java`, `grammar.java` and `parser.java`. The `main`-class calls the methods and the `grammar`-class parses the input grammar as string into a format that then can be processed in the `parser`-class. This `parser`-class has three different parsing methods, which are tested and compared against each other. The function and implementation is former described in Section 3. Also in Section 3 the runtimes in O-notation are calculated for each approach and modifications for the parsing of linear grammars are described.

In Section 4 the runtimes and experiments of the different algorithms are compared and differences in efficiency are shown. The results from the experiments are compared to the calculated runtimes in O-notation from Section 3 and those differences are explained.

Concepts, ideas and first steps for the error correction of input words with deletion and exchange are presented in Section 5.

In the final part (Section 6) the results and future possibilities are discussed.

2 Background

In this section background information, definitions and examples on formal languages, alphabets and Kleene star (Section 2.1), formal grammars (Section 2.2), Chomsky-Normal-Form (Section 2.3), CYK-algorithm (Section 2.5), dynmaic programming (Section 2.4), recursion (Section 2.6) and divide and conquer (Section 2.7) are presented.

2.1 Formal language

Formal languages are abstract languages, which define the syntax of the words that get accepted by that language. It is a set of words that get accepted by the language and has a set of symbols that is called alphabet, which contains all the possible characters of the words. Those characters are called nonterminal symbols. [1, 8]

Definition (Kleene star). *The Kleene star Σ^* of an alphabet Σ is the set of all words that can be created through concatenation of the symbols of the alphabet Σ. The empty word ϵ is included.*

Definition (Formal language). *A formal language L over an alphabet Σ is a subset of the Kleene star of the alphabet: $L \subseteq \Sigma^*$*

Example (Formal language). [1] *The language accepts words that contain the same amount of as and bs, while the a has to be left of the b. The alphabet Σ of this language looks like this:*

$$\Sigma = \{a, b\}$$

The Kleene star Σ^* of the alphabet Σ looks like this:

$$\Sigma^* = \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, abb, bbb, bba, baa, aba, bab, ...\}$$

The language definition L is the following one:

$$L = \{(a^n b^n)^m\} \ with \ n, m \in \mathbb{N}$$

2.2 Formal grammar

A formal grammar describes how to form strings with correct syntax from a language's alphabet. A grammar does not describe the meaning of the strings or any semantics — only their syntax is defined. The grammar is a set of rules, which define which words are accepted by a formal language. Those rules consist of terminal and nonterminal symbols. The terminal symbols are the characters of the alphabet of the language and the nonterminal symbols are used to build the rules of the language – they get replaced by terminal symbols. [1, 8]

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[1]The following examples show the *Well-Balanced Parantheses* example from the assignment task sheet with the alphabet $\{a, b\}$ instead of $\{(,)\}$.

Definition (Formal grammar)**.** *A formal grammar G is defined as a 4-tuple:*

$$G = (V, T, P, S)$$

The set V contains the nonterminal symbols of the grammar and the set T the terminal symbols, which is the alphabet of a language. This assumes that $V \cap T = \emptyset$. The set P is the set of production rules, where an element of V points to an element of $V^* \times T^*$. The symbol $S \in V$ represents the start symbol.

The production rules in the set P have the format `head → body`. *The head is always a nonterminal symbol and the body can be a combination of terminal and nonterminal symbols. A nonterminal symbol* `A` *in the body of a rule can be replaced by the body of a rule with the form* `A → body`. *[9]*

A formal grammar defines which words over the alphabet $T/Sigma$ are contained in the associated laguage.

Example (Formal grammar)**.** *The example grammar G for the previous example language L over the alphabet Σ is the following:*

$$G = (\{S\}, \{a, b\}, \{S \to SS, S \to aSb, S \to ab\}, \{S\})$$

The rules of the grammar G over the alphabet $\{a, b\}$ with the start symbol S and the nonterminal symbols $\{S\}$ can also be written in the following form:

$$S \ \to \ SS \ | \ aSb \ | \ ab$$

The start symbol `S` *can for example be replaced with its body* `SS`. *If then both symbols* `S` *are replaced with the body* `ab` *the resulting word is* `abab`. *It is part of the language, because it was build with the grammar G.*

2.3   Chomsky-Normal-Form

The *Chomsky-Normal-Form* (short: *CNF*) is a grammar which is formatted in a specific way. If the head of a rule (nonterminal symbol) is not generating the empty word (`S` $\to \epsilon$) it either generates two nonterminal symbols or one terminal symbol for the grammar to be in *CNF*. [1]

Example (Chomsky-Normal-Form)**.** *This is the previous example in CNF. How it was built can be seen in Appendix C.*

$$S \to SS \mid LA \mid LR$$
$$A \to SR$$
$$L \to a$$
$$R \to b$$

2.4 Dynamic programming

The technique of dynamic programming can be used to solve problems, which can be divided into smaller subproblems. The solutions of the subproblems are saved (for example in a multi-dimensional array) and referenced later. [3]

Example (Knapsack problem)**.** *As an example the table for the knapsack problem is filled out, because it is a really common example for the concept of dynamic programming.*

Given is a set of items with a weight and a value. The task is to choose which items to include so that the total weight is less than the given limit of the knapsack and the total value is as large as possible.

The formula with which the dynamic programming works is the following:

$$Opt(i,j) = \begin{cases} 0, & \text{for } 0 \leq j \leq size \\ Opt(i-1,j), & \text{for } j < w[i] \\ max\{Opt(i-1,j), v[i] + Opt(i-1,j-w[i])\}, & else \end{cases}$$

If $0 \leq j \leq w$ then there are no objects which could be put into the bag. The second case acts when object i does not fit into the bag and the optimal solution is found with the objects from index 1 to $i-1$. Else the object i is either part of the optimal solution or it consists out of the objects 1 to $i-1$.

The table of the values and weight of each item with index i is shown on the left and on the right is the table that gets filled in with a dynamic programming approach of the formula above. The maximum bag size is 7.

i	$w[i]$	$v[i]$
1	1	1
2	3	4
3	2	3
4	4	6
5	6	8

5	0	1	3	4	6	7	9	10
4	0	1	3	4	6	7	9	10
3	0	1	3	4	5	7	8	8
2	0	1	1	4	5	5	5	5
1	0	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7

The first column in the table represent the objects and the last row the weight. The entries in the table show the maximum value. In this example the maximum value for the size 7 is 10.

2.5 CYK-algorithm

Cocke-Younger-Kasami-algorithm (short: *CYK*-algorithm) takes a grammar $G = (V, T, P, S)$ in *CNF* and a word $w = w_1, w_2, ..., w_n \in T^*$ as an input. It then examines if the word follows the rules of the grammar. Then for every substring $w_{i,j} = w_i, ..., w_{i+j-1}$ (begins at index i and has the length j) of the word w the set of nonterminal symbols that lead to $w_{i,j}$ gets calculated and saved as $V_{i,j}$ to access it in later steps. [10]

Example (Chomsky-Normal-Form). *The following table shows the CYK-algorithm with the previous grammar example and the input word aaabbb. The word is accepted by the grammar rules, because the initiating nonterminal symbol S can be filled into the lowest field. The table is filled in step by step and the rules that are used in each step are marked, for a better understanding on how the algorithm works. [4]*

$S \to SS \mid LA \mid LR$

$A \to SR$

$L \to a$

$R \to b$

$S \to SS \mid LA \mid LR$

$A \to SR$

$L \to a$

$R \to b$

$S \to SS \mid LA \mid LR$

$A \to SR$

$L \to a$

$R \to b$

$S \to SS \mid LA \mid LR$

$A \to SR$

$L \to a$

$R \to b$
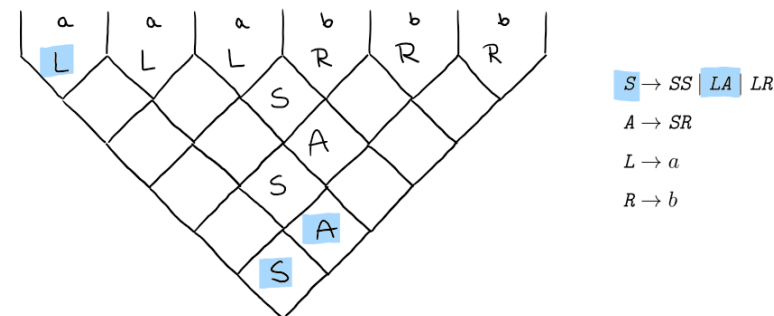
2.6   Recursion

A function that calls itself is called recursive function. In computer science recursive functions use base cases to terminate the program and stopping it from going on forever. Base cases are problems that can be solved without any more recursive calls. [5]

Example (Recursion). *For definition and examples for recursion read Section 2.6.*

Example (Recursion). *Building a sum for the numbers $1 + 2 + ... + n$.*

Nonrecursive approach:

$$f(n) = 1 + 2 + ... + n$$

Recursive approach:

$$f(n) = 1 \qquad \text{if } n = 1$$
$$f(n) = n + f(n-1) \quad \text{if } n > 1$$

2.7 Divide and conquer

The divide and conquer method is used to design efficient algorithms. In this method, the problem is divided recursively into smaller and simpler subproblems, until those subproblems can be solved. Then those subproblems are combined again to give the solution of the original problem.

Example (Divide and conquer). *A problem that can be solved with divide and conquer is the sorting of an array. The array is then divided in smaller arrays, until the array size is one. Then the subarrays are merged together in the correct order.*

The unsorted array for the example is the following:

| 31 | 7 | 97 | 11 | 13 | 2 | 19 |

The array gets divided into two smaller subarray, which are sorted with the same technique individually and later merged together again:

| 31 | 7 | 97 | 11 | | 13 | 2 | 19 |

Those subarrays get divided into even smaller arrays again:

| 31 | 7 | | 97 | 11 | | 13 | 2 | | 19 |

The array size of the subproblems is now 1:

| 31 | | 7 | | 97 | | 11 | | 13 | | 2 | | 19 |

The subarrays get merged together in the right order:

| 7 | 31 | | 11 | 97 | | 2 | 13 | | 19 |

For each merge of the subarrays only the smallest symbols of each array need to be compared. The smaller one is then sorted into the merged array and again the smallest symbols are compared:

7	11	31	97		2	13	19

The array is complete and sorted:

2	7	11	13	19	31	97

3   Three variants of CYK algorithm

The implementation was done in Java and three different classes were implemented: `main.java` (described in Section 3.1), `grammar.java` (described in Section 3.2) and `parser.java` (described in Section 3.3). The `main`-class calls the methods and the `grammar`-class parses the input grammar and string into a format that then can be processed in the `parser`-class.

3.1   Main

The `main`-class takes the input grammar and word and parses them into `String[]` and `String`. A detailed description on how to run the code can be found in Appendix A.

3.2   Grammar

The `grammar`-class assigns the nonterminal symbols to integers and builds arrays with them. The arrays have the type `Integer[][][]`. The start symbol for example is then assigned with the integer zero and the bodies of that rule are at the index zero of the three-dimensional array. The first dimension represents the head of a rule, the second dimension represents the body which is divided into two arrays – for each symbol of the body one array.

For faster and easier access, three arrays are built: One array that only contains nonterminal symbols and rules, one array that contains only terminal rules and one array that contains both.

3.3   Parser

The `parser`-class contains three different parsing methods: `parseNaive` (described in Section 3.3.1), `parseTopDown` (described in Section 3.3.3) and `parseBottomUp` (described in Section 3.3.7). Also there exists a modified method of the `parseTopDown`, which takes linear grammars (which are translated into *CNF*) and parses them faster (described in Section 3.3.5).

Each method has a counter as `long` which counts the number of iterations and a timer as `long` in *ms* which measures the runtime of the method. In the following sections the algorithms are described, presented as pseudo code and the runtime is analysed.

3.3.1   Naive description

The naive algorithm is a recursive divide-and-conquer algorithm. The problem gets divided into smaller subproblems and then the method is called again for each subproblem. It returns a boolean and has an initial method to start the recursion with the start values:

- The start symbol of the grammar is first parameter. It is an integer called `indexNT` and initialized with 0, because the start symbol is assigned to the index 0.

- The second parameter is the integer 0 as a start index of the input word.

- The third parameter is the integer n, which is the length of the input word.

The start values get assigned in the recursion call which can be seen in the following pseudo code:

Algorithm 1 Recursion call: Boolean parseNaive()

1: *counter* $\leftarrow 0$
2: **return** parseNaive(0, 0, inputWord.length)

The naive approach does not use dynamic programming. Instead it checks for each call `parseNaive(indexNT, i, j)` first if $i = j - 1$ and checks if the nonterminal symbol head of `indexNT` leads to a body of a rule with `s[i]`. This is the base case of the recursion which then returns true or false depending if the rule `indexNT` $\rightarrow$ `s[i]` exists. This base case can be seen in line 2-7 in the pseudo code below.

If i is not equal to $j - 1$ it loops for the integer k from $i + 1$ to $j - 1$ and checks for all rules `A` $\rightarrow$ `BC` if both calls `parseNaive(B, i, k)` and `parseNaive(C, k, j)` return true. This recursive call applies the function on the substrings of the input word. If such a pair of substrings is found, the function returns true, because the recursive call in combination with the "**and**" leads to the result of the complete word. If such a pair cannot be found the function returns false after looping through all rule bodies and values for k. This part of the recursion can be seen in the pseudo code below in line 9-21.

Algorithm 2 Boolean parseNaive(int indexNT, int i, int j)

1: *counter* $\leftarrow$ *counter* $+ 1$
2: **if** $i == (j - 1)$ **then**
3:      **for** $l \leftarrow 0$ **to** $ruleset[0].length$ **do**
4:          **if** $ruleset[indexNT][l][0] == inputWord[i]$
                or $ruleset[indexNT][l][1] == inputWord[i]$ **then**
5:             **return** true
6:          **end if**
7:      **end for**
8: **else**
9:      **for** $bodyIndex \leftarrow 0$ **to** $ruleset[indexNT].length$ **do**
10:          **if** $ruleset[indexNT][bodyIndex][0] \: != null$
                and $ruleset[indexNT][bodyIndex][1] \: != null$ **then**
11:             $first \leftarrow ruleset[indexNT][bodyIndex][0]$
12:             $second \leftarrow ruleset[indexNT][bodyIndex][1]$
13:             **for** $k \leftarrow i + 1$ **to** j **do**
14:                 **if** $parseNaive(first, i, k)$ **and** $parseNaive(second, k, j)$ **then**
15:                    **return** true
16:                 **end if**
17:             **end for**
18:          **end if**
19:      **end for**
20: **end if**
21: **return** false

3.3.2   Naive runtime

In the following table the upper bound runtime of the second method (algorithm 2) is listed for each line. The variable n represents the length of the input word and k the dimension of the rule array.

Line	Runtime	Type
1	1	Assignment
2	1	Comparison
3	k	Loop
4	$1 \cdot k$	Comparison
5	$1 \cdot k$	Return statement
9	k	Loop
10	$1 \cdot k$	comparison
11	$1 \cdot k$	Assignment
12	$1 \cdot k$	Assignment
13	$n \cdot k$	Loop
14	$n!$	Recusrsive call
15	$1 \cdot n \cdot k$	Return statement
21	1	Return statement

Calculating the runtime:

$$1 + 1 + k + k + k + k + k + k + k$$
$$+ n \cdot k + n^n + n \cdot k$$
$$= 2 + 7k + 2(n \cdot k) + n!$$
$$= 2 + 7k + 2n \cdot 2k + n!$$
$$\in O(n!)$$

The runtime of the naive method is in $O(n!)$.

3.3.3   Top-Down description

The top-down method is an improved version of the naive method (Section 3.3.1). This algorithm works recursive and with the concept of divide and conquer, too. In this algorithm another global array is used, which contains the values `true`, `false` and `null`. The array gets initialized with `null` in each field. That happens in the recursion call method below.

Algorithm 3 Recursion call: Boolean parseTD()

1:   $counter \leftarrow 0$
2: **for** $i \leftarrow 0$ **to** $table.length$ **do**
3:   **for** $j \leftarrow 0$ **to** $table[i].length$ **do**
4:    **for** $k \leftarrow 0$ **to** $table[i][j].length$ **do**
5:     $table[i][j][k] \leftarrow null$
6:    **end for**
7:   **end for**
8: **end for**
9: **return** parseTD(0, 0, inputWord.length)

Additional to the naive algorithm, the recursion in the topdown approach starts with another condition: If one of the values in the global table is not `null`, the next recursive call is not executed and this value gets returned. This if-condition can be seen in the following part of the pseudo code. The rest of the topdown approach is a similar approach to the naive method that was already described in Section 3.3.1.

Algorithm 4 Additional condition in Boolean parseTD(int indexNT, int i, int j)

1: **if** table[indexNT][i][j] != null **then**
2:     **return** table[indexNT][i][j]
3: **end if**

The complete topdown algorithm is shown as pseudo code below. In the inner for-loop (line 18) the boolean value of the next method calls get assigned into the global table. If there is assigned a true (line 19) the value true gets returns (lime 20). The general topdown approach is the same approach as the naive method that is described in Section 3.3.1.

Algorithm 5 Boolean parseTD(int indexNT, int i, int j)

1: $counter \leftarrow counter + 1$
2: $rulesetLength \leftarrow ruleset[0].length$
3: **if** $table[indexNT][i][j] \,!= null$ **then**
4:     **return** $table[indexNT][i][j]$
5: **end if**
6: **if** $i == (j - 1)$ **then**
7:     **for** $l \leftarrow 0$ **to** $ruleset[0].length$ **do**
8:         **if** $ruleset[indexNT][l][0] == inputWord[i]$
              or $ruleset[indexNT][l][1] == inputWord[i]$ **then**
9:             **return** true
10:        **end if**
11:    **end for**
12: **else**
13:    **for** $bodyIndex \leftarrow 0$ **to** $ruleset[indexNT].length$ **do**
14:        **if** $ruleset[indexNT][bodyIndex][0] \,!= null$
              and $ruleset[indexNT][bodyIndex][1] \,!= null$ **then**
15:            $first \leftarrow ruleset[indexNT][bodyIndex][0]$
16:            $second \leftarrow ruleset[indexNT][bodyIndex][1]$
17:            **for** $k \leftarrow i + 1$ **to** j **do**
18:                $table[indexNT][i][j] \leftarrow (parseTD(first, i, k) \text{ and } parseTD(second, k, j))$
19:                **if** $table[indexNT][i][j] == true$ **then**
20:                    **return** true
21:                **end if**
22:            **end for**
23:        **end if**
24:    **end for**
25: **end if**
26: **return** false

3.3.4   Top-Down runtime

In this part the runtime of the topdown method is calculated. The variable n represents the length of the input word and k the dimension of the rule array. The first part of the calculation shows the recursion call method.

Line	Runtime	Type
1	1	Assignment
2	k	Loop
3	$k \cdot n$	Loop
4	$k \cdot n \cdot n$	Loop
5	$k \cdot n \cdot n \cdot 1$	Assignment
9	...	Function call

Calculating the runtime of the method call:

$$1 + k + k \cdot n + k \cdot n \cdot n + k \cdot n \cdot n$$
$$= 1 + k + k \cdot n + 2(n^2 \cdot k)$$
$$\in O(n^2)$$

The runtime analysis for the upper bound of the `parseTD` method is analogue to the `parseNaive` runtime analysis. According to this the topdown algorithm has an upper bound runtime in $O(n!)$, too.

3.3.5 Top-Down modified for linear grammars

For the parsing of linear grammars, the topdown approach (Section 3.3.3) was modified to reduce the amounts of recursive calls. The topdown approach was chosen, because it seems to have the best performance of the different parsing methods, considering the experiments.

In the modified version for linear grammars, the grammar gets changed into CNF first. The modified algorithm can now work more efficient, because for each rule is only one recursive call of the method instead of two necessary.

For each rule it is checked if the first or the second element of the body is the head of a terminal rule. Then it is checked if the head of the terminal rule is the head of the current position on the input word. For the other symbol the methos gets called recursively. This part is shown in the pseudo code below:

Algorithm 6 boolean parseLinearTD(int indexNT, int i, int j)

1: $counter \leftarrow counter + 1$
2: $rulesetLength \leftarrow ruleset[0].length$
3: **if** $table[indexNT][i][j] \; != null$ **then**
4:      **return** $table[indexNT][i][j]$
5: **end if**
6: **if** $i == (j - 1)$ **then**
7:      **for** $l \leftarrow 0$ **to** $ruleset[0].length$ **do**
8:          **if** $ruleset[indexNT][l][0] == inputWord[i]$
                 and $rulesetLinear[indexNT][l][1] == null$ **then**
9:              $tableLinear[indexNT][i][j] \leftarrow true$
10:             **return** true
11:         **end if**
12:     **end for**

13:      **return** false
14: **end if**
15: **for** $bodyIndex \leftarrow 0$ **to** $ruleset[indexNT].length$ **do**
16:      **if** $ruleset[indexNT][bodyIndex][0] \, ! = null$
             and $ruleset[indexNT][bodyIndex][1] \, ! = null$ **then**
17:        $first \leftarrow ruleset[indexNT][bodyIndex][0]$
18:        $second \leftarrow ruleset[indexNT][bodyIndex][1]$
19:        **for** $k \leftarrow i + 1$ **to** j **do**
20:          **if** $second$ is head of a terminal rule **then**
21:            $table[indexNT][i][j] = parseLinearTD(first, i, k)$ **and** $second$ is head of terminal rule with body $inputAsInt[k]$
22:          **end if**
23:          **if** $first$ is head of a terminal rule **then**
24:            $table[indexNT][i][j] = parseLinearTD(second, k, j)$ **and** $first$ is head of terminal rule with body $inputAsInt[i]$
25:          **end if**
26:        **end for**
27:      **end if**
28: **end for**
29: **if** $tableLinear[indexNT][i][j]! = null$ **and** $tableLinear[indexNT][i][j] == true$ **then**
30:      **return** true
31: **end if**
32: $tableLinear[indexNT][i][j] \leftarrow false$
33: **return** false

3.3.6   Top-Down modified for linear grammars runtime

The runtime of the modified version is $O(n^2)$, because the recursive call is not in $O(n!)$ anymore like in Section 3.3.2 but $O(n)$. Instead the method is in $O(n^2)$.

3.3.7   Bottom-Up description

The `parseBU` method works with dynamic programming. First a table DP with the size $n \times n$ gets constructed (line 2) – n represents the input word length. In the first step the integers that represent the nonterminal symbols that are head of a terminal rule get assigned (line 3 - 12). For the character at index i of the input word the value of $DP[i][i]$ gets the nonterminal symbol that leads to the character of the word. The approach of dynamic programming calculates the smallest substrings first and saves them for the efficient processing of the next bigger substrings. The smallest substrings are the single characters.

Algorithm 7 Boolean parseBU()

1: $wordlength \leftarrow word.length$
2: $Integer[][][]DP \leftarrow newInteger[wordLength][wordLength][wordlength]$
3: **for** $i \leftarrow 0$ **to** $wordlength$ **do**
4:      **if** $ruleset\ contains\ word[i]$ **then**
5:         assign nonterminal symbols of $inputword[i]$ to $DP[i][i]$
6:      **end if**
7: **end for**
8: **for** $l \leftarrow 0$ **to** $wordlength$ **do**
9:      **for** $i \leftarrow 0$ **to** $wordlength - l$ **do**
10:         $j \leftarrow i + l$
11:         **for** $k \leftarrow 0$ **to** j **do**
12:            **for** $head \leftarrow 0$ **to** $ruleset.length$ **do**
13:               **for** $body \leftarrow 0$ **to** $ruleset[head].length$ **do**
14:                  $conter \leftarrow counter + 1$
15:                  **if** $ruleset[indexNT][bodyIndex][0]\ != null$
              and $ruleset[indexNT][bodyIndex][1]\ != null$ **then**
16:                     $first \leftarrow ruleset[head][body].charAt[0]$
17:                     $second \leftarrow ruleset[head][body][1]$
18:                     **if** $DP[i][k]\ contains\ first$ **and** $DP[k + 1][j]\ contains\ second$ **then**
19:                       assign head to $DP[i][j][c]$[2]
20:                     **end if**
21:                  **end if**
22:               **end for**
23:            **end for**
24:         **end for**
25:      **end for**
26: **end for**
27: **if** $DP[0][wordlength - 1]\ contains\ 0$ **then**
28:      **return** true
29: **end if**
30: **return** false

In line 13 to 32 the *CYK*-algorithm gets executed. It is described in section 2.5. For each field the two fields *leading* to it get compared, to then fill in the head of a rule that concludes into the both compared nonterminal symbols.

In the last lines (line 33-35) the last field of the *DP*-array gets checked. If it containes the start symbol of the grammar, the word is contained in the language and the algorithm returns true. Else false is returned.

3.3.8 Bottom-Up runtime

In the following part the upper bound runtime of the bottom up algorithm gets analysed. The variable n represents the length of the input word and k the dimension of the rule array.

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[2]c is the first empty position in $DP[i][j]$

Line	Runtime	Type
1	1	Assignment
2	1	Assignment
3	n	Loop
4	n	Comparison
5	n	Assignment
6	n	Comparison
7	n	Assignment
9	n	Assignment
13	n	Loop
14	n · n	Loop
15	n · n	Assignment
16	n · n · n	Loop
17	n · n · n · k	Loop
18	n · n · n · k · k	Loop
19	n · n · n · k · k	Assignment
20	n · n · n · k · k	Comparison
21	n · n · n · k · k	Assignment
22	n · n · n · k · k	Assignment
23	n · n · n · k · k	Comparison
24	n · n · n · k · k	Assignment
25	n · n · n · k · k	Assignment
33	1	Comparison
34	1	Return statement
36	1	Return statement

Calculating the runtime:

$$5 + 7n + 2n^2 + n^3 + n^3 \cdot k + 8 \cdot n^3 \cdot k^2$$
$$5 + 7n + 2n^2 + (k+1)n^3 + 8 \cdot n^3 \cdot k^2$$
$$\in O(n^3)$$

The runtime of the `parseBU` method is in $O(n^3)$.

4   Evaluation

In this section the calculated runtimes in *O*-notation from Section 3 and experiments are compared. More information and definitions on the *O*-notation can be read in *Big O Notation* from P. Danziger [2].

4.1   Experiments

The following tables show some tests that were run with the different parsing methods and grammars. The first of each column for the method shows the truth value that is returned (as T for true and F for false), C represents the counter and T the time in ms.

Well-Balanced-Parantheses:

`"SSS" "SLA" "SLR" "ASR" "L(" "R)" "(*)*"`

Word	Length	N	NC	NT	B	BC	BT	T	TC	TT
()	2	T	6	5ms	T	0	2ms	T	6	1ms
(())	4	T	33	4ms	T	84	3ms	T	28	1ms
((()))	6	T	212	6ms	T	360	6ms	T	84	1ms
(((())))	8	T	1295	6ms	T	924	9ms	T	190	1ms
((((()))))	10	T	7666	9ms	T	1872	11ms	T	362	1ms
((...))	20	T	51.863.993	1.049ms	T	15.732	23ms	T	2.772	3ms
((...))	30	T	348.838.486.712	3.699.571ms	T	102.660	32ms	T	9.232	5ms
((...))	40				T	127.452	50ms	T	21.743	4ms

The input word with a length of 40 did not terminate over night for the naive approach. The corresponding graphs to the data in the table are shown here:
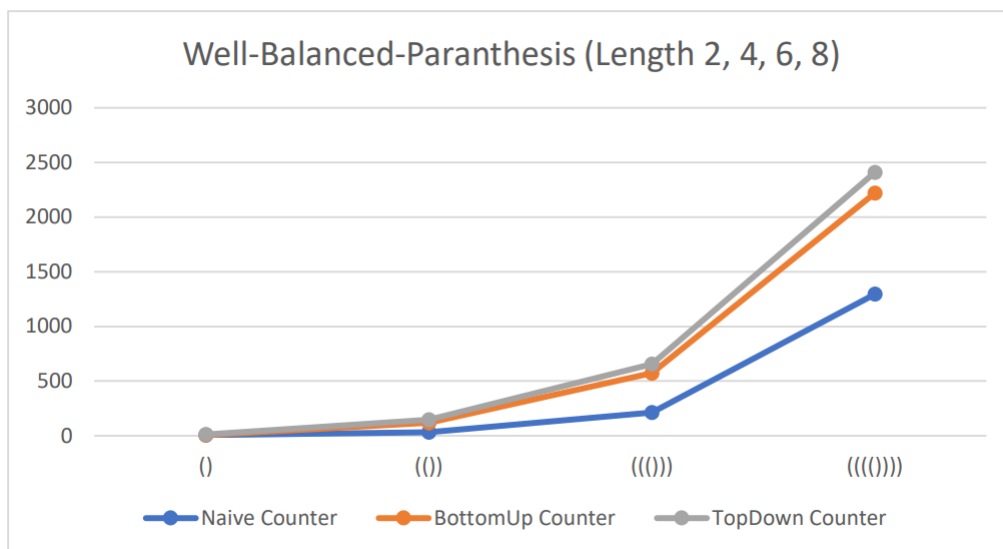

Figure 1: Well-Balanced-Parantheses with input (), (()), ((())) and (((())))
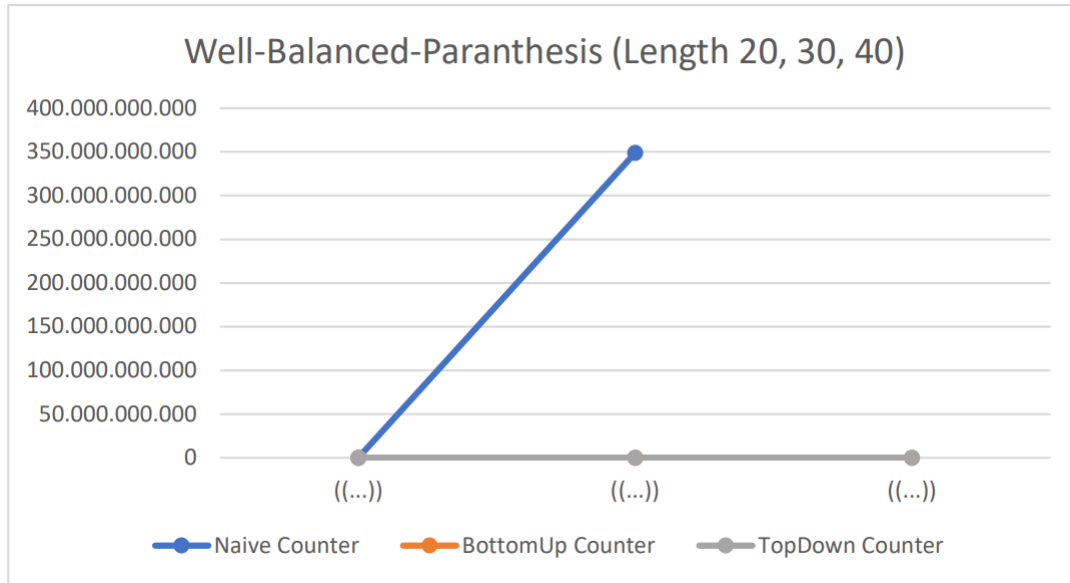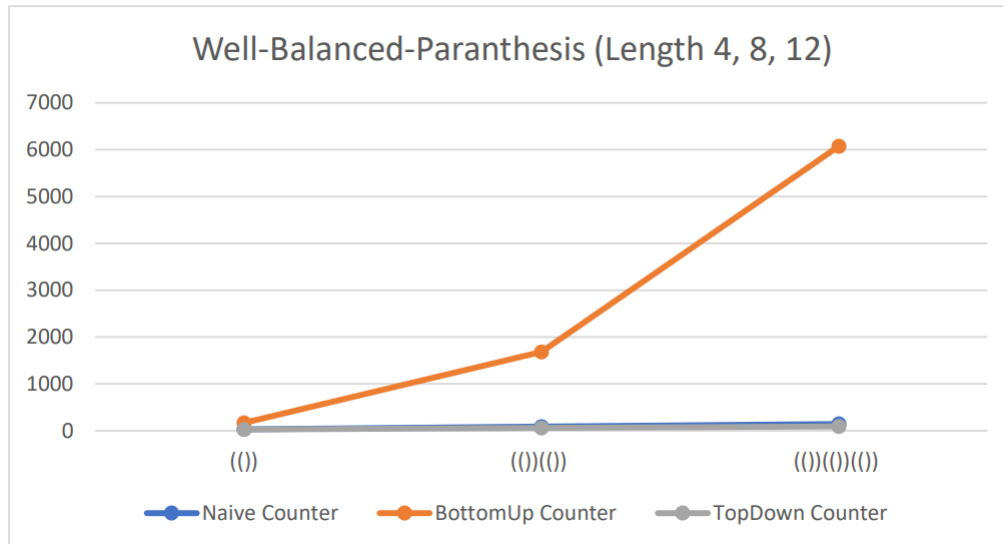
Figure 2: Well-Balanced-Parantheses with input of length 20, 30 and 40

Word	Length	N	NC	NT	B	BC	BT	T	TC	TT
(())	4	T	33	0ms	T	168	7ms	T	28	5ms
(())(())	8	T	89	1ms	T	1680	7ms	T	60	2ms
(())(())(())	12	T	145	1ms	T	6072	14ms	T	92	3ms
(())(())(())(())	16	T	201	15ms	T	14.880	13ms	T	124	3ms
(())(())(())(())(())	20	T	257	15ms	T	29.640	21ms	T	156	4ms
(())(())(())(())(())(())	24	T	313	16ms	T	51.888	22ms	T	188	5ms

Figure 3: Well-Balanced-Parantheses with input (()), (())(()) and (())(())(())
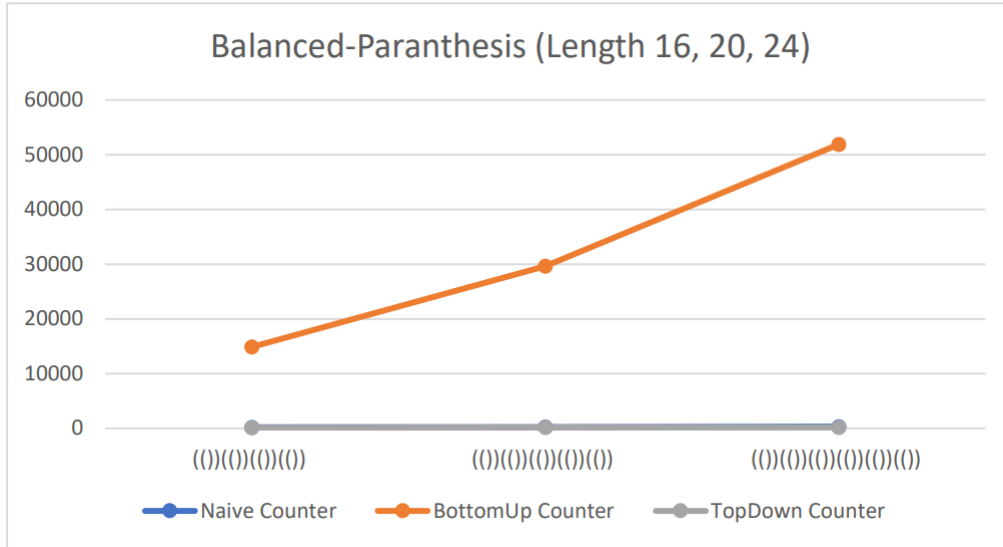
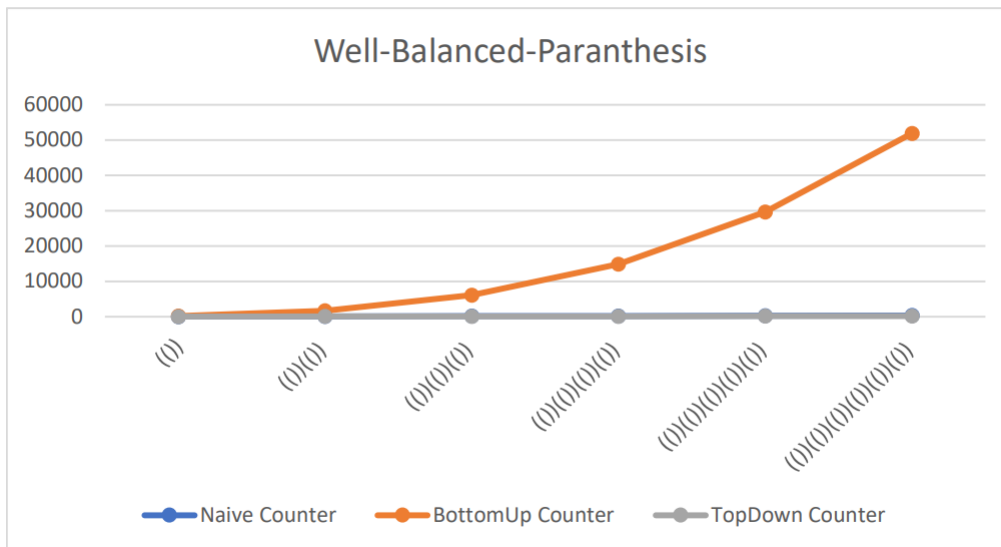Figure 4: Well-Balanced-Parantheses with input of length 16, 20 and 24

Figure 5: Well-Balanced-Parantheses with input of length 4, 8, 12, 16, 20 and 24

Stupid grammar:

"SST" "STS" "Ta"

Word	Length	N	NC	NT	B	BC	BT	T	TC	TT
a	1	F	1	1ms	F	0	6ms	F	1	5ms
aa	2	F	4	1ms	F	4	8ms	F	4	2ms
aaaa	4	F	33	5ms	F	28	6ms	F	27	1ms
aaaaaaaa	8	F	1.596	8ms	F	308	5ms	F	197	1ms
aaaaaaaaaaaaaaaa	16	F	3.524.577	89ms	F	2.660	15ms	F	1.481	1ms
aaaaaaa...aaaaaaaaa	32				F	41.664	25ms	F	11.409	4ms

The input word with a length of 32 did not terminate for the naive approach. The corresponding graphs to the data in the table are shown below.
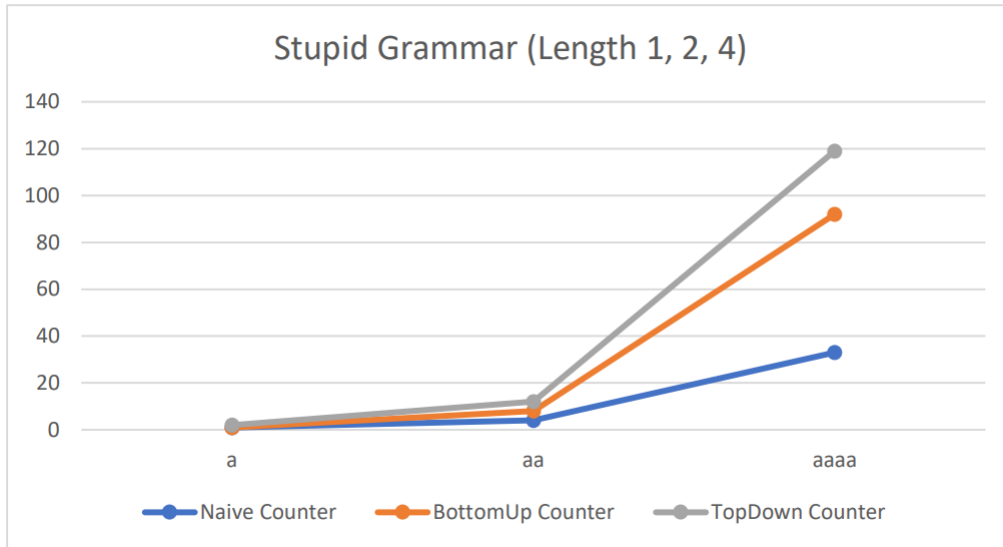

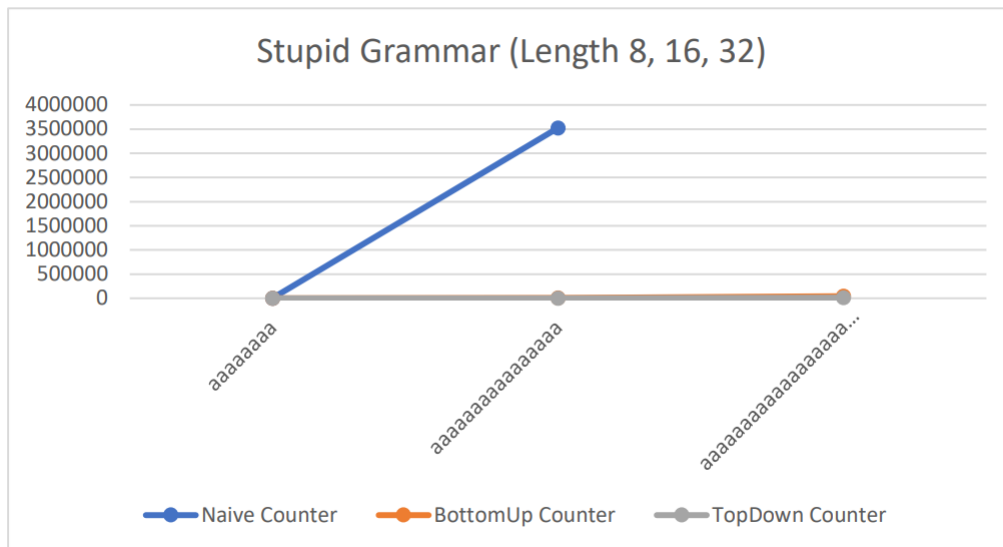Figure 6: Stupid grammar with input `a`, `aa` and `aaaa`

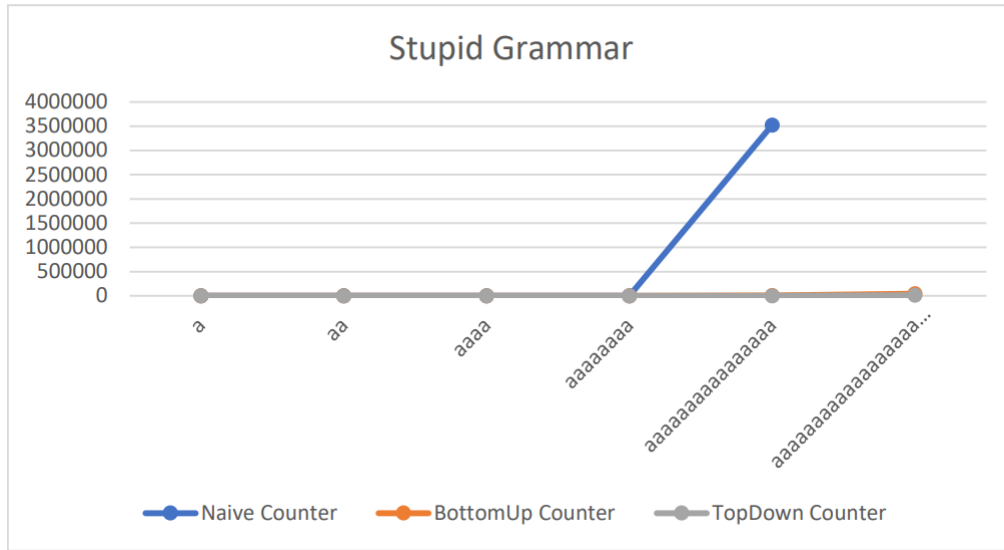Figure 7: Stupid grammar with input of length 8, 16 and 32

Figure 8: Stupid grammar with input of length 1, 2, 4, 8, 16, and 32

Linear grammar example (abc-grammar):

`"SAc" "Sb" "AaS" "AaB" "BbS" "a*b*c*"`

Word	Length	NC	NT	BC	BT	TC	TT	LC	LT
abc	3	6	1ms	60	5ms	6	3ms	5	0ms
aabbcc	6	51	1ms	660	5ms	49	3ms	51	1ms
aaaabbbbcccc	12	1.457	1ms	6.072	9ms	518	3ms	650	1ms
a...b...c...	24	481.557	9ms	51.888	20ms	4.312	4ms	6.732	3ms

The incrementations of the input word length was chosen, to show the difference in the performance of the parsing methods for similar structured input but different input length.

4.2   *O*-notation and runtime comparison

The runtimes of the methods are calculated in section 3. The results are the following:

Method	Runtime
`parseNaive`	$O(n!)$
`ParseTD`	$O(n!)$
`ParseBU`	$O(n^3)$
`ParseLinearTD`	$O(n^2)$

Seeing those runtimes one could think that the naive and the topdown approach are equally efficient. Considering the differences in the algorithms and the results of the experiments it gets shown that this is not the case. The O-notation runtimes are the upper bounds for the runtime.

Regarding the experiments that were run on the code, the topdown method seems to be the most efficient, followed by the bottomup method. The least efficient method is in these cases the naive approach.

The linear version of the topdown is more efficient than the original paring method, because it is restricted to linear grammars and with this restriction the recursive calls can be reduced from two to one in each call.

It is to consider that the O-notation represents the upper bound of the runtime. Algorithms can run faster than the runtime in O-notation predicts.

To confirm the calculated runtimes from Section 3, the results of the experiments are calculated, considering the O-notation runtime. For the input length (n) the actual results and O-notation results as upper bound are compared.

Naive approach ($O(n!)$):

Well-Balanced-Parantheses:

n	$n!$	counter
2	2	6
4	24	33
6	720	212
8	40.320	1295
10	3.628.800	7666
20	2.432.902.008.176.640.000	51.863.993
30	265.252.859.812.191.058.636.308.480.000.000	348.838.486.712

Abc-grammar:

n	$n!$	counter
3	6	6
4	24	51
12	479.001.600	1.457
24	620.448.401.733.239.439.360.000	481.557

The results of the experiments confirm the runtime calculation of $O(n!)$ in Section 3.3.2. Small deviations for smaller input numbers can happen, because the constants are not considered.

Topdown approach ($O(n!)$):

Well-Balanced-Parantheses:

n	$n!$	counter
2	2	6
4	24	28
6	720	84
8	40.320	190
10	3.628.800	362
20	2.432.902.008.176.640.000	2.772
30	265.252.859.812.191.058.636.308.480.000.000	9.232

Abc-grammar:

n	$n!$	counter
3	6	6
4	24	49
12	479.001.600	518
24	620.448.401.733.239.439.360.000	6.732

The results of the experiments confirm the runtime calculation of $O(n!)$ in Section 3.3.4.

Bottomup approach ($O(n^3)$):

Well-Balanced-Parantheses:

n	n^3	counter
2	8	6
4	64	28
6	216	84
8	512	190
10	1.000	362
20	8.000	2.772
30	27.000	21.743

Abc-grammar:

n	n^3	counter
3	27	6
4	64	49
12	1.728	518
24	13.824	4.312

The results of the experiments confirm the runtime calculation of $O(n^3)$ in Section 3.3.8.

Linear topdown approach ($O(n^2)$):

Abc-grammar:

n	n^2	counter
3	9	5
4	16	51
12	144	650
24	576	6.732

The results of the experiments do not confirm the runtime calculation of $O(n^2)$ in Section 3.3.5. This is further discussed in Section 4.4.

4.3 Results

The O-notation results from Section 3 show, that the naive and the topdown approach have the same upper bound runtime of $O(n!)$ and the bottomup parser has an upper bound runtime in $O(n^3)$. The experiments on the other hand show, that the topdown approach is a lot

Method	Runtime
`parseNaive`	$O(n!)$
`ParseTD`	$O(n!)$
`ParseBU`	$O(n^3)$
`ParseLinearTD`	$O(n^3)$

faster than the naive parser. Considering the experiments, the topdown parser seems to be the fastest, followed by the bottomup approach and the naive is by far the slowest. The parser for linear grammars has different restrictions for the input and its results are discussed in Section 4.4.

The reason for the faster results of the topdown parser is the global boolean table, which saves results from previous function calls and can cause less recursive calls of the function to compare the values while the naive approach calls the recursion for every single possibility. The actual runtime of the naive approach is often a lot closer to the upper bound than

the actual runtime of the topdown approach, which seems to perform more efficient in the experiments.

In the experiments, the topdown performed even better than the bottomup approach, even though the bottomup parser is in $O(n^3)$. Because the bottomup fills in the CYK-table iteratively, the upper bound runtime is similar to the actual runtime.

For these results it needs to be considered, that the O-notation stes the bound from above. An algorithm does not run worse than the depending O-notation runtime, but it can be better. It grows at most as much as the O-notation function. [2]

4.4 Results for linear grammars

The upper bound runtime of the modified topdown parser for linear grammars is faster than the runtimes of the parser for grammars in CNF. The topdown runtime is in $O(n!)$ and the modified topdown approach should be in $O(n^2)$. In the linear approach are one instead of two recursive call per execution made, because the linear grammars have at most one nonterminal symbol in the body of each rule. The recursive call needs only to be made for the nonterminal symbols. This is why the upper bound runtime if the topdown approach for linear grammars is in $O(n^2)$, while the topdown approach for grammars in CNF is in $O(n!)$.

Surprisingly, the experiments with the modified parsing method do not show the expected results. The linear topdown approach seems to perform faster than the not modified approach, but with more function calls. This seems unlikely, because the recursive calls of the function are reduced from two to one in the modified approach. The faster performance can be due to hidden background processes on the computer. The amount of calls on the other hand should be lower than the amount of calls in the not modified topdown parser. This leads to the assumption, that the modified topdown parser is not working correctly.

The parsing from a linear grammar into CNF works correctly. As seen in the experiments, the input of linear grammars or grammars in CNF have no significantly influence on the runtime of the parsing methods. The naive parser is in both cases called for every option recursively. The bottomup parser iterates in both cases over the whole CYK-table and the topdown parser makes two recursive calls per call.

5 Error correction

Apart from checking if an input word is part of a language, the amount of errors and the correction of those can be interesting. Two options for the error correction are considered in this section: A symbol can be exchanged or symbols can be deleted. The output for some examples of the error correction can be seen in Appendix A.

For the error correction the bottomup parser modified, because the CYK-table shows, which substrings lead to which nonterminal symbols, for example the start symbol.

5.1 Deletion

For the deletion the table of the CYK-algorithm is used. This is described in Section 2.5 and the parsing method for it is explained in Section 3.3.7. The CYK-table has to contain the start symbol in the entry on the right top corner to conclude that the word is part of the language. To find an accepted word with deletion, the entry with the start symbol which is closest to the right top is searched.

The concept on how to find the entry in the array that is further on the top right is described here and shown in the pseudo code of the function `findBestIndeces` in algorithm 8. The function takes two array positions as (i, j) as input and returns the better position in an int array of the length 2.

Algorithm 8 int[] findBestIndeces(int i_a, int j_a, int i_b, int j_b)

1: $int[]\ betterIndeces \leftarrow new\ int[2]$
2: $int\ indicator_i \leftarrow i_a - i_b$
3: $int\ indicator_j \leftarrow j_b - j_a$
4: **if** $indicator_i + indicator_j) == 0$ **then**
5:   ▷ both are equal far away
6:     $betterIndeces[0] \leftarrow i_a$
7:     $betterIndeces[1] \leftarrow j_a$
8: **end if**
9: **if** $indicator_i + indicator_j) > 0$ **then**
10:   ▷ b is closer
11:     $betterIndeces[0] \leftarrow i_b$
12:     $betterIndeces[1] \leftarrow j_b$
13: **else**
14:   ▷ a is closer
15:     $betterIndeces[0] \leftarrow i_a$
16:     $betterIndeces[1] \leftarrow j_a$
17: **end if**
18: **return** $betterIndeces$

The parameters for the functions are the i and j values for the two positions A and B in the array as A_i, A_j, B_i, B_j. The searched position in the array is a small i with a large j.

Two values are stored for later comparisons: $x = A_i - B_i$ and $y = B_j - A_j$. If x and y are equal, the positions are equally far away from the top right array entry. If $x + y$ is positive, B is the preferred position, else it is A.

With those indices the part of the word that is accepted and the part that needs to be deleted can be calculated. This is done in the method `solveErrorWithDeletion` which is shown as pseudo code in algorithm 9. This concludes in the longest accepted word with deletion, which is then returned.

It can happen that no accepted word is found with deletion, if no subword concluded into the start symbol in the CYK-table.

The distance of the entry in the CYK-table that contains the start symbol and the top right corner can also be returned as the amount of symbols that need to be deleted to receive an accepted word. This error counter works only for the deletion method.

Algorithm 9 Integer[] solveErrorWithDeletion(Integer[][][] CYK, int amountOfErrors)

1: $int\ i \leftarrow -1$

2: $int\ j \leftarrow -1$

3: $int[]\ betterIndeces \leftarrow new\ int[2]$

4: **for** $index \leftarrow 0$ **to** $CYK.length$ **do**

5:     **for** $entries \leftarrow 0$ **to** $CYK[amountOfErrors][index].length$ **do**

6:         **if** $CYK[amountOfErrors][index][entries] == 0$ **then**

7:             $betterIndeces \leftarrow findBestIndeces(i, j, amountOfErrors, index)$

8:             $i \leftarrow betterIndeces[0]$

9:             $j \leftarrow betterIndeces[1]$

10:         **end if**

11:         $temp \leftarrow CYK.length - 1 - amountOfErrors$

12:         **if** $CYK[index][temp][entries] == 0$ **then**

13:             $betterIndeces \leftarrow findBestIndeces(i, j, index, temp)$

14:             $i \leftarrow betterIndeces[0]$

15:             $j \leftarrow betterIndeces[1]$

16:         **end if**

17:     **end for**

18: **end for**

19: **if** $i == -1$ **then**

20:     $Integer[]\ noAcceptedWord \leftarrow new\ Integer[1]$

21:     $noAcceptedWord[0] \leftarrow -1$

22:     **return** $noAcceptedWord$

23: **end if**

24: $int\ from \leftarrow Math.min(i, j)$

25: $int\ to \leftarrow Math.max(i, j)$

26: $Integer[]acceptedWord \leftarrow new\ Integer[to - from + 1]$

27: **if** $from == to$ **then**

28:     **for** $k \leftarrow 1$ **to** $inputAsInt.length$ **do**

29:         **if** $inputAsInt[k]! = null$ **and** $inputAsInt[k] == 0$ **then**

30:             $acceptedWord[k] \leftarrow inputAsInt[k]$

31:             $break$

32:         **end if**

33:     **end for**

34:     **return** $acceptedWord$

35: **end if**

36: **for** $k \leftarrow 1$ **to** $to - from$ **do**

37:     **if** $inputAsInt[k + from]! = null$ **then**

38:         $acceptedWord[k] \leftarrow inputAsInt[k + from]$

39:     **end if**

40: **end for**

41: **return** $acceptedWord$

5.2 Exchange

The exchange method works iterative and iterates over every symbol of the input word and tries ever terminal symbol on each index. The method can only exchange one symbol. First, the symbol and each entry in the CYK-table that concluded from this symbol in the input word are deleted. For this, the corresponding entries are replaced with -1. Then the fields that are assigned with -1 in the CYK-table get calculated with a new input symbol, which replaces the original one. If a success is found, the accepted word is returned.

In algorithm 10 the adapted version of the bottomUp parser can be seen as pseudo code. In this methos, a symbol from the input word got replaced by a new symbol (which is the parameter `newSymbol`). All the entries, that relied on this symbol were replaced by -1 before and this version of the bottomUp parser examines the new entries of these field to produce a full CYK-table for the new input word without calculating the already known parts again.

Algorithm 10 parseBUnewSymbol(Integer[][][] DP, int newSymbol)

1: $wordlength \leftarrow word.length$

2: **for** $i \leftarrow 0$ **to** $wordlength$ **do**

3:     **if** $DP[i][i][0]! = null$ **and** $DP[i][i][0] == -1$ **then**

4:         $DP[i][i][0] \leftarrow newSymbol$

5:     **end if**

6: **end for**

7: **for** $l \leftarrow 0$ **to** $wordlength$ **do**

```
 8:     for i ← 0 to wordlength − l do
 9:         j ← i + l
10:         if DP[i][j][0]! = null and DP[i][j][0] == −1 then
11:             for k ← 0 to j do
12:                 for head ← 0 to ruleset.length do
13:                     for body ← 0 to ruleset[head].length do
14:                         conter ← counter + 1
15:                         if ruleset[indexNT][bodyIndex][0] ! = null
            and ruleset[indexNT][bodyIndex][1] ! = null then
16:                             first ← ruleset[head][body].charAt[0]
17:                             second ← ruleset[head][body][1]
18:                             if DP[i][k] contains first and DP[k + 1][j] contains second
    then
19:                                 assign head to DP[i][j][c]
20:                             end if
21:                         end if
22:                     end for
23:                 end for
24:             end for
25:         end if
26:     end for
27: end for
28: if DP[0][wordlength − 1] contains 0 then
29:     return true
30: end if
31: return false
```

6   Conclusion and Future Work

6.1   Conclusion

Regarding the experiments from section 4, the topdown method seems to be the most efficient, followed by the bottomup method. The least efficient method is in these cases the naive approach. The *O*-notation showed different results considering the upper bounds of the methods. This shows, that the *O*-notation bound is an upper bound and an algorithm can perform faster in experiments, depending on the input stream.

These algorithms show differences in efficiency but also limits for the examination of words with the *CYK*-algorithm. The amount of calls raised especially with the Well-Balanced-Parantheses language exponentially. This can also be seen in the graphs in Appendix B.

The linear parsing method on the other hand worked with a different restriction on the input stream, which can lead into even higher performances.

6.2   Future work

Formal languages and grammars can be used in different use cases. For exampel for AI learning methods. Real languages like English can be defined as a formal language, too. But it is important to consider that formal languages only define the syntax and not the semantics. In addition to this has english a lot of rules and many exceptions. [6]

The future work for this code is to optimize the error correction. The combination of deletion and exchange is not implemented yet. It would be interesting to find a more efficient way to calculate the exchange of symbols, which would enable the exchange of more than one symbol. In addition to this an accurate error counter for the symbols that need to be exchanged could be implemented.

Another extension to the code would be a method, that translated every kind of grammar into *CNF*, if possible. Then input words for every grammar could be parsed with the different parsing approaches and possibilities to modify the parsing methods for different input grammars appear, similar to the modifications that were done for the parsing of linear grammars.

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A How to use the code?

The code can be run in the terminal and input is expected as Strings in quotation marks. The first rule begins with the startsymbol of the grammar.

First: Rules without arrows (one rule as one String)

Last: The last argument is the input word

Input example (*Well-Balanced-Parantheses*):

`java Main "SSS" "SLA" "SLR" "ASR" "L(" "R)" "(())"`

for the grammar S → SS | LA | LR, A → SR, L → (, R →) and the input word (()). Other example:

`java Main "SAc" "Sb" "AaS" "AaB" "BbS" "abc"`

Output example:

The first part of the output shows the arrays, which get generated in the `Grammar.java` class.

The first array contains all rules.

```
Matrix all rules:
[SS, LA, LR]
[SR, , ]
[(, , ]
[), , ]
```

The second array contains only the terminal rules.

```
Matrix T rules:
[]
[]
[(]
[)]
```

The third array contains only the nonterminal rules.

```
Matrix NT rules:
[SS, LA, LR]
[SR, , ]
```

Then it is shown which symbols are represented by which integers. Later the symbols can be referred to with those integers.

```
Symbols as Integers:
0 1 2 3 4 5
S A L R ( )
```

After this the mentioned arrays are shown again but the nonterminal symbols got replaced with the according integers.

```
Integer-Matrix all rules:
[0 0, 2 1, 2 3]
[0 3, , ]
[4, , ]
[5, , ]
```

Then the input word is shown with the symbols replaced by their integers.

```
Inputword as Integers:
4 4 5 5
```

```
[2, , , 0]
[, 2, 0, 1]
[, , 3, ]
[, , , 3]
BottomUp: true    Amount of calls: 168    Amount of errors: 0
BottomUp runtime: 7ms

TopDown: true    Amount of calls: 28
TopDown runtime: 3ms

Naive: true    Amount of calls: 33
Naive runtime: 1ms
```

Then the results, counter and runtime in *ms* is shown for each parsing method. For the `BottomUp` method is the CYK algorithm table printed. The error counter is printed, too.

In this case the error correction shows that no errors where found and this leads to no symbols being exchanged or deleted.

```
Error correction

Error correction with exchange:
No exchange option for 1 symbol found/necessary


Error counter for deletion: 0
Error correction with deletion
amount of errors: 0
Accepted word:
( ( ) )
```

The following example shows the error correction output for the input `java Main "SSS" "SLA" "SLR" "ASR" "L(" "R)" "((()"`.

In this case the error correction shows the solution with exchange (one symbol is exchanged) and for deletion (two symbols are deleted).

```
Error correction with exchange:
1 symbol was exchanged.
Accepted word:
( ) ( )

Error counter for deletion: 2
Error correction with deletion
amount of errors: 2
Accepted word:
( )
```

```
BottomUp: true    Amount of calls: 440    Amount of errors: 0
BottomUp runtime: 6ms

TopDown: true    Amount of calls: 21
TopDown runtime: 3ms

Naive: true    Amount of calls: 21
Naive runtime: 1ms

LinearTopDown: true    Amount of calls: 30
LinearTopDown runtime: 1ms
```

If the input grammar is a linear grammar, the program checks if it is in *CNF*, build *CNF* out of the linear grammar and runs the optimized version of the TopDown parser for linear grammars.

B   Graphs and additional results

The following tables show some tests that were run with the different parsing methods and grammars. The first of each column for the method shows the truth value that is returned (as T for true and F for false), C represents the counter and T the time in ms. Those tests were run before the BottomUp method was working correctly.

Well-Balanced-Parantheses:

Word	Length	N	NC	NT	B	BC	BT	T	TC	TT
()	2	T	6	5ms	T	0	2ms	T	6	1ms
(())	4	T	33	4ms	T	84	3ms	T	28	1ms
((()))	6	T	212	6ms	T	360	6ms	T	84	1ms
(((())))	8	T	1295	6ms	T	924	9ms	T	190	1ms
((((()))))	10	T	7666	9ms	T	1872	11ms	T	362	1ms
((...))	20	T	51.863.993	1.049ms	T	15.732	23ms	T	2.772	3ms
((...))	30	T	348.838.486.712	3.699.571ms	T	102.660	32ms	T	9.232	5ms
((...))	40				T	127.452	50ms	T	21.743	4ms

The input word with a length of 40 did not terminate over night for the naive approach.

Stupid grammar:

Word	Length	N	NC	NT	B	BC	BT	T	TC	TT
aaaa	4	F	33	5ms	F	28	6ms	F	27	1ms
aaaaaaaa	8	F	1.596	8ms	F	308	5ms	F	197	1ms
aaaaaaaaaaaaaaaa	16	F	3.524.577	89ms	F	2.660	15ms	F	1.481	1ms

Simple grammar:
`S → aA    A → Sb    A → b`

Word	Length	NC	NT	BC	BT	TC	TT	LC	LT
ab	2	3	0ms	8	7ms	3	3ms	2	0ms
aabb	4	8	1ms	112	7ms	8	3ms	8	0ms
aaaabbbb	8	58	0ms	1.120	7ms	50	2ms	84	1ms
a...b	16	3.144	1ms	9.920	11ms	486	3ms	940	1ms
a...b	32	7.048.930	69ms	83.328	26ms	4.558	6ms	9.052	2ms

Graphs:

Well-Balanced-Parantheses:

Stupid Grammar:
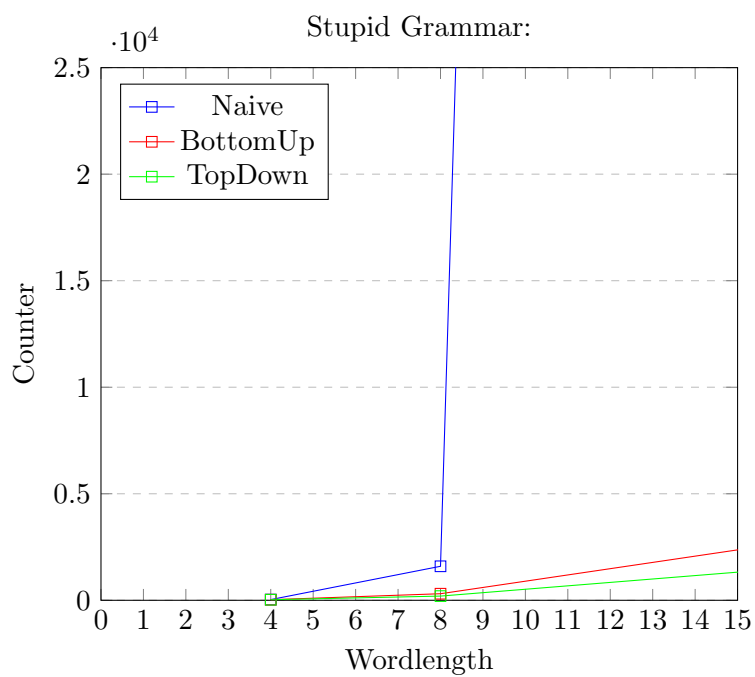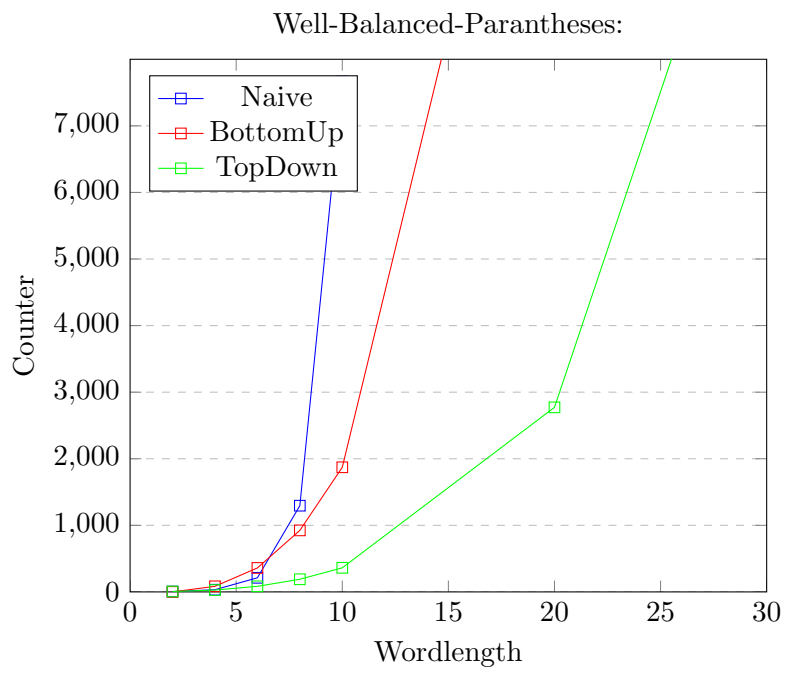
C   CNF Algorithm on an example

In this part, the following ruleset of a grammar is translated into *CNF*.

$$S \rightarrow SS \mid aSb \mid ab$$

1. Remove every nonterminal symbol that cannot be reached or is not generating another symbol:
   S is the only nonterminal symbol and does not need to be removed.

2. Remove all symbols that cannot be reached. (All symbols can be reached.)

3. Replace the terminal symbols in the body of other rules with new nonterminal symbols to not have bodies which contain terminal and nonterminal symbols:

$$S \rightarrow SS \mid LSR \mid LR$$
$$L \rightarrow a$$
$$R \rightarrow b$$

4. On the right side of the rules are only two nonterminal symbols allowed:

$$S \rightarrow SS \mid LA \mid LR$$
$$A \rightarrow SR$$
$$L \rightarrow a$$
$$R \rightarrow b$$

5. Remove all ϵ-rules and paste in the start symbol what can be generated by them. (This grammar does not have ϵ-rules.)

6. Check on transitivity and remove those dependencies. (There are no transitive rules here.)

This is the grammar in CNF:

$$S \rightarrow SS \mid LA \mid LR$$
$$A \rightarrow SR$$
$$L \rightarrow a$$
$$R \rightarrow b$$