Department of Computer Science
UNIVERSITY OF COLORADO **BOULDER**

# Topic Models


Advanced Machine Learning for NLP
Jordan Boyd-Graber
SLIDES ADAPTED FROM DAVID MIMNO

## Learning the Hidden Space

- Two major tools:
  - Gibbs Sampling: Easier to implement, easier to understand
  - Variational Inference: faster, harder to implement
- Variational shows the connections to “deep” models better, so it’s the focus
- However, would be injustice to not at least discuss Gibbs sampling

## Inference

- We are interested in posterior distribution

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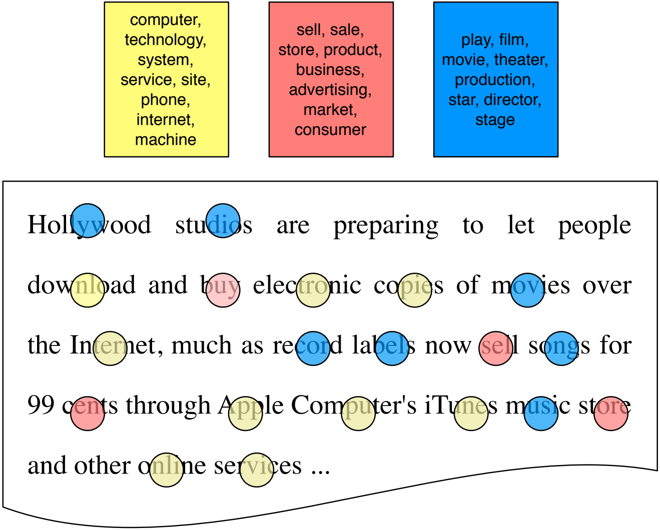
$$p(\boldsymbol{z},\boldsymbol{\beta},\boldsymbol{\theta}|\boldsymbol{w},\alpha,\lambda) \tag{2}$$

$$p(\boldsymbol{w},\boldsymbol{z},\boldsymbol{\theta},\boldsymbol{\beta}|\alpha,\lambda) = \prod_k p(\beta_k|\lambda)\prod_d p(\theta_d|\alpha)\prod_n p(z_{d,n}|\theta_d)p(w_{d,n}|\beta_{z_{d,n}})$$

## Gibbs Sampling

- A form of Markov Chain Monte Carlo
- Chain is a sequence of random variable states
- Given a state $\{z_1, \dots z_N\}$ given certain technical conditions, drawing $z_k \sim p(z_1, \dots z_{k-1}, z_{k+1}, \dots z_N | X, \Theta)$ for all $k$ (repeatedly) results in a Markov Chain whose stationary distribution *is* the posterior.
- For notational convenience, call $\boldsymbol{z}$ with $z_{d,n}$ removed $\boldsymbol{z}_{-d,n}$

## Inference


computer, technology, system, service, site, phone, internet, machine
sell, sale, store, product, business, advertising, market, consumer
play, film, movie, theater, production, star, director, stage
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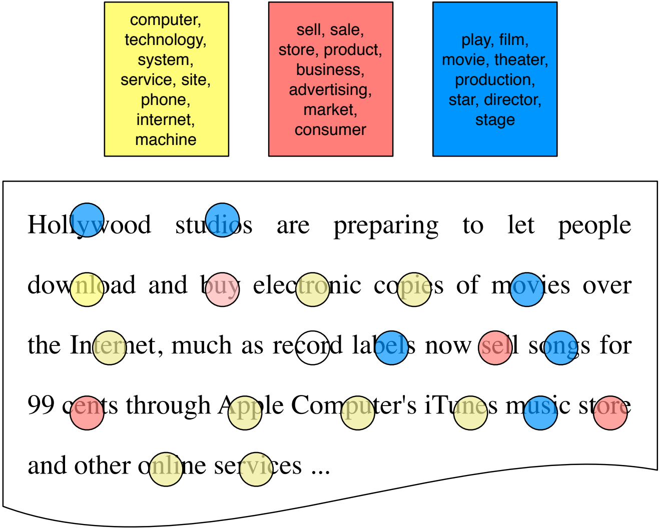

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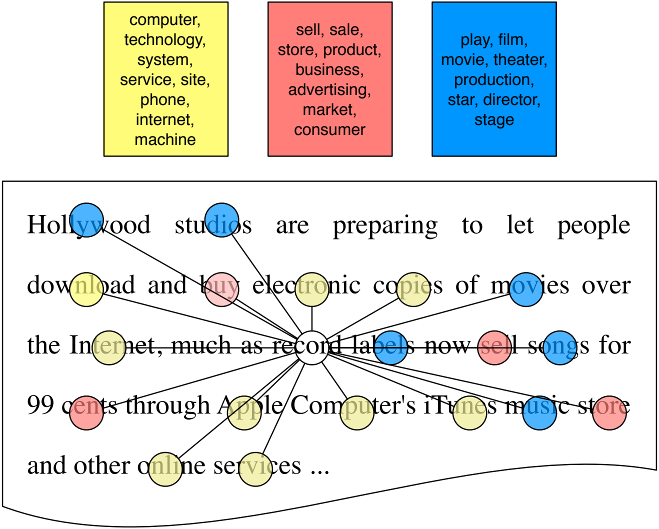

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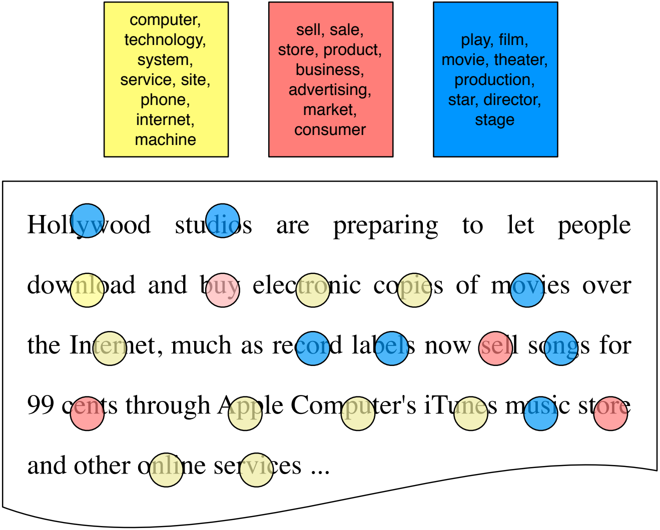
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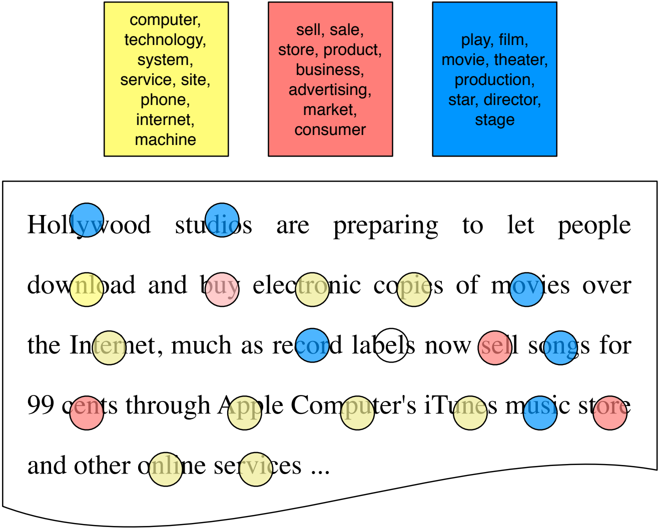
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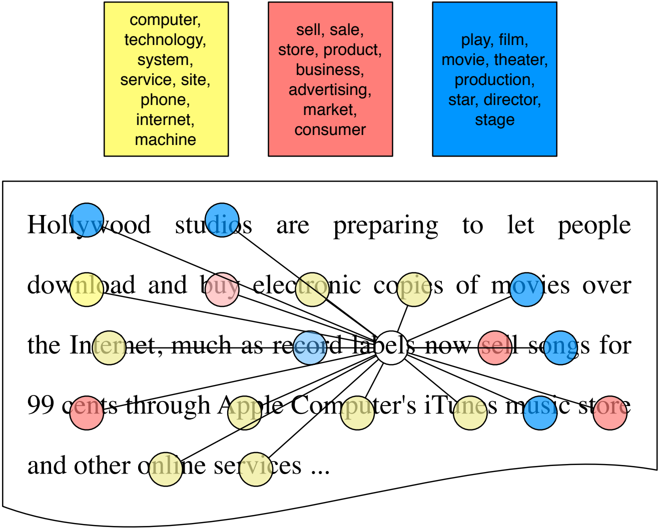
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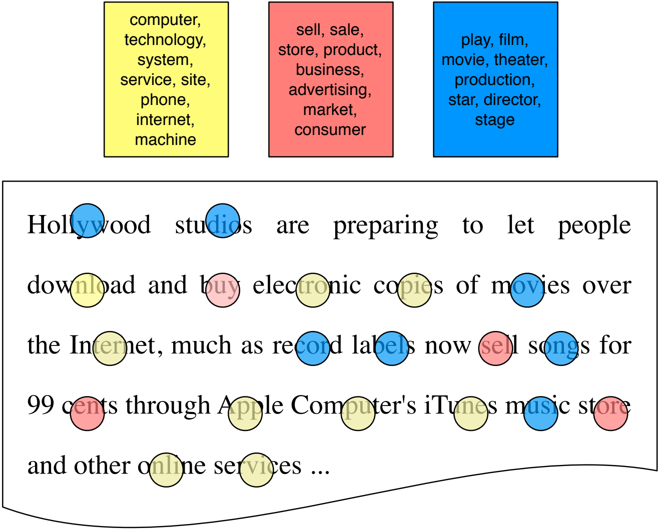
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## Gibbs Sampling

- For LDA, we will sample the topic assignments
- Thus, we want:

$$p(z_{d,n} = k | \boldsymbol{z}_{-d,n}, \boldsymbol{w}, \alpha, \lambda) = \frac{p(z_{d,n} = k, \boldsymbol{z}_{-d,n} | \boldsymbol{w}, \alpha, \lambda)}{p(\boldsymbol{z}_{-d,n} | \boldsymbol{w}, \alpha, \lambda)}$$

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- The topics and per-document topic proportions are integrated out / marginalized
- Let $n_{d,i}$ be the number of words taking topic $i$ in document $d$. Let $v_{k,w}$ be the number of times word $w$ is used in topic $k$.

$$= \frac{\int_{\theta_d} \left(\prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1}\right) \theta_d^{\alpha_k + n_{d,i}} d\theta_d \int_{\beta_k} \left(\prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1}\right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,i}} d\beta_k}{\int_{\theta_d} \left(\prod_i \theta_d^{\alpha_i + n_{d,i} - 1}\right) d\theta_d \int_{\beta_k} \left(\prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1}\right) d\beta_k}$$

## Gibbs Sampling

- Integral is normalizer of Dirichlet distribution

$$\int_{\beta_k} \left( \prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k = \frac{\prod_i^V \Gamma\left(\beta_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \beta_i + v_{k,i}\right)}$$

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- So we can simplify

$$\frac{\int_{\theta_d} \left( \prod_{i \neq k} \theta_d^{\alpha_i + n_{d,i} - 1} \right) \theta_d^{\alpha_k + n_{d,i}} d\theta_d \int_{\beta_k} \left( \prod_{i \neq w_{d,n}} \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) \beta_{k,w_{d,n}}^{\lambda_i + v_{k,i}} d\beta_k}{\int_{\theta_d} \left( \prod_i \theta_d^{\alpha_i + n_{d,i} - 1} \right) d\theta_d \int_{\beta_k} \left( \prod_i \beta_{k,i}^{\lambda_i + v_{k,i} - 1} \right) d\beta_k} =$$

$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma\left(\alpha_k + n_{d,k}\right)}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma\left(\lambda_k + v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

## Gamma Function Identity

$$z = \frac{\Gamma(z+1)}{\Gamma(z)} \tag{3}$$

$$\frac{\frac{\Gamma(\alpha_k + n_{d,k} + 1)}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i} + 1\right)} \prod_{i \neq k}^K \Gamma\left(\alpha_k + n_{d,k}\right)}{\frac{\prod_i^K \Gamma(\alpha_i + n_{d,i})}{\Gamma\left(\sum_i^K \alpha_i + n_{d,i}\right)}} \frac{\frac{\Gamma\left(\lambda_{w_{d,n}} + v_{k,w_{d,n}} + 1\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i} + 1\right)} \prod_{i \neq w_{d,n}}^V \Gamma\left(\lambda_k + v_{k,w_{d,n}}\right)}{\frac{\prod_i^V \Gamma\left(\lambda_i + v_{k,i}\right)}{\Gamma\left(\sum_i^V \lambda_i + v_{k,i}\right)}}$$

$$= \frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i}$$

## Gibbs Sampling Equation

$$\frac{n_{d,k} + \alpha_k}{\sum_i^K n_{d,i} + \alpha_i} \frac{v_{k,w_{d,n}} + \lambda_{w_{d,n}}}{\sum_i v_{k,i} + \lambda_i} \tag{4}$$

- Number of times document $d$ uses topic $k$
- Number of times topic $k$ uses word type $w_{d,n}$
- Dirichlet parameter for document to topic distribution
- Dirichlet parameter for topic to word distribution
- How much this document likes topic $k$
- How much this topic likes word $w_{d,n}$

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## Sample Document

| | | | | |
|---|---|---|---|---|
| | | | | |
| Etruscan | trade | price | temple | market |

## Sample Document

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|---|---|---|---|---|
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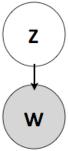
## Randomly Assign Topics




| 3 | 2 | 1 | 3 | 1 |
| --- | --- | --- | --- | --- |
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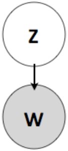
## Randomly Assign Topics

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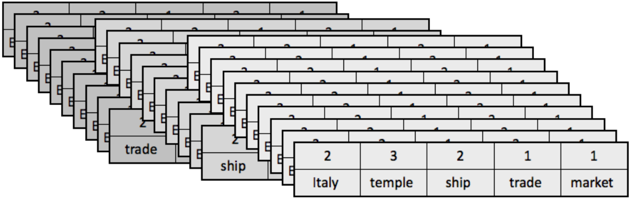

## Total Topic Counts

| 3 | 2 | 1 | 3 | 1 |
|---|---|---|---|---|
| Etruscan | trade | price | temple | market |

Total counts from **all** docs

| | 1 | 2 | 3 |
|---|---|---|---|
| Etruscan | 1 | 0 | 35 |
| market | 50 | 0 | 1 |
| price | 42 | 1 | 0 |
| temple | 0 | 0 | 20 |
| trade | 10 | 8 | 1 |
| ... | | | |

## Total Topic Counts

| | | | | |
|---|---|---|---|---|
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### Sampling Equation

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## We want to sample this word . . .

| 3 | **2** | 1 | 3 | 1 |
|---|---|---|---|---|
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## Decrement its count

| | | | | |
|---|---|---|---|---|
| 3 | ? | 1 | 3 | 1 |
| Etruscan | trade | price | temple | market |

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|---|---|---|---|
| Etruscan | 1 | 0 | 35 |
| market | 50 | 0 | 1 |
| price | 42 | 1 | 0 |
| temple | 0 | 0 | 20 |
| trade | 10 | **7** | 1 |
| ... | | | |

## What is the conditional distribution for this topic?

| | | | | |
|---|---|---|---|---|
| 3 | ? | 1 | 3 | 1 |
| Etruscan | trade | price | temple | market |

## Part 1: How much does this document like each topic?

| | | | | |
|---|---|---|---|---|
| 3 | ? | 1 | 3 | 1 |
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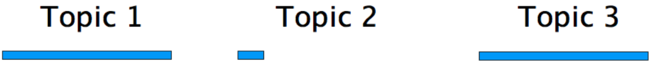

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Topic 1 Topic 2 Topic 3

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## Part 2: How much does each topic like the word?

| | | | | |
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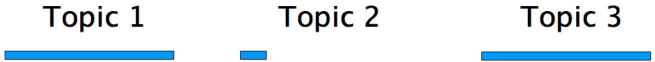


| | 1 | 2 | 3 |
|---|---|---|---|
| trade | 10 | 7 | 1 |

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| | | | |
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## Geometric interpretation

| | | | | |
|---|---|---|---|---|
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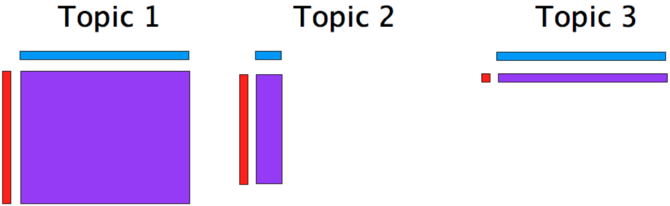

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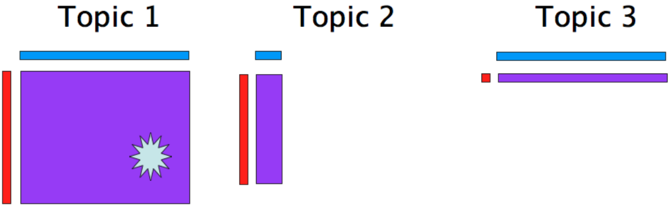

## Update counts

| | | | | |
|---|---|---|---|---|
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| | 1 | 2 | 3 |
|---|---|---|---|
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| market | 50 | 0 | 1 |
| price | 42 | 1 | 0 |
| temple | 0 | 0 | 20 |
| trade | **10** | 7 | 1 |
| ... | | | |

## Update counts

| | | | | |
|---|---|---|---|---|
| 3 | **1** | 1 | 3 | 1 |
| Etruscan | trade | price | temple | market |

| | 1 | 2 | 3 |
|---|---|---|---|
| Etruscan | 1 | 0 | 35 |
| market | 50 | 0 | 1 |
| price | 42 | 1 | 0 |
| temple | 0 | 0 | 20 |
| trade | **11** | 7 | 1 |
| ... | | | |

## Update counts

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 1 | 3 | 1 |
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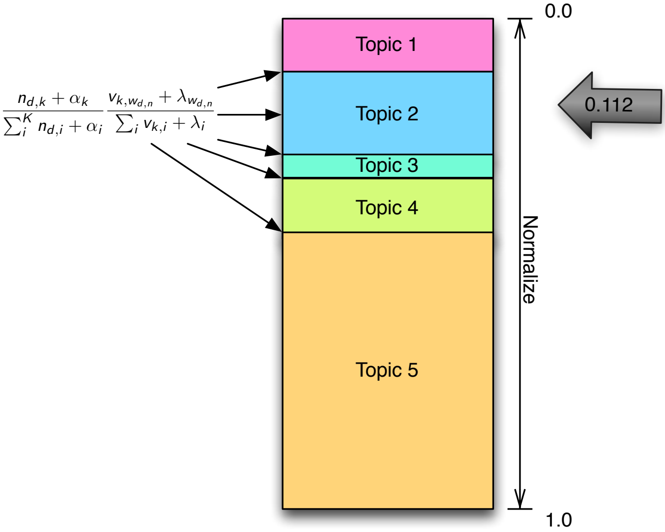
## Details: how to sample from a distribution



## Algorithm

1. For each iteration $i$:
   1. For each document $d$ and word $n$ currently assigned to $z_{old}$:
      1. Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
      2. Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
      3. Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

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   1. For each document $d$ and word $n$ currently assigned to $z_{old}$:
      1. Decrement $n_{d,z_{old}}$ and $v_{z_{old},w_{d,n}}$
      2. Sample $z_{new} = k$ with probability proportional to $\frac{n_{d,k}+\alpha_k}{\sum_i^K n_{d,i}+\alpha_i} \frac{v_{k,w_{d,n}}+\lambda_{w_{d,n}}}{\sum_i v_{k,i}+\lambda_i}$
      3. Increment $n_{d,z_{new}}$ and $v_{z_{new},w_{d,n}}$

# Desiderata

- Hyperparameters: Sample them too (slice sampling)
- Initialization: Random
- Sampling: Until likelihood converges
- Lag / burn-in: Difference of opinion on this
- Number of chains: Should do more than one

## Available implementations

- Mallet (http://mallet.cs.umass.edu)
- LDAC (http://www.cs.princeton.edu/ blei/lda-c)
- Topicmod (http://code.google.com/p/topicmod)