

HW12 - 2

$$\frac{d^2 y}{dt^2} - (1-y^2) \frac{dy}{dt} + y = 0$$

$$\frac{d}{dt} \left( \frac{dy}{dt} \right) - (1-y^2) \frac{dy}{dt} + y = 0, \text{ let } \frac{dy}{dt} = v$$

$$\frac{d}{dt} (v) - (1-y^2) v + y = 0$$

$$\frac{dv}{dt} - (1-y^2) v + y = 0$$

$$\text{at } t_0, t_0 = 0$$

$$v = \frac{dy}{dt}$$

$$v(t_0) = \frac{dy}{dt}(t_0)$$

$$v(t_0) = v(0) = 1$$

$$\frac{dy}{dt} = v, y'(0) = 1, \phi_{y,0} = \phi_{v,0}$$

$$\frac{dv}{dt} = (1-y^2) v - y, v(0) = 1, \phi_{v,1} = (1-y_1^2) v_1 + y_1$$



$i$	$t_i$	$y_i$	$v_i$	$\phi y_i$	$\phi v_i$
0	0	1	1	1	-1
1	0.25	1.25	0.75	0.75	-1.6718
2	0.50	1.4375	0.33203	0.33203	-1.7915
3	0.75	1.5205	-0.11586	-0.11586	-1.36844
4	1.0	1.491335	-0.4547	-0.4547	-0.93466

$$y_1 = y_0 + \phi y_0 h = 1 + (1)(0.25) = 1.25$$

$$v_1 = v_0 + \phi v_0 h = 1 + (-1)(0.25) = 0.75$$

$$\phi y_1 = v_1 = 0.75$$

$$\phi v_1 = (1 - y_1^2) v_1 - y_1 = (1 - (1.25)^2)(0.75) - (1.25) = -1.6718$$

$$y_2 = y_1 + \phi y_1 h = (1.25) + (0.75)(0.25) = 1.4375$$

$$v_2 = v_1 + \phi v_1 h = (0.75) + (-1.6718)(0.25) = 0.33203$$

$$\phi y_2 = v_2 = 0.33203$$

$$\phi v_2 = (1 - y_2^2) v_2 - y_2 = (1 - (1.4375)^2)(0.33203) - (1.4375) = -1.7915$$

$$y_3 = y_2 + \phi y_2 h = (1.4375) + (0.33203)(0.25) = 1.5205$$

$$v_3 = v_2 + \phi v_2 h = (0.33203) + (-1.7915)(0.25) = -0.11586$$

$$\phi y_3 = v_3 = -0.11586$$

$$\phi v_3 = (1 - y_3^2) v_3 - y_3 = (1 - (1.5205)^2)(-0.11586) - (1.5205) = -1.36844$$



$$y_4 = y_0 + \phi y_3(h) = (1.5205) - 0.11586 * 0.25 \\ = 1.491535$$

$$v_4 = v_3 + \phi v_3(h) = -0.11586 + (-1.36844)(0.25) \\ = -0.45471$$

$$\phi y_4 = v_4 = -0.45471$$

$$\phi v_4 = (1 - y_4^2) v_4 - y_4 = (1 - (1.491535)^2)(-0.45471) \\ - 1.491535 = -0.93466$$