Topic: Finite Difference – numerical differentiation
Read: Chapter 4: 4.3.4. Chapter 21: intro & sec 21.1, 21.2, 21.4, 21.7
See figures 21.3, 21.4 & 21.5 for the 2nd order approximations.

Handwork problem:

HW11_1 Let $f(x) = x\cos(x) - x^2\sin(x)$.

a) determine the true value of $f'(x)$ at $x = 2.9, 3.0, 3.1, 3.2$
b) evaluate the function $f(x)$ at the same 4 points
c) Use the results from b) to estimate the derivative at all points using forward (O(h)), backward (O(h)) and centered ($O(h^2)$) formulas (where possible)
d) compute et, the true relative errors

Present your results in a table

Coding problems:

HW11_2 (textbook 21.14) A plane is being tracked by radar, and data are taken every second in polar coordinates θ and r

t (**s**)	200	202	204	206	208	210
θ (**rad**)	0.75	0.72	0.70	0.68	0.67	0.66
r (**m**)	5120	5370	5560	5800	6030	6240

At 206 seconds, use the centered finite-difference (second order correct) to find the vector expressions for velocity $\vec{v}$ and $\vec{a}$. The velocity and acceleration given in polar coordinates are

$$v = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta \quad \text{and} \quad a = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

Write a MATLAB script to implement the above and use `fprintf` to display the magnitudes of velocity and acceleration at 206 seconds.

HW11_3 Chemical reactions often follow the model:

$$\frac{dc}{dt} = -kc^n$$

t	10	20	30	40	50	60
c	3.52	2.48	1.75	1.23	0.87	0.61

where c is the concentration, t is time, k is the reaction rate, n is the reaction order. Given the data in the table at right, first estimate dc/dt using $O(h^2)$ formulas (use `gradient`, followed by end corrections), then use these values to solve for k and n by linear regression. Plot the curve fit.

HW11_4 A uniform beam is simply supported at both ends and is subjected to a load. The deflection of the beam is given by the differential equation

$$\frac{d^2y}{dx^2} = -\frac{M(x)}{EI}$$

x (m)	0	0.2	0.4	0.6	0.8	1.0
y (cm)	0	7.78	10.68	8.38	3.97	0

where y is the deflection, x is the coordinate measured along the length of the beam, $M(x)$ is the bending moment, and $EI = 1.2 \times 10^7 N \cdot m^2$ is the flexural rigidity of the beam. The data shown in the table is obtained from measuring the deflection of the beam vs position. Find the bending moment $M(x)$ at each location x from this given data. Print the results in a table.