4.1 $A = \begin{bmatrix} -6 \\ 9 \\ 9 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 8 \\ 6 & 6 \\ -6 & 9 \\ -1 & 3 \end{bmatrix}$ $C = [-8 \ \ 7 \ \ 9]$ $D = \begin{bmatrix} 4 & 3 & -4 & 4 \\ 5 & -6 & -8 & -3 \\ 5 & 4 & -7 & 9 \\ -2 & -8 & 7 & -8 \end{bmatrix}$

① $e = C * A - 10$

$e(1,1) = C_{11}A_{11} + C_{12}A_{21} + C_{13}A_{31} - 10$

$= (-8)(-6) + (7)(9) + (9)(9) - 10 = \boxed{182}$

② $f = D \wedge 2$

$f(4,2) = D_{41}D_{12} + D_{42}D_{22} + D_{43}D_{32} + D_{44}D_{42}$

$= (-2)(3) + (-8)(-6) + (7)(4) + (-8)(-8) = \boxed{134}$

③ $g = D.*D$

$g(4,2) = D_{42} \cdot D_{42} = (-8)(-8) = \boxed{64}$

④ $h = D(2:3, :) * B$

Here, $D(2:3, :) = \begin{bmatrix} 5 & -6 & -8 & -3 \\ 5 & 4 & -7 & 9 \end{bmatrix} = P$ (say)

$h(2,2) = P_{21}B_{12} + P_{22}B_{22} + P_{23}B_{32} + P_{24}B_{42}$

$= (5)(8) + (4)(6) + (-7)(9) + (9)(3) = \boxed{28}$

4-2 **Gaussian Elimination (Gauss Naive)**

Augmented A matrix =
$$\left[\begin{array}{cccc|c} 3 & 1 & 5 & 5 & 42 \\ 4 & -4 & 5 & 0 & -9 \\ -4 & -2 & -4 & 3 & -3 \\ 5 & 1 & -5 & -4 & -5 \end{array}\right]$$

Forward Elimination of unknowns:

1st elimination:

new R2 = R2 − (4/3) R1 :
$$\begin{array}{r} [4 \quad -4 \quad 5 \quad 0 \mid -9] \\ -\,[4 \quad 4/3 \quad 20/3 \quad 20/3 \mid 56] \\ \hline [0 \quad -5.3333 \quad -1.6667 \quad -6.6667 \mid -65] \end{array}$$

new R3 = R3 − (−4/3) R1 :
$$\begin{array}{r} [-4 \quad -2 \quad -4 \quad 3 \mid -3] \\ -\,[-4 \quad -4/3 \quad -20/3 \quad -20/3 \mid -56] \\ \hline [0 \quad -0.6667 \quad 2.6667 \quad 9.6667 \mid 53] \end{array}$$

new R4 = R4 − (5/3) R1 :
$$\begin{array}{r} [5 \quad 1 \quad -5 \quad -4 \mid -5] \\ -\,[5 \quad 5/3 \quad 25/3 \quad 25/3 \mid 70] \\ \hline [0 \quad -0.6667 \quad -13.3333 \quad -12.3333 \mid -75] \end{array}$$

After 1st elimination:
$$\left[\begin{array}{cccc|c} 3 & 1 & 5 & 5 & 42 \\ 0 & -5.3333 & -1.6667 & -6.6667 & -65 \\ 0 & -0.6667 & 2.6667 & 9.6667 & 53 \\ 0 & -0.6667 & -13.3333 & -12.3333 & -75 \end{array}\right]$$

2nd elimination:

new R3 = R3 − $\left(\dfrac{-0.6667}{-5.3333}\right)$ R2 :
$$\begin{array}{r} [0 \quad -0.6667 \quad 2.6667 \quad 9.6667 \mid 53] \\ -\,[0 \quad -0.6667 \quad -0.2083 \quad -0.8334 \mid -8.1255] \\ \hline [0 \quad 0 \quad 2.875 \quad 10.5001 \mid 61.1255] \end{array}$$

new R4 = R4 − $\left(\dfrac{-0.6667}{-5.333}\right)$ R2 :
$$\begin{array}{r} [0 \quad -0.6667 \quad -13.3333 \quad -12.3333 \mid -75] \\ -\,[0 \quad -0.6667 \quad -0.2083 \quad -0.8334 \mid -8.1255] \\ \hline [0 \quad 0 \quad -13.125 \quad -11.4999 \mid -66.8745] \end{array}$$

After 2nd Elimination:

$$\left[\begin{array}{cccc|c} 3 & 1 & 5 & 5 & 42 \\ 0 & -5.3338 & -1.6667 & -6.6667 & -65 \\ 0 & 0 & 2.875 & 10.5001 & 61.1255 \\ 0 & 0 & -13.125 & -11.4999 & -66.8745 \end{array}\right]$$

3rd elimination:

$$newR4 = R4 - \left(\frac{-13.125}{2.875}\right) R3 : \quad \begin{array}{l} [0 \quad 0 \quad -13.125 \quad -11.4999 \mid -66.8745] \\ -[0 \quad 0 \quad -13.125 \quad -47.9352 \mid -279.0512] \\ \hline [0 \quad 0 \quad 0 \quad 36.4353 \mid 212.1767] \end{array}$$

After 3rd elimination (upper triangular matrix):

$$\left[\begin{array}{cccc|c} 3 & 1 & 5 & 5 & 42 \\ 0 & -5.3333 & -1.6667 & -6.6667 & -65 \\ 0 & 0 & 2.875 & 10.5001 & 61.1255 \\ 0 & 0 & 0 & 36.4353 & 212.1767 \end{array}\right]$$

Back substitution:

$$36.4353\, x_4 = 212.1767 \Rightarrow x_4 = \frac{(212.1767)}{(36.4353)} = 5.8234$$

$$2.875\, x_3 + 10.5001\,(5.8234) = 61.1255$$

$$\Rightarrow 2.875\, x_3 = 61.1255 - 61.1463 = -0.0208 \Rightarrow x_3 = \frac{-0.0208}{2.875} = -0.0072$$

$$-5.3333 x_2 - 1.6667(-0.0072) - 6.6667(5.8234) = -65$$

$$\Rightarrow -5.3333 x_2 = -65 - 0.012 + 38.8229 = -26.1891 \Rightarrow x_2 = \frac{-26.1891}{-5.3333} = 4.9105$$

$$3x_1 + (4.9105) + 5(-0.0072) + 5(5.8234) = 42$$

$$\Rightarrow 3x_1 = 42 - 4.9105 + 0.036 - 29.1170 = 8.0085 \Rightarrow x_1 = \frac{8.0085}{3} = 2.6695$$

$$\boxed{\begin{array}{l} x_1 = 2.6695 \\ x_2 = 4.9105 \\ x_3 = -0.0072 \\ x_4 = 5.8234 \end{array}}$$