HW5

5-1 Augmented A matrix :

$$\left[\begin{array}{cccc|c} 3 & 1 & 5 & 5 & 42 \\ 4 & -4 & 5 & 0 & -9 \\ -4 & -2 & -4 & 3 & -3 \\ -5 & 1 & -5 & -4 & -5 \end{array}\right]$$

As -5 is the largest element by magnitude of the potential pivots of 1st elimination, swap R1 & R4

$$\left[\begin{array}{cccc|c} -5 & 1 & -5 & -4 & -5 \\ 4 & -4 & 5 & 0 & -9 \\ -4 & -2 & -4 & 3 & -3 \\ 3 & 1 & 5 & 5 & 42 \end{array}\right]$$

1st Elimination :

$$\text{new}R2 = R2 - \left(\frac{4}{-5}\right)R1 : \quad \begin{array}{r} [4 \;\; -4 \;\; 5 \;\; 0 \;|\; -9] \\ -\,[4 \;\; -0.8 \;\; 4 \;\; 3.2 \;|\; 4] \\ \hline [0 \;\; -3.2 \;\; 1 \;\; -3.2 \;|\; -13] \end{array}$$

$$\text{new}R3 = R3 - \left(\frac{-4}{-5}\right)R1 : \quad \begin{array}{r} [-4 \;\; -2 \;\; -4 \;\; 3 \;|\; -3] \\ -\,[-4 \;\; 0.8 \;\; -4 \;\; -3.2 \;|\; -4] \\ \hline [0 \;\; -2.8 \;\; 0 \;\; 6.2 \;|\; 1] \end{array}$$

$$\text{new}R4 = R4 - \left(\frac{3}{-5}\right)R1 : \quad \begin{array}{r} [3 \;\; 1 \;\; 5 \;\; 5 \;|\; 42] \\ -\,[3 \;\; -0.6 \;\; 3 \;\; 2.4 \;|\; 3] \\ \hline [0 \;\; 1.6 \;\; 2 \;\; 2.6 \;|\; 39] \end{array}$$

After 1st elimination :

$$\left[\begin{array}{cccc|c} -5 & 1 & -5 & -4 & -5 \\ 0 & -3.2 & 1 & -3.2 & -13 \\ 0 & -2.8 & 0 & 6.2 & 1 \\ 0 & 1.6 & 2 & 2.6 & 39 \end{array}\right]$$

No swapping needed as the largest element by magnitude is already in pivot position

2nd Elimination

new $R3 = R3 - \left(\frac{-2.8}{-3.2}\right) R2$:

$$\begin{array}{rrrrr|r} & [0 & -2.8 & 0 & 6.2 & 1] \\ - & [0 & -2.8 & 0.875 & -2.8 & -11.375] \\ \hline & [0 & 0 & -0.875 & 9 & 12.375] \end{array}$$

new $R4 = R4 - \left(\frac{1.6}{-3.2}\right) R2$:

$$\begin{array}{rrrrr|r} & [0 & 1.6 & 2 & 2.6 & 39] \\ - & [0 & 1.6 & -0.5 & 1.6 & 6.5] \\ \hline & [0 & 0 & 2.5 & 1 & 32.5] \end{array}$$

After 2nd elimination:

$$\left[\begin{array}{rrrr|r} -5 & 1 & -5 & -4 & -5 \\ 0 & -3.2 & 1 & -3.2 & -13 \\ 0 & 0 & -0.875 & 9 & 12.375 \\ 0 & 0 & 2.5 & 1 & 32.5 \end{array}\right] \xrightarrow{\text{swap } R2 \,\&\, R3} \left[\begin{array}{rrrr|r} -5 & 1 & -5 & -4 & -5 \\ 0 & -3.2 & 1 & -3.2 & -13 \\ 0 & 0 & 2.5 & 1 & 32.5 \\ 0 & 0 & -0.875 & 9 & 12.375 \end{array}\right]$$

3rd Elimination :

new $RA = RA - \left(\frac{-0.875}{2.5}\right) R3$:

$$\begin{array}{rrrrr|r} & [0 & 0 & -0.875 & 9 & 12.375] \\ - & [0 & 0 & -0.875 & -0.35 & -11.375] \\ \hline & 0 & 0 & 0 & 9.35 & 23.75] \end{array}$$

After 3rd elimination:

$$\left[\begin{array}{rrrr|r} -5 & 1 & -5 & -4 & -5 \\ 0 & -3.2 & 1 & -3.2 & -13 \\ 0 & 0 & 2.5 & 1 & 32.5 \\ 0 & 0 & 0 & 9.35 & 23.75 \end{array}\right] \longrightarrow \text{Upper triangular}$$

Back substitution : $9.35x_4 = 23.75 \Rightarrow x_4 = \frac{23.75}{9.35} = 2.5401$

$2.5x_3 + 2.5401 = 32.5 \Rightarrow x_3 = \frac{29.9599}{2.5} = 11.9840$

$-3.2x_2 + 11.9840 - 3.2(2.5401) = -13 \Rightarrow x_2 = \frac{-16.8557}{-3.2} = 5.2674$

$-5x_1 + 5.2674 - 5(11.984) - 4(2.5401) = -5$

$\Rightarrow x_1 = \frac{59.8130}{-5} = -11.9626$

$$\boxed{\begin{array}{ll} x_1 = -11.9626, & x_2 = 5.2674 \\ x_3 = 11.9840, & x_4 = 2.5401 \end{array}}$$