Analytically

HW10_1 $\int_0^3 \frac{dx}{(3x+1)^3}$:

$U = 3x+1$

$dU = 3dx$

$$\int_0^3 \frac{dU}{(U)^3 * 3} = \frac{1}{3}\left[\frac{-1}{2U^2}\right]_0^3 = \cancel{-\frac{1}{6}\left[\frac{1}{2(3)^2}\right]}$$

$$= -\frac{1}{6}\left[\frac{1}{(3*3+1)^2} - \frac{1}{(1)}\right] = -\frac{1}{6}\left[\frac{-99}{100}\right] = 0.165$$

(a) Trapezoidal rule, $n=6$, $a=0$, $b=3$

$$I = \frac{(b-a)}{n} * \left[\frac{f(a)+f(b)}{2}\right] = \frac{h}{2}\left[f(a)+f(b)\right]$$

$h = \frac{3-0}{6} = 0.5$, $f(a) = f(0) = 1$, $f(b) = f(3) = \frac{1}{(3\times3+1)^3} = \frac{1}{1000}$

$$I = \frac{0.5}{2}\left(1 + \frac{1}{1000}\right) = 0.25025$$

(b) Composite trapezoid rule, $n=6$, $h=0.5$

$$I = \frac{h}{2}\left[f(x_0) + f(x_n) + 2(f(x_1) + f(x_2) + \dots + f(x_{n-1}))\right]$$

I

$f(x_0) = f(0) = 1$, $f(x_1) = f(0.5) = \frac{1}{(3*0.5+1)^3} = 0.064$

$f(x_2) = f(1) = \frac{1}{(3*1+1)^3} = 0.015625$, $f(x_3) = f(1.5) = 6.01\times10^{-3}$

$f(x_4) = f(2) = 2.91 * 10^{-3}$ $f(x_5) = f(2.5) = 1.628\times10^{-3}$

$f(x_6) = f(3) = 10^{-3}$

$$I = \frac{0.5}{2}\left[1 + 1\times10^{-3} + 2\left(0.064 + 0.015625 + 6.01\times10^{-3} + 2.91\times10^{-3} + 1.628\times10^{-3}\right)\right]$$

$= 0.365649$

(d) Simpson's $\frac{1}{3}$ rd rule , $n = 6$; $h = 0.5$

$$I = \frac{h}{3}\left(f(x_0) + f(x_n)\right)$$

$f(x_0) = f(0) = 1$

$f(x_n) = f(3) = \frac{1}{1000}$

$$I = \frac{0.5}{3} * \left(1 + \frac{1}{1000}\right) = 0.16683$$

(e) Composite Simpson's Rule , $n = 6$

$$I = \frac{h}{3} \times \left(f(x_0) + f(x_6) + 2\times\left(f(x_2) + f(x_4)\right) + 4\left(f(x_1) + f(x_3) + f(x_5)\right)\right)$$

$$I = \frac{0.5}{2}\left(1 + 1\times10^{-3} + 2\left(0.015625 + 2.91\times10^{-3}\right) + 4\left(0.064 + 6.01\times10^{-3} + 1.62\times10^{-3}\right)\right)$$

$$I = 0.2207$$

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