HW6 Handwork

HW6.1 $10c_1 + 2c_2 - c_3 = 27$
$c_1 + c_2 + 5c_3 = -21$
$-3c_1 - 6c_2 + 2c_3 = -60$

$$\Rightarrow A = \begin{bmatrix} \textcircled{10} & 2 & -1 \\ 1 & \textcircled{1} & 5 \\ -3 & -6 & \textcircled{2} \end{bmatrix}$$

$|10| > |2| + |-1|$
$|1| < |1| + |5|$
$|2| < |-3| + |-6|$

Not diagonally dominant
$\Rightarrow$ convergence not guaranteed

Reordered for diagonal dominance:

$$\begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

$|10| > |2| + |-1|$
$|-6| > |-3| + |2|$
$|5| > |1| + |1|$

$$c_1 = \frac{(27 - 2c_2 + c_3)}{10}$$

$$c_2 = \frac{(-60 + 3c_1 - 2c_3)}{-6}$$

$$c_3 = \frac{(-21 - c_1 - c_2)}{5}$$

Initial values : $c_1 = c_2 = c_3 = 0$

it = 1

$c_1 = \frac{(27 - 2(0) + 0)}{10} = 2.7$ $\quad ea_1 = \left|\frac{2.7 - 0}{2.7}\right| \times 100\% = 100\%$

$c_2 = \frac{(-60 + 3(2.7) - 2(0))}{-6} = 8.65$ $\quad ea_2 = \left|\frac{8.65 - 0}{8.65}\right| \times 100\% = 100\%$

$c_3 = \frac{(-21 - 2.7 - 8.65)}{5} = -6.47$ $\quad ea_3 = \left|\frac{-6.47 - 0}{-6.47}\right| \times 100\% = 100\%$

it = 2

$c_1 = \frac{(27 - 2(8.65) - 6.47}{10} = 0.323$ $\quad ea_1 = \left|\frac{0.323 - 2.7}{0.323}\right| \times 100\% = 735.913\%$

$c_2 = \frac{(-60 + 3(0.323) - 2(-6.47))}{-6} = 7.68183$ $\quad ea_2 = \left|\frac{7.68183 - 8.65}{7.68183}\right| \times 100\% = 12.603\%$

$c_3 = \frac{(-21 - 0.323 - 7.68183)}{5} = -5.80097$ $\quad ea_3 = \left|\frac{-5.80097 - (-6.47)}{-5.80097}\right| \times 100\% = 11.533\%$

it = 3

$$c_1 = \frac{(27 - 2(7.68183) - 5.80097)}{10} = 0.58354$$

$$c_2 = \frac{(-60 + 3(0.58354) - 2(-5.80097))}{-6} = 7.77457$$

$$c_3 = \frac{(-21 - 0.58354 - 7.77457)}{5} = -5.87162$$

$$ea_1 = \left|\frac{0.58354 - 0.323}{0.58354}\right| \times 100\% = 44.65\%$$

$$ea_2 = \left|\frac{7.77457 - 7.68183}{7.77457}\right| \times 100\% = 1.19\%$$

$$ea_3 = \left|\frac{-5.87162 - (-5.80097)}{-5.87162}\right| \times 100\% = 1.20\%$$

it = 4

$$c_1 = \frac{(27 - 2(7.77457) - 5.87162)}{10} = 0.55792$$

$$c_2 = \frac{(-60 + 3(0.55792) - 2(-5.87162))}{-6} = 7.76383$$

$$c_3 = \frac{(-21 - 0.55792 - 7.76383)}{5} = -5.86435$$

$$ea_1 = \left|\frac{0.55792 - 0.58354}{0.55792}\right| \times 100\% = 4.59\%$$

$$ea_2 = \left|\frac{7.76383 - 7.77457}{7.76383}\right| \times 100\% = 0.14\%$$

$$ea_3 = \left|\frac{-5.86435 - (-5.87162)}{-5.86435}\right| \times 100\% = 0.12\%$$

As $ea_1 < 5\%$, $ea_2 < 5\%$, $ea_3 < 5\%$, we stop here

($e_s = 5\%$)

Final answer:

$c_1 = 0.55792$

$c_2 = 7.76383$

$c_3 = -5.86435$

HW6-2 $\quad x^2+y-x=0.75 \\ x^2-y^2=5xy$ $\Rightarrow$ $f_1(x,y)=x^2+y-x-0.75 \\ f_2(x,y)=x^2-y^2-5xy$

Vector-valued function, $f(\bar{x})=\begin{bmatrix} x^2+y-x-0.75 \\ x^2-y^2-5xy \end{bmatrix}$

Jacobian, $J=\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}=\begin{bmatrix} (2x-1) & 1 \\ (2x-5y) & (-2y-5x) \end{bmatrix}$

Initial values: $x=1.2, \quad y=1$

it=1

$f=\begin{bmatrix} 1.2^2+1-1.2-0.75 \\ 1.2^2-1^2-5(1.2)(1) \end{bmatrix}=\begin{bmatrix} 0.49 \\ -5.56 \end{bmatrix}$, $J=\begin{bmatrix} (2(1.2)-1) & 1 \\ (2(1.2)-5(1)) & (-2(1)-5(1.2)) \end{bmatrix}=\begin{bmatrix} 1.4 & 1 \\ -2.6 & -8 \end{bmatrix}$

$|J|=(1.4)(-8)-(1)(-2.6)=-8.6$

$J_1=\begin{bmatrix} 0.49 & 1 \\ -5.56 & -8 \end{bmatrix}$, $\therefore |J_1|=(0.49)(-8)-(1)(-5.56)=1.64$

$J_2=\begin{bmatrix} 1.4 & 0.49 \\ -2.6 & -5.56 \end{bmatrix}$, $\therefore |J_2|=(1.4)(-5.56)-(0.49)(-2.6)=-6.51$

$\therefore s_1=\frac{|J_1|}{|J|}=\frac{1.64}{-8.6}=-0.1907$

$s_2=\frac{|J_2|}{|J|}=\frac{-6.51}{-8.6}=0.757$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix}_{new}=\begin{bmatrix} 1.2 \\ 1 \end{bmatrix}-\begin{bmatrix} -0.1907 \\ 0.757 \end{bmatrix}=\begin{bmatrix} 1.3907 \\ 0.243 \end{bmatrix}$

$\varepsilon_{a_1}=\left|\frac{1.3907-1.2}{1.3907}\right|\times 100\%=13.71\%$

$\varepsilon_{a_2}=\left|\frac{0.243-1}{0.243}\right|\times 100\%=311.48\%$

it = 2

$x = 1.3907, \quad y = 0.243$

$$f = \begin{bmatrix} (1.3907)^2 + 0.243 - 1.3907 - 0.75 \\ (1.3907)^2 - (0.243)^2 - 5(1.3907)(0.243) \end{bmatrix} = \begin{bmatrix} 0.0364 \\ 0.1851 \end{bmatrix}$$

$$J = \begin{bmatrix} 2(1.3907) - 1 & 1 \\ 2(1.3907) - 5(0.243) & -2(0.243) - 5(1.3907) \end{bmatrix} = \begin{bmatrix} 1.7814 & 1 \\ 1.5663 & -7.4395 \end{bmatrix}$$

$$|J| = (1.7814)(-7.4395) - (1)(1.5663) = -14.819$$

$$J_1 = \begin{bmatrix} 0.0364 & 1 \\ 0.1851 & -7.4395 \end{bmatrix}, \quad |J_1| = (0.0364)(-7.4395) - (1)(0.1851) = -0.4557$$

$$J_2 = \begin{bmatrix} 1.7814 & 0.0364 \\ 1.5663 & 0.1851 \end{bmatrix}, \quad |J_2| = (1.7814)(0.1851) - (0.0364)(1.5663) = 0.2728$$

$$S_1 = \frac{|J_1|}{|J|} = \frac{-0.4557}{-14.819} = 0.0307$$

$$S_2 = \frac{|J_2|}{|J|} = \frac{0.2728}{-14.819} = -0.0184$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_{new} = \begin{bmatrix} 1.3907 \\ 0.243 \end{bmatrix} - \begin{bmatrix} 0.0307 \\ -0.0184 \end{bmatrix} = \begin{bmatrix} 1.3599 \\ 0.2614 \end{bmatrix}$$

$$ea_1 = \left| \frac{1.3599 - 1.3907}{1.3599} \right| \times 100\% = 2.26\%, \quad ea_2 = \left| \frac{0.2614 - 0.243}{0.2614} \right| \times 100\% = 7.04\%$$

it = 3

$x = 1.3599, \quad y = 0.2614$

$$f = \begin{bmatrix} 0.0009 \\ 0.0034 \end{bmatrix}, \quad J = \begin{bmatrix} 1.7199 & 1 \\ 1.4127 & -7.3226 \end{bmatrix} \quad \therefore |J| = -14.0069$$

$$J_1 = \begin{bmatrix} 0.0009 & 1 \\ 0.0034 & -7.3226 \end{bmatrix}, \quad |J_1| = -0.0104 \quad \therefore S_1 = \frac{|J_1|}{|J|} = 0.00074$$

$$J_2 = \begin{bmatrix} 1.7199 & 1 \\ 1.4127 & -7.3226 \end{bmatrix}, \quad |J_2| = 0.0046 \quad \therefore S_2 = \frac{|J_2|}{|J|} = -0.00033$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_{new} = \begin{bmatrix} 1.3599 \\ 0.2614 \end{bmatrix} - \begin{bmatrix} 0.00074 \\ -0.00033 \end{bmatrix} = \begin{bmatrix} 1.3592 \\ 0.2617 \end{bmatrix}$$

$$ea_1 = \left| \frac{1.3592 - 1.3599}{1.3592} \right| \times 100\% = 0.05\%$$

$$ea_2 = \left| \frac{0.2617 - 0.2614}{0.2617} \right| \times 100\% = 0.12\%$$

As both $ea_1 < 0.5\%$, $ea_2 < 0.5\%$, stopping condition met.

Final answer: $x = 1.3592$, $y = 0.2617$