ME3215 Numerical Methods Spring 2022
HW #6 due 03/02/2022

Topic: Iterative methods: Gauss – Seidel (linear); successive substitution, Newton-Raphson-(non-linear)
Read: Chapter 12: 12.1 (omit 12.1.3) 12.2.
Remember to press **Cntl C** if your computer becomes unresponsive

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Handwork problems:

HW 6_1 Before you begin, verify if this system will converge for Gauss-Seidel method. If yes, explain why you think so. If not, rearrange to take the system to a form so that convergence is assured.

System:
$$\begin{aligned} 10c_1 + 2c_2 - c_3 &= 27 \\ c_1 + c_2 + 5c_3 &= -21 \\ -3c_1 - 6c_2 + 2c_3 &= -60 \end{aligned}$$

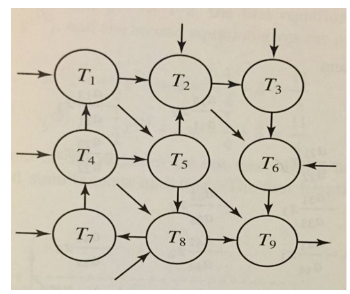
Finally, solve using Gauss-Seidel to $e_s = 5\%$.

HW6_2 Solve the system of equations at right using the Newton-Raphson method. Start with x = 1.2 and y = 1 and use es = 0.5%. Show your work & describe your steps. Use Cramer's rule to solve the Jacobian equation.

$$\boxed{\begin{aligned} x^2 + y - x &= 0.75 \\ x^2 - y^2 &= 5xy \end{aligned}}$$

Coding problems:

HW6_3 For a system of nine interconnected tanks (shown at right), the mass-balance equations are:



$$\begin{aligned}
r_{01}c_{01} + r_{41}c_4 &= (r_{12} + r_{15})c_1 \\
r_{02}c_{02} + r_{12}c_1 + r_{52}c_5 &= (r_{23} + r_{26})c_2 \\
r_{03}c_{03} + r_{23}c_2 &= r_{36}c_3 \\
r_{04}c_{04} + r_{74}c_7 &= (r_{41} + r_{48} + r_{45})c_4 \\
r_{15}c_1 + r_{45}c_4 &= (r_{52} + r_{58} + r_{59})c_5 \\
r_{06}c_{06} + r_{26}c_2 + r_{36}c_3 &= r_{69}c_6 \\
r_{07}c_{07} + r_{87}c_8 &= r_{74}c_7 \\
r_{08}c_{08} + r_{58}c_5 + r_{48}c_4 &= (r_{87} + r_{89})c_8 \\
r_{59}c_5 + r_{69}c_6 + r_{89}c_8 &= rc_9
\end{aligned}$$

The concentration of chemical inflow into tank T_i is designated c_i and the initial concentration in tank T_i is designated c_{0i}. The flow rate from tank T_i to T_j is designated r_{ij}. Furthermore, the total flow into each tank must equal the flow out, r (from tank 9).

Given initial concentrations:

c_{01}=1, c_{02}=2, c_{03}=1, c_{04}=2, c_{06}=1, c_{07}=1 and c_{08}=1, in g/L

given flow rate between tanks:

r_{12}=6, r_{15}=2, r_{23}=5, r_{26}=5, r_{36}=8, r_{41}=5, r_{45}=1, r_{48}=1, r_{52}=1, r_{58}=2, r_{59}=1, r_{69}=15, r_{74}=4, r_{87}=2, r_{89}=3, in L/min

and given external flow rates (i.e., inflow into the tanks from outside):

r_{01}=3, r_{02}=3, r_{03}=3, r_{04}=3, r_{06}=2, r_{07}=2, r_{08}=3, in L/min

<u>Note:</u> $r = r_{01} + r_{02} + r_{03} + r_{04} + r_{06} + r_{07} + r_{08}$

With these given parameters we can solve for the concentrations in the tanks. Write a MATLAB script to solve for these concentrations using Gauss-Seidel, to an accuracy of 5 sig figs. Use `fprintf` statement to print the final answer.

HW6_4: Write code to solve the following system of equations using the Newton-Raphson method.

Let $x = 1, y = 1, z = 1$ for starting guesses and determine the solution to 4 sig figs. Display the final answers on screen using an `fprintf` statement.

$$f(x,y,z) = x^3 - 10x + y - z = -3$$
$$g(x,y,z) = y^3 + 10y - 2x - 2z = 5$$
$$h(x,y,z) = x + y - 10z + 2\sin(z) = -5$$