HW9_1 (handwork)

Linearization :

$$y = \alpha x e^{\beta x}$$

$$\Rightarrow \frac{y}{x} = \alpha e^{\beta x}$$

$$\Rightarrow \ln(y/x) = \ln(\alpha) + \ln(e^{\beta x})$$

$$\Rightarrow \ln(y/x) = \ln(\alpha) + \beta x$$

$$\Rightarrow Y = a_0 + a_1 X$$

where, $Y = \ln(y/x)$, $X = x$
$a_0 = \ln(\alpha)$, $a_1 = \beta$

Linear regression :

x	y
0.1	0.75
0.2	1.25
0.4	1.45
0.6	1.25
0.9	0.85
1.3	0.55
1.5	0.35
1.7	0.28
1.8	0.18

$\Rightarrow$

	X	Y	XY	X^2
	0.1	2.0149	0.2015	0.01
	0.2	1.8826	0.3665	0.04
	0.4	1.2879	0.5151	0.16
	0.6	0.7340	0.4404	0.36
	0.9	−0.0572	−0.0514	0.81
	1.3	−0.8602	−1.1183	1.69
	1.5	−1.4553	−2.1829	2.25
	1.7	−1.8036	−3.0661	2.89
	1.8	−2.3026	−4.1447	3.24
$\Sigma \rightarrow$	8.5	−0.6095	−9.0399	11.45

data points, $n = 9$

slope, $a_1 = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} = \dfrac{9(-9.0399) - (8.5)(-0.6095)}{9(11.45) - (8.5)^2} = -2.4733$

intercept, $a_0 = \bar{y} - a_1 \bar{x} = \dfrac{\sum y_i}{n} - a_1 \dfrac{\sum x_i}{n} = \dfrac{-0.6095}{9} - (-2.4733)\left(\dfrac{8.5}{9}\right) = 2.2682$

Transformation back to original model :

$$\left.\begin{array}{l} \ln(\alpha) = a_0 \Rightarrow \alpha = e^{a_0} = e^{2.2682} = 9.6618 \\ \beta = a_1 = -2.4733 \end{array}\right\}$$

Original model

$$\boxed{y = 9.6618\, x e^{-2.4733x}}$$