ME-2315 HW-01

1) $$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{1+2n} x^{1+2n} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots \infty$$

True Value $= \tan^{-1}(0.75) = 0.64350$

Term's	Result	ϵ_t (%)	ϵ_a (%)
1	0.75	16.54992	
2	0.60938	5.30319	23.07692
3	0.65684	2.0724	7.2255
4	0.63777	0.89111	2.99
5	0.6461127	0.4060	1.29124
6	0.642273	0.19131	0.597812
7	0.644100	0.093	0.28365

Calculations:-

Formula:-

$$\Rightarrow \epsilon_t = \left| \frac{\text{True} - \text{Approx}}{\text{True}} \right| \times 100\% \quad ; \quad \epsilon_a = \left| \frac{\text{current Approx} - \text{Previous Approx}}{\text{current Approx}} \right| \times 100$$

$\Rightarrow$ iteration 1

$$\arctan(0.75) = 0.75$$

$$\epsilon_t = \left| \frac{0.6435 - 0.75}{0.6435} \right| \times 100\% = 16.54\%$$

iteration 2:

$$\arctan(0.75) = 0.75 - \frac{0.75^3}{3} = 0.609375$$

$$\epsilon^t = \left| \frac{0.6435 - 0.60938}{0.6435} \right| \times 100\% = 5.30319\%$$

$$e_a = \left| \frac{0.60938 - 0.75}{0.60938} \right| \times 100\% = 23.07592\%$$

iteration 3:

$$\arctan(0.75) = 0.75 - \frac{0.75^3}{3} + \frac{0.75^5}{5} = 0.609375 + 0.04746$$
$$= 0.65684$$

$$e_t = \left| \frac{0.6435 - 0.65684}{0.6435} \right| \times 100\% = 2.0724\%$$

$$e_a = \left| \frac{0.65684 - 0.609375}{0.65684} \right| \times 100\% = 7.2255\%$$

iteration 4:-

$$\arctan(0.75) = 0.75 - \frac{0.75^3}{3} + \frac{0.75^5}{5} - \frac{0.75^7}{7}$$
$$= 0.65684 - 0.019069 = 0.63777$$

$$e_t = \left| \frac{0.6435 - 0.63777}{0.6435} \right| \times 100\% = 0.89111\%$$

$$e_a = \left| \frac{0.63777 - 0.65684}{0.63777} \right| \times 100\% = 2.99\%$$

iteration 5:

$$\arctan(0.75) = 0.75 - \frac{0.75^3}{3} + \frac{0.75^5}{5} - \frac{0.75^7}{7} + \frac{0.75^9}{9}$$
$$= 0.63777 + 8.34 \times 10^{-3} = 0.6461127$$

$$e_t = \left| \frac{0.6435 - 0.6461127}{0.6435} \right| \times 100\% = 0.4060\%$$

$$e_a = \left| \frac{0.6461127 - 0.63777}{0.6461127} \right| \times 100\% = 1.291214\%$$

Iteration 6:

$$\arctan(0.75) = 0.75 - \frac{0.75^3}{3} + \frac{0.75^5}{5} - \frac{0.75^7}{7} + \frac{0.75^9}{9} - \frac{0.75^{11}}{11}$$

$$= 0.6461127 - 3.839557 \times 10^{-3} = 0.642273$$

$$\varepsilon_t = \left| \frac{0.6435 - 0.642273}{0.6435} \right| \times 100\% = 0.1913\%$$

$$\varepsilon_a = \left| \frac{0.64227 - 0.6461127}{0.64227} \right| \times 100\% = 0.597812\%$$

Iteration 7:

$$\arctan(0.75) = 0.75 - \frac{0.75^3}{3} + \frac{0.75^5}{5} - \frac{0.75^7}{7} + \frac{0.75^9}{9} - \frac{0.75^{11}}{11} + \frac{0.75^{13}}{13}$$

$$= 0.642273 + 1.8275 \times 10^{-3} = 0.644100$$

$$\varepsilon_t = \left| \frac{0.6435 - 0.644100}{0.6435} \right| \times 100\% = 0.093\%$$

$$\varepsilon_a = \left| \frac{0.6441 - 0.642273}{0.6441} \right| \times 100\% = 0.2836\%$$