Chapra 15.2

Zero intercept quadratic model : $y = a_2 x^2 + a_1 x$

For the ith data point :

$y_i = a_2 x_i^2 + a_1 x_i + e_i$

Solve for e_i :

$e_i = y_i - a_2 x_i^2 - a_1 x_i$

$S_r = \sum_i e_i^2 = \sum_i (y_i - a_2 x_i^2 - a_1 x_i)^2$

Minimize S_r w.r.t. a_1 & a_2 by taking partial derivatives and set $=0$:

$$\left.\begin{aligned} \frac{\partial S_r}{\partial a_2} &= \sum_i 2(y_i - a_2 x_i^2 - a_1 x_i)(-x_i^2) = 0 \\ \frac{\partial S_r}{\partial a_1} &= \sum_i 2(y_i - a_2 x_i^2 - a_1 x_i)(-x_i) = 0 \end{aligned}\right\} \text{normal equations}$$

Simplifying :

$$\left.\begin{aligned} a_2 \sum_i x_i^4 + a_1 \sum_i x_i^3 &= \sum_i x_i^2 y_i \\ a_2 \sum_i x_i^3 + a_1 \sum_i x_i^2 &= \sum_i x_i y_i \end{aligned}\right\} \Rightarrow \begin{bmatrix} \sum_i x_i^4 & \sum_i x_i^3 \\ \sum_i x_i^3 & \sum_i x_i^2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum_i x_i^2 y_i \\ \sum_i x_i y_i \end{bmatrix}$$

Solve by Cramer's rule :

$$a_2 = \frac{\sum x_i^2 y_i \sum x_i^2 - \sum x_i y_i \sum x_i^3}{\sum x_i^4 \sum x_i^2 - \left(\sum x_i^3\right)^2}$$

$$a_1 = \frac{\sum x_i^4 \sum x_i y_i - \sum x_i^3 \sum x_i^2 y_i}{\sum x_i^4 \sum x_i^2 - \left(\sum x_i^3\right)^2}$$

Using the given data :

$\sum x_i^2 = 20400$

$\sum x_i^3 = 129000$

$\sum_i x_i^4 = 8772000$

$\sum x_i y_i = 312850$

$\sum x_i^2 y_i = 20516500$

Plug in to solve :

$a_2 = 0.119075$, $a_1 = 7.771024$

$\therefore$ model : $y = 0.119075 x^2 + 7.771024 x$