Homework 10, ME3215 Spring 2022

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Numerical Integration: Newton-Cotes formulas
```

HW10_1: Comparing numerical methods to analytical

```
clear;clc
%first, define the function to integrate
f=@(x) 1./(3*x+1).^3;
a=0;b=3;%integral endpoints

%Exact integral
exact=@(x)-1./(6*(3*x + 1).^2);
I = exact(3)-exact(0);

%Part a handwork, Trapezoid Rule ------ n=6 ------

%set up variables for the computation
h6=(b-a)/6;   %for n=6
x6=[a:h6:b];  %set the x-values x0 to xn (for n=6)
y6=f(x6);    %set the f(x) values for x0 to xn (for n=6)

I1_area = (x6(2)-x6(1))*(y6(1)+y6(2))/2
I2_area = (x6(3)-x6(2))*(y6(2)+y6(3))/2
I3_area = (x6(4)-x6(3))*(y6(3)+y6(4))/2
I4_area = (x6(5)-x6(4))*(y6(4)+y6(5))/2
I5_area = (x6(6)-x6(5))*(y6(5)+y6(6))/2
I6_area = (x6(7)-x6(6))*(y6(6)+y6(7))/2

I6 = I1_area+I2_area+I3_area+I4_area+I5_area+I6_area
```

```
I1_area =

    0.2660


I2_area =

    0.0199
```

```
I3_area =

    0.0054


I4_area =

    0.0022


I5_area =

    0.0011


I6_area =

   6.5708e-04


I6 =

    0.2953

%Part b & c Trapezoid Rule ----n=6----
%now apply the composite rule, eq 19.17
I6=h6/2*(y6(1)+2*(y6(2)+y6(3)+y6(4)+y6(5)+y6(6))+y6(7))


I6 =

    0.2953


%----n=12----
h12=(b-a)/12;   %for n=12
x12=[a:h12:b];  %set the x-values x0 to xn (for n=12)
y12=f(x12);     %set the f(x) values for x0 to xn (for n=12)

%now apply the composite rule, eq 19.17
I12=h12/2*(y12(1)+2*sum(y12(2:end-1))+y12(end))


I12 =

    0.2058


%-----n=24-----
h24=(b-a)/24;   %for n=24
x24=[a:h24:b];  %set the x-values x0 to xn (for n=24)
y24=f(x24);     %set the f(x) values for x0 to xn (for n=24)

%now apply the composite rule, eq 19.17
```

```
I24=h24/2*(y24(1)+2*sum(y24(2:end-1))+y24(end))


I24 =

    0.1762


%Part d handwork, Simpson's 1/3 rule, multiple applications, ----- n =
 6 -----

I1_area = h6/3*(y6(1) + 4 * y6(2) + y6(3))
I2_area = h6/3*(y6(3) + 4 * y6(4) + y6(5))
I3_area = h6/3*(y6(5) + 4 * y6(6) + y6(7))
I6s1 = I1_area+I2_area+I3_area


I1_area =

    0.2119


I2_area =

    0.0071


I3_area =

    0.0017


I6s1 =

    0.2208


%Part e & f Simpson's 1/3 rule ----- n=6 -------
%apply the composite rule, eq 19.26
n=6;
I6s1=h6/3*(y6(1)+4*(y6(2)+y6(4)+y6(6))+2*(y6(3)+y6(5))+y6(7))


I6s1 =

    0.2208


%------ n= 12 --------
%now apply the composite rule, eq 19.26
n=12;
I12s1=h12/3*(y12(1)+4*sum(y12(2:2:end-1))+2*sum(y12(3:2:end-2))+y12(end))


I12s1 =
```

```
    0.1760

%------ n = 24 ------
%now apply the composite rule, eq 19.26
n=24;
I24s1=h24/3*(y24(1)+4*sum(y24(2:2:end-1))+2*sum(y24(3:2:end-2))+y24(end))


I24s1 =

    0.1664


%compute the errors & report results
fprintf('Exact integral = %6.4f\n\n',I);
fprintf('Trapezoid rule:\n')
fprintf(' for n=6:   %6.6f, error %6.4f%%\n',I6,abs((I-I6)/I)*100)
fprintf(' for n=12:  %6.6f, error %6.4f%%\n',I12,abs((I-I12)/I)*100)
fprintf(' for n=24:  %6.6f, error %6.4f%%\n',I24,abs((I-I24)/I)*100)
fprintf('\nSimpson''s 1/3 rule:\n')
fprintf(' for n=6:   %6.6f, error %6.4f%%\n',I6s1,abs((I-I6s1)/I)*100)
fprintf(' for n=12:  %6.6f, error %6.4f%%\n',I12s1,abs((I-I12s1)/
I)*100)
fprintf(' for n=24:  %6.6f, error %6.4f%%\n',I24s1,abs((I-I24s1)/
I)*100)

Exact integral = 0.1650

Trapezoid rule:
 for n=6:   0.295340, error 78.9937%
 for n=12:  0.205809, error 24.7329%
 for n=24:  0.176231, error 6.8067%

Simpson's 1/3 rule:
 for n=6:   0.220773, error 33.8016%
 for n=12:  0.175966, error 6.6460%
 for n=24:  0.166372, error 0.8313%
```

HW10_2: (textbook 20.15) Work done

```
%textbook 20.15
h =0.25;
%make an array for each segment
t1=0:h:5;
t2=5+h:h:15;
v1=4*t1;            %for 0 <= t <= 5
v2=20+(5-t2).^2;    %for 5 < t <= 15
%concatenate
t=[t1 t2];
v=[v1 v2];
%integrate using simpsons 1/3
work = 200*h/3*(v(1)+4*sum(v(2:2:end-1))+2*sum(v(3:2:end-2))+v(end));
```

```
%using trapz
workT=200*trapz(t,v);
fprintf('Work done with Simpsons 1/3rd rule = %.2f Joules\n',work)
fprintf('Work done with "trapz" = %.2f Joules\n',workT)
```

```
Work done with Simpsons 1/3rd rule = 116666.67 Joules
Work done with "trapz" = 116687.50 Joules
```

HW10_3: Pendulum swing

```
L=0.2; %meters
theta0=30; %degrees
theta0_rad=theta0*pi/180;   %convert to radians
k = sin(theta0_rad);
g=9.81;

%compute the integral term
f=@(x)1./(sqrt(1-k^2*sin(x).^2));
I = integral(f,0,pi/2);

%compute T
T = 4*sqrt(L/g)*I;
fprintf('The pendulum period is %.4f sec\n', T)
```

```
The pendulum period is 0.9628 sec
```

HW10_4: (textbook 19.21) Manufactured spherical particle density

```
clear;clc

r=[0 0.12 0.24 0.36 0.49 0.62 0.79 0.86 0.93 1]/10; %cm
rho=[6 5.81 5.14 4.29 3.39 2.7 2.19 2.1 2.04 2];    %g/cm^3

diff(r)
```

```
ans =

  Columns 1 through 7

    0.0120    0.0120    0.0120    0.0130    0.0130    0.0170    0.0070

  Columns 8 through 9

    0.0070    0.0070
```

```
%From this we can tell
%pts 1,2,3,4 : use Simpson's 3/8 rule (3 equal segments  0.012)
%pts 4,5,6 : use Simpson's 1/3 rule (2 equal segments 0.013)
%pts 6,7 : use Trap rule (1 segment 0.17)
```

```
%pts 7,8,9,10 : use Simpson's 3/8 rule (3 equal segments  0.007)

%we are intgrating rho * 4pi r^2, so let's make an array of that
fm = rho *4*pi.*r.^2;

%compute I1 (1-4)
h1=0.012; %also (r(4)-r(1))/3
I1 = 3*h1/8*(fm(1)+3*fm(2)+3*fm(3)+fm(4))
%compute I2 (4-6)
h2=0.013; %also (r(6)-r(4))/2
I2 = h2/3*(fm(4)+4*fm(5)+fm(6))
%compute I3 (6-7)
I3 = (r(7)-r(6))*(fm(6)+fm(7))/2
%compute I4 (7-10)
h4=0.007; %also (r(10)-r(7))/3
I4 = 3*h4/8*(fm(7)+3*fm(8)+3*fm(9)+fm(10))

%result mass
M=I1+I2+I3+I4;

%volume of the sphere
V=4/3*pi*(max(r))^3;

%average density
avg_density = M/V;

fprintf('Particle mass = %7.6f gm; Average density = %5.4f gm/cc
\n',M,avg_density)
```

```
I1 =

   9.5859e-04


I2 =

    0.0026


I3 =

    0.0026


I4 =

    0.0044

Particle mass = 0.010562 gm; Average density = 2.5214 gm/cc
```

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